$H = U + pV$

$G = H - TS$

$A = U - TS$

$dU = đQ - đW$

F1 $dU = dQ - pdV$ with $TdS = dQ_{rev}$

$\boxed{dU = TdS - pdV}$ $\left(\frac{\partial U}{\partial V}\right)_S = -p$; $\left(\frac{\partial U}{\partial S}\right)_V = T$

$dH = dU + pdV + Vdp$

$dH = TdS - \cancel{pdV} + \cancel{pdV} + Vdp$

F2 $\boxed{dH = TdS + Vdp}$ $\left(\frac{\partial H}{\partial p}\right)_S = V$; $\left(\frac{\partial H}{\partial S}\right)_p = T$

$dA = dU - TdS - SdT$

$dA = \cancel{TdS} - pdV - \cancel{TdS} - SdT$

F3 $\boxed{dA = -pdV - SdT}$ $\left(\frac{\partial A}{\partial T}\right)_V = -S$ $\left(\frac{\partial A}{\partial V}\right)_T = -p$

$dG = dH - SdT - TdS$

$dG = \cancel{TdS} + Vdp - SdT - \cancel{TdS}$

F4 $\boxed{dG = -SdT + Vdp}$ $\left(\frac{\partial G}{\partial T}\right)_p = -S$ $\left(\frac{\partial G}{\partial p}\right)_T = V$

Maxwell Relations

$$\frac{\partial}{\partial z}\left(\frac{\partial Y}{\partial x}\right) = \frac{\partial}{x}\left(\frac{\partial Y}{\partial y}\right)$$

1 $-\left(\frac{\partial p}{\partial S}\right)_V = \left(\frac{\partial T}{\partial V}\right)_S$

3* $-\left(\frac{\partial S}{\partial V}\right)_T = -\left(\frac{\partial p}{\partial T}\right)_V$

2 $\left(\frac{\partial V}{\partial S}\right)_p = \left(\frac{\partial T}{\partial p}\right)_S$

4* $-\left(\frac{\partial S}{\partial p}\right)_T = \left(\frac{\partial V}{\partial T}\right)_p$

Definitions

$$\boxed{\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = \beta}$$

Coefficient of Thermal Expansion

$$\boxed{-\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T = \kappa}$$

Isothermal Compression

Cycle Rule

$$\left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V \left(\frac{\partial p}{\partial V}\right)_T = -1$$

$$(\beta V)\left(\frac{\partial T}{\partial p}\right)_V \left(-\frac{1}{V\kappa}\right) = -1$$

$$\boxed{\left(\frac{\partial p}{\partial T}\right)_V = \frac{\beta}{\kappa}}$$

Thus Maxwell Relations

3 $$\boxed{+\left(\frac{\partial S}{\partial V}\right)_T = -\left(\frac{\partial p}{\partial T}\right)_V = -\frac{\beta}{\kappa}}$$

4 $$\boxed{-\left(\frac{\partial S}{\partial p}\right)_T = \left(\frac{\partial V}{\partial T}\right)_p = V\beta}$$

For Gas

$$\eta_J = \left(\frac{\partial T}{\partial V}\right)_U$$

Constant U
free expansion

$$\mu_{JT} \left(\frac{\partial T}{\partial p}\right)_H$$

Constant H (Isolated q & $w = 0$)

Define C_v and C_p

$$\boxed{\left(\frac{\partial Q}{\partial T}\right)_V \equiv C_V}$$

$$dU = dQ - p\,dV$$

$$\left(\frac{dU}{\partial T}\right)_V = \left(\frac{\partial Q}{\partial T}\right)_V - p\cancelto{0}{\left(\frac{dV}{dT}\right)_{V \cdot const}}$$

$$\boxed{\left(\frac{dU}{dT}\right)_V = C_V}$$

$$\boxed{\left(\frac{\partial Q}{\partial T}\right)_p \equiv C_p}$$

$$dH = \underbrace{dU + p\,dV}_{} + V\,dp$$

$$dH = dQ + V\,dp$$

$$\left(\frac{dH}{dT}\right)_p = \left(\frac{dQ}{dT}\right)_p + 0$$

$$\boxed{\left(\frac{\partial H}{\partial T}\right)_p = C_p}$$

$$dU = dQ - p\,dV$$

divide by dT at constant V

Replace dQ with TdS

$$\left(\frac{\partial U}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V - 0$$

$$\left(\frac{\partial U}{\partial T}\right)_V = C_V = T\left(\frac{\partial S}{\partial T}\right)_V$$

$$\boxed{\left(\frac{\partial S}{\partial T}\right)_V = \frac{C_V}{T}}$$

$$dH = TdS + V\,dp$$

divide by dT at const p

$$\left(\frac{dH}{dT}\right)_p = T\left(\frac{\partial S}{\partial T}\right)_p$$

$$C_p = T\left(\frac{\partial S}{\partial T}\right)_p$$

$$\boxed{\left(\frac{\partial S}{\partial T}\right)_p = \frac{C_p}{T}}$$

Entropy at a function of (T, P) or (T, V)

$$dS(T,p) = \left(\frac{\partial S}{\partial T}\right)_p dT + \left(\frac{\partial S}{\partial p}\right)_T dp$$

