Modeling and Optimizing Binary Prediction Markets: A Sandbox Approach with Cost Function Analysis

Conrad Voigt

November 2024

Abstract

This study explores the modeling and optimization of binary outcome prediction markets using a sandbox approach and cost function analysis, with data sourced from the blockchain-based platform Polymarket. The paper introduces a simplified market model based on the Logarithmic Market Scoring Rule (LMSR) to analyze price dynamics and examines how various cost functions influence market efficiency and price accuracy. A detailed literature review is provided on the optimization of LMSR and Sigmoid functions, highlighting their theoretical foundations and practical applications in prediction markets. The research employs advanced data preprocessing and optimization techniques, including L-BFGS-B, to fit and compare models such as LMSR, Sigmoid, and Exponential Smoothing against a dataset of 428 binary markets. Preliminary results indicate the Sigmoid function outperforms alternatives in predictive accuracy, though its generalizability requires further validation. This paper lays the groundwork for future research on linking theoretical insights from sandbox models with empirical data to enhance the understanding and implementation of prediction markets.

1 Introduction

Prediction markets are innovative tools for aggregating information and forecasting binary outcomes, with applications spanning elections, finance, and public policy. By trading outcome-based contracts, participants reveal collective probabilities, often surpassing traditional forecasting methods in accuracy.

This paper models and optimizes binary prediction markets using a sandbox framework, focusing on the Logarithmic Market Scoring Rule (LMSR) and Sigmoid cost functions. These functions are evaluated against empirical data from Polymarket, a blockchain-based platform, to understand their impact on market efficiency and price accuracy. The study employs advanced optimization techniques, including L-BFGS-B, and benchmarks the models against Exponential Smoothing.

By integrating theoretical insights with real-world data, this research enhances the understanding of prediction market dynamics, providing practical recommendations for designing efficient market mechanisms.

2 Prediction markets and Polymarket

Prediction markets are platforms where participants can trade contracts based on the outcome of future events. These markets aggregate information from diverse participants, effectively transforming collective opinions into probabilistic forecasts represented by market prices. Participants buy and sell contracts, with prices fluctuating based on the perceived likelihood of an event occurring. By design, these markets act as real-time aggregators of public opinion, often outperforming traditional forecasting methods in accuracy.

One prominent example of a prediction market is *Polymarket*, a blockchain-based platform that gained significant traction during major events such as the 2020 U.S. presidential election. *Polymarket* operates by allowing users to bet on binary outcomes, such as "Will candidate X win the election?" or "Will policy Y be enacted by a specific date?" Prices on *Polymarket* reflect the market's consensus probability of these events.

During the 2020 U.S. election, *Polymarket* saw unprecedented engagement, with a betting volume exceeding $3.5 billion dollars globally. This surge highlighted the growing importance of prediction markets as tools for gauging public sentiment and synthesizing complex information. The platform's use of blockchain technology ensures transparency and immutability, making it a reliable source of data for research and analysis.

The data used in this study was sourced from *Polymarket*, capturing detailed price movements for a variety of binary outcome markets. These data reflect the dynamic interplay of market participants' beliefs and provide a rich foundation for evaluating the efficiency and predictive accuracy of cost functions in prediction markets. By leveraging *Polymarket*'s data, this research aims to contribute to a deeper understanding of how market-based mechanisms aggregate and process information.

3 Sandbox Model for Prediction Markets

Prediction markets are designed to aggregate participant beliefs into a single price reflecting the probability of an event occurring. To analyze the behavior of such markets under controlled conditions, this section introduces a sandbox model built upon the Logarithmic Market Scoring Rule (LMSR) as the cost function. The sandbox model provides a simplified, mathematical framework to study how market prices evolve in response to trades and information updates.

3.1 Market Framework

This sandbox model simulates a binary outcome prediction market, where the market price $p_t \in [0, 1]$ at time t represents the aggregated belief about the probability of an event occurring. The model assumes that traders are rational and update their beliefs based on Bayesian inference. Information arrives sequentially, prompting traders to revise their expectations. A fixed liquidity parameter $b > 0$ governs the sensitivity of price changes to trading activity. The LMSR ensures arbitrage-free pricing by design.

