Proof Identity, Invariants and Hyperintensionality

Greg Restall

THE UNIVERSITY OF MELBOURNE

DIP COLLOQUIUM · 7 MARCH 2017

Hyperintensionality

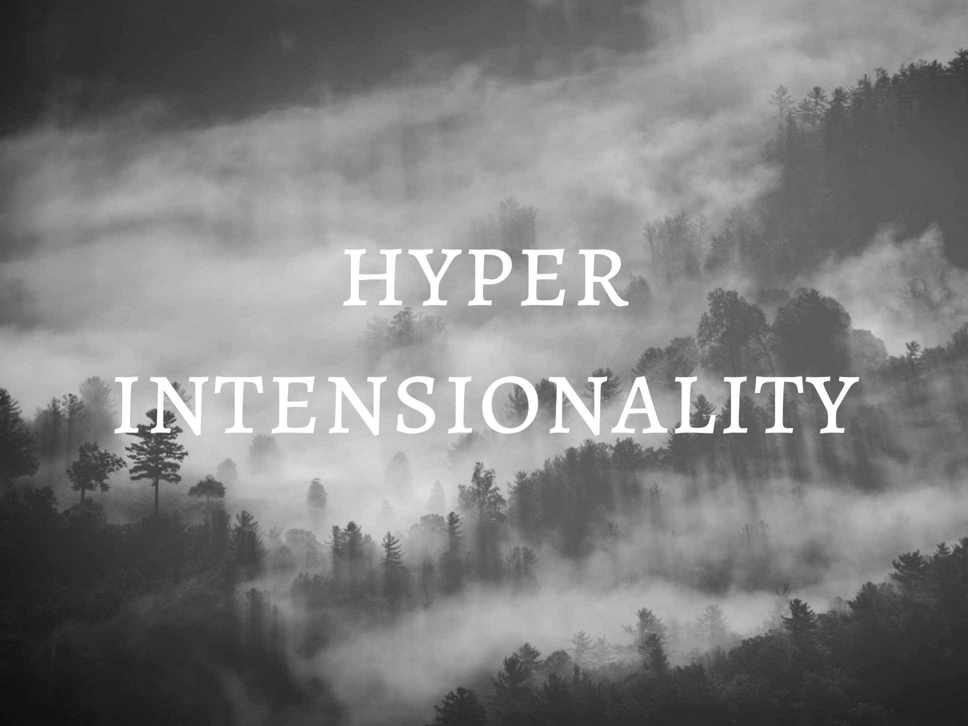
Models and Proofs

Truthmakers

Subject Matter

Proof Invariants

Where we've got, and where to from here



HYPER INTENSIONALITY

Extensionality, Intensionality and Hyperintensionality

- EXTENSIONAL: agree on the same truth value or referent — $\wedge$, $\vee$, $\neg$, $\forall$, $\exists$, $=$.
- INTENSIONAL: agree on the same truth value or referent in every possible world — $\Box$, $\Diamond$, F, P, $\rightarrow$.
- HYPERINTENSIONAL: more finely grained still. *proves that, makes true, because*.
  - Distinguish between logically equivalent propositions. $p \vee \neg p$ and $\top$; $p \vee (p \wedge q)$ and p, etc.
- I want to explore the difference between some *model theoretic* and *proof theoretic* approaches to hyperintensionality.


MODELS AND
PROOFS

MODEL THEORY (1) Define *models* and give recursive conditions explaining when a claim is true in a model.

Two Traditions in Logic and Semantics

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PROOF THEORY (1) Define *proof rules* giving recursive conditions explaining what follows from what.

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Two More Tools for Understanding Aboutness

MODEL THEORY	PROOF THEORY
Situations as Truthmakers	*Proof Invariants*

TRUTHMAKERS

Case 1: $p \vee \neg p$

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$$s \Vdash p \vee \neg p \text{ iff } s \Vdash p \text{ or } s \Vdash \neg p$$

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Many situations are silent on whether p or not.

(*Making* $p \vee \neg p$ *true* is different from *making* $q \vee \neg q$ *true*.)

Case 2: $p \vee (p \wedge q)$

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$$\begin{aligned} s \Vdash p \vee (p \wedge q) \quad & \text{iff} \quad s \Vdash p \text{ or } s \Vdash p \wedge q, \\ & \text{iff} \quad s \Vdash p \text{ or } (s \Vdash p \text{ and } s \Vdash q), \\ & \text{iff} \quad s \Vdash p. \end{aligned}$$

$$p \vee (p \wedge q)$$

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Making $p \vee (p \wedge q)$ true just *is* making p true.

