Exercise Sheet Vectors

1. (a)
$$\begin{pmatrix} 11 \\ 16 \\ 21 \end{pmatrix}$$

   (b)
$$\begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix}$$

2. $\sqrt{14}$

3. $\pm \frac{\sqrt{7}}{3}$

4. $\sqrt{3}$

5.
$$\vec{x} = \begin{pmatrix} 34 \\ -14 \\ 20 \end{pmatrix} \text{ and } \vec{y} = \begin{pmatrix} 18 \\ -48 \\ 1 \end{pmatrix} \text{ and } \vec{z} = \begin{pmatrix} 22 \\ 9 \\ 6 \end{pmatrix}$$

   and
$$|\vec{x}| = \sqrt{1752} = 41,86, \quad |\vec{y}| = \sqrt{2629} = 51,27, \quad |\vec{z}| = \sqrt{601} = 24,52$$

6. $|\vec{a}| = \sqrt{5}, \quad |\vec{b}| = 5$ and $\varphi_a = \tan^{-1}\left(\frac{-2}{1}\right) + 360° = 296,565°$
   and $\varphi_b = \tan^{-1}\left(\frac{4}{-3}\right) + 180° = 126,870°$

7.
$$\vec{F} = \begin{pmatrix} 2,828 - 1,5 + 1,732 \\ 2,828 + 2,598 - 1 \end{pmatrix} = \begin{pmatrix} 3,06 \\ 4,426 \end{pmatrix}$$

   and
$$|\vec{F}| = \sqrt{3,06^2 + 4,426^2} = 5,381$$

   and
$$\varphi_F = \tan^{-1}\left(\frac{4,426}{3,06}\right) = 55,34°$$

8. $\vec{a} \cdot \vec{b} = -1$ and $\vec{a} \cdot \vec{c} = 2$ and $\vec{b} \cdot \vec{c} = -1$

9. $\vec{b}$ and $\vec{c}$ are orthogonal.

10.

$$\vec{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \vec{B} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \vec{C} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \quad \text{and} \quad \vec{D} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix}$$

and $|F| = \sqrt{45} = 6,708$

11. The edges of the middle triangle are parallel to the edges of the triangle and half as long as them.

12. $c = 1$ or $c = -5$.

13.

14. $b = -\frac{40}{11}$ and $a = -\frac{32}{11}$ and $c = -\frac{31}{22}$.

15. $\alpha = 60°$ and area $\frac{\sqrt{3}}{2}$.