3.2 Logarithmic Market Scoring Rule (LMSR)

Agrawal et al. [3] define the cost function $C(q)$ of the LMSR as:

$$C(q) = b \ln(1 + e^{q/b}),$$

where q represents the net position in the market. The price p_t of the "Yes" outcome is derived as the derivative of the cost function:

$$p_t = \frac{\partial C(q)}{\partial q} = \frac{e^{q/b}}{1 + e^{q/b}}.$$

This mapping ensures that p_t lies within the interval $[0, 1]$, making it interpretable as a probability. The liquidity parameter b controls the responsiveness of price changes to trading activity. Smaller values of b result in sharper price adjustments, while larger values produce smoother transitions.

3.3 Information Arrival and Trader Behavior

Traders revise their beliefs upon observing signals s_t about the likelihood of an event occurring. Let $P(\text{Yes})$ and $P(\text{No})$ represent the true probabilities of the outcomes, and $P(s_t \mid \text{Yes})$ and $P(s_t \mid \text{No})$ denote the likelihoods of the signal. A trader's posterior belief $p_j^{\text{posterior}}$ after observing s_t is calculated using Bayes' theorem:

$$p_j^{\text{posterior}} = \frac{P(s_t \mid \text{Yes}) \cdot p_j^{\text{prior}}}{P(s_t \mid \text{Yes}) \cdot p_j^{\text{prior}} + P(s_t \mid \text{No}) \cdot (1 - p_j^{\text{prior}})},$$

where p_j^{prior} is the trader's prior belief.

The trader submits a trade q_j^t based on the deviation of their posterior belief from the current market price:

$$q_j^t = \alpha \left(p_j^{\text{posterior}} - p_t \right),$$

where $\alpha > 0$ is a risk tolerance parameter. Positive trades ($q_j^t > 0$) reflect higher probabilities assigned to "Yes," while negative trades indicate a stronger belief in "No."

3.4 Price Update Mechanism

The LMSR updates the market price dynamically based on aggregate trading activity. After n trades, the total quantity of shares is updated as:

$$q^{(n)} = q^{(n-1)} + \sum_{j=1}^{m} q_j^n,$$

where m is the number of traders in the n-th round. The market price is then recalculated using the LMSR price formula:

$$p_t^{(n)} = \frac{e^{q^{(n)}/b}}{1 + e^{q^{(n)}/b}}.$$

This iterative process reflects how market prices evolve in response to trading activity, with the liquidity parameter b mediating the sensitivity.

3.5 Simulation Framework

To analyze the behavior of the sandbox model, a simulation was designed to evaluate price dynamics. The simulation begins by initializing the price $p_0 = 0.5$, defining the liquidity parameter b, and generating signal distributions $P(s_t \mid \text{Yes})$ and $P(s_t \mid \text{No})$. For each signal s_t, posterior beliefs $p_j^{\text{posterior}}$ are computed, the total position $q^{(n)}$ is updated, and the market price $p_t^{(n)}$ is recalculated. The process is repeated until p_t stabilizes near the true probability $P(\text{Yes})$, as inferred from the signal distribution.

3.6 Analytical Properties

The liquidity parameter b controls the smoothness of price transitions. For small b, prices are highly sensitive to trading activity, exhibiting sharp movements. Larger b values lead to more gradual changes, promoting stability. The convexity of the LMSR cost function guarantees arbitrage-free pricing. Specifically, the convexity condition ensures that:

$$C(q_1 + q_2) \geq C(q_1) + C(q_2).$$

This property prevents traders from exploiting price discrepancies for risk-free profit. Under rational trading and sufficient market activity, the sandbox model converges to an equilibrium price p^* that reflects the true probability of the event:

$$p^* = \frac{P(s_t \mid \text{Yes})}{P(s_t \mid \text{Yes}) + P(s_t \mid \text{No})}.$$

This sandbox model provides a simplified yet rigorous framework for studying prediction market dynamics using the LMSR cost function. By incorporating Bayesian updates, trader decision-making, and LMSR-based price adjustments, the model captures essential features of price formation and information aggregation in prediction markets. This foundation enables deeper exploration of advanced functional forms and optimization techniques in subsequent sections.

4 Data Availabililty and Preprocessing

The dataset used in this study originates from Polymarket, a blockchain-based prediction market platform. These markets are characterized by prices that reflect the implied probability of a particular outcome. The price data was aggregated on an hourly basis to ensure a sufficient frequency of the data for analysis the. Currently, no restrictions have been placed on the themes of the bets, as the objective is to explore general patterns across the dataset and to have a sample as large as possible.

