Exercise Sheet: Vector spaces

1. (a)
   (b) $W = Span\{(6, 1, -7), (-1, 1, 0)\}$
2. $h = 5$
3. b, d, e.
4. (a) $b_3 = 0$
   (b) $b_3 = -b_1, b_2 = 2b_1$, and b_1 is arbitrary, or equivalently $(b_1, b_2, b_3) = span(1, 2, -1)$.
   (c) $b_3 = -b_1, b_2 = -2b_1$, and b_1 is arbitrary, or equivalently (b_1, b_2, b_3)
5. Not/3.
6. (a) No
   (b) No
   (c) Yes
   (d) No
7. (a) T
   (b) F
   (c) F
   (d) T
   (e) F
8. (a)
   (b) No
   (c) $\{(1, 1, 0)\}$.
9. (a) No
   (b) $\{(1, 0, \frac{1}{2}), (0, 1, -\frac{1}{2})\}$.

10. (a) $\{(1, -3, 0), (0, 0, 1)\}$.
    (b) $\{(3, 1, 0), (-2, 0, 1)\}$.

11. A basis: $\{(-\frac{1}{5}, \frac{3}{5}, 1)\}$, its dimension is 1.

12. (a) Linearly Dependent; $v_1 + v_2 + v_3 = 0$
    (b) Linearly Independent

13. (a) $\{(1, 1, 1, 1)\}$
    (b) $\{(1, 0, 0, -1), (0, 1, 0, -1), (0, 0, 1, -1)\}$
    (c) $\{(-1, 1, 1, 0), (-1, 0, 0, 1)\}$.
    (d)
    - Column space of U: A basis: $\{(1, 0), (0, 1)\}$. Its dimension is 2.
    - Null space of U: A basis $\{(-1, 0, 1, 0, 0), (0, -1, 0, 1, 0), (-1, 0, 0, 0, 1)\}$ Its dimension is 3.