Negation on the Australian Plan

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THE UNIVERSITY OF MELBOURNE

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My Aim

To explain the difference between *Australian Plan* and *American Plan* models for negation,

and to defend the Australian Plan against recent criticisms.

Setting the Scene

The American Plan

The Australian Plan

Grounding?

Compatibility?

Pluralism?

What is the *Point*?


SETTING THE SCENE

Negation Principles

$$\neg A \wedge \neg B \succ \neg(A \vee B) \qquad \neg A \vee \neg B \succ \neg(A \wedge B)$$

$$\neg(A \vee B) \succ \neg A \wedge \neg B \qquad \neg(A \wedge B) \succ \neg A \vee \neg B$$

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TV

classical

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TV	K3
classical	gaps

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$$\neg A \land \neg B \succ \neg(A \lor B) \qquad \neg A \lor \neg B \succ \neg(A \land B)$$

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TV	K3	LP	FDE	J
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- J: Heyting algebras, such as algebras of open subsets of a topological space. (Kripke frames included.)

Algebras are…

… not *great* for explaining what the connectives *mean*.

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They give *truth conditions* only when
we connect algebraic values with *truth*.


THE AMERICAN PLAN

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- J: not possible in the American Plan.

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- It's a natural picture for logics generated by 3- and 4-valued algebras, like K3, LP and FDE.
- It's given various interpretations (chiefly in the work of Dunn, Belnap, and Priest).


THE AUSTRALIAN PLAN

- $x \Vdash \neg A$ iff $x^* \nVdash A$

Frame Semantics and Compatibility

- $x \Vdash \neg A$ iff $(\forall y)(xCy \Rightarrow y \nVdash A)$

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- FDE: Arbitrary Routley star.
- J: xCy iff $(\exists z)(x \sqsubseteq z \wedge y \sqsubseteq z)$

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  - Two worlds: *here* and *there*. Everything **true** in the interpretation holds *here*, and everything not **false** in the interpretation holds *there*.
- From any world x in an Australian plan interpretation using the Routley star, we can define an American plan interpretation.
  - Something is **true** in the interpretation if it holds at x, and **false** if it doesn't hold at x^*.

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There is More to Negation than Modality

Michael De[1] · Hitoshi Omori[2]

Received: 17 February 2016 / Accepted: 20 January 2017

Abstract There is a relatively recent trend in treating negation as a modal operator. One such reason is that doing so provides a uniform semantics for the negations of a

Objections

- It's wrong to attempt to ground *negation* in a notion of *incompatibility*
- The notion of compatibility is unclear.
- The Australian Plan does not pin down a single logic of negation.


QUESTIONS ABOUT
GROUNDING

Problem 1

Isn't there a problem with grounding negation in a primitive notion of incompatibility?

No, there isn't

No, but you need to understand how models work, and in particular, the metaphysics of points.

Deflationary Accounts

$$x \Vdash \Box A \text{ iff } (\forall y)(xRy \Rightarrow y \Vdash A)$$

Here's Prior

. . . possible worlds, in the sense of possible states of affairs are not really individuals (just as numbers are not really individuals). To say that a state of affairs obtains is just to say that something is the case; to say that something is a possible state of affairs is just to say that something could be the case; and to say that something is the case 'in' a possible state of affairs is just to say that the thing in question would necessarily be the case if that state of affairs obtained, i.e. if something else were the case . . . We understand 'truth in states of affairs' because we understand 'necessarily'; not vice versa.

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The direction of grounding can be the reverse,
just as it is for modal semantics.

Especially if we are attempting to understand
incompatibility *and* the notion of *point* in a frame.

Metaphysically Robust Accounts

If you have some independent metaphysical
account of what these *points* are,
there may be a deep connection between
compatibility between points
and the semantics of negation.


QUESTIONS ABOUT COMPATIBILITY

Problem 1: How does compatibility work?

It's true that Sam is not a gram heavier than she actually is, even though she easily could have been. And since she could easily have been a gram heavier, there are worlds where she is that are very similar to our own. Indeed, these worlds seem compatible with ours, if we are going by our intuitive notion of compatibility. And yet, on the modal account of negation, all the worlds compatible with ours are ones where Sam is not a gram heavier than she actually is, no matter how similar they are to ours.

How does compatibility work? (cont.)

Why are all the compatible worlds like this? To emphasize, our intuitive understanding of (in)compatibility does not tell us that these worlds are incompatible with ours. If there is any kind of explanation as to why these worlds should be incompatible with ours, we can only see that it must ultimately appeal to negation.

Worlds where it is true that we are a gram heavier than we actually are are incompatible with our own because they make true the negation of a sentence that is here true.

(De and Omori, p. 6)

No

- A situation (or world) where Sam weighs exactly n grams is incompatible with a situation (or world) where Sam weighs exactly $n + 1$ grams.

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- (Whether this is further articulated using the concept of negation is irrelevant.)

Problem 2: Compatibility is inconsistent with Paraconsistency

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But then, $x \sqsubseteq x$ gives xCx.

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- xCy doesn't mean $(\exists z)(x \sqsubseteq z \wedge y \subseteq z)$.
- At least, not if states can be self-incompatible.
- We can even have states x, y where $x \not\!C x$ but xCy.

QUESTIONS ABOUT PLURALISM

We also do not find the arguments in favor of or against various constraints on incompatibility compelling enough to allow us to comfortably say that such and such are the Laws of Negation. If something is to qualify as an adequate account of negation, it should be fairly clear according to that account what the laws of negation are.

(De and Omori, p. 3)

So?

The same can be said for the American Plan, of course.

. . . at least if you're a logical pluralist like me.

… but if you're not a pluralist

Fix on a particular notion of *point*,
and the appropriate notion of
compatibility will be easier to settle on.


WHAT IS THE POINT?

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- This makes point-based semantics for other notions *much* more complex. In that you need to hand-tune the relationship between any newly defined notion ($\Box$, $\Diamond$, *counterfactuals*, *quantifiers*, etc.) and **falsity**, as well as their relationship to **truth**.

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- This makes point-based semantics for other notions *much* more complex. In that you need to hand-tune the relationship between any newly defined notion ($\Box$, $\Diamond$, *counterfactuals*, *quantifiers*, etc.) and **falsity**, as well as their relationship to **truth**.
- An Australian-plan semantics is more *modular*, in that once you determine truth-at-a-point, for the new notion, its interaction with negation is fixed — each notion is given independent truth conditions.

THANK YOU!

http://consequently.org/presentation/2017/negation-on-the-australian-plan

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