The raw dataset comprises price data for 120 distinct bets, each representing an independent outcome markets. These distinct bets were further split up to only have binary outcome bets: For example one market was on the presidential race, which had more than one candidate. These markets were split up to create binary outcome markets to have separate one s for each candidate. This processing step not only increased the sample size significantly to 428 but also ensured that we can approach all with the same framework for binary outcomes. The average duration of these bets spans 246 observations, allowing for meaningful time-series analysis. However, the dataset presented several challenges, particularly in cleaning and standardizing the data. One major issue was the presence of low-volume trades, which often introduced noise into the price series. To address this, a volume cutoff threshold was applied to exclude markets with less than 1 million of total trading volume, ensuring that the analysis focuses on reliable market data.

The raw dataset is still undergoing cleaning, as inconsistencies and missing data are being addressed to enhance its usability. Labels for the binary outcomes are also being adjusted to ensure consistency across all bets, and any mislabeled or ambiguous data points are being corrected. The end goal is to produce a clean and reliable dataset that accurately reflects the structure and behavior of binary prediction markets, providing a solid foundation for the application and evaluation of predictive models.

5 Literature Review on Cost Function Optimization

The optimization of cost functions in prediction markets has been a central area of research, with various functional forms proposed to improve market efficiency and enhance predictive accuracy. These cost functions are designed to aggregate market participants' information into a single price that reflects the probability of an event. While many cost functions consider multiple data inputs, such as price and volume, this study is limited to using price data alone, which imposes certain constraints on the selection of functional forms.

Initially, nine cost functions were identified from the literature as potential candidates for implementation. However, after careful consideration, the list was narrowed to five based on their reliance on price data. Of these, three functions—focused on regret minimization, smoothness conditions, and convex optimization—were excluded after further analysis. These approaches rely heavily on assumptions about expected prices, which, when applied to price-only data, reduce to simple variance calculations. This reduction limits their practical utility for the current study, as they fail to capture the dynamic market behavior intended to be modeled.

As a result, two cost functions were selected for implementation: the Logarithmic Market Scoring Rule (LMSR) and the Sigmoid Function. Both functions are well-suited for price-only datasets and have distinct theoretical foundations that align with the study's objectives. The LMSR is a widely used cost function in prediction markets, characterized by its ability to dynamically adjust market prices based on trades. Its logarithmic structure ensures that price updates are proportional to the size of the trade, balancing market liquidity and stability. The Sigmoid function, on the other hand, offers a flexible framework for mapping prices into probabilities, with parameters that control the steepness and midpoint of the curve. This function is particularly useful for capturing non-linear relationships in the data.

By focusing on these two cost functions, the study aims to leverage their complementary properties to evaluate market behavior effectively. This selection reflects both the practical constraints of the dataset and the theoretical strengths of these models in aggregating and interpreting market information.

5.1 Logarithmic Market Scoring and Sigmoid

Agrawal et al. [3] introduce the **Logarithmic Market Scoring Rule (LMSR)** as a foundational cost function for prediction markets. The LMSR is mathematically defined as:

$$C(q) = b \ln(1 + e^{q/b}),$$

where q represents the total number of shares held, and b is a parameter that determines the liquidity of the market. In this study, the concept of q has been adapted to represent the price of the bet, aligning with the available data. The LMSR offers several significant advantages. Its logarithmic structure ensures that price updates are proportional to the size of the trades, which maintains market efficiency while avoiding arbitrage opportunities. This property makes LMSR particularly appealing for markets characterized by high levels of uncertainty, as it balances stability with responsiveness to new information.

A key strength of LMSR lies in its ability to continuously adjust prices in response to market activity. This feature not only prevents market manipulation but also promotes more accurate aggregation of information from diverse participants. The function's dependence on the liquidity parameter b allows for customization based on market conditions. For instance, a higher b results in less volatile price adjustments, fostering stability in markets with fewer participants or less trading activity. Conversely, a smaller b creates more dynamic price updates, which may be suitable for highly liquid markets with frequent trading. These properties have established LMSR as one of the most widely used cost functions in prediction market research and practical applications.