Case 3: $(p \vee \neg p) \vee (q \vee \neg q)$ and $(p \vee \neg p) \wedge (q \vee \neg q)$

$$(p \vee \neg p) \vee (q \vee \neg q)$$

Either make p true, or make p false,
or make q true, or make q false.

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$$(p \vee \neg p) \vee (q \vee \neg q)$$

Either make p true, or make p false,
or make q true, or make q false.

$$(p \vee \neg p) \wedge (q \vee \neg q)$$

Either make p and q true, or make p true and q false,
or make p false and q true, or make p and q false.


SUBJECT MATTER

"The [positive] subject matter of S
is the relation m such that
worlds are m-dissimilar iff
S is differently true in them."

Positive Subject Matter

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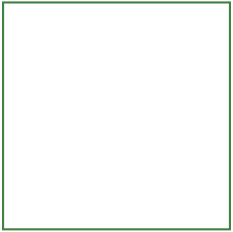
This is: how S is true

Picturing Positive Subject Matter



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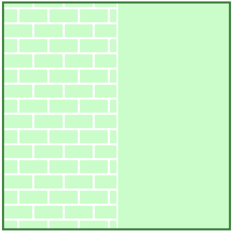

The negative subject matter of S is the relation m′ such that worlds are m′-dissimilar iff S is differently false in them.”

The negative subject matter of S
is the relation m′ such that
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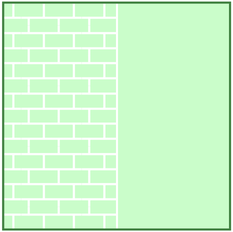
This is: how S is false

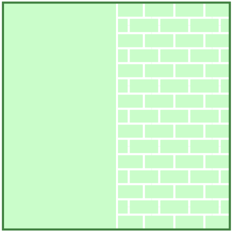
$\{$how S is true, how S is false$\}$

Aboutness, Chapter 2

Picturing Subject Matter



how S is true



how S is false

The subject matter of $p \vee \neg p$

how $p \vee \neg p$ is true

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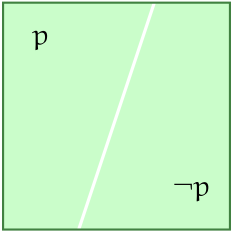
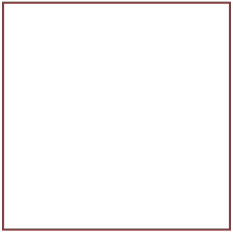
The subject matter of $p \vee \neg p$



how $p \vee \neg p$ is true



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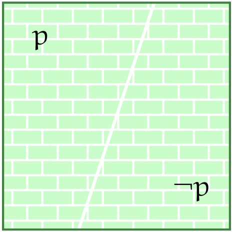
The subject matter of $p \vee \neg p$



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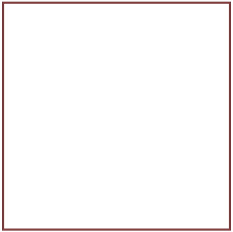

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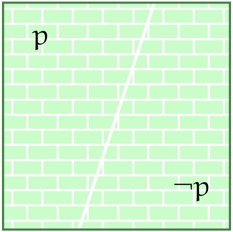
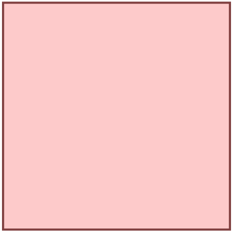
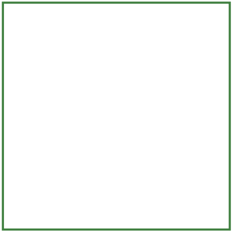




how $p \vee \neg p$ is true

how $p \vee \neg p$ is false

The subject matter of $p \vee (p \wedge q)$



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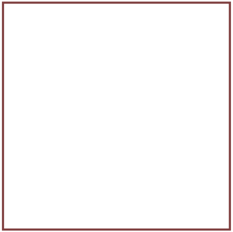
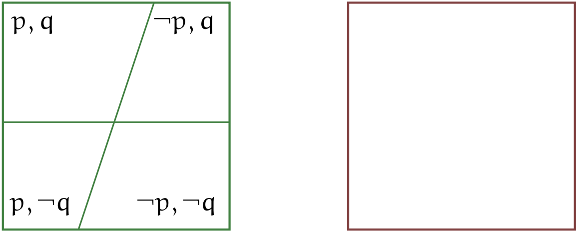


