Homework #11.

# 2 (7.45)

$$\frac{U}{V} = \frac{8\pi^5 k^4}{15(hc)^3} \cdot T^4 = \alpha T^4. \qquad U = \alpha T^4 \cdot V.$$

entropy? $\quad dU = TdS - PdV$

V = const. $\quad \frac{dU}{T} = dS. \Rightarrow \frac{4\alpha T^3 dT \cdot V}{T} = dS.$

$$S = \int_0^S dS = \int_0^T \frac{V \cdot 4\alpha T^3 dT}{T} = \frac{4}{3}\alpha T^3 \cdot V$$

Pressure.

$$dU = TdS - PdV \qquad P = -\left(\frac{\partial U}{\partial V}\right)_S.$$

$$P = -\frac{\partial}{\partial V}\Big|_S \left(\alpha T^4 \cdot V\right) = -\frac{\partial}{\partial V}\Big|_S \left(\frac{4}{3}\alpha T^3 \cdot V \cdot \frac{3}{4} T\right) =$$

$$= -\frac{\partial}{\partial V}\Big|_S \left(S \cdot \frac{3}{4} T\right) \quad \text{but entropy is kept constant.}$$

$$P = -S \cdot \frac{3}{4} \cdot \left(\frac{\partial T}{\partial V}\right)_S = -\frac{4}{3}\alpha T^3 \cdot V \cdot \frac{3}{4} \cdot \left(\frac{\partial T}{\partial V}\right)_S = -U \cdot \left(\frac{\partial T}{\partial V}\right)_S$$

$S = \frac{4}{3}\alpha T^3 \cdot S$. if $S$ = const $\Rightarrow T^3 \cdot V$ = const.

$$\left[ d(T^3 \cdot V) = 0 \right].$$

$$3 V \cdot T^2 \cdot dT + T^3 dV = 0. \qquad \frac{dT}{dV} = -\frac{1}{3}\frac{T}{V}$$

$$P = -U \cdot \left(-\frac{1}{3}\frac{T}{V}\right) = +\frac{1}{3}\alpha T^4 = +\frac{1}{3}\frac{U}{V}$$

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1500k $P_r = 0.0013$ Pa $\qquad P_g = 1$ atm. $= 10^5$ Pa.

Sun $T = 15 \cdot 10^6$ k. $\frac{T_s}{1500\text{k}} = 10^4 \quad P_s = 1.3 \times 10^{13}$ Pa

$$P = \frac{NkT}{V} \cdot 2 = 2.5 \times 10^{16} \text{ Pa}.$$

? $\quad$ proton + electron

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#1 7.44 (a) To compute the number of ~~modes~~ photons, we can simply sum the Plank distribution over all "modes" in the cube (see lecture), including factor of 2 to account for 2 polarization.

$$N = 2\sum_{n_x}\sum_{n_y}\sum_{n_z} \overline{n}_{PL}(\varepsilon) = 2\sum_{n_x n_y n_z} \frac{1}{\exp\left(\frac{hcn}{2LkT}\right)-1}$$

Where $n = (n_x^2+n_y^2+n_z^2)^{1/2}$.

We can transform $\Sigma$ to $\int$ by usual means.

$$N = 2\sum \frac{1}{\exp\left(\frac{hcn}{2LkT}\right)-1} \cdot \frac{\Delta n_x \Delta n_y \Delta n_z}{\Delta n_x \Delta n_y \Delta n_z} = \qquad \left|\begin{array}{l}\Delta n_x = 1\\ \Delta n_y = 1 \\ \Delta n_z = 1\end{array}\right.$$

$$= 2\int_0^\infty\int_0^\infty\int_0^\infty \frac{1}{\exp\left(\frac{hcn}{2LkT}\right)-1} \cdot dn_x dn_y dn_z = \qquad \Big\} \text{transforming to spherical coordinates.}$$

$$= 2\,\frac{1}{8}\int_0^\infty \frac{1}{\exp\left(\frac{hcn}{2LkT}\right)-1} \cdot 4\pi n^2 dn =$$

$$= 8V\pi\left(\frac{kT}{hc}\right)^3 \underbrace{\int_0^\infty \frac{x^2}{\exp(x)-1}dx}_{2.4.}$$

(b) $S = \frac{32\pi^5}{45}V\left(\frac{kT}{hc}\right)^3 k.$ (eq. 7.89 in text book) $\frac{S}{N} = 3.60k.$

(c) At room temperature.

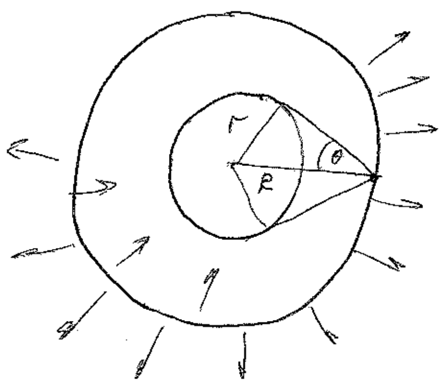
$$\frac{N}{V} = 2.4\times 8\pi\left(\frac{kT}{hc}\right)^3 = 5.8\times 10^{14}\,m^{-3} \qquad T = 300K.$$

$$\frac{N}{V} = 6.8\times 10^{16}\,m^{-3} \qquad T = 1500K$$

$$\frac{N}{V} = 4.1\times 10^{8}\,m^{-3} \qquad T = 2.73K.$$

Homework #11

#3



1) Let's write a power balance for the shell.

absorbtion. emission.

$$\sigma \cdot 2\pi r \cdot L \cdot T_0^4 + L\sigma 2\pi R T_1^4 (1-\beta) = \sigma 2\pi L R T_1^4 + \sigma 2\pi L R \cdot T_1^4 \quad (1)$$

all radiation emitted by the cylinder is captured by the shell. | $\beta$ - fraction of radiation emitted by inner surface of the shell but captured by the cylinder | radiation emitted by outer surface | radiation emitted by inner surface.

$\beta$ -? every point on the inner surface of the shell emits uniformly in $\pi$ angle. =>

$\beta = \frac{2\theta}{\pi}$, where $\theta = \arcsin\frac{r}{R}$

We have from (1)

$$rT^4 = RT_1^4\beta = RT_1^4$$

$$\boxed{T_1^4 = \frac{T_0^4 \cdot r}{R(1+\beta)} = T_0^4 \frac{r}{R}\frac{1}{1+\beta}}$$

if $r \approx R$ $\quad \theta = \frac{\pi}{2}$.

$$T_1^4 = T_0^4 \cdot \frac{1}{1+\frac{2}{\pi}\frac{\pi}{2}} = T_0^4 \cdot \frac{1}{2}$$ — that is the result we had for two parallel plates.