

# Generality & Existence II

## Modality & Quantifiers

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ASL / CENTRAL APA · FEBRUARY 2019

To analyse the *quantifiers*

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(including their interactions with *modals*)

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using the tools of *proof theory*

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(including their interactions with *modals*)  
using the tools of *proof theory*  
in order to better understand  
*quantification, existence and modality*.

Sequents & Defining Rules

Hypersequents & Defining Rules

Quantification & the Barcan Formula

Positions & Models

Consequences & Questions

# SEQUENTS & DEFINING RULES

## Sequents

$$\Gamma \supset \Delta$$

Don't assert each element of  $\Gamma$   
and deny each element of  $\Delta$ .



*Identity:*  $A \succ A$

## Structural Rules

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*Weakening:*  $\frac{\Gamma \succ \Delta}{\Gamma, A \succ \Delta} \quad \frac{\Gamma \succ \Delta}{\Gamma \succ A, \Delta}$

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Structural rules govern declarative sentences *as such*.

## Giving the Meaning of a Logical Constant

With Left/Right rules?

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \wedge B \succ \Delta} [\wedge L] \qquad \frac{\Gamma \succ A, \Delta \quad \Gamma \succ B, \Delta}{\Gamma \succ A \wedge B, \Delta} [\wedge R]$$

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$$\frac{\Gamma \succ A, \Delta}{\Gamma \succ A \text{ tonk } B, \Delta} [\text{tonk}R]$$

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Use  $\succ_{\mathcal{L}}$  to *define*  $\succ_{\mathcal{L}'}$ .



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*Desideratum #1:*  $\succ_{\mathcal{L}'}$  is conservative:  $(\succ_{\mathcal{L}'})|_{\mathcal{L}}$  is  $\succ_{\mathcal{L}}$ .

*Desideratum #2:* Concepts are defined *uniquely*.

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*Identity* and *Cut* determine the behaviour of conjunctions on the *right*.

# From $[\wedge Df]$ to $[\wedge L/R]$

$$\begin{array}{c}
 \frac{\frac{\frac{}{A \wedge B \succ A \wedge B} [Id]}{A, B \succ A \wedge B} [\wedge Df]}{\Gamma \succ B, \Delta \quad A, B \succ A \wedge B} [Cut] \\
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 \end{array}$$

$$\frac{\Gamma \succ A, \Delta \quad \Gamma \succ B, \Delta}{\Gamma \succ A \wedge B, \Delta} [\wedge R]$$

## And Back

$$\frac{\frac{A \succ A \quad B \succ B}{A, B \succ A \wedge B} [\wedge R] \quad \Gamma, A \wedge B \succ \Delta}{\Gamma, A, B \succ \Delta} [Cut]$$

I want to see how this works  
for modal operators, and  
examine their interaction  
with the quantifiers.

## Why this is important

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Explaining *why* the modal operators have the logical properties they exhibit is an open question.

... possible worlds, in the sense of possible states of affairs are not *really* individuals (just as numbers are not *really* individuals).

To say that a state of affairs obtains is just to say that something is the case; to say that something is a possible state of affairs is just to say that something could be the case; and to say that something is the case 'in' a possible state of affairs is just to say that the thing in question would necessarily be the case if that state of affairs obtained, i.e. if something else were the case ...

We understand 'truth in states of affairs' because we understand 'necessarily'; not *vice versa*.

— "Worlds, Times and Selves"  
(1969)



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---

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- ▶ (Why does possibility distribute over disjunction, necessity over disjunction? Why do many modalities *work* in the way modelled by normal modal logics?)
- ▶ If modality is *primitive* we have no explanation.
- ▶ If modality is governed by the rules introduced here, then we can see *why* possible worlds are useful, and model the behaviour of modal concepts.

# HYPERSEQUENTS & DEFINING RULES

## Modal Reasoning involves *Shifts*

Suppose it's necessary that  $p$  and necessary that  $q$ .

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Suppose it's necessary that  $p$  and necessary that  $q$ .  
Is it necessary that both  $p$  and  $q$ ?

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Suppose it's necessary that p and necessary that q.

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Could we *avoid* p and q?

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Is it necessary that both  $p$  and  $q$ ?

Could we *avoid*  $p$  and  $q$ ?

