STAT 350 Lecture 7: Hypothesis Testing

Proof by Contradiction

(Chapter 10 of WMMY)

Outline

Motivating example 1 – fair/unfair coin

- Suppose you want to know if a coin is fair – here the null hypothesis is that the coin is fair and the research hypothesis/alternative hypothesis is that the coin is unfair.

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- How to carry out the study?

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- Make data-driven decision!

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- You collect data by flipping the coin 30 times and counting Heads, say 20 Heads.

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- How to make use of data to make decisions?

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  - Confidence Interval for the chance to be Heads, say p.

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- How to make use of data to make decisions?
    - Confidence Interval for the chance to be Heads, say p.
    - Any other procedures?

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  - Confidence Interval for the chance to be Heads, say p.
  - Any other procedures? —**Proof by contradiction idea**

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- What if this event happened? —you'd be suspicious about the assumption that the coin is fair.
- In order to make decision, different people have different judgments based on their own threshold chance to define small chance event, which is called the significance level.

Motivating example 2 – criminal court trials

- In criminal court cases, the defendant is assumed innocent.
- Data, in the form of physical evidence and testimony, is collected to determine whether the defendant is guilty.
- The reasoning behind a guilty verdict would be something like

  If the defendant is innocent, then it is unlikely he would have so much evidence stacked against him; therefore, he must be guilty.

Introduction

- Hypothesis testing is probably the most frequently used statistical procedure.
- The basic idea is as follows:
  - Start with some hypothesis about the population parameter of interest.
  - Collect data by sampling or experimentation.
  - Evaluate the amount of evidence in the data for/against the hypothesis.
- Just as with CIs, the actual calculations are very simple, but we have to be careful about the logic involved and interpretation of the results.

Outline

Null and alternative hypotheses

- The problem begins with a claim about a population parameter to be tested.
- This claim is formulated into a set of two hypotheses:
  - Null hypothesis, H_0: What we believe to be true by default, before seeing data.
  - Alternative hypothesis, H_1: The "complement" of H_0.
- In the coin example:

$$H_0 : p = 0.5 \quad \text{vs.} \quad H_1 : p \neq 0.5.$$

- In the criminal trial example:

$$H_0 : \text{the defendant is innocent} \quad \text{vs.} \quad H_1 : \text{the defendant is guilty.}$$

Types of null and alternative hypotheses

- Suppose the parameter μ is of interest.
- There are two types of hypotheses about μ we will encounter.
- *One-sided hypothesis.*

$$\begin{cases} H_0 : \mu = \mu_0 \\ H_1 : \mu > \mu_0 \end{cases} \quad \text{or} \quad \begin{cases} H_0 : \mu = \mu_0 \\ H_1 : \mu < \mu_0 \end{cases}$$

- *Two-sided hypothesis.*

$$\begin{cases} H_0 : \mu = \mu_0 \\ H_1 : \mu \neq \mu_0 \end{cases}$$

Examples

State the null and alternative hypotheses.

1. Census data shows that the mean household income in the area served by a shopping mall is \$72,500 per year. A market research firm questions shoppers at the mall to find out whether the mean household income of mall shoppers is higher than that of the general population.
2. Last year, a company's service technicians took an average of 1.8 hours to respond to trouble calls from business customers who had purchased service contracts. Do this year's data show a different average response time?

Decisions

- After the evidence is measured (more on that later) we ultimately must decide between H_0 and H_1.
- The choices are "Reject H_0" and "Accept H_0" – but keep in mind that **accepting H_0 does not imply H_0 has been proven. We just do not have enough evidence to reject it.**
- There are two possibilities for the truth and two decisions, which makes four possible outcomes:

	H_0 is true	H_1 is true
Reject H_0	**Type I error**	Correct decision!
Accept H_0	Correct decision!	**Type II error**

Making decisions

- Two possible errors:

$$\textbf{Type I error} = \text{Rejecting } H_0 \text{ when } H_0 \text{ is true}$$
$$\textbf{Type II error} = \text{Accepting } H_0 \text{ when } H_1 \text{ is true}$$

- Consider the two probabilities:

$$\alpha = \mathrm{P}(\text{Type I error}) = \mathrm{P}(\text{Rejecting } H_0 \mid H_0 \text{ is true})$$
$$\beta = \mathrm{P}(\text{Type II error}) = \mathrm{P}(\text{Accepting } H_0 \mid H_1 \text{ is true})$$

- α is called the *significance level* of the test; $1 - \beta$, which represents the probability of correctly detecting H_1, is called the *power*.
- The goal is to design a test that controls the type I error first and then minimizes type II error (maximizes power).

