Solution to HW4

5.8 For $n = 8$ and $p = 0.6$, we have

(a) $P(X = 3) = b(3; 8, 0.6) = P(X \leq 3) - P(X \leq 2) = 0.1737 - 0.0498 = 0.1239$.

(b) $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.4059 = 0.5941$.

5.12 From Table A.1 with $n = 9$ and $p = 0.25$, we have $P(X < 4) = 0.8343$.

5.32 (a) Probability that all 4 fire $= h(4; 10, 4, 7) = \frac{1}{6}$.

(b) Probability that at most 2 will not fire $= \sum_{x=0}^{2} h(x; 10, 4, 3) = \frac{29}{30}$.

5.36 (a) $P(X = 0) = h(0; 25, 3, 3) = \frac{77}{115}$.

(b) $P(X = 1) = h(1; 25, 3, 1) = \frac{3}{25}$.

5.67 (a) $P(X > 10 | \lambda t = 14) = 1 - 0.1757 = 0.8243$.

(b) $\lambda t = 14$.

6.6 (a) The area to the left of z is $1 - 0.3622 = 0.6378$ which is closer to the tabled value 0.6368 than to 0.6406. Therefore, we choose $z = 0.35$.

(b) From Table A.3, $z = -1.21$.

(c) The total area to the left of z is $0.5000 + 0.4838 = 0.9838$. Therefore, from Table A.3, $z = 2.14$.

(d) The distribution contains an area of 0.025 to the left of $-z$ and therefore a total area of $0.025 + 0.95 = 0.975$ to the left of z. From Table A.3, $z = 1.96$.

6.8 (a) $z = (17 - 30)/6 = -2.17$. Area$= 1 - 0.0150 = 0.9850$.

(b) $z = (22 - 30)/6 = -1.33$. Area$= 0.0918$.

(c) $z_1 = (32 - 30)/6 = 0.33$, $z_2 = (41 - 30)/6 = 1.83$. Area $= 0.9664 - 0.6293 = 0.3371$.

(d) $z = 0.84$. Therefore, $x = 30 + (6)(0.84) = 35.04$.

(e) $z_1 = -1.15$, $z_2 = 1.15$. Therefore, $x_1 = 30 + (6)(-1.15) = 23.1$ and $x_2 = 30 + (6)(1.15) = 36.9$.

6.14 (a) $z = (10.075 - 10.000)/0.03 = 2.5$; $P(X > 10.075) = P(Z > 2.5) = 0.0062$. Therefore, 0.62% of the rings have inside diameters exceeding 10.075 cm.

(b) $z_1 = (9.97 - 10)/0.03 = -1.0$, $z_2 = (10.03 - 10)/0.03 = 1.0$;
$P(9.97 < X < 10.03) = P(-1.0 < Z < 1.0) = 0.8413 - 0.1587 = 0.6826$.

(c) $z = -1.04$, $x = 10 + (0.03)(-1.04) = 9.969$ cm.

6.45 $P(X < 3) = \frac{1}{4}\int_0^3 e^{-x/4}\,dx = -e^{-x/4}\Big|_0^3 = 1 - e^{-3/4} = 0.5276$.
Let Y be the number of days a person is served in less than 3 minutes. Then
$P(Y \geq 4) = \sum_{x=4}^{6} b(y; 6, 1 - e^{-3/4}) = \binom{6}{4}(0.5276)^4(0.4724)^2 + \binom{6}{5}(0.5276)^5(0.4724)$
$+ \binom{6}{6}(0.5276)^6 = 0.3968$.

Homework problems from the textbook:

5.8, 5.12, 5.32, 5.36, 5.67, 6.6, 6.8, 6.14, 6.45

Extra problems:

Problem 1

A satellite system consists of 4 components and can function adequately if at least 2 of the 4 components are in working condition. If each component is, independently, in working condition with probability 0.6, what is the probability that the system functions adequately?

