# PH1140 OSCILLATION PROJECT

GROUP BETA

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(NOT IN ORDER OF CONTRIBUTION)

# TABLE OF CONTENT

1) Objectives
2) Division of Labour
3) Theory
4) The Setup
5) Experimental Procedure
6) Results and Discussion
7) Conclusion

Aside: Generalisation to Fractional Order Oscillators

# OBJECTIVES

Objectives:

- To investigate several harmonic oscillators including:
  - Simple Harmonic Oscillator
  - Damped Oscillator
  - Damped Driven Oscillators
- Compare how well our theoretical models fit the experimental results
  - Experimentally verify theory

# DIVISION OF LABOUR

- Alex
  - Carry Out Experimental Procedure
  - Measurement
- Araaf
  - Carry Out Experimental Procedure
  - Measurement
- Conrad
  - Theory
  - Plotting
  - Write Up

# THEORY

Recipe:

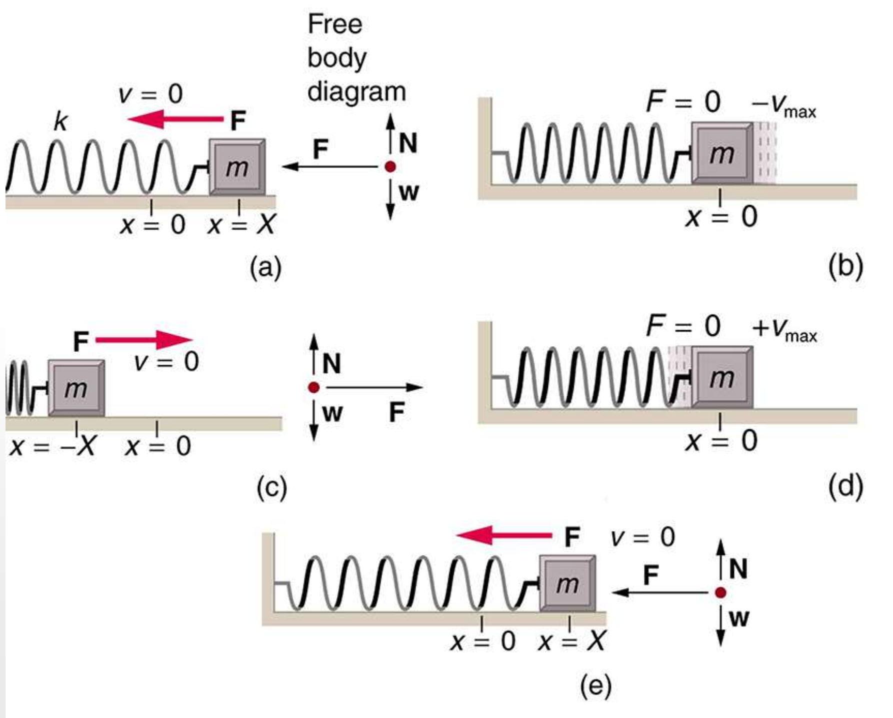
- Identity Set Up
- Consider its Lagrangian
- Euler-Lagrange Equation
- Solve the ODEs
- Theoretical Models

Let’s consider see an example for Simple Harmonic Oscillator (SHO)

## THEORY: SIMPLE HARMONIC OSCILLATOR

Set Up:

- Consider a point mass attached to a spring
- Mass, m (kg)
- Spring Constant, k (N/m)
- Move along the x-axis




https://courses.lumenlearning.com/atd-austincc-physics1/chapter/16-3-simple-harmonic-motion-a-special-periodic-motion/

# THEORY: SIMPLE HARMONIC OSCILLATOR

- Lagrangian

$$L = T - V$$

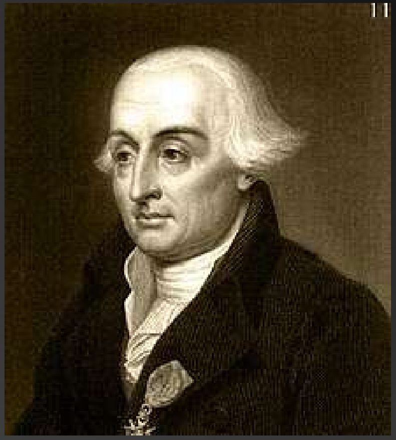
$$L = \frac{1}{2}mx'^2 - \frac{1}{2}kx^2$$

- Euler-Lagrange Equation

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial x'}\right) = 0$$

Fun Fact:
Lagrange was only 19 when he collaborated with Euler for this equation!