Decision rules – the rejection region approach

- Data will be summarized by a *test statistic*, say W.
- Rejection region procedure:
    - Decision rule will be of the form

$$\text{rule} = \begin{cases} \text{Reject } H_0 & \text{if } W \in R(\alpha) \\ \text{Accept } H_0 & \text{if } W \notin R(\alpha) \end{cases}$$

    - $R(\alpha)$ is called the rejection region, chosen based on the given significance level α to control the type I error.
    - In other words, reject H_0 if the test statistic falls in the rejection region.
- There is another (equivalent) approach based on what's called a *p-value* – we'll discuss this more later.

Doing the test

1. State the null and alternative hypotheses.
2. Determine the significance level (generally given in problem).
3. Calculate the test statistic.
4. Identify the rejection region.
5. Make decision, with justification based on a comparison of the test statistic and the rejection region.

Outline

Design of Test Procedure

Design a test procedure for the population mean μ. Assume population std dev σ is *known* and sample size large enough.

- Step 1: Determine the two hypotheses by picking one out of the three: $H_0 : \mu = \mu_0$ vs. $H_1 : \mu \neq \mu_0$.
- Step 2: Choose the appropriate test statistic based on the given information: $Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$.
- Step 3: Figure out the sampling distribution of your test statistic **based on** H_0 **is true**:

$$Z \overset{H_0}{\sim} N(0, 1).$$

Design of Test Procedure

- Step 4: Determine the rejection region in terms of your test statistic in two steps:
  1. Direction of the region: determined by the direction of H_1 (**larger or smaller**). It would make sense to reject H_0 (i.e. conclude H_1) only when $\overline{X}$ is **too large or too small**, relative to μ_0 and its variability $\sigma/\sqrt{n}$. That is exactly saying Z is too small or too large.
  2. Bounds of the region: determined by controlling the probability of making Type I error.

$$\mathrm{P}(\text{Type I error}) = \mathrm{P}\left(\frac{|\overline{X} - \mu_0|}{\sigma/\sqrt{n}} > c \,\middle|\, \mu = \mu_0 \right) = \alpha.$$

  - Since when H_0 is true, CLT tells us that $Z = \frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}} \overset{H_0}{\sim} \mathrm{N}(0,1)$, therefore, choose $c = z^*_{\alpha/2}$=invNorm($1-\alpha/2$,0,1).
  - Some slight modifications required for one-sided tests.

Design of Test Procedure

- Step 5: Make decision based on checking whether the observed test statistic $z_{obs} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ falls in the rejection region.
  1. If z_{obs} is in the rejection region, reject H_0.
  2. If z_{obs} is out of the rejection region, accept H_0.
- Step 6: Make your conclusion in terms of the context.

Note that in Step 4, by controlling the probability to make type I error at α level, it is actually defining a small chance event in terms of $\bar{X}_n$ or Z under H_0 holds true. And this small chance event is then defined as the rejection region. If the observed test statistic falls in this small chance event, we get the contradiction since we should have a large chance to observe the observed test statistic. This makes us to reject the null hypothesis by proof of contradiction principle.

Rejection Region Corresponding to Three H_1

According to the above 6 steps for conducting hypothesis testing, the thing needs to notice is the construction of the rejection region (direction and bounds) in terms of three different alternative hypothesis.

- Case 1: $H_1 : \mu \neq \mu_0$.
  reject H_0 if test statistic $z_{\text{obs}} < -z^*_{\alpha/2}$ or $z_{\text{obs}} > z^*_{\alpha/2}$
- Case 2: $H_1 : \mu < \mu_0$.
  reject H_0 if test statistic $z_{\text{obs}} < -z^*_{\alpha}$
- Case 3: $H_1 : \mu > \mu_0$.
  reject H_0 if test statistic $z_{\text{obs}} > z^*_{\alpha}$

Example 1

A biologist is studying the cellulose content of a variety of alfalfa hay. Suppose the cellulose content comes from a Normal population with unknown mean but known standard deviation $\sigma = 8$mg. A previous study claimed that the mean cellulose content was 140 mg. In a sample of 15 cuttings, the mean cellulose content of 145 mg. Perform a hypothesis test to determine if the mean cellulose content is different from 140 mg. (Use $\alpha = 0.05$.)

