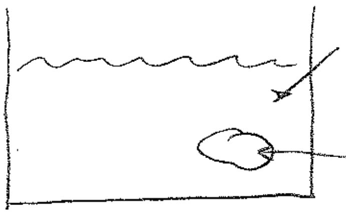
Homework #5 3760

Problem #1



$T_{ib} = 50^{\circ}C$

benzene $50^{\circ}C$, $m_b = 2$ kg. $C_b = 1.75 \frac{kJ}{kg \cdot K}$

ice $m_i = 0.1$ kg. $T_{ii} = 0^{\circ}C$

ice latent heat $L = 333$ kJ/kg.

water $C_w = 4.2$ kJ/kg·K.

1) benzene and water do not mix → there is no contribution to entropy from mixing.

2) Final temperature of the solution.

$$L \cdot m_i + C_w m_i (T_f - T_{ii}) + C_b m_b (T_f - T_{ib}) = 0$$

(heat from benzene is transfered to ice and water)

$$T_f = \frac{L \cdot m_i - C_w m_i T_{ii} - C_B m_b T_{ib}}{-(C_w m_i + C_b m_b)} =$$

$$= \frac{333 \cdot 0.1 - 4.2 \cdot 0.1 \cdot 273 - 1.75 \cdot 2 \cdot (50 + 273)}{-(4.2 \cdot 0.1 + 1.75 \cdot 2)} \simeq 309 \text{ K}.$$

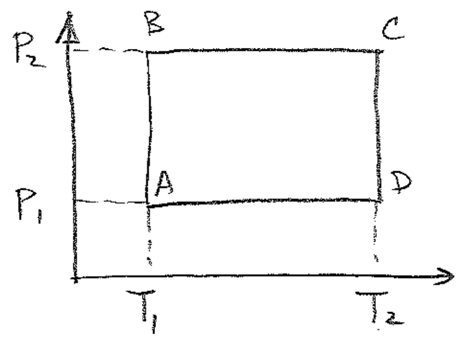
$$dS = \frac{dQ}{T}$$

$$\Delta S = \frac{L m_i}{273 K} + \int_{T_{ii}}^{T_f} C_w \cdot m_i \cdot \frac{dT}{T} + \int_{T_{ib}}^{T_F} \cdot C_b \cdot m_b \cdot \frac{dT}{T} =$$

$$= \frac{333 \cdot 0.1}{273} + 4.2 \cdot 0.1 \cdot \ln\left(\frac{309}{273}\right) + 1.75 \cdot 2 \cdot \ln\left(\frac{309}{323}\right) =$$

$$= 0.12 + 0.05 - 0.155 \approx 0.015 \text{ J/K}.$$

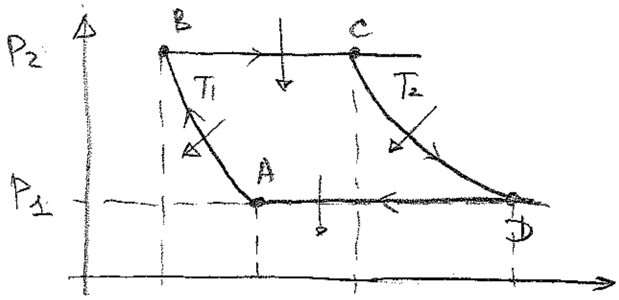
Problem #2



$$\eta = \frac{W}{Q_{in}}.$$

$$W = \int p\,dV.$$

We need to draw our cycle in P-V coordinates.



$$PV = nRT \qquad V = \frac{nRT}{P}$$

$$V_c = \frac{nRT_2}{P_2} \qquad V_d = \frac{nRT_2}{P_1}$$

Heat goes into the system at B-C and C-D parts of the cycle.

$$dQ = dU - PdV$$

heat $$\Delta Q_{BC} = \frac{3}{2} nR\cdot(T_2 - T_1) - \int_{V_b}^{V_c} PdV = \frac{3}{2} nR(T_2 - T_1) - (P_2 V_c - P_2 V_B) =$$

$$= \frac{3}{2} nR(T_2 - T_1) - nR(T_2 - T_1)$$

$$\Delta Q_{CD} = \int_{V_c}^{V_d} PdV = nRT_2 \cdot \int_{V_c}^{V_d} \frac{dV}{V} = nRT_2 \ln\left(\frac{V_c}{V_d}\right) = nRT_2 \ln\left(\frac{P_2}{P_1}\right)$$

Work. $W = \oint PdV = \oint Vdp$ — we may choose any way to compute the area of the cycle.

$$W = \int_{CD} VdP + \int_{AB} VdP = \int_{P_2}^{P_1} \frac{nRT_2}{P}\,dp + \int_{P_1}^{P_2} \frac{nRT_1}{P}\,dP =$$

$$= nR(T_2 - T_1)\ln\left(\frac{P_2}{P_1}\right)$$

$$\eta = \frac{(T_2 - T_1)\ln\left(\frac{P_2}{P_1}\right)}{T_2 \ln\left(\frac{P_2}{P_1}\right) + \frac{5}{2}(T_2 - T_1)}$$