Lagrange
(1736-1813)



Euler
(1703-1783)

# THEORY: SIMPLE HARMONIC OSCILLATOR

- Then we obtain the following expression:

$$mx'' + kx = 0$$

$$x'' = -\frac{k}{m}x$$

- Since the natural frequency $\omega = \sqrt{\frac{k}{m}}$. Hence:

$$x'' = -\omega^2 x$$

# THEORY: SIMPLE HARMONIC OSCILLATOR

- Solve the homogenous ODE

We use the Ansatz x = $e^{rt}$ where x, $r \in \mathbb{C}$

Recall the ODE:

$$x'' = -\omega^2 x$$

By substituting in our ansatz:

$$r^2 + \omega^2 = 0$$

Solving the characteristic polynomial as above

# THEORY: SIMPLE HARMONIC OSCILLATOR

- Characteristic Polynomial

$$r^2 + \omega^2 = 0$$
$$r = \pm\omega i$$

- By substituting into our general form of a 2nd order ODE solution:

$$x(t) = c_0 cos(\omega t) + c_1 sin(\omega t)$$

- At time t = 0, x has to be the amplitude:

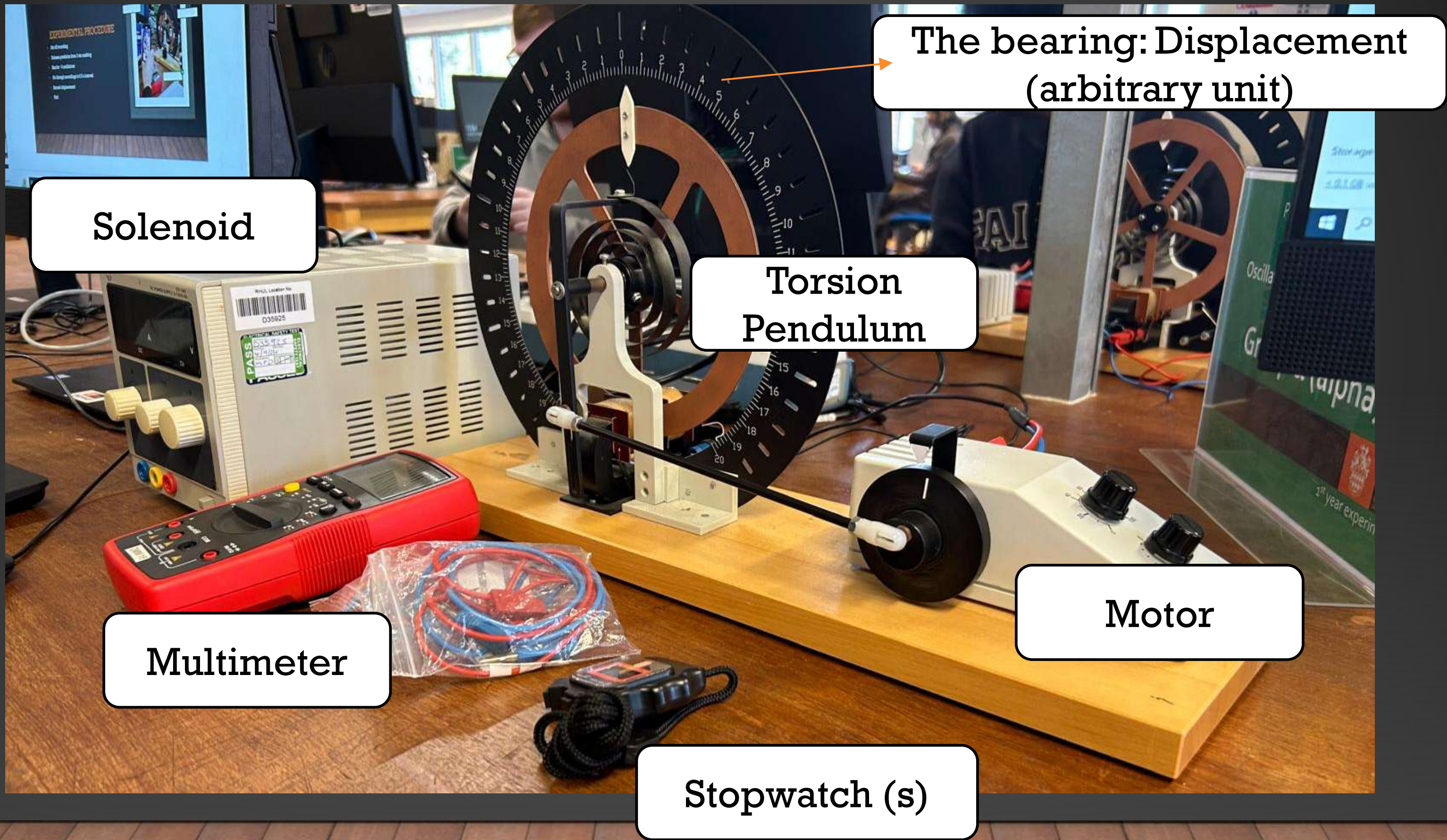
$$x(0) = A$$

- Hence:

