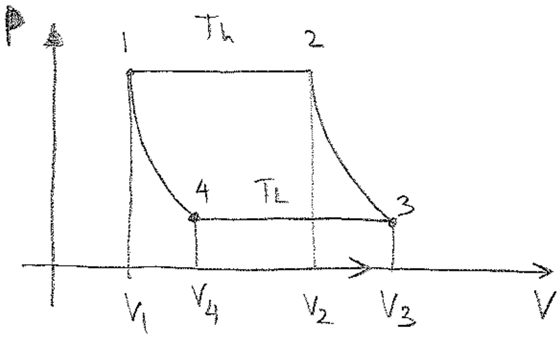
Extra credit

#1.



$P = \frac{1}{3}\alpha T^4.$

The pictures in the statement of the problem are for ideal gas.

(a) $T_h^3 V_1 = T_L^3 V_4 \qquad V_4 = V_1 \dfrac{T_h^3}{T_L^3}$

$T_h^3 V_2 = T_L^3 V_3 \qquad V_3 = V_2 \dfrac{T_h^3}{T_L^3}$

(b) 1-2 $\quad T_h = \text{const} \;=\; P = \text{const}.$

$W_{12} = \frac{1}{3}\alpha T_h^4 (V_2 - V_1) \qquad U_1 = \alpha T_h^4 V_1 \qquad U_2 = \alpha T_L^4 V_2$

$Q_{in} = W_{12} + (U_2 - U_1) = \frac{4}{3}\alpha T_h^4 (V_2 - V_1)$

(c) $W_{23} = (U_2 - U_3) = -(U_3 - U_2) = \alpha T_h^4 V_2 - \alpha T_L^4 \cdot \underbrace{V_2 \cdot \frac{T_h^3}{T_L^3}}_{V_3} =$

$= \alpha\left(T_h^4 - T_h^3 T_L\right)\cdot V_2.$

$W_{41} = -\alpha\left(T_h^4 - T_h^3 T_L\right) V_1$

W_{23} and W_{41} do not cancel each other.

(d) $W_{34} = \frac{1}{3}\alpha T_L^4 \cdot (V_4 - V_3) = \frac{1}{3}\alpha T_L^4\left(V_1 \frac{T_h^3}{T_L^3} - V_2 \frac{T_h^3}{T_L^3}\right) = \frac{1}{3}\alpha (V_1 - V_2)\cdot T_L\, T_h^3.$

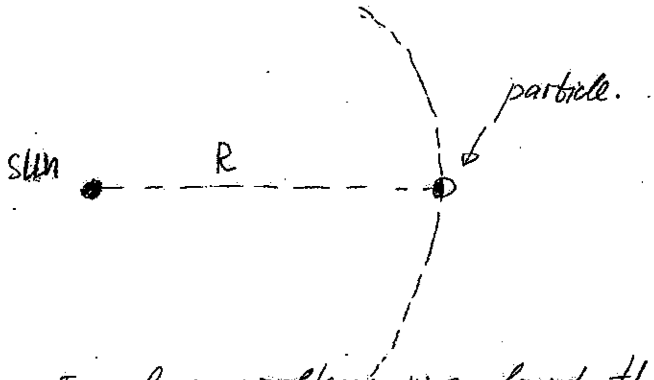
$$\eta = \frac{\frac{1}{3}\alpha T_h^4 (V_2 - V_1) - \frac{1}{3} T_L T_h^3 (V_2 - V_1) + \alpha(T_h^4 - T_h^3 T_L)\cdot(V_2 - V_1)}{\frac{4}{3}\alpha T_h^4 (V_2 - V_1)} =$$

$$= \frac{\frac{1}{3} T_h^4 - \frac{1}{3} T_L T_h^3 + T_h^4 - T_h^3 T_L}{\frac{4}{3} T_h^4} = 1 - \frac{T_L}{T_h}$$

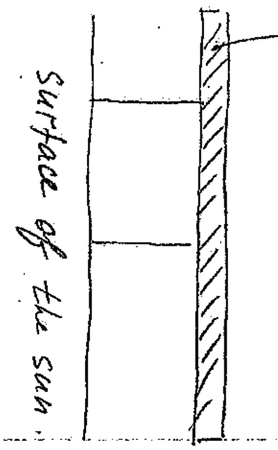
The Carnot cycle.

