

```

    ↳ Code contains syntax errors
    ↳ List of arguments should be separated by commas
    ↳ The code will show what error for
      ↳ a flat x.
      ↳ NameError for 'p.polyfit'
      ↳ Attribute error
      ↳ SyntaxError for 'from m' (missing colon)
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1. Import numpy as np
   def lagrange_interpolation(x_points, y_point):
       def L(k, x):
           terms = []
           for i in range(len(x_points)):
               if i != k:
                   terms.append((x - x_points[i]) / (x - x_points[k]))
           return np.prod(terms, axis=0)
       def p(x):
           total_points = [(1, 1), (2, 4), (3, 9), (4, 16)]
           X_points, Y_points = zip(*total_points)
           lagrange_poly = lagrange_interpolation(X_points, Y_points)
           print("Lagrange polynomial at x = 2.5: ", lagrange_poly(2.5))
       return p
2. Import numpy as np
   def newton_interpolation(x, y):
       def divided_diff(x, y):
           n = len(x)
           cof = np.zeros((n, n))
           cof[0, 0] = y
           for j in range(n-1):
               for i in range(n-1-j):
                   cof[i, j] = (cof[i+1, j-1] + cof[i, j-1]) / (x[i+1] - x[i])
           return cof
       def P(c):
           result, prod = cof[0, 0], 1
           for i in range(1, len(x)):
               prod *= (x[i] - x[i-1])
               result += cof[0, i] * prod
           return result
       cof = divided_diff(x, y)
       return lambda xi: P(cof)
   X, Y = [1, 2, 3, 4], [1, 4, 9, 16]
   p = newton_interpolation(X, Y)
   print("Newton polynomial at x = 2.5: ", p(2.5))

```

```

1. import numpy as np
def powerIteration (A, num_iterations:int):
    A = np.array (A)
    b_k = np.random.rand (A.shape [1])
    for i in range (num_iterations):
        b_k = norm = np.linalg.norm (b_k)
        eigenvalue = np.dot (b_k.T, np.dot (A, b_k)) / np.dot (b_k.T, b_k)
        return eigenvalue, b_k
    A = [[4, 1, 1], [1, 3, -1], [-1, -1, 2]]
    print eigenvalue, b_k
    dominant_eigenvalue, dominant_eigenvector = powerIteration (A,1000)
    print ("Dominant Eigenvalue : ", dominant_eigenvalue)
    print ("Dominant Eigenvector : ", dominant_eigenvector)
    print ("Q^-1, Q = np.linalg.Qr (A) \n", Q_1, Q)

2. def qr_algorithm (A, num_iterations:int):
    A = np.array (A)
    n = A.shape [0]
    Q = np.eye (n)
    for i in range (num_iterations):
        Q, R = np.linalg.QR (A)
        A = np.dot (Q, R)
        eigenvalue = np.diag (A)
        return eigenvalue, Q
    A = [[4, 1, 1], [1, 3, -1], [-1, -1, 2]]
    print eigenvalue, Q
    eigenvalues, eigenvectors = qr_algorithm (A)
    print ("Eigenvalues : ", eigenvalues)
    print ("Eigenvectors : ", eigenvectors)

```

3. Comparison.  
Both methods effectively identifies the dominant eigenvalue, and the QR algorithm additionally provides all eigenvalues and their corresponding eigenvectors, demonstrating a comprehensive solution compared to the power iteration method.

```
# import numpy as np
def gradient_descent ( learning_rate = 0.1 , initial_guess = (0,0) , num_iterations = 1000 ) :
    X, Y = initial_guess
    for _ in range (num_iterations):
        grad_X = 2*X - Y + 1
        grad_Y = 2*Y - X - 1
        X = learning_rate * grad_X
        Y = learning_rate * grad_Y
    return X, Y
min_X, min_Y = gradient_descent ()
print(f'minimum found at: X = {min_X} y = {min_Y}')
```

