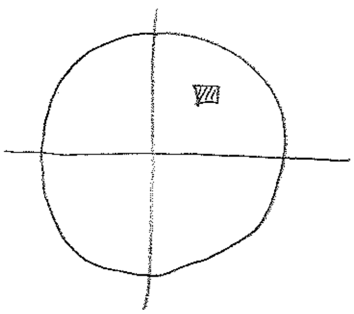
Homework N10

Pr #1

a) $P(x,y,z) = A\cdot \exp\left(-\frac{U(x,y,z)}{kT}\right) dx\,dy\,dz$ — probability to find a molecule in the volume element between x $x+dx$, y $y+dy$, z $z+dz$.



$$n(x,y,z) = \frac{N\cdot P(x,y,z)}{dx\,dy\,dz}$$

N - total number of molecules in a centrifuge n - concentration.

Centrifugal force $F = m\omega^2 r$; Potential corresponding to that force can be found from $F = -\nabla U$

$$\frac{d}{dR}U = -m\omega^2 r \qquad U(r) = U(0) - m\omega^2 r^2 \cdot \frac{1}{2}.$$

$$n(r) = \frac{A\cdot N\cdot P(x,y,z)}{dx\,dy\,dz} = A\cdot N\cdot e^{-\frac{U(0)}{kT}}\cdot \exp\left(\frac{m\omega^2 r^2}{2kT}\right)$$

when $r=0$ $\exp\left(\frac{m\omega^2 r^2}{2kT}\right) = 0 \Rightarrow n(r) = n(0)\exp\left(\frac{m\omega^2 r^2}{2kT}\right)$

(b) Total number of particle in cylinder.

$$N(R) = \int_0^Z \int_0^{2\pi} \int_0^R n(0)\exp\left(\frac{m\omega^2 r^2}{2kT}\right)\cdot r\,d\varphi\,dr =$$

$$= Z\cdot n(0)\cdot 2\pi\cdot \frac{2\,kT}{m\omega^2}\cdot\frac{1}{2}\int_0^G \exp(g^2)\,dg \qquad g = \frac{r^2 m\omega^2}{2kT}$$

$$= Z\cdot n(0)\cdot 2\pi\,\frac{2kT}{m\omega^2}\cdot\frac{1}{2}\cdot\left(\exp\left(\frac{m\omega^2 R^2}{2kT}\right) - 1\right)$$

$$N\left(\frac{R}{2}\right) = 2\cdot n(0)\cdot 2\pi\cdot\frac{2kT}{m\omega^2}\cdot\frac{1}{2}\cdot\left(\exp\left(\frac{m\omega^2 R^2}{8kT}\right) - 1\right)$$

$$\frac{N(R/2)}{N(R)} = \frac{\exp\left(\frac{m\omega^2 R^2}{8kT}\right) - 1}{\exp\left(\frac{m\omega^2 R^2}{8kT}\right) - 1}$$

#2

$m_{CO} = 4.65 \times 10^{-26}$ kg

$m_{NO} = 4.98 \times 10^{-26}$ kg

$m_1 = m_{C_2H_3NO_5} = 2.01 \times 10^{-25}$ kg.

$h_{SLC} = 1320$ m

$h_{Alta} = 2600$ m

$H \cong 1300$m.

$$X = \frac{n_{Ata}}{n_{SLC}} = \exp\left(-\frac{mgH}{2kT}\right)$$

CO $X = 87\,\%$

NO $X = 86\,\%$

$C_2H_3NO_5$ $X = 54\%$

The concentration ratios above give us equilibrium distribution of molecules. Clearly during Inversion we have much larger ratio. Inversion is an equilibrium phenomena. It stays for long time because with no wind or convection, the only mechanism to establish equilibrium concentration in diffusion. But diffusion is very slow process.

Problem #3 HW #10.

(a) $P(v_x, v_y) = A \exp\left(-\frac{m(v_x^2+v_y^2)}{2kT}\right) dv_x dv_y$

$$\int P(v_x, v_y) = 1 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dv_x dv_y \; A \cdot \exp\left(-\frac{m(v_x^2+v_y^2)}{2kT}\right)$$

$$= A \cdot \left(\frac{2kT\pi}{m}\right) = 1 \qquad A = \left(\frac{m}{2\pi kT}\right)$$

(b) $$P(v) = \int_0^{2\pi} d\varphi \cdot dv \cdot v \cdot \left(\frac{m}{2\pi kT}\right) \cdot \exp\left(-\frac{mv^2}{2kT}\right)$$

$$P(v) = 2\pi v \left(\frac{m}{2\pi kT}\right) \cdot \exp\left(-\frac{mv^2}{2kT}\right) \cdot dv$$

(c) $$\langle dN(v)\rangle = N \cdot P(v) = N \cdot 2\pi v \cdot \left(\frac{m}{2\pi kT}\right) \cdot \exp\left(-\frac{mv^2}{2kT}\right) dv$$

(d) average energy of a molecule $\langle E\rangle = \frac{m}{2}\langle v_x^2 + v_y^2\rangle$

$$\langle E\rangle = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{m}{2}(v_x^2+v_y^2) \cdot \left(\frac{m}{2\pi kT}\right) \cdot \exp\left(-\frac{m(v_x^2+v_y^2)}{2kT}\right) dv_x dv_y =$$

$$= \frac{m}{2}\left(\frac{m}{2\pi kT}\right) \cdot \left[\int_{-\infty}^{+\infty} dv_x \cdot v_x^2 \cdot \exp\left(-\frac{mv_x^2}{2kT}\right) \times \int_{-\infty} dv_y \exp\left(-\frac{mv_y^2}{2kT}\right) + \right.$$

$$\left. + \int_{-\infty}^{+\infty} dv_x \cdot \exp\left(-\frac{mv_x^2}{2kT}\right) \times \int_{-\infty}^{+\infty} dv_y \cdot v_y^2 \cdot \exp\left(-\frac{mv_y^2}{2kT}\right) \right] =$$

$$= \frac{m}{2}\left(\frac{m}{2\pi kT}\right)\left[\cdot \frac{1}{2} \cdot \sqrt{\pi} \cdot \left(\frac{2kT}{m}\right)^{3/2} \cdot \sqrt{\pi} \cdot \left(\frac{2kT}{m}\right)^{1/2} \times 2\right] =$$

same Integral for dv_y

$$= \frac{m}{2} \frac{m}{2kT} \cdot 2^2 \cdot \left(\frac{kT}{m}\right)^2 = kT$$

Heat capacity $= \frac{\partial}{\partial T} \cdot \langle E\rangle \cdot N = k \cdot N$