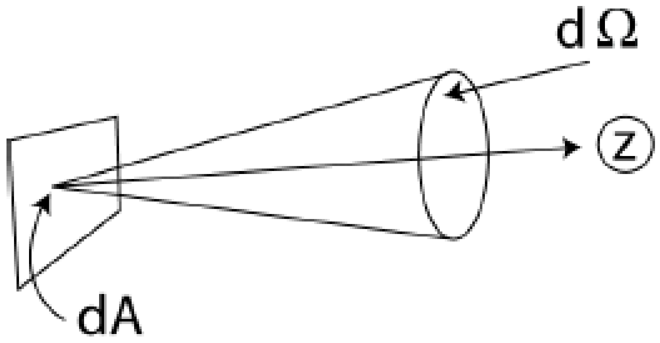
Collection of particles

- “Particle distribution function” or “Phase space density”

$$f(\boldsymbol{x},\boldsymbol{v},t)=\frac{\#}{d^3\boldsymbol{x}\;d^3\boldsymbol{v}}$$

It satisfies, $n(\boldsymbol{x},t)=\int f\left(\boldsymbol{x},\boldsymbol{v},t\right)d^3\boldsymbol{v}$

Thus, n has units: $\frac{1}{m^3}$ f has units: $\frac{1}{m^3}\frac{s^3}{m^3}$



Relate f to what a particle detector measures in space j (flux):

j measured as function of position, energy, look direction, time

$$j=\frac{\#}{dA\,d\Omega\,dK\,dt}:\qquad \text{units}\ \frac{\#}{\mathrm{m}^2\text{ - s - ster - joule}}$$ (“keV” typically used, not Joule)

Collection of particles

Spherical coords: **(Non-relativistic!)**

$$d^3\mathbf{v} = v^2\, dv \sin\theta\, d\theta\, d\Psi = v^2 dv\, d\Omega; \qquad d^3\mathbf{x} = dx\, dy\, dz = dA\, v dt \qquad (dA \perp \mathbf{v})$$

$$\therefore f = \frac{\#}{v^3 dA d\Omega dt dv}$$

$$K = \tfrac{1}{2} m v^2, \text{ so that } dK = mvdv \qquad \text{Thus } v^3 dv = v^2 dK/m = 2K\, dK/m^2$$

$$\therefore f = \frac{\#\, m^2}{2\, dA\, d\Omega\, K dK dt} = \frac{m^2 j}{2K} = \frac{m}{v^2} j \qquad j = \frac{\#}{dA\, d\Omega\, dK\, dt}$$

$$= 5.45\, x 10^{-19} \frac{j(cm^2 - s - ster - keV)^{-2}}{K(keV)} \quad \text{for protons}$$

$$= 1.62\, x 10^{-25} \frac{j(cm^2 - s - ster - keV)^{-2}}{K(keV)} \quad \text{for electrons}$$

$$\boxed{f = \frac{m}{v^2} j}$$