

Answer Key Sheet: Relations

1. $\mathbb{Z} \times \mathbb{Z}$ is the set of all ordered pairs of integer numbers.
2. 2^{mn} .
3. Yes
4. $a \neq 0$
5. Reflexive, symmetric, not antisymmetric, transitive.
6. Yes
7. No
8. No
9. (a) 15.
(b)

$$B \cup C = \{6, 8, 12, 16, 18, 24, 30\},$$

$$B \cap C = \{24\},$$

Binary string representation for B : 001001001001001

Binary string representation for C : 000100010001000

Binary string representation for $B \cup C$: 001101011001001

Binary string representation for $B \cap C$: 0000000000001000

Binary string representation for \bar{B} : 110110110110110

10.

11. (a) Yes
(b) Yes
(c) No
(d) No
(e) No
(f) No

12. (a) $M_R = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, $|R| = 9$ which is the number of occurrences of 1 in M_R .

(b) not symmetric, antisymmetric, transitive.

13. (a) $R^{-1} = R = \{(a,b), (a,c), (b,a), (b,b), (c,a), (c,c)\}$ as the matrix
 M_R is symmetric. $M_{R^{-1}} = M_R^T = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

(b) $M_{\bar{R}} = \bar{M}_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, and $\bar{R} = \{(a,a), (b,c), (c,b)\}$.

(c)

$$M_{R^2} = M_R \odot M_R = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

and $R^2 = A \times A$.

- (d) $R \cup S = \{(a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c)\}$.

$$R \circ S =$$

$$S \circ R =$$

14. R_1 is Not as it is not symmetric. R_2 is an equivalence relation.

15. (a) the set of city pairs (a,b) such that there is connecting flight from city a to city b with 1 transit.
(b) the set of city pairs (a,b) such that there is connecting flight from city a to city b with 2 transits.
(c) the set of city pairs (a,b) such that there is a direct non-stop flight from city b to city a .

16. $\mathbb{Z} \times \mathbb{Z}$.

17. $\{(a,a) : a \in \mathbb{Z}\} \cup \{(a,-a) : a \in \mathbb{Z}\}$.

18. (a) is the set of student pairs (a,b) such that they share a common classmate.

(b)

(c)

19. $M_{R^*} = W_n = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

20. (a) $R \cup \{(1, 1), (2, 2), (3, 3), (4, 4)\}$.
(b) $R \cup \{(3, 2), (4, 3), (1, 4)\}$.
(c) $\{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$
(d) $\{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$
(e) $\{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$

21. Similar to previous question

22.

23. (a)
(b) $\{1100, 1010, 1001, 0110, 0101, 0011\}$
(c) $\frac{n(n-1)}{2}$.

24.

25. (a) No
(b) Yes
(c) Yes

26.

27. $\{0, 0001, 001, 01, 010, 0101, 011, 11\}$

28.