Example 2

Suppose the mean income of 35-year-olds in the U.S. is \$24,000. A random sample of 100 35-year-olds in North Dakota results in a sample mean income of \$23,500. Assume the standard deviation σ is known and equal to \$4,000.

1. At a 5% level of significance, should we conclude that mean income for 35-year-olds in North Dakota is lower than the national average?
2. Based on your conclusion, what's the type of error you could make?
3. Suppose that the true mean income of 35-year-olds in North Dakota is actually \$23,600. For the test in part (1), find the probability β of comitting a Type II error.
4. What's the power to correctly detect the true mean income of 35-year-olds in North Dakota, which is actually \$23,600, based on your procedure in part (1)?

Example 3

All cigarettes presently on the market have a mean nicotine content of 1.6 mg per cigarette. A firm that produces cigarettes claims that it has discovered a new way to cure tobacco leaves that will result in the mean nicotine per cigarette being less than 1.6 mg. To test this claim, a sample of 36 of the firm's cigarettes were analyzed. Assume that the standard deviation of the cigarettes nicotine content is 0.8 mg.

1. What conclusions can be drawn, at the 5% level, if the average nicotine content of the 36 cigarettes is 1.54 mg?
2. If the truth is 1.5 mg per cigarette, what is the minimum sample size to have at least 90% power to detect it for such 5% level significance test?

Example 3: solution

solution:

1. Denote μ as the mean nicotine per cigarette produced by the firm in the new way.
   $\sigma = 0.8, \mu_0 = 1.6, n = 36, \bar{x} = 1.54, \alpha = 0.05.$
   1. Step 1: $H_0 : \mu = 1.6, H_1 : \mu < 1.6.$
   2. Step 2 and 3: $Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{\bar{X} - 1.6}{0.8/6} \overset{H_0}{\sim} N(0, 1).$
   3. Step 4: Rejection region in terms of Z:
      $Z < -z^*_\alpha$ =-invNorm(1-0.05, 0, 1)=-1.64.
   4. Step 5: Since $z_{obs} = \frac{\bar{x} - 1.6}{0.8/6} = \frac{1.54 - 1.6}{0.8/6} = -0.45$ is outside of the rejection region $(-\infty, -1.64)$, accept H_0.
   5. Step 6: We do not have strong evidence to make conclusion that the mean nicotine per cigarette produced in the new way is reduced.

Example 3: solution

solution:

2. Denote μ as the mean nicotine per cigarette produced by the firm in the new way. $\sigma = 0.8, \mu_1 = 1.5, \alpha = 0.05$. We are looking for the sample size to control the probability to make type I error at 5% and also to control the probability to make type II error at most 10% (1-power).
   - Due to the construction of the rejection region $Z = \frac{\bar{X}_n - \mu_0}{\sigma/\sqrt{n}} = \frac{\bar{X}_n - 1.6}{0.8/\sqrt{n}} < -z^*_\alpha = -1.64$, we guaranteed the probability of making type I error is controlled at 5%. Write the rejection region in terms of $\bar{X}_n$: $\bar{X}_n < 1.6 - 1.64\frac{0.8}{\sqrt{n}}$.
   - How to choose n to get the probability to make type II error controlled at most 10%:

$$\beta = 0.1 = \mathrm{P}(\text{Accept } H_0|H_1) = \mathrm{P}(\bar{X}_n \geq 1.6 - 1.64\frac{0.8}{\sqrt{n}}|\mu = \mu_1 = 1.5)$$

$$= \mathrm{P}(Z = \frac{\bar{X}_n - 1.5}{0.8/\sqrt{n}} \geq \frac{(1.6 - 1.64\frac{0.8}{\sqrt{n}}) - 1.5}{0.8/\sqrt{n}}) = \mathrm{P}(Z \geq \frac{\sqrt{n}}{8} - 1.64)$$

Thus $\frac{\sqrt{n}}{8} - 1.64 =$invNorm(0.9,0,1)=1.28, i.e. $n = (8 \times (1.28 + 1.64))^2 = 545.69$, and thus $n = 546$.

p-value approach

- You'll notice that the rejection region approach gives no indication of the strength of evidence for/against H_0.
- The p-value approach gives a measure of strength of evidence.
- **Definition. Assuming H_0 is true**, the p-value is the probability that the test statistic W takes a value more "extreme" (towards H_1) than what actually observed w_{obs}.
- If H_0 is true and our sample is "representative" of the population, then we expect the p-value to be large.
- Therefore, a small p-value is evidence against H_0. The p values is also called **observed significance**.
- The p-value approach rejects H_0 if p-value $< \alpha$.

p-value approach – cont.

