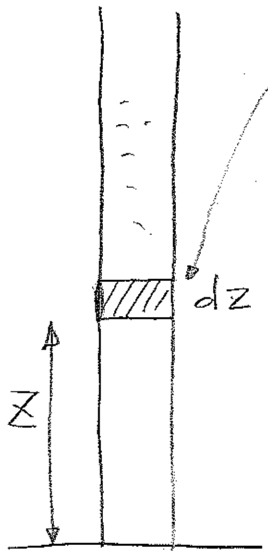
Homework #3 3760

Pr #1.

Isentropic equilibrium of the atmosphere.

1) Condition of mechanical equilibrium




Pressure in volume $dz \cdot A$ is coused by the weight of the column of air on top of it.

$$P(z) = \int_z^{\infty} \rho(z) \cdot g \cdot dz$$

$$\frac{dP(z)}{dz} = \frac{d}{dz} \int_z^{\infty} \rho(z) \cdot g \, dz = -\rho(z) g \quad (1)$$

2) The atmosphere is adiabatic that is $PV^{\gamma} = const$. We need to find what is

$\frac{dT}{dz}$ -?

2) From ideal gas law $PV = \frac{m}{\mu} RT \Rightarrow \rho = \frac{P}{T} \cdot \frac{\mu}{R}$ (2)

3) from law of mass conservation.

$$\left.\begin{array}{l} PV^{\gamma} = const. \\ \frac{PV}{T} = R\frac{m}{\mu} = const. \Rightarrow \frac{P^{\gamma} V^{\gamma}}{T^{\gamma}} = const \end{array}\right\} \Rightarrow P^{1-\gamma} \cdot T^{\gamma} = const. \quad (3)$$

in equation (3) both P and T are functions of z

Let's take derivative of the equation (3) over z

$$\frac{d}{dz}\left(P^{1-\gamma} T^{\gamma}\right) = (1-\gamma) P^{-\gamma} T^{\gamma} \frac{dP}{dz} + P^{1-\gamma} \cdot \gamma \, T^{\gamma-1} \frac{dT}{dz} = 0$$

$$\left.\begin{array}{l} \frac{dP}{dz} = \frac{dT}{dz} \cdot \frac{\gamma}{1-\gamma} \cdot \frac{P}{T} \\ \text{for air, } \gamma = \frac{7}{5} \end{array}\right\} \Rightarrow \frac{dP}{dz} = \frac{7}{2} \cdot \frac{P}{T} \frac{dT}{dz} \quad (4)$$

Let's now put together equations (1), (2) and (4)

$$\frac{dT}{dz}\cdot\frac{7}{2}\cdot\frac{P}{T}=\frac{dP}{dz}=-\rho(z)\cdot g=\underbrace{\frac{-P}{T}\cdot\frac{\mu}{R}}_{\rho}\cdot g$$

$$\frac{dT}{dz}=-\frac{2}{7}\cdot\frac{\mu}{R}\cdot g=-\frac{2}{7}\cdot\frac{29\cdot10^{-3}}{8.31}\cdot 9.8=-10^{-2}\frac{K}{m}=-10\cdot\frac{K}{km}$$

molecular mass of air is 29 g (Google) Kilometer

$$\frac{dT}{dz}=-10\frac{^\circ C}{km}$$

(c) show that $P\propto\rho^{\gamma}$

equation (2) $\rho=\frac{P}{T}\cdot\frac{\mu}{R}$ can be re-written $\rho^{\gamma}=\frac{P^{\gamma}}{T^{\gamma}}\cdot\frac{\mu^{\gamma}}{R^{\gamma}}$

equation (3) $P^{1-\gamma}\cdot T^{\gamma}=const=C$

combining (2) (3) we have.

$$\rho^{\gamma}\cdot C=P^{1-\gamma}T^{\gamma}\cdot\frac{P^{\gamma}}{T^{\gamma}}\cdot\frac{\mu^{\gamma}}{R^{\gamma}}=\frac{\mu^{\gamma}}{R^{\gamma}}\cdot P \quad \text{or}$$

$$P\propto\rho^{\gamma}$$

HW #3

Pr # 2.