$$x(t) = Acos(\omega t)$$

Theoretical Model

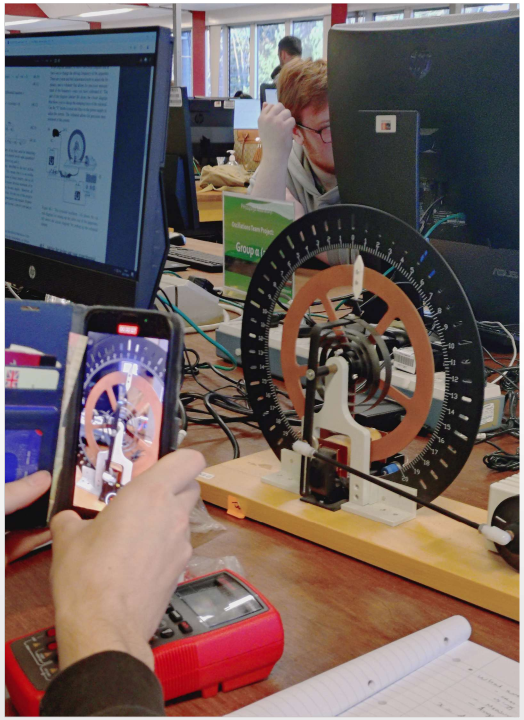
# THE SET-UP



The bearing: Displacement (arbitrary unit)
Solenoid
Torsion Pendulum
Motor
Multimeter
Stopwatch (s)

# EXPERIMENTAL PROCEDURE

- Set off recording
- Release pendulum from the marking at 3 arbitrary units:
- For the DDHO:
  - On top of our manual release, we also used a motor to drive the oscillator
  - Angular frequency : 2.42 rads^-1 from a measured period of 2.60s.
- Run for ~5 oscillations
  - To record the time of oscillations, use the iPhone software to record and slow down the footage of the oscillation to make accuracy and precision of measurements better
- Go through recordings in 0.5 s interval
- Record displacement for the respective time
- Plot!

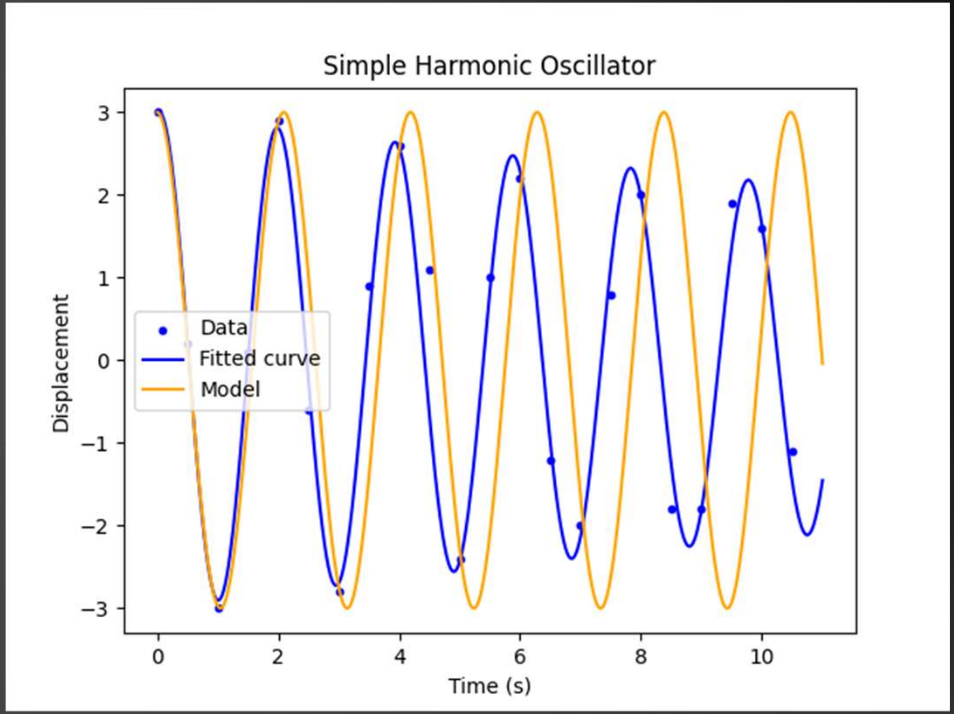


# SIMPLE HARMONIC OSCILLATOR

- The theoretical model is given to be:

  $x(t) = A\cos(\omega t)$

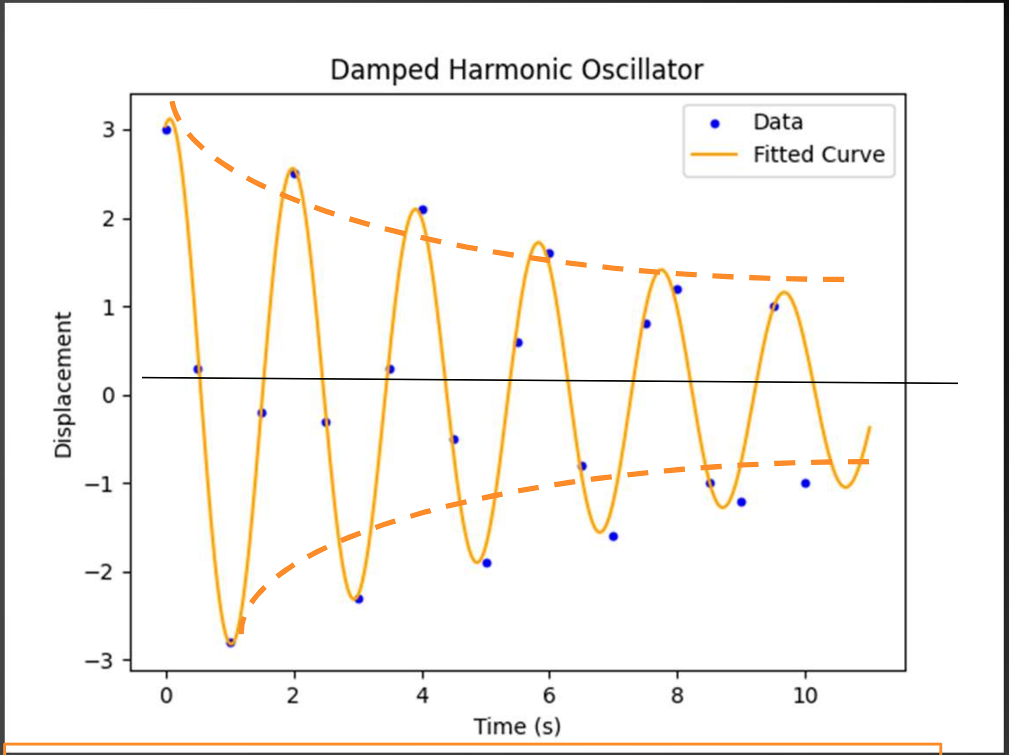
- For small time, t, the model fits the theoretical-model but as time, t , we cam see that the displacement from the equilibrium is not constant, hence suggesting a dampening effect.
- This occurs, as a direct consequence to the frictional force as no system is frictionless, thus explaining why some minimal damping occurs.




See the appendix A for SHO data.

# DAMPED HARMONIC OSCILLATOR

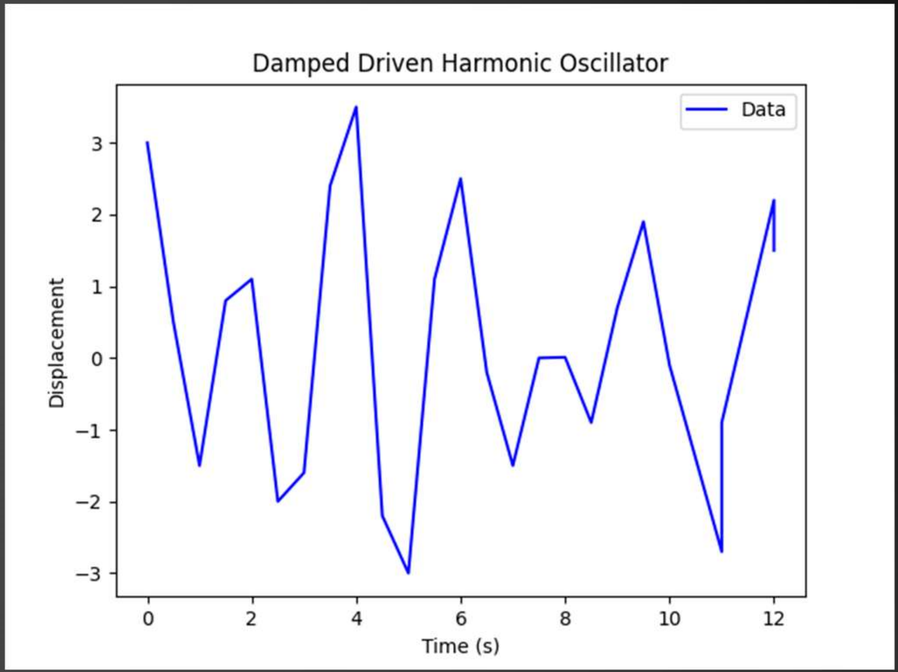
- The theoretical model is given to be:
- $x(t) = A\cos(\omega t)\exp(-\gamma t/2)$
- Unlike the damping in SHO, the damping in DHO is generated due to the magnetic field in the solenoid of component B, the denser the B-field, the greater the damping force.
- Outlier caused by non-uniform B-field?
- The initial amplitude, A was 3 (arbitrary units)




See Appendix B for DHO data.

