Solution to HW2

2.4 (a) $S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$.

(b) $S = \{(x,y) \mid 1 \le x, y \le 6\}$.

2.8 (a) $A = \{(3,6),(4,5),(4,6),(5,4),(5,5),(5,6),(6,3),(6,4),(6,5),(6,6)\}$.

(b) $B = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(2,1),(2,3),(2,4),$
$(2,5),(2,6)\}$.

2.33 (a) With $n_1 = 4$ possible answers for the first question, $n_2 = 4$ possible answers for the second question, and so forth, the generalized multiplication rule yields $4^5 = 1024$ ways to answer the test.

(b) With $n_1 = 3$ wrong answers for the first question, $n_2 = 3$ wrong answers for the second question, and so forth, the generalized multiplication rule yields

$$n_1 n_2 n_3 n_4 n_5 = (3)(3)(3)(3)(3) = 3^5 = 243$$

ways to answer the test and get all questions wrong.

2.36 (a) Any of the 6 nonzero digits can be chosen for the hundreds position, and of the remaining 6 digits for the tens position, leaving 5 digits for the units position. So, there are $(6)(6)(5) = 180$ three digit numbers.

(b) The units position can be filled using any of the 3 odd digits. Any of the remaining 5 nonzero digits can be chosen for the hundreds position, leaving a choice of 5 digits for the tens position. By Theorem 2.2, there are $(3)(5)(5) = 75$ three digit odd numbers.

(c) If a 4, 5, or 6 is used in the hundreds position there remain 6 and 5 choices, respectively, for the tens and units positions. This gives $(3)(6)(5) = 90$ three digit numbers beginning with a 4, 5, or 6. If a 3 is used in the hundreds position, then a 4, 5, or 6 must be used in the tens position leaving 5 choices for the units position. In this case, there are $(1)(3)(5) = 15$ three digit number begin with a 3. So, the total number of three digit numbers that are greater than 330 is $90 + 15 = 105$.

2.59 (a) $\frac{\binom{4}{3}\binom{48}{2}}{\binom{52}{5}} = \frac{94}{54145}$.

(b) $\frac{\binom{13}{4}\binom{13}{1}}{\binom{52}{5}} = \frac{143}{39984}$.

2.66 (a) $0.02 + 0.30 = 0.32 = 32\%$;

(b) $0.32 + 0.25 + 0.30 = 0.87 = 87\%$;

(c) $0.05 + 0.06 + 0.02 = 0.13 = 13\%$;

(d) $1 - 0.05 - 0.32 = 0.63 = 63\%$.

2.76 Consider the events:

A: a person is experiencing hypertension,
B: a person is a heavy smoker,
C: a person is a nonsmoker.

(a) $P(A \mid B) = 30/49$;

(b) $P(C \mid A') = 48/93 = 16/31$.

2.85 Consider the events:

A: the doctor makes a correct diagnosis,
B: the patient sues.
$P(A' \cap B) = P(A')P(B \mid A') = (0.3)(0.9) = 0.27$.

2.95 Consider the events:

C: an adult selected has cancer,
D: the adult is diagnosed as having cancer.
$P(C) = 0.05$, $P(D \mid C) = 0.78$, $P(C') = 0.95$ and $P(D \mid C') = 0.06$. So, $P(D) = P(C \cap D) + P(C' \cap D) = (0.05)(0.78) + (0.95)(0.06) = 0.096$.

2.97 $P(C \mid D) = \frac{P(C \cap D)}{P(D)} = \frac{0.039}{0.096} = 0.40625$.

Problem 1:

Of three cards, one is painted red on both sides; one is painted black on both sides; and one is painted red on one side and black on the other. A card is randomly chosen and placed on a table. If the side facing up is red, what is the probability that the other side is also red?

A: facing up side is red
B: the other side is red

We're looking for $Pr(B|A)$.

~~Sample space method:~~

~~the new sample space = {Card 1, Card 3}~~

$$Pr(B|A) = \frac{Pr(B \cap A)}{Pr(A)} = \frac{Pr(\text{Card } 1)}{Pr(A)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

Problem 2:

Police often set up sobriety checkpointsroadblocks where drivers are asked a few brief questions to allow the officer to judge whether or not the person may have been drinking. If the officer does not suspect a problem, drivers are released to go on their way. Otherwise, drivers are detained for a Breathalyzer test that will determine whether or not they will be arrested. The police say that based on the brief initial stop, trained officers can make the right decision 80% of the time. Suppose the police operate a sobriety checkpoint after 9 p.m. on a Saturday night, a time when national traffic safety experts suspect that about 12% of drivers have been drinking.

a) Draw a tree diagram to describe the information of this problem.

b) You are stopped at the checkpoint and, of course, have not been drinking. What's the probability that you are detained for further testing?

c) What's the probability that any given driver will be detained?

d) What's the probability that a driver who is detained has actually been drinking?

e) What's the probability that a driver who was released had actually been drinking?

a) driver

$\xrightarrow{0.12}$ D $\xrightarrow{0.8}$ P $\rightarrow Pr(D \cap P) = 0.12 \times 0.8 = 0.096$

D $\xrightarrow{0.2}$ N $\rightarrow Pr(D \cap N) = 0.12 \times 0.2 = 0.024$

$\xrightarrow{0.88}$ ND $\xrightarrow{0.2}$ P $\rightarrow Pr(ND \cap P) = 0.88 \times 0.2 = 0.176$

ND $\xrightarrow{0.8}$ N $\rightarrow Pr(ND \cap N) = 0.88 \times 0.8 = 0.704$

b) $Pr(P \mid ND) = 0.2$

c) $Pr(P) = Pr(P \cap D) + Pr(P \cap ND) = 0.096 + 0.176 = 0.272$

d) $Pr(D \mid P) = \frac{Pr(D \cap P)}{Pr(P)} = \frac{0.096}{0.272} = 0.353$

e) $Pr(D \mid N) = \frac{Pr(D \cap N)}{Pr(N)} = \frac{0.024}{1 - 0.272} = 0.033$

Problem 3:

Your roommate, who's a bit of a slacker, is trying to convince you that money can't buy happiness, citing a Harvard study showing that only 10% of happy people are rich.

After giving it some thought, it occurs to you that this statistic isn't very compelling. What you really want to know is what percent of rich people are happy. This would give a better idea of whether becoming rich might make you happy.

Bayes' Theorem (The key idea of Bayes' theorem is reversing the statistic using the overall rates.) tells you how to calculate this other, reversed statistic using two additional pieces of information:

1. The percent of people overall who are happy: 40% of people are happy; H
2. The percent of people overall who are rich: 5% of people are rich. R

So if the Harvard study is correct, then what is the fraction of rich people who are happy? Does it give a pretty strong evidence that majority of rich people are happy?

$$\Pr(H|R) = \frac{\Pr(H \cap R)}{\Pr(R)} = \frac{\Pr(R|H)\Pr(H)}{\Pr(R)}$$

$$= \frac{0.1 \times 0.4}{0.05} = 0.8$$

Yes.