1. (a) a linear map.
   (b) not a linear map.
   (c) not a linear map.
   (d) a linear map.
   (e) not a linear map.
   (f) not a linear map.
   (g) a linear map

2. (a) $T(2,2) = (4,4)$.
   (b) $T(3,1) = (2,2)$.
   (c) $\begin{pmatrix} 0 & 2 \\ 0 & 2 \end{pmatrix}$.
   (d) $T(a,b) = (2b, 2b)$.
   (e) $T^2(2,2) = (8,8)$.

3. (a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$
   (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$.
   (c) $\begin{pmatrix} 2 & -1 \\ 3 & 4 \\ 1 & -5 \end{pmatrix}$.
   (d) $\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$.

4. (a) $\operatorname{im}(T) = \mathbb{R}^2$ and $\ker(T) = \{(0,0)\}$.
   (b) $\operatorname{im}(T)$ is the $x - y$ plane and $\ker(T)$ is the z-axis.
   (c) $\operatorname{im}(T) = \{(0,0)\}$ and $\ker(T) = \mathbb{R}^2$.
   (d) $\operatorname{im}(T)$ is the line $y = x$, and $\ker(T)$ is the line $x = 0$.
   (e) There may have different ways to describe its range and kernel. For instance, $\operatorname{im}(T) = \mathit{span}\{(2,3,1), (-1,4,5)\}$, and $\ker(T) = \{(0,0)\}$.

5. (a)

(b) the set of all symmetric matrices of $M_{n\times n}$ and $\dim(\ker(L)) = \frac{n(n+1)}{2}$.

(c) the set of matrices of $A = (a_{ij}) \in M_{n\times n}$ such that $a_{ij} = -a_{ij}$ and $\dim(\mathrm{im}(L)) = \frac{n(n-1)}{2}$.

6. (a) $T^2(v) = v$.

   (b) $T^2(v) = v + (2, 2)$.

   (c) $T^2(v) = -v$.

7.

8.