how $p \vee (p \wedge q)$ is false

The subject matter of $p \lor (p \land q)$



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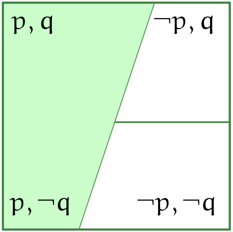

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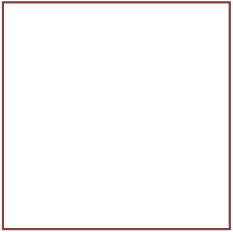

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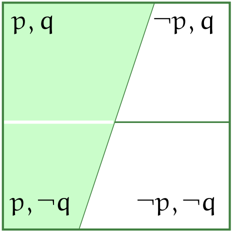

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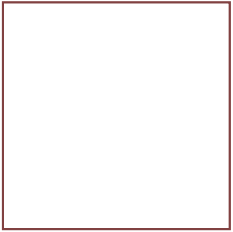

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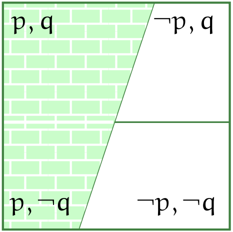

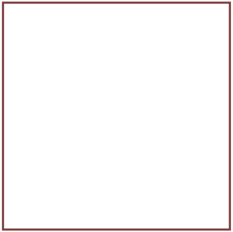

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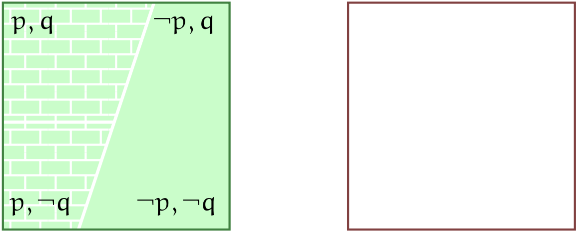

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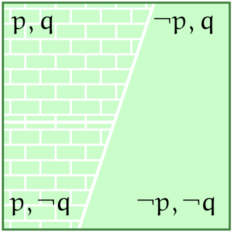

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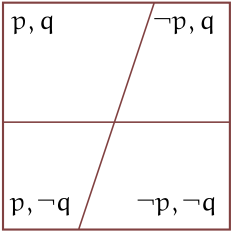

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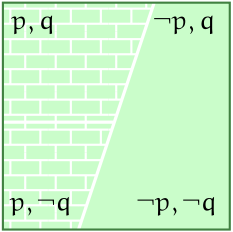

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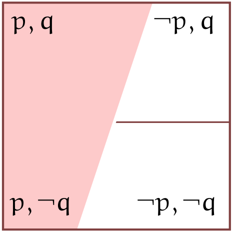

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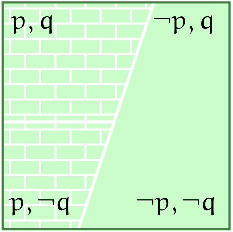

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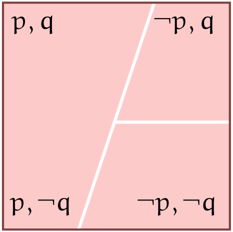

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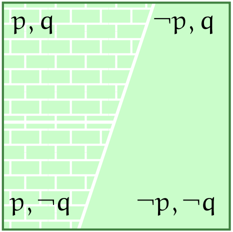

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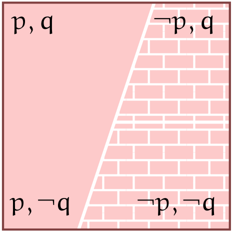

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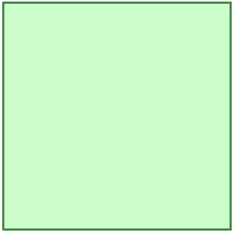

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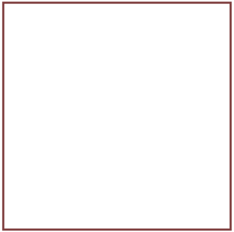

