IMSE866: Homework #2, Fall 2018

Due by 11:59 pm Friday Nov 30, 2018 (submit electronically)

Unless otherwise specified, please turn in your answers to the problems in a single file as either *Hw2F18.docx* or *Hw2F18.pdf* file with each problem clearly labeled. This homework requires you to create model, perform analysis and write standard report (Executive summary, introduction, problem description, model, analysis, results, and conclusion). You will submit your work to the class Canvas site. This is an individual homework.

Problem 1 (100 points)

To cut costs and reduce lead times, many manufacturers design their products and processes so that the final product can be quickly assembled from its components. In the literature, these systems are commonly referred to as assemble-to-order (ATO) systems.

Figure 1 illustrates an ATO system that assembles a single product with two components. Component $k, k = 1,2$; can be manufactured by the in-house manufacturing facility M_k and the local subcontractor S_k. The components are stored at inventory location L_k and are assembled at station A to satisfy the demand for the final product. We assume that the customer orders for the final product arrive according to a Poisson process $N(t), t \geq 0$ with rate λ and are satisfied on a first-come-first serve (FCFS) basis at assembly station A. Assembly operations of this station are instantaneous, i.e. if both components are available at the demand arrival epoch, then the demand for the final product is immediately satisfied. If one or more component is unavailable, then the demand for the final product is backordered and the customer order stays in the queue at station A. We model the local subcontractor S_k and the in-house manufacturing facility M_k, as a single server queue with exponentially distributed service time with mean $\mu_{s,k}^{-1}$ and $\mu_{m,k}^{-1}$ respectively. The production cost per unit is $c_{s,k}$ and $c_{m,k}$ for component k at S_k and M_k respectively. Assume that $\mu_{s,k} < \mu_{m,k}$ and $c_{s,k} < c_{m,k}$.

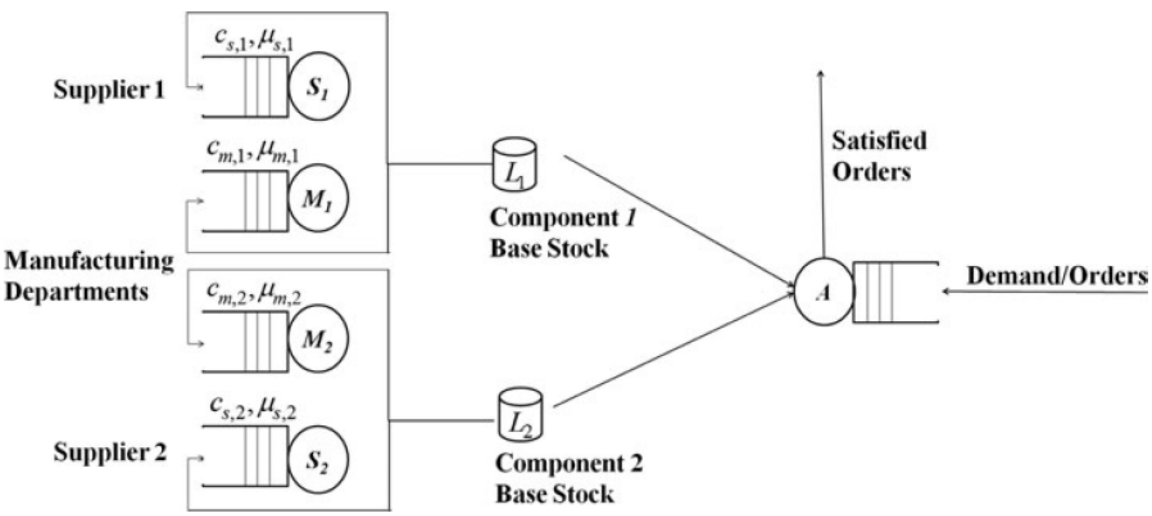

Figure 1: Supply chain of an ATO system with two components

We assume that the system maintains a base stock level z_k, for each component \$k\$, i.e., we ensure that the net inventory position is z_k through orders for replenishing inventory placed at demand arrival epochs. Let $O_{m,k}(t), O_{s,k}(t), I_k(t), B_k(t)$ denote in-house manufacturer's on-orders, subcontractor's on-orders, on-hand inventory quantity and backorders for component k respectively at decision epoch t. Then, since the system maintains a base stock policy for each component, the following equation holds.

$$z_k = O_{m,k}(t) + O_{s,k}(t) + I_k(t) - B_k(t), \qquad \forall k, t$$

Note that at anytime $t, I_k(t)B_k(t) = 0$. For this system, we analyze system performance under dual base stock policy.

Under the dual base stock policy, if at any instant t corresponding to a demand arrival, $I_k(t) < e_k$ (where e_k is a predefined inventory threshold limit), then the manufacturer uses all available capacity at its internal manufacturing facility, M_k and the local subcontractor, S_k to replenish the inventory for component k. If instead at the demand arrival epoch t, $z_k > I_k(t) \geq e_k$, the manufacturer places an order to replenish inventory for component k only to its local subcontractor S_k. If $I_k(t) = z_k$; at demand arrival epoch, no replenishment order is placed for component k.

Question 1 (20 points): Develop a CTMC model, define state space, transition diagram.

Question 2 (20 points): Write the steady state equations.

Question 3 (20 points): Use the following parameters to solve the steady state equations and find the cost. Use $e_k = 5$.

System parameters		*Costs*	
λ	1.5	$c_{m,k}, k = 1, 2$	10
$z_k, k = 1, 2$	10	$c_{s,k}, k = 1, 2$	5
$\mu_{m,k}, k = 1, 2$	2	$b_k, k = 1, 2$	20
$\mu_{s,k}, k = 1, 2$	1	$h_k, k = 1, 2$	1

Question 4 (40 points): Write a report (executive summary, problem description, model, analysis, results and conclusions).