# DAMPED DRIVEN HARMONIC OSCILLATOR

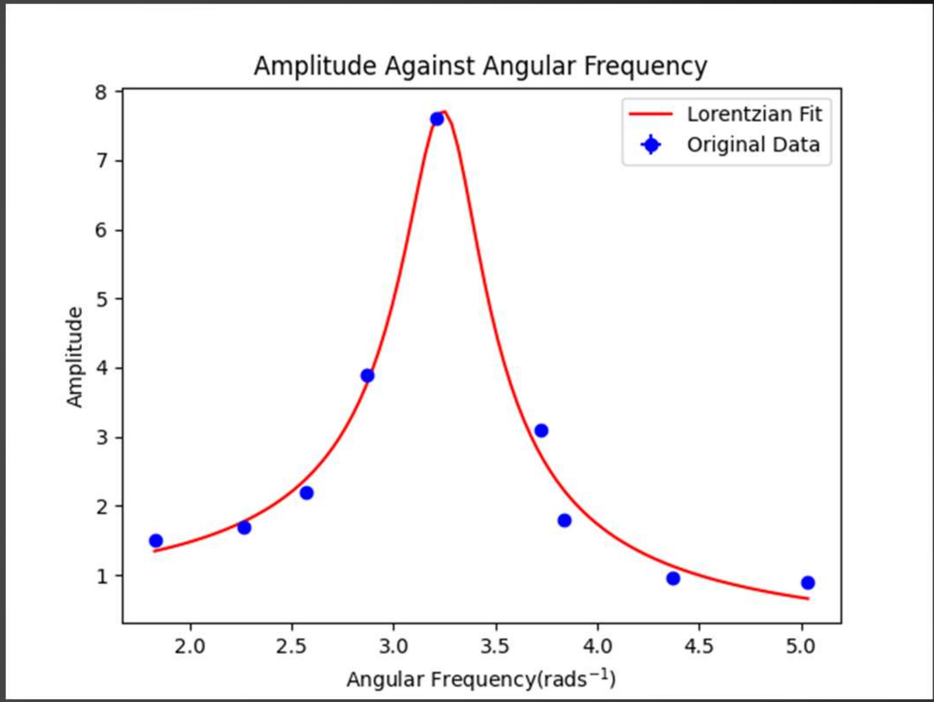
- For this system, we did not fit it against a model
  - $x(t) = A\cos(\omega_D t+\phi)$
- However, we can still see the amplitude changing in relatively irregular manner (decreasing and increasing) compared to previous systems
- Therefore, we can confirm, there exists application of external force (i.e. the driving force from the motor.)
- The time period (T) and angular frequency ($\omega_0$):
  - **T = 3.34 s ±0.001 s**
  - **$\omega_0$ = 2.51 ± 0.001 rad/s (using uncertainty formula)**




See Appendix C for DDHO data.

# AMPLITUDE-ANGULAR FREQUENCY

- Identify time of 1 full period of the motor.
- Use 2π/T to get angular frequency
- Recorded the associated max. displacement
- By increasing the setting on the motor, we obtained different value of angular frequency and displacement.
- Plotting max displacement against ω, we get the resonance graph (on the right)
- The result shows a Cauchy distribution




See Appendix D for x-omega data.

# CONCLUSION

We discussed:

- Theoretical Model
- Experimental Set Up
- Experimental Procedure
- Results
- Relevant Uncertainties and their Effects on Results

Based on experimental results, it matches and verifies our theories!

# ASIDE
# GENERALISATION TO FRACTIONAL ORDER OSCILLATORS

CONRAD HO

(PLEASE FEEL FREE TO TALK TO ME IF YOU FIND THIS INTERESTING! WOULD BE HAPPY GO INTO MORE DEPTH!)

# ASIDE: GENERALISATION TO FRACTIONAL ORDER OSCILLATORS

- Replace the order of the ODEs into fractional order $\alpha$
- Note:

$$D_t^{\alpha} \leftrightarrow \frac{d^{\alpha}}{dt^{\alpha}}, \qquad 0 < \alpha < 1\,, \alpha \in \mathbb{C}$$