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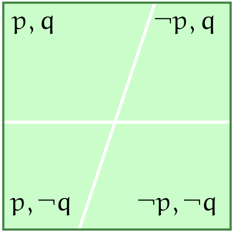

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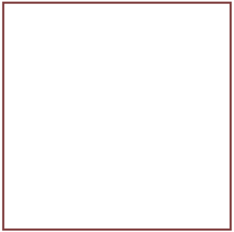

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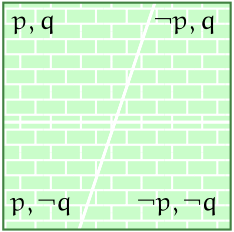

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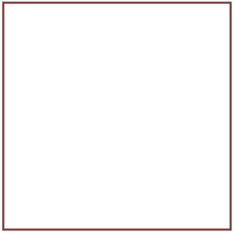

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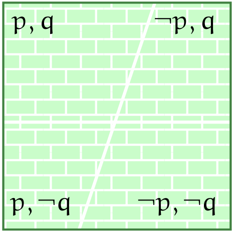

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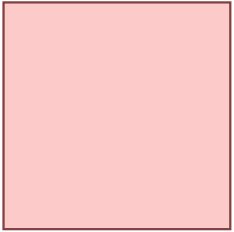

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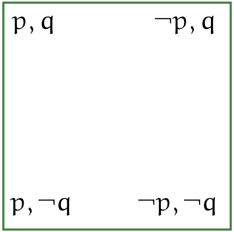

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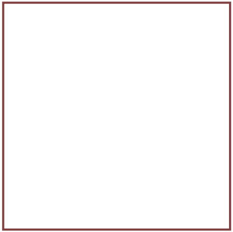

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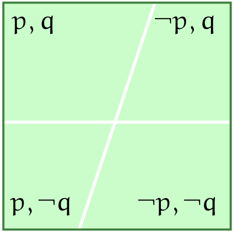

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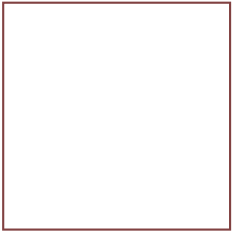

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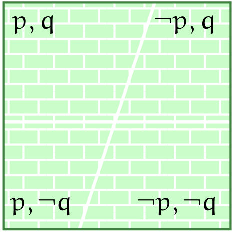

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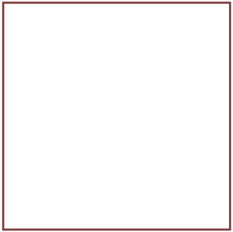

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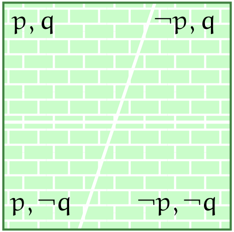

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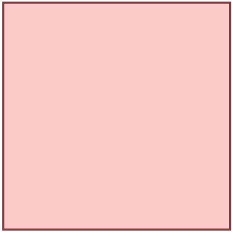

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PROOF INVARIANTS

When is π_1 the same proof as π_2?

$$\dfrac{\dfrac{p \land q}{p}\land E}{p \lor q}\lor I \qquad\qquad \dfrac{\dfrac{p \succ p}{p \succ p \lor q}\lor R}{p \land q \succ p \lor q}\land L$$

When is π_1 the same proof as π_2?

$$\dfrac{\dfrac{p \wedge q}{p}\wedge E}{p \vee q}\vee I$$

$$\dfrac{\dfrac{p \succ p}{p \succ p \vee q}\vee R}{p \wedge q \succ p \vee q}\wedge L$$

When is π_1 the same proof as π_2?

$$\cfrac{\cfrac{p \land q}{p}\,{\land}E}{p \lor q}\,{\lor}I$$

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When is π_1 the same proof as π_2?