For the mean problem, the test statistic is $Z = \frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}}$, and the observed value is $z_{\text{obs}} = \frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}}$.

- If $H_1 : \mu > \mu_0$, then p-value $= \mathrm{P}(Z \geq z_{\text{obs}})$.
- If $H_1 : \mu < \mu_0$, then p-value $= \mathrm{P}(Z \leq z_{\text{obs}})$.
- If $H_1 : \mu \neq \mu_0$, then p-value $= 2\mathrm{P}(Z \geq |z_{\text{obs}}|)$.

Example 4

Bolts used to assemble certain engine blocks that should average 1.50 inches in length. Problems arise if the machine produces bolts that are either too long or too short on average. A random sample of 400 bolts are taken, and they show an average of 1.504 inches. Assume $\sigma = 0.075$ inches. Does the machine need adjustments? Compute the p-value for the test and make your conclusion at significance level $\alpha = 0.1$ and $\alpha = 0.3$.

Example 4: solution

solution: denote μ as the average length in inches for the bolts used to assemble certain engine blocks.
$n = 400, \sigma = 0.075, \bar{x} = 1.504$.

- Step 1: $H_0 : \mu = 1.5, H_1 : \mu \neq 1.5$.
- Step 2 and 3: $Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} = \frac{\bar{X}-1.5}{0.075/\sqrt{400}} \overset{H_0}{\sim} N(0,1)$.
- Step 4: p-value=P($|Z| > z_{\text{obs}} = \frac{1.504-1.5}{0.075/\sqrt{400}} = 1.07) =$ 2×normalcdf(1.07, 9^9, 0, 1)=2×0.142=0.284.
- Step 5: Since the 0.1¡p-value=0.284¡0.3, we reject H_0 at the significance level $\alpha = 0.3$ and accept H_0 at the significance level $\alpha = 0.1$.
- Step 6: With $\alpha = 0.3$, we have strong evidence that the machine needs adjustments. With $\alpha = 0.1$, we do not have strong evidence to make adjustments for the machine.

Outline

5 Different Cases

1. without normality, $n \geq 30$ and σ is given
2. without normality, $n \geq 30$ and s is given
3. X_i is Normal, σ is given
4. X_i is Normal, s is given
5. X_i is Bernoulli, $n \geq 30$

sampling distribution for the 5 cases:

1) $Z = \frac{(\overline{X}_n - \mu)}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

2) $Z = \frac{(\overline{X}_n - \mu)}{s/\sqrt{n}} \sim \mathrm{N}(0,1)$

3) $Z = \frac{(\overline{X}_n - \mu)}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

5) $Z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \sim \mathrm{N}(0,1)$

$$\Rightarrow \begin{cases} \boxed{H_1 : \mu > \mu_0} : & R(\alpha) = \{Z > z^*_\alpha\}; \text{p-value} = P(Z > z_{obs}) \\ \boxed{H_1 : \mu < \mu_0} : & R(\alpha) = \{Z < z^*_\alpha\}; \text{p-value} = P(Z < z_{obs}) \\ \boxed{H_1 : \mu \neq \mu_0} : & R(\alpha) = \{|Z| > z^*_{\alpha/2}\}; \text{p-value} = 2P(Z > |z_{obs}|) \end{cases}$$

5 Different Cases

4) $T = \frac{(\overline{X}_n - \mu)}{s/\sqrt{n}} \sim \mathrm{t}(n-1)$

$$\Rightarrow \begin{cases} \boxed{H_1 : \mu > \mu_0} : & R(\alpha) = \{T > t^*_\alpha\}; \text{p-value} = P(T > t_{obs}) \\ \boxed{H_1 : \mu < \mu_0} : & R(\alpha) = \{T < t^*_\alpha\}; \text{p-value} = P(T < t_{obs}) \\ \boxed{H_1 : \mu \neq \mu_0} : & R(\alpha) = \{|T| > t^*_{\alpha/2}\}; \text{p-value} = 2P(T > |t_{obs}|) \end{cases}$$

Note t^*_β, the $100(1-\beta)$ percentile of a Student-t distribution with $n-1$ degrees of freedom. Sometimes it is written as $t^*_\beta(n-1)$.