Consider any way it could go:

Since it's necessary that  $p$ , here we have  $p$ .

## Modal Reasoning involves *Shifts*

Suppose it's necessary that p and necessary that q.

Is it necessary that both p and q?

Could we *avoid* p and q?

Consider any way it could go:

Since it's necessary that p, here we have p.

Since it's necessary that q, here we have q.

## Modal Reasoning involves *Shifts*

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Since it's necessary that p, here we have p.

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So, we have both p and q.

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So, we have both p and q.

So, no matter how things go, we have p and q.

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Suppose it's necessary that p and necessary that q.

Is it necessary that both p and q?

Could we *avoid* p and q?

Consider any way it could go:

Since it's necessary that p, here we have p.

Since it's necessary that q, here we have q.

So, we have both p and q.

So, no matter how things go, we have p and q.

So the conjunction p and q is necessary.

# Exposing the Structure of that Deduction

$$\begin{array}{c}
 \frac{\Box p, \Box q \succ \Box p}{\Box p, \Box q \succ \quad | \quad \succ p} [\Box Df] \qquad \frac{\Box p, \Box q \succ \Box q}{\Box p, \Box q \succ \quad | \quad \succ q} [\Box Df] \\
 \hline
 \Box p, \Box q \succ \quad | \quad \succ p \wedge q \qquad \qquad \qquad [\wedge R] \\
 \hline
 \Box p, \Box q \succ \quad | \quad \succ p \wedge q \\
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 \Box p, \Box q \succ \Box(p \wedge q) \qquad \qquad \qquad [\Box Df] \\
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$$\Box p, \Box q \succ \quad | \quad \succ p \wedge q$$

Don't assert  $\Box p$  and  $\Box q$  in one 'zone'  
and deny  $p \wedge q$  in another.

$$\Gamma \succ \Delta \mid \Gamma' \succ \Delta'$$

Don't assert each member of  $\Gamma$   
and deny each member of  $\Delta$  in one 'zone'  
and assert each member of  $\Gamma'$   
and deny each member of  $\Delta'$  in another.

## Two Kinds of Zone Shift

**INDICATIVE:** suppose I'm wrong and that. . .

**SUBJUNCTIVE:** suppose things go differently.  
or *had gone* differently.



# Two Kinds of Zone Shift



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- Suppose Oswald *didn't* shoot JFK.

## Two Kinds of Zone Shift



- ▶ Suppose Oswald *didn't* shoot JFK.
- ▶ Suppose Oswald *hadn't* shot JFK.

*Freedom, oh freedom, well that's just some people talkin'.*  
— The Eagles

# STEREOSCOPIC VISION:

## Persons, Freedom, and Two Spaces of Material Inference

*Mark Lance*  
Georgetown University

*W. Heath White*  
University of North Carolina at Wilmington

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<[www.philosophersimprint.org/007004/](http://www.philosophersimprint.org/007004/)>

WHAT IS A PERSON, as opposed to a non-person? One might begin to address the question by appealing to a second distinction: between *agents*, characterized by the ability to act freely and intentionally, and mere patients, caught up in events but in no sense authors of the happenings involving them. An alternative way to address the question appeals to a third distinction: between *subjects*—bearers of rights and responsibilities, commitments and entitlements, makers of claims, thinkers of thoughts, issuers of orders, and posers of questions—and mere objects, graspable or evaluable by subjects but not themselves graspers or evaluators.

We take it as a methodological point of departure that these three distinctions are largely coextensive, indeed coextensive in conceptually central cases. Granted, these distinctions can come apart. One might think that 'person' applies to anything that is worthy of a distinctive sort of moral respect and think this applicable to some fetuses or the deeply infirm elderly. Even if the particular respect due such beings is importantly different from "what we owe each other", such respect could still be thought to be of the kind distinctively due people, and think this even while holding that such people lack agentive or subjective capacity. Similarly, one might think dogs or various severely impaired humans to be attenuated subjects but not agents.

Without taking any particular stand on such examples, our methodological hypothesis is that such cases, if they exist, are understood as persons (agents, subjects) essentially by reference to paradigm cases and, indeed, to a single paradigm within which person/non-person, subject/object, and agent/patient are conceptually connected.<sup>1</sup> Stated

1. For one detailed development of this sort of paradigm-riff structure, and a defense of the possibility of concepts essentially governed by such a structure, see Lance and Little (2004). Discussions with Hilda Lindeman have helped

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  - (*Analogies:*  $\forall x$  from first order logic and natural language's 'all.' Frictionless planes. etc.)