Let $X =$ # functioning components out of 4

Then $X \sim \text{binom}(n=4, p=0.6)$

$\Pr\{\text{System functions}\}$

$= \Pr\{X \geq 2\} = \binom{4}{2}\cdot .6^2 \cdot .4^2 + \binom{4}{3}\cdot .6^3 \cdot .4 + \binom{4}{4}\cdot .6^4$

$= .3456 + .3456 + .1296 = .8208$

alternatively,

$\hookrightarrow = 1 - \Pr\{X=0\} - \Pr\{X=1\} = 1 - \binom{4}{0}\cdot .6^0 \cdot .4^4 - \binom{4}{1}\cdot .6 \cdot .4^3$

$= 1 - .0256 - .1536 = .8208$

$= 1 - \text{binomcdf}(4, .6, 1) = .8208$

Problem 2

Suppose that a particular personal trait — like eye colour or handedness — is classified on the basis of one pair of genes. Let d denote a dominant gene and r a recessive one. A person with dd genes is "purely dominant", one with rr genes "purely recessive", and one with rd genes a "hybrid". Purely dominant and hybrid individuals are alike in appearance with respect to the trait in question. Children receive one gene from each parent. If, with respect to a particular trait, two hybrid parents have a total of four children, what is the probability that exactly three of the four children have the outward appearance of the dominant gene? Assume that each child is equally likely to inherit either of two genes from each parent.

genotype	Mother (dr) d	r
Father (dr) d	1 dd	dr 1
r	1 rd	rr 2

phenotype $X=1$ if dd, dr, rd

$X=2$ if rr

$\Pr\{X=1\} = \Pr\{dd, dr, rd\} = \frac{3}{4}$

$\Pr\{X=2\} = \Pr\{rr\} = \frac{1}{4}$

$Y =$ # children with $\{X=1\}$ out of 4 children

Then $Y \sim \text{binom}(n=4, p=3/4)$

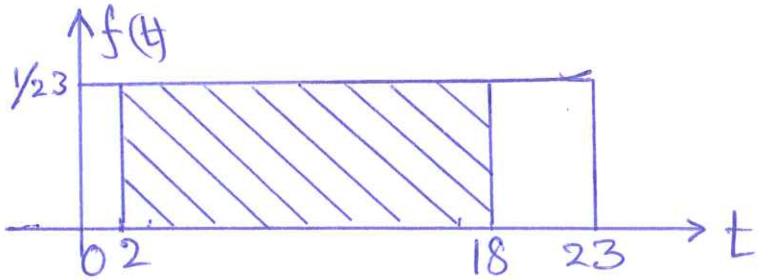
$\Pr\{Y=3\} = \binom{4}{3}\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^1 = 4\cdot\frac{27}{64}\cdot\frac{1}{4} = \frac{27}{64} = .4219$

$= \text{binompdf}(4, \frac{3}{4}, 3) = .421875$

Problem 3

We will assume that the smiling times, in seconds, for an eight-week old baby, follow a uniform distribution between 0 and 23 seconds, inclusive.

1. What is the probability that a randomly chosen eight-week old baby smiles between 2 and 18 seconds?
2. Find the 90% percentile for an eight-week old baby's smiling time.
3. Find the probability that a random eight-week old baby smiles more than 12 seconds knowing that the baby smiles more than 8 seconds.



$$f(t) = \begin{cases} \frac{1}{23}, & 0 \le t \le 23 \\ 0, & \text{otherwise} \end{cases}$$

1. $Pr\{2 \le T \le 18\} = \int_2^{18} f(t)\,dt = \int_2^{18} \frac{1}{23}\,dt = \frac{18-2}{23} = \frac{16}{23}$

$= 0.6957$

2. Solve: $Pr\{T \le t\} = .90$

$\frac{t}{23} = .90$

$t = 23(.90) = 20.7$

$t_{.90} = 20.7$ sec

3. $Pr\{T > 12 \mid T > 8\} = \frac{Pr(\{T > 12\} \cap \{T > 8\})}{Pr\{T > 8\}}$

$= \frac{Pr\{T > 12\}}{Pr\{T > 8\}} = \frac{23-12}{23-8} = \frac{11}{15} = .7333$

Problem 4

Assume jobs arrive every 15 seconds on average.