Hypothesis Test for Means — General

$$
\boxed{\text{Normal}}
\begin{cases}
\boxed{\text{Yes}}
\begin{cases}
\boxed{\sigma \text{ given}} : \boxed{\text{ZTest}} \\
\boxed{s \text{ given}} : \boxed{\text{TTest}}
\end{cases} \\
\boxed{\text{No}}
\begin{cases}
\boxed{\text{Bernoulli}}
\begin{cases}
\boxed{n \geq 30} : \boxed{\text{1PropZTest}} \\
\boxed{n < 30} : \boxed{\text{No method}}
\end{cases} \\
\boxed{\text{Not Bernoulli}}
\begin{cases}
\boxed{n \geq 30} : \boxed{\text{ZTest}} \ (\boxed{\text{no matter } s \text{ or } \sigma}) \\
\boxed{n < 30} : \boxed{\text{No method}}
\end{cases}
\end{cases}
\end{cases}
$$

Example 5

A sample of size $n = 18$ is selected from a normal population with unknown mean μ and unknown std dev σ. The sample has mean $\overline{x} = 16.3$ and std dev $s = 3.32$. Test the one-sided hypothesis that $\mu > 15$ using both the rejection region and the p-value methods. Assume $\alpha = 0.05$.

Example 6

Information on a large packet of seeds indicates that the germination rate is p_0=92%. Some researchers claim that the proportion of germination should be higher than that.

1. For testing this claim, what is the appropriate null hypothesis and alternative hypothesis?
2. Now 300 seeds are chosen randomly with 290 germinated to test this claim. Compute the test statistic and get the rejection region at the significance level $\alpha = 0.05$.
3. Report the p value.
4. Given the significance level $\alpha = 0.05$, make a conclusion about your test. Please interpret the conclusion.
5. Based on the conclusion you made in last step, what possible error could you make? Type I or Type II?
6. If the true germination rate $p^* = 94\%$, what's the power of the test?

Outline

Two-sample Hypothesis Testing

1. Matched two samples: by taking difference, it becomes one sample problem;
2. Independent two samples, $X_{11}, \ldots, X_{1n_1} \sim \mathrm{N}(\mu_1, \sigma_1^2)$ and $X_{21}, \ldots, X_{2n_2} \sim \mathrm{N}(\mu_2, \sigma_2^2)$ with $H_0 : \mu_1 - \mu_2 = \mu_d$:
   - If σ_1 and σ_2 are known, we would use the following test statistic
     $$[(\overline{X}_1 - \overline{X}_2) - \mu_d] / \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \overset{H_0}{\sim} \mathrm{N}(0, 1).$$
   - If σ_1 and σ_2 are unknown but equal, with the pooled standard deviation we use the following statistic
     $$[(\overline{X}_1 - \overline{X}_2) - \mu_d] / \left\{ s_{\mathrm{pool}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right\} \overset{H_0}{\sim} \mathrm{t}(n_1 + n_2 - 2).$$

Two independent sample Hypothesis Testing — General

$$
\boxed{\text{Normal}}
\begin{cases}
\boxed{\text{Yes}} \begin{cases} \boxed{\sigma_1 \text{ and } \sigma_2 \text{ given}} : \boxed{\text{2-SampZTest}} \\ \boxed{\sigma_1 = \sigma_2 \text{ unknown}} : \boxed{\text{2-SampTTest}} \ (\boxed{\text{Pooled}}) \end{cases} \\
\boxed{\text{No}} \begin{cases} \boxed{n_1, n_2 \geq 30} : \boxed{\text{2-SampZTest}} \ (\boxed{s_1, s_2 \text{ or } \sigma_1, \sigma_2 \text{ given}}) \\ \boxed{n < 30} : \boxed{\text{No method}} \end{cases}
\end{cases}
$$

Example 7

A group of college seniors are selected to participate in a manual dexterity skill test against a group of 20 industrial workers. Skills are assessed by scores (normally distributed with common variance) obtained on a test taken by both groups. Descriptive statistics are below.

Group	n	$\overline{x}$	s
Students	15	35.12	4.31
Workers	20	37.32	3.83

Conduct a hypothesis test to determine whether the industrial worker had better manual dexterity skills than the students at the 5% level.

Example 8

A vote is to be taken among the residents of a town and the surrounding county to determine whether a proposed chemical plant should be constructed. The construction site is within the town limits, and for this reason many voters in the county believe that the proposal will pass because of the large proportion of town voters who favor the construction. To determine if there is a significant difference in the proportions of town voters and county voters favoring the proposal, a poll is taken. If 120 of 200 town voters favor the proposal and 240 of 500 county residents favor it, would you agree that the proportion of town voters favoring the proposal is higher than the proportion of county voters? Use an $\alpha = 0.05$ level of significance.