Shamsi and Cuffe [10] propose the use of a **sigmoid function** to model the relationship between market prices and probabilities in binary outcome prediction markets. Their cost function is given by:

$$P(x) = \frac{1}{1 + e^{-k(x-x_0)}}$$

where k controls the steepness of the curve, and x_0 determines the midpoint. The sigmoid function is particularly effective in capturing the non-linear dynamics of market prices, making it a natural fit for binary outcomes where probabilities transition smoothly between extremes. The authors applied this formulation to renewable energy production forecasts, demonstrating its ability to translate market prices into probabilistic predictions.

The sigmoid function's flexibility allows it to adapt to various market behaviors through its parameters. The steepness parameter k can be adjusted to reflect the sensitivity of price changes to new information, while the midpoint x_0 provides a reference point for interpreting price-probability relationships. This adaptability is particularly advantageous in markets where price movements may not directly correspond to linear changes in probability. By providing a continuous and differentiable mapping of prices to probabilities, the sigmoid function enhances interpretability and predictive accuracy in binary markets.

Both the LMSR and sigmoid functions are designed to address the unique challenges of prediction markets, particularly those involving binary outcomes. While LMSR emphasizes market stability and arbitrage-free pricing, the sigmoid function excels in scenarios requiring smooth transitions and probabilistic interpretations of prices. Together, these functions provide robust tools for understanding and modeling market dynamics, with applications ranging from financial forecasting to renewable energy predictions.

5.2 Modifications of the functions

Modified LMSR for Predicted Prices

The traditional LMSR relies on cumulative market activity, quantified as the total number of shares traded, to calculate prices. However, in this study, the LMSR was reformulated to work directly with observed market prices. The modified LMSR is expressed as:

$$p^t = b \ln(1 + e^{(p_t - p_0)/b}),$$

where:

- p^t is the predicted price at time t,
- p_t is the observed price at time t,
- p_0 is the baseline or initial observed price, and

- b is the liquidity parameter to be optimized.

This modification involves several key changes to the original LMSR. First, the cumulative shares (q) were replaced with the observed price (p_t), reflecting the nature of the dataset. Second, the inclusion of p_0 serves as a reference point to center the transformation and standardize the calculation of predicted prices. Lastly, the reformulation focuses on predicting prices (p^t) rather than cumulative costs, making the model's outputs directly comparable to the observed prices.

The modified LMSR retains its foundational strengths, including the ability to dynamically adjust prices based on the liquidity parameter b. This parameter governs the responsiveness of price changes to new information, with larger values of b resulting in smoother transitions and smaller values leading to sharper updates. These properties make the LMSR suitable for capturing market dynamics while ensuring the prices remain within the range of probabilities, a critical requirement in this study.

Modified Sigmoid for Predicted Prices

The Sigmoid function provides a flexible and probabilistic framework for modeling price dynamics. In its original form, the sigmoid maps an input variable, such as share quantities or uncertainty levels, to a probability between 0 and 1. The function was adapted for this study to predict market prices directly, resulting in the following formulation:

$$p^t = \frac{1}{1 + e^{-k(p_t - x_0)}},$$

where:

- p^t is the predicted price at time t,
- p_t is the observed price at time t,
- k is the steepness of the sigmoid curve, to be optimized, and
- x_0 is the midpoint price, representing the reference point of the sigmoid curve.

This adaptation involves replacing the generic input variable (x) with the observed price (p_t), allowing the model to leverage the inherent probability-like nature of the dataset. The sigmoid's parameters, k and x_0, were optimized to best fit the observed price trends. The steepness parameter (k) controls how rapidly the prices transition between extremes, while the midpoint parameter (x_0) serves as the pivot point around which the sigmoid curve is centered.

One of the advantages of the Sigmoid function is its smooth and continuous mapping of prices to probabilities. This feature aligns well with the normalized price data, which already lies within the [0, 1] range, simplifying implementation and interpretation. Additionally, the function's flexibility allows it to capture non-linear relationships in price data, making it a valuable tool for modeling binary outcome markets.

The choice of LMSR and Sigmoid functions was motivated by their ability to model probabilities directly, making them inherently compatible with the normalized price data in this study. The LMSR excels in balancing market stability with dynamic responsiveness through its liquidity parameter, while the Sigmoid function provides a probabilistic interpretation of prices with adjustable steepness and midpoint parameters. Together, these models offer complementary approaches to understanding and predicting market dynamics in binary outcome markets. Their adaptations ensure that they are well-suited for price-only datasets, providing a robust foundation for further analysis and evaluation.

