Midterm # 2
Part A (closed books)

1) Write down one of the possible definition of chemical potential. (1 point)

$\mu = \frac{G}{N}$ $\mu = \left(\frac{\partial G}{\partial N}\right)_{P,T}$ $\mu = \left(\frac{\partial F}{\partial N}\right)_{V,T}$

2) Write down the thermodynamic identity. What physical law does it represent? (1 point) Using the identity write down the definition of temperature in statistical mechanics. (1 point)

$dU = TdS - PdV$ (The energy conservation, 1st Law) $T = \left(\frac{\partial U}{\partial S}\right)_V$

3) A thermally isolated chamber is separated by a partition into two parts each with volume V_0. In the left compartment there is 1 mole of monoatomic ideal gas at temperature T_0. The partition is removed, gas expands. Is this process reversible or irreversible? (1 point) What work does the gas do in this expansion into vacuum? (1 point) The process is irreversible.

$W = 0$. gas expanding into vacuum does not do any work.

4) What substance has the highest chemical potential: saturated water vapor at temperature 50°C, ice at -10°C, water at 100°C? (1 point)

Water at 100°C. The saturated water vapor at 50°C has the same chemical potential as water at 50°C. $\mu(\text{water}, 50°C) < \mu(\text{water}, 100°C)$

5) The Gibbs free energy is defined as $G=U-TS+PV$. Using the thermodynamic identity write down the total differential of G. (1 point)

$dG = dU - TdS - SdT + PdV + VdP = -SdT + VdP$.

$dU = TdS - PdV$

6) A container is separated into two connected compartments kept at the same temperature. The compartment A has volume 1 cm^3 and contains 10^{10} molecules of nitrogen. The compartment B has volume 1 m^3 and has 10^{13} molecules of nitrogen. In which compartment is the chemical potential higher? (1 point for answer, plus 1 point for argumentation) compartmet A. molecules flow from high μ to low μ. The concentration of molecules in A compartment is higher.

7) A gas is kept a container at constant fixed temperature T. Half of the gas molecules is pumped out. What is the change, if any, of the chemical potential of the gas molecules that are left in the container? (2 point)

The chemical potential decreases.

Argumentation

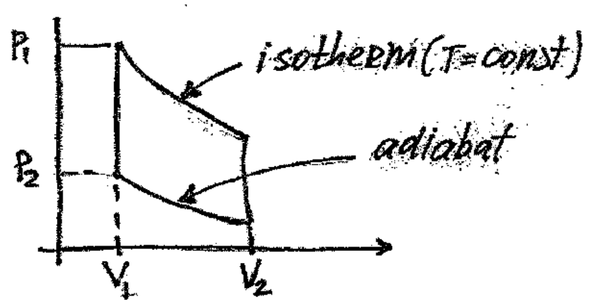
1) $\mu = \frac{G}{N}$ $d\mu = V_m dP = \frac{kT}{P} dP$, after half of the molecules is pumped out pressure drops twice.

$\mu = \mu_0 + kT \ln\left(\frac{P_f}{P_i}\right) = \mu_0 + kT \ln\left(\frac{1}{2}\right) = \mu_0 - kT \ln 2$. decrease

2) arguments similar to the answer to #6. The final state after the pumping has lower pressure and lower concentration. If the initial and final states are connected, molecules would flow from the former. $\Rightarrow$ μ has decreased.

Midterm #2
Part B (open books)
Problem #1
An *ideal* heat engine has an efficiency η when it operates in normal forward cycle. When it operates in reverse cycle mechanical work A provided by an external source is used to remove heat Q from a refrigerator. Assuming that the operation of the engine in the reverse cycle is also ideal find the ratio Q/A.

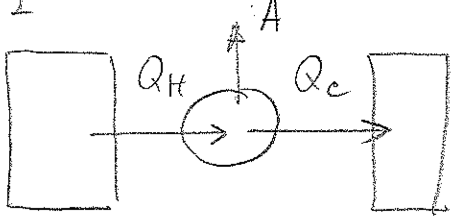
Problem #2
Find the efficiency of the heat engine that uses 1 mole of monoatomic ideal gas as working substance and operates along the following cycle.



Problem #3
A hypothetical liquid boils at 27 °C in atmospheric pressure and at 35 °C when the pressure is raised to 1.1 atm. Estimate the latent heat of this liquid.