## Example Subjunctive Shifts

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$$[oSk : ]_{@} \mid [@oSk : oSk]$$

We open up a zone for consideration, in which we deny  $oSk$ , while keeping track of the initial zone where we assert it.

(And if we like, we can assert  $@oSk$  in the zone under the counterfactual supposition.)

## Disagreement and Indicative Shifting

I think that Oswald shot JFK, but you don't.

I consider what it would mean for you to be right.

If you're right, Oswald *actually didn't* shoot JFK.

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$$[oSk : ]_{@} \Rightarrow [ : oSk]_{@}$$

## Indicative Shifting

*I think that Hesperus is Phosphorous, but I recognise that you don't. I don't take you to be inconsistent or misusing names.*

## Indicative Shifting

I think that Hesperus is Phosphorous, but I recognise that *you don't*. I don't take *you* to be *inconsistent* or misusing names.

We *don't* have this:

$$a = b \succ \quad \Longrightarrow \quad Fa \succ Fb$$

It's coherent for you to assert  $Fa$  and deny  $Fb$  even if I take it that  $a = b$ , and it's coherent for me to consider an alternative in which  $a \neq b$  even if I don't agree.



## *Idealised* Indicative Shifts

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- ▶ This is as *liberal* as possible about what counts as an *alternative* from any alternative zone.
- ▶ This gives us a motivation for a richer family of hypersequents.

## Two Dimensional Hypersequents

$$\begin{array}{ccccccc}
 X_1^1 \succ_{@} Y_1^1 & | & X_2^1 \succ Y_2^1 & | & \cdots & | & X_{m_1}^1 \succ Y_{m_1}^1 & || \\
 X_1^2 \succ_{@} Y_1^2 & | & X_2^2 \succ Y_2^2 & | & \cdots & | & X_{m_2}^2 \succ Y_{m_2}^2 & || \\
 \vdots & & \vdots & & & & \vdots & \\
 X_1^n \succ_{@} Y_1^n & | & X_2^n \succ Y_2^n & | & \cdots & | & X_{m_n}^n \succ Y_{m_n}^n & 
 \end{array}$$

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Think of these as *scorecards*, keeping track of assertions and denials.

# Notation

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$$\mathcal{H}[\Gamma \succ \Delta]$$

# Notation

---

$$\mathcal{H}[\Gamma' \succ \Delta']$$



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$$\mathcal{H}[\Gamma' \succ \Delta']$$

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$$\mathcal{H}[\Gamma' \succ \Delta']$$

$$\mathcal{H}[\Gamma \succ \Delta \mid \Gamma' \succ \Delta']$$

$$\mathcal{H}[\Gamma \succ \Delta \parallel \Gamma' \succ \Delta']$$

## Defining Rule for $\Box$

$$\frac{\mathcal{H}[\Gamma \succ \Delta \mid \quad \succ A]}{\mathcal{H}[\Gamma \succ \Box A, \Delta]} [\Box Df]$$

## Defining Rule for @

$$\frac{\mathcal{H}[\Gamma, A \succ_{@} \Delta \mid \Gamma' \succ \Delta']}{\mathcal{H}[\Gamma \succ_{@} \Delta \mid \Gamma', @A \succ \Delta']} \text{[@Df]}$$

## Defining Rule for $[e]$

$$\frac{\mathcal{H}[\Gamma \succ \Delta \parallel \succ_{@} A]}{\mathcal{H}[\Gamma \succ [e]A, \Delta]} \text{ } [[e]Df]$$

## Example Derivation

$$\begin{array}{c}
 \begin{array}{c}
 \succ \quad | \quad [e]A \succ [e]A \\
 \hline
 \succ \quad | \quad [e]A \succ \quad || \quad \succ_{@} A
 \end{array} \quad [e]Df \\
 \hline
 \begin{array}{c}
 \succ \quad | \quad [e]A \succ \quad || \quad \succ_{@} A \\
 \hline
 \succ \quad | \quad [e]A \succ [e]@A
 \end{array} \quad [e]Df \\
 \hline
 \begin{array}{c}
 \succ \quad | \quad \succ [e]A \supset [e]@A \\
 \hline
 \succ \quad \square([e]A \supset [e]@A)
 \end{array} \quad \begin{array}{l} \supset Df \\ \square Df \end{array}
 \end{array}$$