In conclusion, the optimization of cost functions in prediction markets encompasses a diverse array of functional forms, including logarithmic, regret-minimizing, sigmoid, and convex functions. Each of these forms serves specific purposes, such as enhancing market efficiency, minimizing regret, and accommodating real-world uncertainties. The integration of these functional forms into market design is crucial for achieving accurate predictions and maintaining robust market dynamics.

6 Implementation and Optimization of the Cost Functions

To fit the prediction models, a combination of advanced optimization techniques and robust validation metrics was utilized to ensure reliable and interpretable results. Specifically, the Limited-memory Broyden–Fletcher–Goldfarb–Shanno with Box constraints (L-BFGS-B) optimization algorithm was employed. This method is a well-established gradient-based optimization technique that is particularly effective for problems involving continuous variables and bound constraints. In the context of the models, the L-BFGS-B algorithm efficiently minimized the Residual Sum of Squares (RSS), a measure of the total squared difference between observed and predicted prices.

The L-BFGS-B method has several advantages that make it suitable for this application. First, it approximates second derivative (Hessian) information, which significantly reduces memory usage and computation time compared to methods that compute the full Hessian. This feature is particularly beneficial for larger datasets, where computational efficiency is critical. Second, the algorithm supports constraints on parameter values, ensuring that parameters remain within meaningful ranges. For example, in the Modified LMSR model, the liquidity parameter b was constrained to be positive, as negative liquidity values are not meaningful in the context of market modeling. Similarly, in the Modified Sigmoid model, the steepness parameter k and midpoint parameter $x0$ were constrained to ensure valid and interpretable predictions. These constraints helped maintain the mathematical and conceptual integrity of the models.

In addition to fitting the Modified LMSR and Modified Sigmoid models, exponential smoothing was implemented as a benchmark to provide a baseline for comparison. Exponential smoothing is a widely used method in time series forecasting and is particularly effective in capturing short-term trends while filtering out noise. For each dataset, the smoothing parameter was optimized iteratively to achieve the best possible fit. The smoothing parameter controls the weight given to recent versus older data, and optimizing it allowed a balance to be achieved between responsiveness to new information and the stability of the trend.

To evaluate the performance of these models and compare their predictive capabilities, a set of widely accepted validation metrics was employed. The Mean Squared Error (MSE) was calculated to measure the average squared difference between observed and predicted prices. This metric provides a clear sense of the overall prediction error across the dataset. To complement this, the Root Mean Squared Error (RMSE) was also computed, which is the square root of the MSE. Unlike MSE, RMSE is expressed in the same units as the observed prices, making it more interpretable for understanding the magnitude of prediction errors.

Another key metric was the Coefficient of Determination R^2, which measures how well the model explains the variability in the observed prices. This metric ranges from 0 to 1, where values closer to 1 indicate a stronger fit, and negative values suggest that the model performs worse than a simple mean prediction. The metric is particularly useful for comparing the explanatory power of different models. Finally, the Residual R^2 Sum of Squares (RSS), which represents the total squared error between observed and predicted values, was directly minimized during the optimization process. Minimizing RSS ensures that the model closely approximates the observed data within the constraints and assumptions of the model.

7 Preliminary Results

The implementation of the Logarithmic Market Scoring Rule (LMSR), Sigmoid function, and Exponential Smoothing models across 428 binary prediction market bets provided meaningful insights into their predictive performance. The average performance metrics—Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and R2R2̂R2—are summarized in Table 1.

Exponential Smoothing, an established and widely used technique for modeling price movements, served as a benchmark to compare the performance of the LMSR and Sigmoid models. This method achieved an average MSE of 0.0022754, RMSE of 0.0313441, and an R^2 value of 0.6776. These results reflect its ability to effectively smooth price trends while maintaining reasonable predictive accuracy. As a benchmark, Exponential Smoothing provides a baseline to assess the relative strengths of the newer approaches explored in this study.