Problem #3 (homeword 12)

to compute the size of Al particle we need to balance attractive gravitational force and force due to radiation pressure exciting by the sun.



In class problem we found that radiation pressure on walls of a cavity at temperature T is $P = \frac{1}{3} \cdot w$, where $w = \frac{U}{V}$ - is energy density of radiation. We assume that the sun is black body radiator, The properties of photons it emits are the same the blackbody in equilibrium with radiation at temperature T absorbs.



black. body at short distance that we may neglect curvature of the sun. will reseive the pressure. as if it is a wall. of cavity with temperature T

That is $P = \frac{1}{3} \cdot \frac{U_{eff}}{V}$

U_{eff} - radiation enerdy of the cavity that has the size of the sun and temperature equal to the sun surface temperature.

$$P_s = \frac{1}{3}\frac{\pi^2}{15\hbar^3 c^3}(kT_s)^4$$ T_s - surface temperature of the sun.

Pressure is carried by photons. The number of photon crossing the unit area at large distance from the sun will behave like

$N = N_s \cdot \frac{R_s^2}{R^2}$, where N_s is the number at the surface of the sun.

Same relation is true for pressure

$$P = P_s \cdot \frac{R_s^2}{R^2}$$

The force acting on Al particle from radiation (assuming blackbody) is.

$F = P \cdot S = P \cdot \pi r^2$ (r - radius of particle)

it should be larger than gravitational force.

$$P_s \cdot \frac{R_s^2}{R^2} \cdot \pi r^2 > G \cdot \left(\frac{4}{3}\pi r^3 \cdot \rho\right) \cdot M_s / R^2$$, where

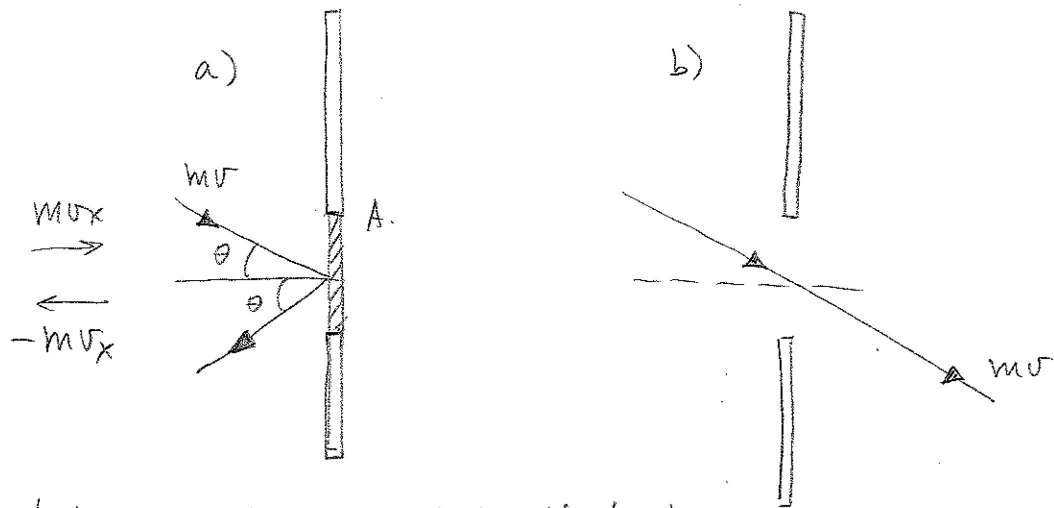
M_s - mass of the sun, ρ - density of Al. as you see R - distance from particle to the sun drops out of the problem.

$$r < \left(\frac{1}{3} \cdot \rho \cdot M_s\right)\left(\frac{1}{3}\frac{\pi^2}{15}\frac{(kT_s)^4 \cdot R_s^2}{\hbar^3 c^3}\right)$$

after plugging numbers you should get.

$(\approx 1\ \mu m)$

#3



a) Let's consider a particle that bounces off the wall. It transfers momentum $= 2mv_x = 2mv\cos\theta$. Averaging over particles and angles (done in class) gives force. $F = P \cdot A$ (A - area, P pressure)

b) When a particle leaves through a hole there is no bouncing back. It takes from a vessel momentum $p_x = mv_x$, a half of what we had in (a) $\Rightarrow$

$F = \frac{1}{2} P \cdot A$. You may also prove it from the Maxwell distribution.