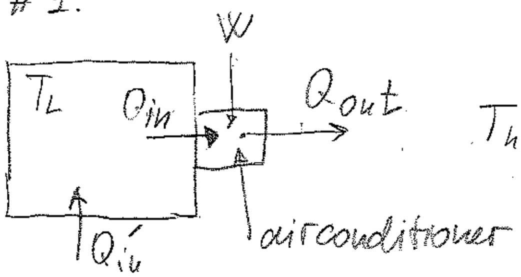
Homework # 6

Pr # 1.



Q_{in} – heat removed by AC from a room

Q'_{in} – heat entering the room through walls.

W – work done by AC.

① AC efficiency $COP = \frac{Q_{in}}{W} = \frac{Q_{in}}{Q_{out} - Q_{in}} = \eta$

From 2d law $\frac{Q_{in}}{T_L} = \frac{Q_{out}}{T_h} \Rightarrow COP = \frac{Q_{in}}{\frac{T_h}{T_L} Q_{in} - Q_{in}} = \frac{T_L}{T_h - T_L} = \eta$

② Let F_{in} be Q_{in}/t – heat flow, amount of heat per second removed by AC from the room.

$F_{in} = P \cdot \eta$, where P is electrical power consumption

$F_{in} = P \cdot \frac{T_L}{T_h - T_L}$

③ $F'_{in} = A(T_h - T_L)$ – heat flow into the room through walls.

steady state. is when $F'_{in} = F_{in}$

$P \cdot \frac{T_L}{T_h - T_L} = A(T_h - T_L)$, we want to know T_L

$$A T_L^2 - (2AT_h + P) T_L + A T_h^2 = 0.$$

$$T_L = \frac{(2AT_h + P) - \sqrt{(2AT_h + P)^2 - 4A^2T_h^2}}{2A} =$$

$$= \left(T_h + \frac{P}{2A}\right) - \left[\left(T_h + \frac{P}{2A}\right)^2 - T_h^2\right]^{1/2}$$

Comment "+" sign in the root of this quadratic equation is not physical.

$$A = \frac{P \cdot T_L}{(T_h - T_L)^2} = \frac{2 \cdot 10^3 \cdot 290}{(20)^2} = \frac{290}{2} = 1450 \, \frac{W}{K}.$$

HW #6

Problem #3

There were two mistakes in the problem statement.

$P_{st} = 3\ kPa$ (not $34\ kPa$)

$L = 2260\ J/g$ (not $2.5\ kJ/kg$)

Reasoning: When air goes up water condenses. As a result of this vapor → water phase transformation heat is released and is absorbed by air. So we need to find out how much water is in $1\ m^3$ of air (initially) and make an energy balance.

(a) Partial pressure of vapor $P = h \cdot 3kPa = 1.8 \times 10^3\ Pa$.

$PV = \frac{m}{\mu} RT \qquad m = \frac{PV\mu}{RT} = \frac{1.8 \cdot 10^{+3} \cdot 1 \cdot 18g}{8.31 \cdot 300} = 13g.$

(b) The heat released as a result of vapor condensation.

$Q = 13 \cdot 2260 = 29,380\ J.$

(c) Number of moles in $1\ m^3$ of air

$PV = \nu RT \qquad \nu = \frac{PV}{RT} = \frac{10^5}{8.31 \cdot 300} = 40$ moles.

(d) $\Delta Q = \Delta U$ – we assume that there is no volume change.

$\frac{5}{2} R \cdot \nu \Delta T = 29 \times 10^3 J \qquad \Delta T = 34\ K.$

Comments: Actual difference in temperature between Santa Cruz and Fresno (they have about the same latitude) is 12 K. Possible reasons for disagreement:

1. Not all water condenses. (relative humidity in Fresno ~25% in afternoon. This alone will bring our estimate down to $\Delta T = 20K$.
2. The air coming to Fresno not only from Sierra Nevada.
3. Process is not completely adiabatic. Air picks up some cold from tops of mountains.

Homework # 6

PR # 2

(a) $\Delta G = 2(-237.13\ kJ) + (-394.36\ kJ) - (-50.72\ kJ) = -817.9\ kJ$

$\Delta H = 2(-285.83\ kJ) + (-393.51\ kJ) - (-74.81)\ kJ = -890.36\ kJ$

(b) Under ideal condition all of the decrease in G comes out as electrical work $\Rightarrow W = 818\ kJ$.

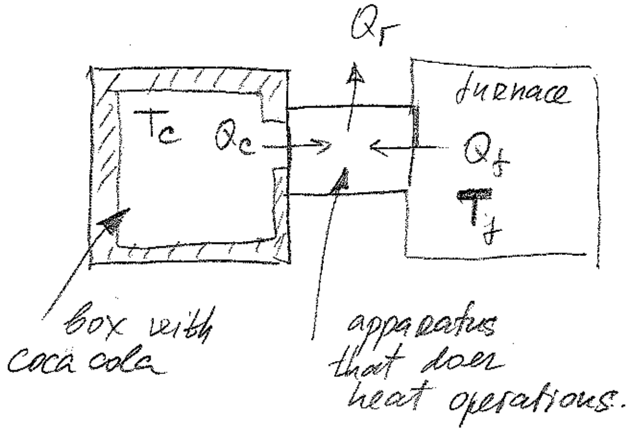
(c) Wast heat $Q = \Delta H - \Delta G = -72\ kJ$ (negative sign means that heat goes out of the fuel cell.

(d) $$\text{Voltage} = \frac{\text{electrical work done}}{\text{total charg}} =$$

$$= \frac{818\ kJ}{(8)\cdot(6.02 \times 10^{+23})(1.6 \times 10^{-19} c)} = 1.06\ V$$

Homework # 6

Problem # 4 extra credit.



(a) $Q_c + Q_f = Q_r$ $\quad cop = \dfrac{Q_c}{Q_f} = \eta = \dfrac{Q_c}{Q_r - Q_c}$

(c) 2nd Law $\rightarrow$ ideal operation $\rightarrow$ no entropy production.

$$S_c + S_f = S_R \qquad \frac{Q_c}{T_c} + \frac{Q_f}{T_f} = \frac{Q_r}{T_r} = \frac{Q_f + Q_c}{T_r}$$

$$\eta \cdot \frac{1}{T_c} + \frac{1}{T_f} = \frac{1}{T_r} + \frac{1}{T_r}\,\eta$$

$$\eta = \frac{\frac{1}{T_r} - \frac{1}{T_c}}{\frac{1}{T_f} - \frac{1}{T_r}} = \frac{T_c (T_f - T_r)}{T_f (T_r - T_c)}$$

(b) Coefficient of performance can be larger than 1