Answer Keys: Matrices

1.
$$\begin{pmatrix} 4 & 3 \\ 8 & 16 \\ 5 & 21 \end{pmatrix}$$

2. Not possible

3. Not possible

4.
$$\begin{pmatrix} 1 & 1 & 2 & 1 \\ 4 & 7 & 5 & 2 \\ 8 & 17 & 7 & 2 \end{pmatrix}$$

5.
$$\begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & 3 \end{pmatrix}$$

6. 2

7. (a) 5

   (b)
   $$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$

   (c)
   $$\begin{pmatrix} 2 & 0 & 1 \\ 16 & 3 & 11 \end{pmatrix}$$

   (d) Not possible

   (e)
   $$\begin{pmatrix} 2 & 16 \\ 0 & 3 \\ 1 & 11 \end{pmatrix}$$

8. (a) $A^2 + AB + BA + B^2$

(b) $A^2 - AB - BA + B^2$

(c) $A^2 - AB + BA - B^2$

(d) $A^2 + AB - BA - B^2$

9. The transition matrix:

$$X = \begin{pmatrix} 0,85 & 0,15 & 0,05 \\ 0,05 & 0,75 & 0,05 \\ 0,10 & 0,10 & 0,90 \end{pmatrix}$$

The market shares at the end of the first year: $\begin{pmatrix} 0,39 \\ 0,19 \\ 0,42 \end{pmatrix}$.

10. $$c = -13 \text{ and } b = -9 \text{ and } a = 11$$

11. $$\begin{pmatrix} 21 & -10 \\ 11 & 13 \\ 0 & 2 \end{pmatrix}$$

12. $$(AB)^T = B^T A^T = \begin{pmatrix} -9 & 0 & -3 \\ -2 & 4 & 17 \\ 5 & 13 & 47 \end{pmatrix}$$

13.

14.

15. (a) F

(b) T

(c) T

16.

(a) 7.3^3 (c) $\frac{1}{56}$ (e) 49

(b) $\frac{8}{7}$ (d) 7 (f) -7

17.

18. (a) 52

(b) 0

(c) -192

(d) 61.

(e) $3abc - a^3 - b^3 - c^3$

(f) f_n, where f_n is the nth Fibonacci.

(g)
$$\prod_{i>j}(x_i - x_j).$$

19.
$$x = 1 \text{ or } y = 1 \text{ or } z = 1$$

20.
$$A^{-1} = \begin{pmatrix} 4 & -2 & 1 \\ -5 & 3 & -2 \\ 2 & -1 & 1 \end{pmatrix}$$

21. 1/2.

22. $(x_1, x_2, x_3, x_4) == (-2, -1/2, 1, 0)t + (3, 1/2, 0, 1)s + (0, 1/2, 0, 0)$, where s and t are arbitrary.

23. If $a = 2$:
$x_1 = 2 - 2t, x_2 = -\frac{1}{2}t, x_3 = t$, where t is arbitrary.
Otherwise: No solution.

24. For $a \neq 1$:
$(x, y, z) = (a + 3, 2, -1)$

For $a = 1$:
$(x, y, z) = (2 - 2t, 1 - t, t)$ for $t \in \mathbb{R}$

25. (a)
$$\frac{1}{5}\begin{pmatrix} 4 & 3 & 2 & 1 \\ 3 & 6 & 4 & 2 \\ 2 & 4 & 6 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

(b)
$$\frac{1}{5}\begin{pmatrix} 18 \\ 21 \\ 24 \\ 12 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 8 \\ 6 \\ -1 \\ -3 \end{pmatrix}$$

26. (a)
$$A^{-1} = \frac{1}{12}\begin{pmatrix} -9 & 5 & 1 \\ 6 & -2 & 2 \\ 9 & -1 & -5 \end{pmatrix}$$

and
$$B^{-1} = \frac{1}{6}\begin{pmatrix} -5 & 1 & 1 \\ 1 & -2 & 1 \\ 3 & 1 & -1 \end{pmatrix}$$

(b) i.
$$\begin{pmatrix} 0 & 0 & 2 \\ 0 & -1 & 4 \\ 6 & 5 & 10 \end{pmatrix}$$

ii.

$$\frac{1}{2}\begin{pmatrix} 3 & 0 & 2 \\ 2 & 4 & 4 \\ -5 & -4 & -4 \end{pmatrix}$$

iii.

$$\frac{1}{6}\begin{pmatrix} 15 & -1 & -5 \\ 15 & 5 & -11 \\ 33 & 5 & -11 \end{pmatrix}$$

iv.

$$\frac{1}{2}\begin{pmatrix} 1 & 0 & -2 \\ -2 & 0 & -4 \\ 5 & 4 & 8 \end{pmatrix}$$

v.

$$\frac{1}{12}\begin{pmatrix} -9 & 5 & 1 \\ 6 & -2 & 2 \\ 9 & -1 & -5 \end{pmatrix}$$

vi.

$$\frac{1}{12}\begin{pmatrix} 1 & 3 & -1 \\ 4 & 2 & 0 \\ 3 & -3 & -3 \end{pmatrix}$$

27. (a)

$$\begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b)

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{16} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{32} \end{pmatrix}$$

(c)
$$\begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

(d) Not regular

(e)
$$\begin{pmatrix} 1 & -2 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(f)
$$\begin{pmatrix} 2-n & 1 & 1 & \dots & 1 \\ 1 & -1 & 0 & \dots & 0 \\ 1 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & 0 & 0 & \dots & -1 \end{pmatrix}$$

28. (a) $D = 6$ and $x_1 = -8$ and $x_2 = -3$ and $x_3 = 3$.

    (b) $D = 12$ and $x_1 = -65/12$ and $x_2 = 19/6$ and $x_3 = 73/12$.

29.
$$\vec{x} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

30. (a) 2.

    (b) 70.

    (c) 11.