Exact expression is $P(215, 400) = \frac{400!}{215!\,(400-215)!} \cdot \frac{1}{2^{400}}$ (1)

We use the Gaussian function to make an approximation to the above expression. In lecture notes for spin system we have.

$$P(S,N) = \frac{1}{2^N} \cdot \sqrt{\frac{2}{\pi N}} \cdot 2^N \cdot \exp\left(-\frac{2S^2}{N}\right) = \sqrt{\frac{2}{\pi N}} \cdot \exp\left(-\frac{2S^2}{N}\right) \quad (2)$$

Where $S$ is the spin excess $2S = N_\uparrow - N_\downarrow$. The equation (2) needs to be modified to be applicable to our problem. Let's re-write (2) using variable $t = 2S = N_\uparrow - N_\downarrow$.

$$P(t,N) = \sqrt{\frac{2}{\pi N}} \cdot \exp\left(-\frac{t^2}{2N}\right) \quad (3)$$

Equation (3) is what we need for our head and **tail** problem if we set $t = N_{head} - N_{tail} = 30$.

$$P(215, 400) = \sqrt{\frac{2}{\pi \cdot 400}} \cdot \exp\left(-\frac{900}{2 \cdot 400}\right) = 0.04 \cdot 0.324 = \underline{0.013}$$

HW #3

Pr #3

A) Lets, using the method of symbolic multiplication list all possibilities for 1 ball

$1_R + 2_R + 3_R$.

here, for example $1_R$ means red ball in 1st basket.

all possible combination can be generated by multiplication

$(1_R+2_R+3_R)\cdot(1_B+2_B+3_B)\cdot(1_Y+2_Y+3_Y)(1_G+2_G+3_G)$

So we have in total $3^4$ - different combinations

Probability of a particular combination is $1/3^4$

(B) Using symbolic multiplication we may list all combination as a product

$$(1_R+2_R+3_R)\underbrace{(1_R+2_R+3_R)}_{\text{the second red ball}}(1_B+2_B+3_B)(1_Y+2_Y+3_Y)\underbrace{(1_Y+2_Y+3_Y)}_{\text{the second yellow ball}}$$

total number of combinations is $3^5$

if all balls are different, probability of a particular combination is $1/3^5$. But since we have 2 red balls and 2 yellow balls, any combination with a particular arrangment of red balls appears twice in our set. The same is said for red balls. Therefore the probability of $1_R\, 2_B\, 2_Y\, 3_Y\, 3_R$ is $4/3^5$

Problem #4. HW #3.

$N = 10$ ; $2S = N_\uparrow - N_\downarrow$

Lets make a table.

| $N_\uparrow$ | $S$ | $S^2$ | $g = \frac{N!}{N_\uparrow ! N_\downarrow !}$ | $S^2 \cdot g$ |
|---|---|---|---|---|
| 0 | -5 | 25 | 1 | 25 |
| 1 | -4 | 16 | 10 | 160 |
| 2 | -3 | 9 | 45 | 405 |
| 3 | -2 | 4 | 120 | 480 |
| 4 | -1 | 1 | 210 | 210 |
| 5 | 0 | 0 | 252 | 0 |
| 6 | 1 | 1 | 210 | 210 |
| 7 | 2 | 4 | 120 | 480 |
| 8 | 3 | 9 | 45 | 405 |
| 9 | 4 | 16 | 10 | 160 |
| 10 | 5 | 25 | 1 | 25 |

$$\langle S^2 \rangle = \sum_{N_\uparrow = 0}^{N_\uparrow = 10} S^2 \cdot g \cdot \frac{1}{2^{10}} = \frac{1}{2^{10}} \cdot \sum S^2 \cdot g = 2.5$$