1. Specify an appropriate probability model to model the number of jobs arrived during a given minute.
2. What is the probability to get exactly 4 jobs during a given minute?
3. Specify an appropriate probability model to model the waiting time (in minute) for a job to arrive.
4. What is the probability of waiting less than or equal to 30 seconds, i.e .5 min, to get a job?

Let X = # jobs arriving per minute

1. Given $\mathbb{E}[X] = \frac{60 \text{ sec}}{15 \text{ sec}} = 4$, we model

$X \sim \text{Poisson}(\lambda = 4)$

i.e. $\Pr\{X = k\} = e^{-4} \frac{4^k}{k!}$ for $k = 0, 1, 2, \cdots$

2. $\Pr\{X=4\} = e^{-4}\frac{4^4}{4!} = (0.018316)\frac{64}{6} = .1954$

$= \text{Poissonpdf}(4,4) = .195366 8148$

3. Let T = waiting time until a job arrives

We model $T \sim \text{Exponential}(\alpha = 4)$

rate per minute

note $\mathbb{E}[T] = \frac{1}{\alpha} = \frac{1}{4}$ min

4. $30 \text{ sec} = \frac{1}{2}$ min

$\Pr\{T \le \frac{1}{2}\} = \int_0^{1/2} 4e^{-4t}\,dt$

$= \int_0^2 e^{-y}\,dy = 1 - e^{-2} = 1 - .1353 = .8647$

Problem 5

1. For $X \sim \mathsf{N}(\mu, \sigma^2)$, and $Z \sim \mathsf{N}(0, 1)$, is
$$\Pr(a \leq X \leq b) = \Pr(\frac{a-\mu}{\sigma} \leq Z \leq \frac{b-\mu}{\sigma})?$$
Verify it by choosing a particular pair of (μ, σ) and some reasonable a, b.
2. Given $X \sim \mathsf{N}(\mu, \sigma^2 = 9)$, and $\Pr(X \leq 4) = 0.6$, what is μ?
3. Given $X \sim \mathsf{N}(\mu = 10, \sigma^2)$, and $\Pr(X \leq 14) = 0.6$, what is σ^2?

normalcdf(-1,1,0,1)

normalcdf(80,100,90,10)

1. Yes, true. Let us pick $\mu = 90$, $\sigma^2 = 100 = 10^2$

(a, b)	$\Pr\{a \leq X \leq b\}$	$\Pr(\frac{a-\mu}{\sigma} \leq Z \leq \frac{b-\mu}{\sigma})$
$(80, 100)$	$\Pr\{80 \leq X \leq 100\}$	$\Pr(-1 \leq Z \leq 1) = .68269$
$(90, 110)$	$\Pr\{90 \leq X \leq 110\}$	$\Pr(0 \leq Z \leq 2) = .47725$
$(120, \infty)$	$\Pr\{120 \leq X\}$	$\Pr(3 \leq Z) = .00135$

normalcdf(120, 10^{10}, 90, 10)

2. $X \sim N(\mu, \sigma^2 = 9)$

$0.6 = \Pr(X \leq 4) = \Pr(Z \leq \frac{4-\mu}{3})$

$\Rightarrow \frac{4-\mu}{3} = Z_{0.6} = .25334710 =$ invNorm(.6,0,1)

$\Rightarrow \mu = 4 - 3(.2533) = 3.24$

3. $X \sim N(\mu = 10, \sigma^2)$

$0.6 = \Pr(X \leq 14) = \Pr(Z \leq \frac{14-10}{\sigma})$

$\Rightarrow \frac{14-10}{\sigma} = Z_{0.6} = .25334710 =$ invNorm(.6,0,1)

$\Rightarrow \sigma = \frac{4}{.2533} = 15.78$

$\Rightarrow \sigma^2 = 249.28$