Homework #5

Problem # 3

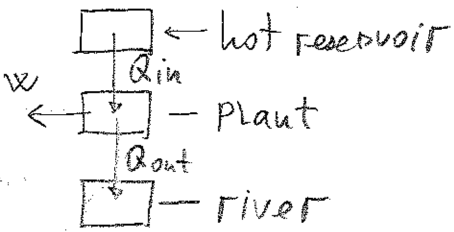
$g(U) = CU^{3N/2}$ Show that $U = \frac{3}{2} k_B N T$

$S = k_B \ln g = k_B \ln CU^{3N/2} = k_B \ln C + \frac{3N}{2} k_B \ln U$

$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right) = \frac{3N}{2} k_B \cdot \frac{1}{U}$ $U = \frac{3}{2} k_B N T$

$\frac{\partial^2 S}{\partial U^2} = \frac{3N}{2} k_B \left(-\frac{1}{U^2}\right) < 0.$

Problem #4.



$\eta = 1 - \frac{T_c}{T_H} = \frac{W}{Q_{in}} = \frac{W}{W + Q_{out}}$

We may introduce power as $A = \frac{W}{t}$ and heat flow in and out as.
$F_{in} = \frac{Q_{in}}{t}$ $F_{out} = \frac{Q_{out}}{t}$, where
t is time of plant operation.

We may than re-write the plant efficiency as

$\eta = \left(1 - \frac{T_c}{T_H}\right) = \frac{A}{A + F_{out}}$

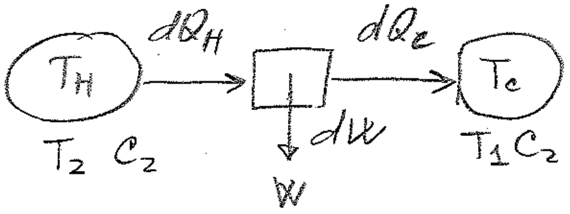
$\frac{T_H}{T_H - T_c} = \frac{A + F_{out}}{A} = 1 + \frac{F_{out}}{A}$

$A = F_{out} * \left(\frac{T_H}{T_H - T_c} - 1\right)^{-1} = F_{out} \cdot \left(\frac{T_H - T_c}{T_c}\right) =$

$A = 1500\,MW \cdot \left(\frac{273 + 500 - (273 + 20)}{273 + 20}\right) = 2457\ kW.$

$T_H = 600°C$ $A = 2970\ kW.$

Problem #5 extra credit



$T_2 > T_1$

T_1 is initial temperature of the hot body. It does change it time. T_c also changes in time and its initial temperature is T_1.

The maximum work will be done if at any stage the engine operates as an ideal heat engine that it does not produce any entropy.

$$\frac{dQ_H}{T_H} + \frac{dQ_c}{T_c} = 0 \Rightarrow c_1 \frac{dT_H}{T_H} + \frac{c_2 \, dT_c}{T_c} = 0 \quad (1)$$

The engine stops to operate when $T_H = T_c = T_f$.

$$c_1 \ln\left(\frac{T_f}{T_1}\right) + c_2 \ln\left(\frac{T_f}{T_2}\right) = 0 \qquad \ln\left[\left(\frac{T_f}{T_1}\right)^{c_1}\right] = \ln\left[\left(\frac{T_2}{T_f}\right)^{c_2}\right]$$

$$\left(\frac{T_f}{T_1}\right)^{c_1} = \left(\frac{T_2}{T_f}\right)^{c_2} \qquad T_f = \left(T_2^{c_2} \, T_1^{c_1}\right)^{\frac{1}{c_1+c_2}} \quad (2)$$

eq. (1) also allows us to find relation between T_H and T_c

$$\left(\frac{T_c}{T_1}\right)^{c_1} = \left(\frac{T_2}{T_H}\right)^{c_2} \qquad T_c = T_H^{c_1/c_2} \cdot T_2^{c_2/c_1} \cdot T_1$$

$$dW = -\left(1 - \frac{T_c}{T_H}\right) \cdot C_2 \cdot dT_H \qquad (- \text{ is becaus } dT_H \text{ is negative})$$

$$W = \int_{T_2}^{T_f} \left(\frac{T_c}{T_H} - 1\right) \cdot c_2 \, dT_H = \int_{T_2}^{T_f} T_2^{c_2/c_1} T_1 \cdot T_H^{c_1/c_2 - 1} \cdot C_2 \, dT_H -$$

$$- \int_{T_2}^{T_f} c_2 \, dT_H = T_2^{c_2/c_1} T_1 \cdot \frac{C_2}{C_1} \cdot \left(T_f^{c_1/c_2} - T_2^{c_1/c_2}\right) +$$

$$+ C_2 \cdot \left(T_2 - T_f\right)$$