- For example, our SHO ODE becomes:

$$D_t^{2\alpha} x(t) = -\omega^2 x$$

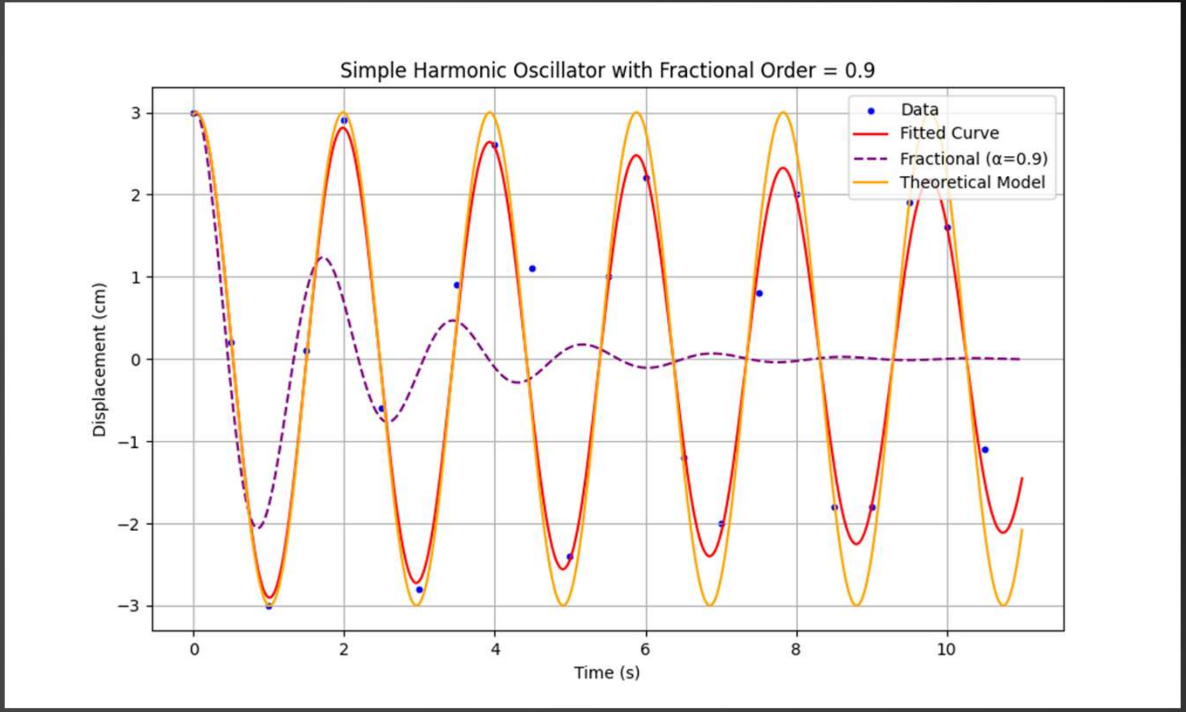
- We solve this Fractional ODE (FODE) numerically using python library *fodeint*

Note: We can solve this analytically but for simplicity's sake and convenience in plotting, we did this numerically

# ASIDE: GENERALISATION TO FRACTIONAL ORDER OSCILLATORS

Result:

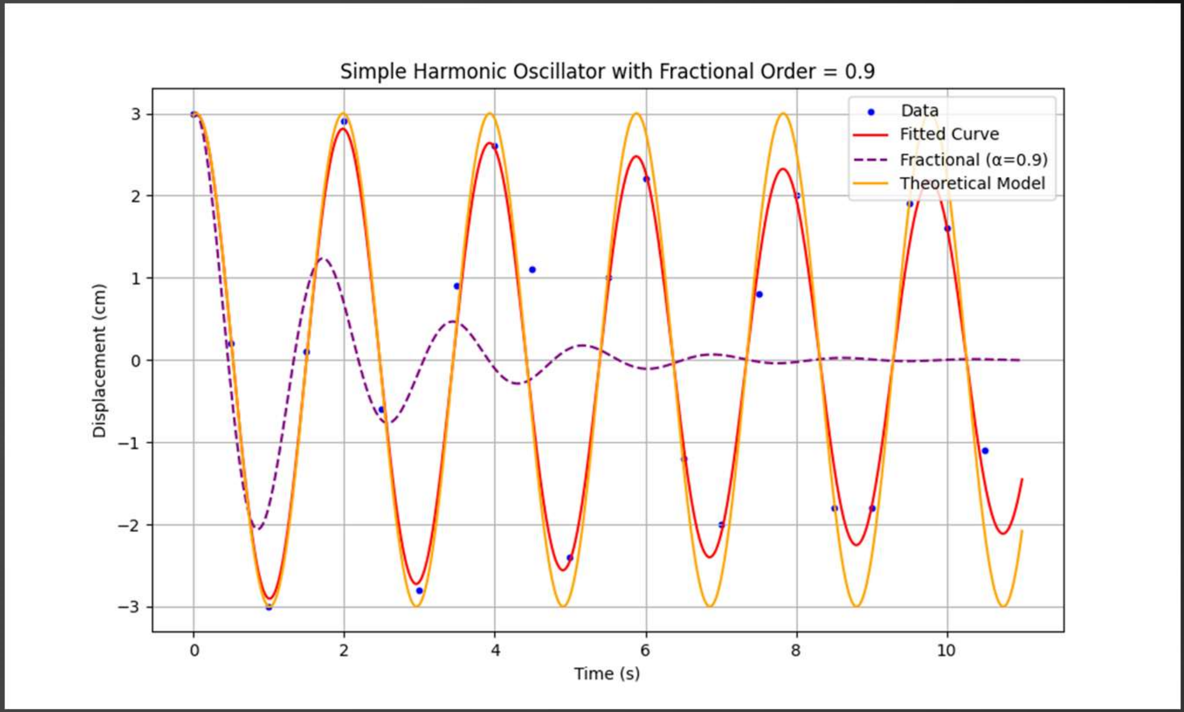
- Plotting α = 0.9 against our data and theoretical model
- As lim α -> 1, we recovers the integer order model
- Not an exponential decay!
- This gives rise to interesting physical investigation