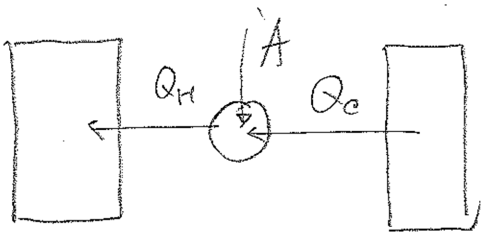
Midteem #2 part B.

PR#1



forward heat engine

$$\eta = \frac{A}{Q_H}$$



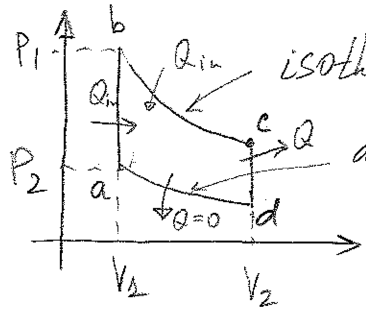
refregirator.

$\frac{Q_c}{A}$ - ?

$$\frac{Q_c}{A} = \frac{Q_H - A}{A} = \left(\frac{1}{\eta} - 1\right)$$

2.

1 mole, gas is monoatomic



η - ?

$$\eta = \frac{W}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Heat goes into the system at branches $a \to b$ and $b \to c$

Heat goes out of the system at $c \to d$.

(a-b) $dU = dQ - PdV \quad dV = 0 \quad dU = dQ$

$$dU = \frac{3}{2} R (T_b - T_a) \qquad T_B = \frac{P_1 V_1}{R} \qquad T_a = \frac{P_2 V_1}{R}$$

$$Q_{ab} = \frac{3}{2}(P_1 V_1 - P_2 V_1)$$

(b-c) $dU = 0 \quad T = \text{const.} \qquad dQ = PdV$

$$Q_{bc} = \int PdV = \int \frac{RT}{V} dV = RT \ln\left(\frac{V_2}{V_1}\right)$$

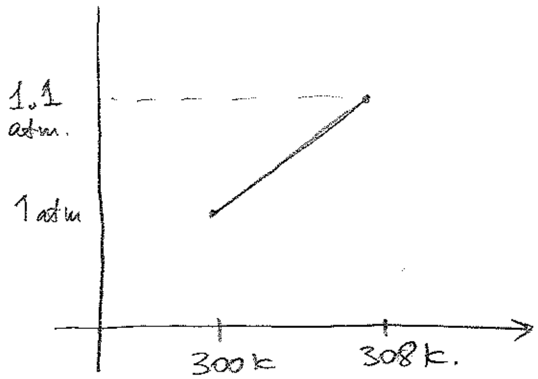
(c-d) $$Q_{cd} = \frac{3}{2} R (T_c - T_d) = \frac{3}{2}(P_c V_2 - P_d V_2) = \frac{3}{2} V_2 \left(P_1 \frac{V_1}{V_2} - P_2 \frac{V_1^\gamma}{V_2^\gamma}\right)$$

$P_c V_2 = P_1 V_1$ (isotherm)

$P_d V_2^\gamma = P_2 V_1^\gamma \qquad \gamma = \frac{5}{3}$

$$\eta = 1 - \frac{\frac{3}{2} V_2 \left(P_1 \frac{V_1}{V_2} - P_2 \frac{V_1^\gamma}{V_2^\gamma}\right)}{RT \ln\left(\frac{V_2}{V_1}\right) + \frac{3}{2} V_1 (P_1 - P_2)}$$

Problem #3   midterm #2.



$$\frac{dP}{dT} = \frac{L}{T\Delta V}$$ $\Delta V \approx V_g$ (because the volume per molecule in a gas is much larger)

$$\frac{dP}{dT} = \frac{L_m}{T V_g}.$$ $PV = NkT$ $\quad V_g = \frac{V}{N} = \frac{kT}{P}$

$$\frac{dP}{dT} = \frac{L_m P}{k T^2}$$ L_m - latent heat per molecule.

$$\frac{dP}{dT} = \frac{L_R P}{R T^2}$$ L_R - latent heat per mole.

$$\frac{dP}{dT} = \frac{1.1 \cdot 10^5 - 1 \cdot 10^5}{8\,k} = \frac{10^4}{8}$$

$$L_R = \frac{dP}{dT} \cdot \frac{R \cdot T^2}{P} = \frac{10^4 \cdot 8.31 \cdot (300)^2}{8 \quad 10^5} = 9.35 \frac{kJ}{mole}.$$