QUANTIFICATION  
& THE BARCAN  
FORMULA

# The Standard Quantifier Rules

$$\frac{\Gamma \succ A(n), \Delta}{\Gamma \succ (\forall x)A(x), \Delta} [\forall Df]$$

$$\frac{\Gamma, A(n) \succ \Delta}{\Gamma, (\exists x)A(x) \succ \Delta} [\exists Df]$$



## Deriving the Barcan Formula

$$\begin{array}{c}
 (\forall x)\Box Fx \succ (\forall x)\Box Fx \\
 \hline
 (\forall x)\Box Fx \succ \Box Fn \quad [\forall Df] \\
 \hline
 (\forall x)\Box Fx \succ \quad | \quad \succ Fn \quad [\Box Df] \\
 \hline
 (\forall x)\Box Fx \succ \quad | \quad \succ (\forall x)Fx \quad [\forall Df] \\
 \hline
 (\forall x)\Box Fx \succ \Box(\forall x)Fx \quad [\Box Df] \\
 \hline
 \succ (\forall x)\Box Fx \supset \Box(\forall x)Fx \quad [\supset Df]
 \end{array}$$

## Where the derivation breaks down

$$\begin{array}{c}
 (\forall x)\Box Fx \succ (\forall x)\Box Fx \\
 \hline
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 \hline
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## *Pro* and *Con* attitudes to Terms

To rule a term *in* is to take it as suitable  
to substitute into a quantifier,  
i.e., to take the term to *denote*.

To rule a term *out* is to take it as unsuitable  
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i.e., to take the term to *not denote*.

We add terms to the LHS and RHS of sequents  $\Gamma \succ \Delta$ .

## Structural Rules remain as before

*Identity:*  $X \succ X$

*Weakening:*  $\frac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma, X \succ \Delta]} \quad \frac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ X, \Delta]}$

*Contraction:*  $\frac{\mathcal{H}[\Gamma, X, X \succ \Delta]}{\mathcal{H}[\Gamma, X \succ \Delta]} \quad \frac{\mathcal{H}[\Gamma \succ X, X, \Delta]}{\mathcal{H}[\Gamma \succ X, \Delta]}$

*Cut:*  $\frac{\mathcal{H}[\Gamma \succ X, \Delta] \quad \mathcal{H}[\Gamma, X \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta]}$

Here  $X$  is either a sentence or a term.

...and there are some more

$$\text{Ext. Weak.:} \quad \frac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta \mid \Gamma' \succ \Delta']} \quad \frac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta \parallel \Gamma' \succ \Delta']}$$

$$\text{Ext. Contr.:} \quad \frac{\mathcal{H}[\Gamma \succ \Delta \mid \Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta]} \quad \frac{\mathcal{H}[\mathcal{S} \parallel \mathcal{S}]}{\mathcal{H}[\mathcal{S}]}$$



## Quantifier Rules, allowing for non-denoting terms

$$\frac{\mathcal{H}[\Gamma, n \succ A(n), \Delta]}{\mathcal{H}[\Gamma \succ (\forall x)A(x), \Delta]} [\forall Df]$$

$$\frac{\mathcal{H}[\Gamma, n, A(n) \succ \Delta]}{\mathcal{H}[\Gamma, (\exists x)A(x) \succ \Delta]} [\exists Df]$$

## Now you *can't* derive the Barcan Formula

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$$(\forall x)\Box Fx \succ \Box(\forall x)Fx \mid \succ (\forall x)Fx$$

## Now you *can't* derive the Barcan Formula

$$(\forall x)\Box Fx \succ \Box(\forall x)Fx \mid \mathbf{b} \succ \mathbf{Fb}, (\forall x)Fx$$

## Now you *can't* derive the Barcan Formula

$$(\forall x)\Box Fx \succ \mathbf{b}, \mathbf{Fb}, \Box(\forall x)Fx \mid \mathbf{b} \succ Fb, (\forall x)Fx$$

## Now you *can't* derive the Barcan Formula

$$\alpha, (\forall x)\Box Fx \succ b, Fb, \Box(\forall x)Fx \mid \alpha, b \succ Fb, (\forall x)Fx$$

## Now you *can't* derive the Barcan Formula

$a, \Box Fa, (\forall x)\Box Fx \succ b, Fb, \Box(\forall x)Fx \mid a, b \succ Fb, (\forall x)Fx$

## Now you *can't* derive the Barcan Formula

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This hypersequent is underivable...