# ASIDE: GENERALISATION TO FRACTIONAL ORDER OSCILLATORS

Theoretical Explanation:

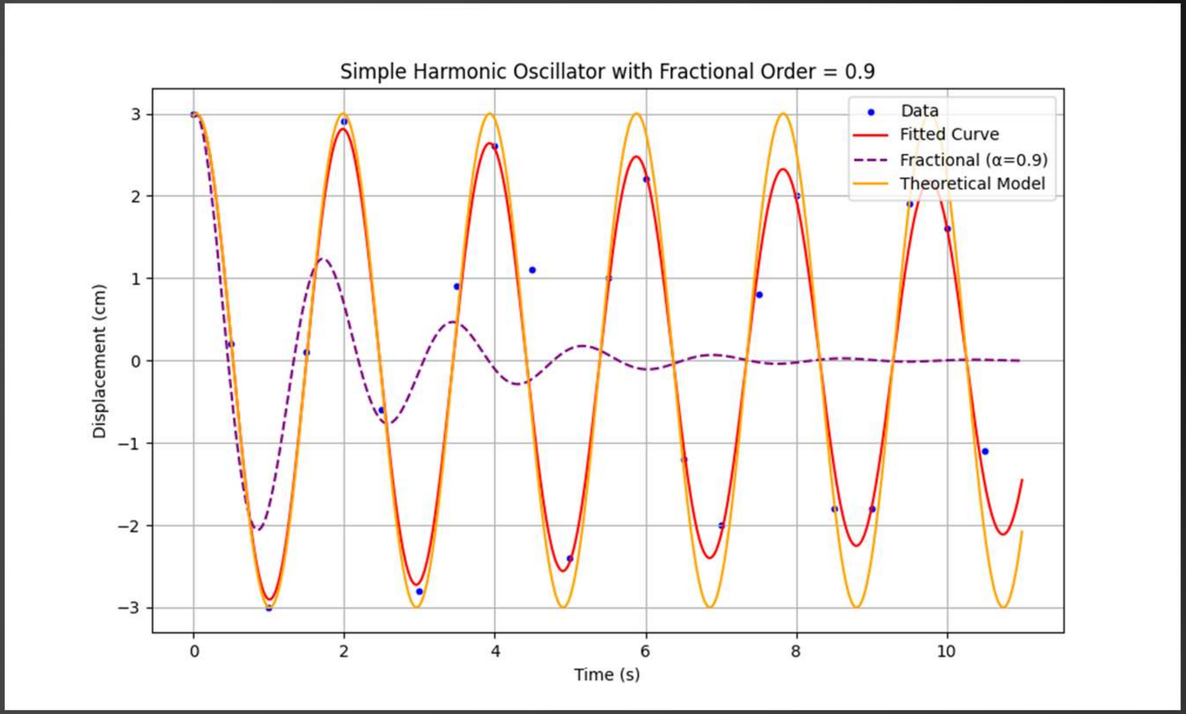
- Integral and Differential is a continuum in fractional order
  - Gives rise to the differintegral operator
- A non-local operator!
- Associated memory effects:
  - Not only instantaneous
  - It considers its history
  - Hence the decay



# ASIDE: GENERALISATION TO FRACTIONAL ORDER OSCILLATORS

Future Work:

- Optimise $\alpha$ to a data set from an experiment
- Investigate non-exponential decay in Oscillators
- Physical implication of fractional order
- Experimental realisation
- Apply to Quantum Harmonic Oscillators?



# ASIDE: GENERALISATION TO FRACTIONAL ORDER OSCILLATORS

Recommended Reading:


[1] Tarasov VE. Fractional dynamics: applications of fractional calculus to dynamics of particles, fields and media. Springer Science & Business Media; 2011 Jan 4.

[2] Jin B. Fractional differential equations. Cham, Switzerland: Springer International Publishing; 2021.

