

Exercise Sheet: Logic

1. Let consider the interpretation v where $v(p) = F, v(q) = T, v(r) = T$. Does v satisfy the following propositional formulas?
 - (a) $(p \Rightarrow \neg q) \vee \neg(r \wedge q)$
 - (b) $(\neg p \vee \neg q) \Rightarrow (p \vee \neg r)$
 - (c) $\neg(\neg p \Rightarrow \neg q) \wedge r$
 - (d) $\neg(\neg p \Rightarrow q \wedge \neg r)$
2. Construct a truth table for the formula $\neg p \wedge (p \Rightarrow q)$.
3. Show that $(p \Rightarrow q) \vee (q \Rightarrow p)$ is a tautology.
4. Construct a truth table for $(p \Rightarrow q) \wedge (q \Rightarrow r)$.
5. Write down the negation of the following statements, simplifying so that only simple statements are negated.
 - (a) $p \vee \neg q$
 - (b) $(p \wedge q) \Rightarrow r$
6. Construct a truth table for $(\neg A \vee B) \Rightarrow (\neg C \wedge D)$.
7. Use a truth table to determine the validity of the argument: *If Tweety is a bat then Tweety can fly. Tweety is not a bat. Therefore, Tweety can not fly.*
8. Use a truth table to determine the validity of the argument:
If Joe studies hard, he will make the dean's list. Joe made the Dean's list. Therefore, Joe studied hard.
9. Determine the validity of the argument:
If Joe can afford it, he will ask Sally to go out on a date. If Bill pays Joe the money he owes him then Joe can afford to go out on a date. Joe does not ask Sally to go out on a date. Therefore, Bill did not pay Joe the money he owes him.
10. Determine the validity of the argument:
If you have high blood pressure then you are at risk for having a stroke. Lynn has high blood pressure. Therefore, Lynn is at risk for having a stroke.

11. Prove the following statements by direct proof
 - (a) The product of two odd number integers is odd.
 - (b) The product of two invertible matrices is invertible.
12. Prove that following statements by contraposition.
 - (a) If a product of two positive real numbers is greater than 100, then at least one of the number is greater than 10.
 - (b) There is no greatest even integer.
13. Use the truth table to determine if $p \wedge \neg q \rightarrow p \wedge q$ is a logical consequence of the formula $\neg p$.
14. Check if each of the following statements is satisfiable or not
 - (a) If new messages are not queued, then they will be sent to the message buffer.
 - (b) $(\neg p \vee q) \wedge (q \rightarrow \neg r \wedge \neg p) \wedge (p \vee r)$
15. Express the negation of these propositions using quanti-fiers, and then express the negation in English.
 - (a) Some old dogs can learn new tricks.
 - (b) No rabbit knows Algebra.
 - (c) Every bird can fly.
 - (d) There is no dog that can talk.
 - (e) There is no one in this class who knows German and Spanish.