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$$\boxed{dS(T,p) = \frac{C_p}{T}\, dT - \left(\frac{\partial V}{\partial T}\right)_p dp}$$

$$\boxed{dS(T,p) = \frac{C_p}{T}\, dT - V\beta\, dp}$$

$$dS(T,V) = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$$

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$$\boxed{dS(T,V) \quad \frac{C_V}{T}\, dT + \left(\frac{\partial p}{\partial T}\right)_V dV}$$

$$\boxed{dS(T,V) = \frac{C_V}{T}\, dT + \beta K\, dV}$$

What about?

$$dS(p,V) = ?$$

Let's look again at $S(V,T)$ but via F1

$$TdS = dU + pdV$$

Expand dU

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

$$dS = \frac{1}{T}\left[\left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] dV\right]$$

$$dS = \frac{1}{T}[C_V]\,dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] dV$$

$$\qquad\;\; \| \qquad\qquad\qquad\qquad \|$$

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$$

$$\boxed{\left(\frac{dS}{dT}\right)_V = \frac{1}{T}C_V} \qquad \boxed{\left(\frac{\partial S}{\partial V}\right)_T = \frac{1}{T}\left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]}$$

$$\boxed{\left(\frac{\partial S}{\partial V}\right)_T \overset{3}{=} \left(\frac{\partial p}{\partial T}\right)_V = \frac{\beta}{K} = \frac{1}{T}\left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]}$$

✓

$$dS = \frac{C_V dT}{T} + \left(\frac{\partial p}{\partial T}\right)_V dV$$

OR ?

Let's Try it for $S(p,T)$ using F2

$$dH = TdS + Vdp$$

$$dS = \frac{dH}{T} - \frac{V}{T}dp$$ now Expand $dH(T,p)$

$$dS = \frac{1}{T}\left[\left(\frac{\partial H}{\partial T}\right)_p dT + \left(\frac{\partial H}{\partial p}\right)_T dp - Vdp\right]$$

Regroup

$$dS(T,p) = \frac{1}{T}\underbrace{\left(\frac{\partial H}{\partial T}\right)_p}_{C_p} dT + \frac{1}{T}\left(\left(\frac{\partial H}{\partial p}\right)_T - V\right)dp$$

$$dS(T,p) = \left(\frac{\partial S}{\partial T}\right)_p dT + \left(\frac{\partial S}{\partial p}\right)_T dp$$

Thus

$$\boxed{\left(\frac{\partial S}{\partial T}\right)_p = \frac{C_p}{T}} \qquad \boxed{\left(\frac{\partial S}{\partial p}\right)_T = \frac{1}{T}\left[\left(\frac{\partial H}{\partial p}\right)_T - V\right]}$$

$$\boxed{\left(\frac{\partial S}{\partial p}\right)_T \overset{4}{=} -\left(\frac{\partial V}{\partial T}\right)_p = -V\beta = \frac{1}{T}\left[\left(\frac{\partial H}{\partial p}\right)_T - V\right]}$$

$$dS(T,p) = \frac{C_p}{T}dT + \left(\frac{\partial V}{\partial T}\right)_p dp$$

OR ?

Enthalpy as function of $H(T,p)$

Just use last Equations

$$\frac{1}{T}\left[\left(\frac{\partial H}{\partial p}\right)_T - V\right] = -\left(\frac{\partial V}{\partial T}\right)_p = -V\beta$$

$$\left(\frac{\partial H}{\partial p}\right)_T - V = -T\left(\frac{\partial V}{\partial T}\right)_p = -TV\beta$$

$$\boxed{\left(\frac{\partial H}{\partial p}\right)_T = V - T\left(\frac{\partial V}{\partial T}\right)_p = V - TV\beta = V(1-T\beta)}$$

In general

$$dH(T,p) = \underbrace{\left(\frac{\partial H}{\partial T}\right)_p}_{\parallel} dT + \left(\frac{\partial H}{\partial p}\right)_T dp$$

$$\boxed{dH = C_p\, dT + \left[V - T\left(\frac{\partial V}{\partial T}\right)_p\right] dp}$$

OR

$$\boxed{dH(T,p) = C_p\, dT + V(1-T\beta)\, dp}$$

Derive $C_p - C_v$

Use 1st Law

$$dQ = \underbrace{dU}_{} + p\,dV$$

$$dQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV + p\,dV$$

Regroup

$$dQ = C_V\,dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] dV$$ ✓

Divide by dT at Constant p

$$\left(\frac{dQ}{dT}\right)_p = C_V \left(\frac{\partial T}{\partial T}\right)_p + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]\left(\frac{\partial V}{\partial T}\right)_p$$ ✓

page 5 $S(V,T)$

$$\boxed{C_p = C_V + T\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_p}$$

$$C_p = C_V + T\left(\frac{\beta}{K}\right)(V\beta)$$

$$\boxed{C_p = C_V + \frac{TV\beta^2}{K}}$$