[3] Herrmann R. Fractional calculus: an introduction for physicists. 2011 Feb 21.

[4] Hilfer R, editor. Applications of fractional calculus in physics. World scientific; 2000 Mar 2.

[5] Diethelm K, Kiryakova V, Luchko Y, Machado JT, Tarasov VE. Trends, directions for further research, and some open problems of fractional calculus. Nonlinear Dynamics. 2022 Mar;107(4):3245-70.

[6] Tarasov VE, editor. Handbook of fractional calculus with applications. Berlin, Germany: de Gruyter; 2019.

# APPENDIX

# APPENDIX A: RAW DATA FOR SHO

| No | Time, t +/- 0.01 s | Displacement , x +/- 0.01 arbitrary units |
|---|---|---|
| 1 | 0.0 | 3.0 |
| 2 | 0.5 | 3.0 |
| 3 | 1.0 | -2.8 |
| 4 | 1.5 | -0.2 |
| 5 | 2.0 | 2.5 |
| 6 | 2.5 | -0.3 |
| 7 | 3.0 | -2.3 |
| 8 | 3.5 | 0.3 |
| 9 | 4.0 | 2.1 |
| 10 | 4.5 | -0.5 |
| 11 | 5.0 | -1.9 |
| 12 | 5.5 | 0.6 |
| 13 | 6.0 | 1.6 |
| 14 | 6.5 | -0.8 |
| 15 | 7.0 | -1.6 |
| 16 | 7.5 | 0.8 |
| 17 | 8.0 | 1.2 |
| 18 | 8.5 | -1.0 |
| 19 | 9.0 | -1.2 |
| 20 | 9.5 | 1.0 |
| 21 | 10.0 | -1.0 |

# APPENDIX B: RAW DATA FOR DHO

| no | t +/- 0.01 s | x +/- 0.01 arbitrary units |
|---|---|---|
| 1 | 0.0 | 3.0 |
| 2 | 0.5 | 0.2 |
| 3 | 1.0 | -3.0 |
| 4 | 1.5 | 0.1 |
| 5 | 2.0 | 2.9 |
| 6 | 2.5 | -0.6 |
| 7 | 3.0 | -2.8 |
| 8 | 3.5 | 0.9 |
| 9 | 4.0 | 2.6 |
| 10 | 4.5 | 1.1 |
| 11 | 5.0 | -2.4 |
| 12 | 5.5 | 1.0 |
| 13 | 6.0 | 2.2 |
| 14 | 6.5 | -1.2 |
| 15 | 7.0 | -2.0 |
| 16 | 7.5 | 0.8 |
| 17 | 8.0 | 2.0 |
| 18 | 8.5 | -1.8 |
| 19 | 9.0 | -1.8 |
| 20 | 9.5 | 1.9 |
| 21 | 10.0 | 1.6 |
| 22 | 10.5 | -1.1 |

# APPENDIX C: RAW DATA FOR DDHO

| No | Time ,t, (s ) +/- 0.01 | Displacement , x  (arbitrary units  ) +/- 0.01 |
|---|---|---|
| 1 | 0.0 | 3.0 |
| 2 | 0.5 | 0.5 |
| 3 | 1.0 | -1.5 |
| 4 | 1.5 | 0.8 |
| 5 | 2.0 | 1.1 |
| 6 | 2.5 | -2.0 |
| 7 | 3.0 | -1.6 |
| 8 | 3.5 | 2.4 |
| 9 | 4.0 | 3.5 |
| 10 | 4.5 | -2.2 |
| 11 | 5.0 | -3.0 |
| 12 | 5.5 | 1.1 |
| 13 | 6.0 | 2.5 |
| 14 | 6.5 | -0.2 |
| 15 | 7.0 | -1.5 |
| 16 | 7.5 | 0.0 |
| 17 | 8.0 | 0.01 |
| 18 | 8.5 | -0.9 |
| 19 | 9.0 | 0.7 |
| 20 | 9.5 | 1.9 |
| 21 | 10.0 | -0.1 |
| 22 | 10.5 | -2.7 |
| 23 | 11.0 | -0.9 |
| 24 | 11.5 | 2.2 |
| 25 | 12.0 | 1.5 |

# APPENDIX D: RAW DATA FOR X-OMEGA

| Period (s) | Angular freq rad/s | Max. Displacement (arbitrary unit) |
|---|---|---|
| 3.44 | 1.83 | 1.5 |
| 2.78 | 2.26 | 1.7 |
| 2.45 | 2.57 | 2.2 |
| 2.19 | 2.87 | 3.9 |
| 1.96 | 3.21 | 7.6 |
| 1.69 | 3.72 | 3.1 |
| 1.64 | 3.84 | 1.8 |
| 1.44 | 4.37 | 0.95 |
| 1.25 | 5.03 | 0.9 |