Homework #4

Problem #1

The Van der Vaals equation for 1 mole of gas can be written as

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT \qquad V \text{ is volume of 1 mole.}$$

Than the pressure is

$$P = \frac{RT}{V-b} - a\frac{1}{V^2}$$

$$W = \int_{V_i}^{V_f} P\,dV = \int_{V_i}^{V_f} \frac{RT\,dV}{(V-b)} - \int_{V_i}^{V_f} a\frac{1}{V^2}\,dV =$$

$$= RT \ln\left(\frac{V_f-b}{V_i-b}\right) + a\left(\frac{1}{V_f} - \frac{1}{V_i}\right)$$

$V_i = 1l = 10^{-3} m^3$

$V_f = 2l = 2 \times 10^{-3} m^3$

$$W = 300 \times 8.31 \cdot \ln\left(\frac{2\cdot 10^{-3} - 3.2\times 10^{-5}}{10^{-3} - 3.2\times 10^{-5}}\right) + 1.38\times 10^{-4}\left(\frac{1}{2\times 10^{-3}} - \frac{1}{10^{-3}}\right) =$$

$$= 1.769 \times 10^3 - 0.069 = 1.77 \times 10^3 J.$$

For an ideal gas we have the same Expression with $a = b = 0$

$W_{id} \cong 1.73 \times 10^3 J$

The difference is about 2%

Problem #2

V_1	V_2	V_3
N_1	N_2	N_3

$$V_1 + V_2 + V_3 = V_0$$

$$N_1 + N_2 + N_3 = N_0$$

$$P(N_1, N_2, N_3) = \frac{N_0!}{N_1!\, N_2!\, N_3!} \cdot \left(\frac{V_1}{V_0}\right)^{N_1} \left(\frac{V_2}{V_0}\right)^{N_2} \left(\frac{V_3}{V_0}\right)^{N_3}$$

Problem # 3

$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O.$

$\Delta H - ?$

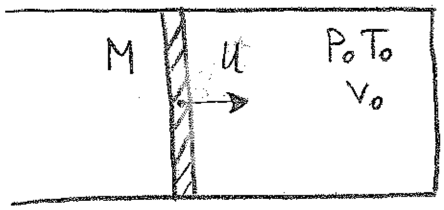
$$\Delta H = \sum_{\text{products}} \Delta H_f - \sum_{\text{reagents}} \Delta H_f =$$

$$= -\underbrace{6 \cdot 393.51}_{CO_2(g)} + \underbrace{6(-285.83)}_{H_2O(l)} - (\underbrace{-1273}_{C_6H_{12}O_6} - \underbrace{6 \times 0}_{\substack{O_2 \\ (\text{standard})}}) = -2803\ (kJ)$$

negative change in enthalpy means release of heat

$Q = 2803\ kJ$

Problem #4 HW #4



in the stament of the problem the initial volume of the gas had to be given.

u – velocity of the piston

U – internal energy of the gas. Gas is monoatomic. The system is thermally isolated.

1) from ideal gas law we find a number of moles of the gas.

$$P_0V_0 = nRT_0 \qquad n = \frac{P_0V_0}{RT_0}$$

2) At maximum compression all kinetic energy of the piston is converted into addition to the internal energy of the gas.

$$\frac{1}{2}Mu^2 = \Delta U = \frac{3}{2} n \cdot R \Delta T \quad \left(\frac{3}{2} \text{ is because the gas is monoatomic}\right)$$

$$T_f = T_0 + \Delta T = T_0 + \frac{Mu^2}{3Rn} = T_0 + \frac{Mu^2}{3P_0V_0}$$

3) (The process is adiabatic $PV^{\gamma} = const$; gas is ideal $\frac{PV}{T} = const.$) $\Rightarrow$ $V^{\gamma-1}T = const$

$$\gamma = \frac{f+2}{f} = \frac{5}{3} \Rightarrow V_f^{2/3} T_f = const. = V_0^{2/3} \cdot T_0$$

$$V_f = V_0 \cdot \left(\frac{T_0}{T_f}\right)^{3/2} = V_0 \left(\frac{T_0}{T_0 + \frac{Mu^2}{3Rn}}\right)^{3/2} = V_0 \left(\frac{T_0}{T_0 + \frac{Mu^2 T_0}{3P_0V_0}}\right)^{3/2} =$$

$$= V_0 \left(\frac{1}{1 + \frac{Mu^2}{3P_0V_0}}\right)^{3/2} = V_0 \left(\frac{3P_0V_0}{3P_0V_0 + Mu^2}\right)^{3/2}$$