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This hypersequent is underivable...

...and it's *fully refined*.

# POSITIONS & MODELS

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- ▶ An arbitrary position is a set (*indicative* alternatives) of sets (*subjunctive* alternatives) of pairs of sets of formulas or terms (*components*), where one component in each indicative alternative is marked with an @.

## Fully Refined Positions

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# Models

*Fully refined positions* are examples of *models*,  
with variable domains and the expected truth conditions for  
the connectives, quantifiers and modal operators.

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- ▶ That fully refined position determines a model in which the hypersequent does not hold.
- ▶ So the models are adequate for the logic.
- ▶ And in the logic, the cut rule is admissible in the cut-free system.

# CONSEQUENCES & QUESTIONS

The structure of modal concepts is explained  
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Worlds (and their domains) are idealised positions.

# Coherent, Well Behaved Contingentism

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# Inner and Outer Quantification

‘Outer’ quantification is an issue for contingentism.  
On most approaches to contingentism, it can be *defined*.



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‘Outer’ quantification is an issue for contingentism.  
On most approaches to contingentism, it can be *defined*.  
This proof theoretical semantics is no different in that regard....

## We have *Outer* Quantification

$$\frac{\mathcal{H}(\mathfrak{n} \succ \mid \Gamma \succ A(\mathfrak{n}), \Delta)}{\mathcal{H}(\Gamma \succ (\forall^\diamond \mathfrak{x}) A(\mathfrak{x}), \Delta)} [\forall^\diamond Df] \qquad \frac{\mathcal{H}(\mathfrak{n} \succ \mid \Gamma, A(\mathfrak{n}) \succ \Delta)}{\mathcal{H}(\Gamma, (\exists^\diamond \mathfrak{x}) A(\mathfrak{x}) \succ \Delta)} [\exists^\diamond Df]$$

for which the substituted term need be defined in *some* zone.

# The Barcan Formula is Derivable

$$\begin{array}{c}
 (\forall^{\diamond}x)\Box A(x) \succ (\forall^{\diamond}x)\Box A(x) \\
 \hline
 n \succ \mid (\forall^{\diamond}x)\Box A(x) \succ \Box A(n) \quad [\forall^{\diamond}Df] \\
 \hline
 n \succ \mid (\forall^{\diamond}x)\Box A(x) \succ \mid \succ A(n) \quad [\Box Df] \\
 \hline
 (\forall^{\diamond}x)\Box A(x) \succ \mid \succ (\forall^{\diamond}x)A(x) \quad [\forall^{\diamond}Df] \\
 \hline
 (\forall^{\diamond}x)\Box A(x) \succ \Box(\forall^{\diamond}x)A(x) \quad [\Box Df]
 \end{array}$$

## But we also have *Way Out* Quantification

$$\frac{\mathcal{H}(\Gamma \succ A(\mathfrak{n}), \Delta)}{\mathcal{H}(\Gamma \succ (\Pi \mathfrak{x})A(\mathfrak{x}), \Delta)} [\Pi Df] \qquad \frac{\mathcal{H}(\Gamma, A(\mathfrak{n}) \succ \Delta)}{\mathcal{H}(\Gamma, (\Sigma \mathfrak{x})A(\mathfrak{x}) \succ \Delta)} [\Sigma Df]$$

for which the term need not be defined *anywhere*.

# Higher Order Contingentism?

$$\forall X \Box \phi(X) \succ \Box \forall X \phi(X)$$

## Higher Order Contingentism?

$$\forall X \Box \phi(X) \succ \Box \forall X \phi(X)$$

What could it mean to rule a *predicate* in or out?

# THANK YOU!

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general-ity-and-existence-2-apa](http://consequently.org/presentation/2019/general-ity-and-existence-2-apa)

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