The Sigmoid function significantly outperformed the benchmark, achieving an exceptionally low MSE of 0.0001349 and RMSE of 0.0082541, with an R^2

	MSE	RMSE	R^2
LSMR	0.00238	0.03231	0.6764
Sigmoid	0.00013	0.00825	0.9760
Exponential Smoothing	0.00227	0.31344	0.6776

Table 1: Comparison of LSMR, Sigmoid and Exponential Smoothing across 428 observations

value of 0.9760. This high level of accuracy demonstrates the Sigmoid function's ability to capture non-linear price dynamics and its suitability for modeling binary outcome markets. The probabilistic nature of the Sigmoid function aligns well with the normalized price data, allowing it to provide a smooth and interpretable mapping of market prices to probabilities. Its flexibility in adjusting to the observed price movements contributes to its outstanding performance.

The LMSR, while less precise than the Sigmoid function, also compared favorably to the benchmark. It achieved an average MSE of 0.0023877, RMSE of 0.0323124, and an R^2 value of 0.6764. The LMSR model's performance is similar to that of Exponential Smoothing, indicating its robustness and utility in capturing market dynamics. LMSR's ability to dynamically adjust prices based on its liquidity parameter makes it a valuable tool for modeling binary prediction markets, particularly in settings where market liquidity plays a central role.

While the Sigmoid function demonstrated superior accuracy, its performance raises the possibility of overfitting. With an R^2 close to 1, there is a risk that the Sigmoid model may have been overly tailored to the training data, potentially reducing its generalizability to new datasets. Ensuring robustness through additional validation, such as cross-validation or testing on independent datasets, is essential to confirm its reliability. In contrast, the LMSR and Exponential Smoothing models, while less precise, offer more conservative fits that may generalize better to other contexts.

Table 1 highlights the comparative performance of these models. The Exponential Smoothing method, as a benchmark, establishes a clear baseline, while the LMSR and Sigmoid function offer valuable insights into novel approaches for modeling price movements in binary prediction markets. These results emphasize the importance of combining established techniques with innovative methods to balance predictive accuracy and generalizability.

8 Future Work

This study provides a foundation for further investigation into the behavior and effectiveness of models used in prediction markets, while emphasizing the need to connect theoretical findings from the sandbox model to empirical observations. Future work will focus on addressing limitations, testing under different conditions, and applying mathematical methods to enhance the understanding of the models' underlying dynamics.

A primary objective will be to link the insights from the sandbox model to the empirical findings derived from Polymarket data. This connection will validate the theoretical properties demonstrated in the sandbox environment—such as equilibrium stability, price responsiveness, and sensitivity to information updates—against real-world market behavior. Out-of-sample testing will be conducted using data not included in the initial analyses, ensuring the models' generalizability. Bootstrapping and Monte Carlo simulations will provide additional assessments of the models' stability and reliability across varied datasets.

The analysis will also extend to examining how the models perform with different data granularities. While daily data has been useful for identifying long-term patterns, incorporating higher-frequency data, such as hourly price movements, will allow for a more detailed investigation of short-term dynamics. This approach will help determine whether the models accurately capture rapid changes in market behavior. Additionally, the models will be evaluated across different market regimes, including periods of high and low volatility and at varying stages in the lifecycle of prediction markets, to test their consistency under diverse conditions.

Addressing potential overfitting, particularly in the Sigmoid function, remains a priority. The high accuracy observed in this study suggests the need for rigorous validation to ensure the models are not overly tailored to the training data. Techniques such as cross-validation and independent dataset testing will help verify the robustness of the results and their applicability to unseen data.

Future work will also involve a deeper exploration of the models' theoretical foundations. Concepts from probability theory and differential equations will be used to better understand how cost functions drive price evolution and aggregate market information. These analyses will focus on key properties such as stability, responsiveness, and parameter sensitivity, providing insights into the mathematical underpinnings of prediction market behavior.

By combining the integration of sandbox-derived findings with empirical data and further theoretical exploration, this research aims to strengthen the reliability and versatility of the models. These efforts will refine their applicability to real-world prediction markets, ensuring a comprehensive understanding of market dynamics while addressing practical and theoretical challenges.

References

[1] Abernethy, J., Chen, Y., & Vaughan, J. (2011). An optimization-based framework for automated market-making. *Proceedings of the 12th ACM conference on Electronic commerce*, 297–306. `https://doi.org/10.1145/1993574.1993621`

[2] Abernethy, J., Frongillo, R., Li, X., & Vaughan, J. (2014). A general volume-parameterized market making framework. *Proceedings of the 15th ACM Conference on Economics and Computation*, 667–684. `https://doi.org/10.1145/2600057.2602900`

[3] Agrawal, S., Delage, E., Peters, M., Wang, Z., & Ye, Y. (2011). A unified framework for dynamic prediction market design. *Operations Research, 59*(3), 550-568. https://doi.org/10.1287/opre.1110.0922

[4] Chen, Y., & Vaughan, J. (2010). A new understanding of prediction markets via no-regret learning. *Proceedings of the 11th ACM conference on Electronic commerce*, 189–198. https://doi.org/10.1145/1807342.1807372

[5] Dudík, M. (2014). Market making with decreasing utility for information. *arXiv preprint arXiv:1407.8161.* https://doi.org/10.48550/arxiv.1407.8161

[6] Frongillo, R. (2017). An axiomatic study of scoring rule markets. *arXiv preprint arXiv:1709.10065*. https://doi.org/10.48550/arxiv.1709.10065

[7] Iyer, K., Johari, R., & Moallemi, C. (2010). Information aggregation in smooth markets. *Proceedings of the 11th ACM conference on Electronic commerce*, 199–206. https://doi.org/10.1145/1807342.1807373

[8] Kroer, C., Dudík, M., Lahaie, S., & Balakrishnan, S. (2016). Arbitrage-free combinatorial market making via integer programming. *Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, 161–178. https://doi.org/10.1145/2940716.2940767

[9] Lekwijit, S., & Sutivong, D. (2018). Optimizing the liquidity parameter of logarithmic market scoring rules prediction markets. *Journal of Modelling in Management, 13*(3), 736-754. https://doi.org/10.1108/jm2-06-2017-0066

[10] Shamsi, M., & Cuffe, P. (2022). Prediction markets for probabilistic forecasting of renewable energy sources. *IEEE Transactions on Sustainable Energy, 13*(2), 1244-1253. https://doi.org/10.1109/tste.2021.3112916

[11] Yamaguchi, N., Hori, M., & Ideguchi, Y. (2018). Minimising the expectation value of the procurement cost in electricity markets based on the prediction error of energy consumption. *Pacific Journal of Mathematics for Industry, 10*(1). https://doi.org/10.1186/s40736-018-0038-7

[12] Bagga, P., Makhesana, M., Darji, P., Patel, K., Pimenov, D., Giasin, K., & Khanna, N. (2022). Tool life prognostics in CNC turning of AISI 4140 steel using neural network based on computer vision. *The International Journal of Advanced Manufacturing Technology, 123*(9-10), 3553-3570. https://doi.org/10.1007/s00170-022-10485-9

[13] Beigi, M., & Ahmadi, I. (2019). Artificial neural networks modeling of kinetic curves of celeriac (*Apium graveolens* L.) in vacuum drying. *Food Science and Technology, 39*(suppl 1), 35-40. https://doi.org/10.1590/fst.35717

[14] Byeon, H., & Kim, S. (2019). Evaluating factors for predicting the life dissatisfaction of South Korean elderly using soft margin support vector machine based on communication frequency, social network, health behavior and depression. *International Journal of Advanced Computer Science and Applications, 10*(9). https://doi.org/10.14569/ijacsa.2019.0100951

[15] Kasnavi, S., Afshar, M., Shariati, M., Kashan, N., & Honarvar, M. (2017). Performance evaluation of support vector machine (SVM)-based predictors in genomic selection. *The Indian Journal of Animal Sciences, 87*(10). https://doi.org/10.56093/ijans.v87i10.75270

[16] Nakshatri, N., Menon, A., Giles, C., Rajtmajer, S., & Griffin, C. (2021). Design and analysis of a synthetic prediction market using dynamic convex sets. https://doi.org/10.48550/arxiv.2101.01787

[17] Venikar, I., Joshi, J., Jalnekar, H., & Raut, S. (2022). Stock market prediction using LSTM. *International Journal for Research in Applied Science and Engineering Technology, 10*(12), 920-924. https://doi.org/10.22214/ijraset.2022.47967

[18] Wu, W., & Liu, H. (2019). Estimation of soil pH with geochemical indices in forest soils. *PLoS One, 14*(10), e0223764. https://doi.org/10.1371/journal.pone.0223764