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$$\cfrac{\cfrac{p \land q}{q}\,{\land E}}{p \lor q}\,{\lor I} \qquad \cfrac{\cfrac{q \succ q}{q \succ p \lor q}\,{\lor R}}{p \land q \succ p \lor q}\,{\land L}$$

When is π_1 the same proof as π_2?

$$\cfrac{\cfrac{p \wedge q}{p}\,{\wedge}E}{p \vee q}\,{\vee}I \qquad\qquad \cfrac{\cfrac{p \succ p}{p \succ p \vee q}\,{\vee}R}{p \wedge q \succ p \vee q}\,{\wedge}L$$

$$\cfrac{\cfrac{p \wedge q}{q}\,{\wedge}E}{p \vee q}\,{\vee}I \qquad\qquad \cfrac{\cfrac{q \succ q}{q \succ p \vee q}\,{\vee}R}{p \wedge q \succ p \vee q}\,{\wedge}L$$

When is π_1 the same proof as π_2?

$$\dfrac{\dfrac{p \land q}{p}\,{\land}E}{p \lor q}\,{\lor}I \qquad\qquad \dfrac{\dfrac{p \succ p}{p \succ p \lor q}\,{\lor}R}{p \land q \succ p \lor q}\,{\land}L$$

$$\dfrac{\dfrac{p \land q}{q}\,{\land}E}{p \lor q}\,{\lor}I \qquad\qquad \dfrac{\dfrac{q \succ q}{q \succ p \lor q}\,{\lor}R}{p \land q \succ p \lor q}\,{\land}L$$

When is π_1 the same proof as π_2?

$$\dfrac{\dfrac{p \land q}{p}\,{\land}E}{p \lor q}\,{\lor}I \qquad\qquad \dfrac{\dfrac{p \succ p}{p \succ p \lor q}\,{\lor}R}{p \land q \succ p \lor q}\,{\land}L$$

$$\dfrac{\dfrac{p \land q}{q}\,{\land}E}{p \lor q}\,{\lor}I \qquad\qquad \dfrac{\dfrac{q \succ q}{q \succ p \lor q}\,{\lor}R}{p \land q \succ p \lor q}\,{\land}L$$

SLOGAN 1: A proof of C from P *shows how* C can obtain in P-circumstances.

SLOGAN 2: Different *proof invariants* give different truth conditions.

$$\dfrac{\dfrac{p \land q}{p}\,{\land}E}{p \lor q}\,{\lor}I \qquad\qquad \dfrac{\dfrac{p \land q}{q}\,{\land}E}{p \lor q}\,{\lor}I$$

Case 1: $p \lor \neg p$

$$\dfrac{p \succ p}{p \succ p \lor \neg p}\,{\lor}R$$

$$\dfrac{\dfrac{p \succ p}{\succ p, \neg p}\,{\neg}R}{\succ p \lor \neg p}\,{\lor}R$$

$$\dfrac{\neg p \succ \neg p}{\neg p \succ p \lor \neg p}\,{\lor}R$$

Case 1: $p \lor \neg p$

$$\dfrac{p \succ p}{p \succ p \lor \neg p}\,\lor R$$

$$\dfrac{\dfrac{p \succ p}{\succ p, \neg p}\,\neg R}{\succ p \lor \neg p}\,\lor R$$

$$\dfrac{\neg p \succ \neg p}{\neg p \succ p \lor \neg p}\,\lor R$$

Holds *when* p

Case 1: $p \lor \neg p$

$$\dfrac{p \succ p}{p \succ p \lor \neg p}\,{\lor}R$$

Holds *when* p

$$\dfrac{\dfrac{p \succ p}{\succ p, \neg p}\,\neg R}{\succ p \lor \neg p}\,{\lor}R$$

$$\dfrac{\neg p \succ \neg p}{\neg p \succ p \lor \neg p}\,{\lor}R$$

Holds *when* $\neg p$

Case 1: $p \lor \neg p$

$$\dfrac{p \succ p}{p \succ p \lor \neg p}\,{\lor}R$$

Holds *when* p

$$\dfrac{\dfrac{p \succ p}{\succ p, \neg p}\,{\neg}R}{\succ p \lor \neg p}\,{\lor}R$$

Holds *independently*

$$\dfrac{\neg p \succ \neg p}{\neg p \succ p \lor \neg p}\,{\lor}R$$

Holds *when* $\neg p$

Case 2: $p \lor (p \land q)$

$$\dfrac{p, q \succ p}{p, q \succ p \lor (p \land q)}\,\lor R$$

$$\dfrac{\dfrac{p \succ p \qquad q \succ q}{p, q \succ p \land q}\,\land R}{p, q \succ p \lor (p \land q)}\,\lor R$$

$$\dfrac{p\,,\, q \succ p}{p\,,\, q \succ p \vee (p \wedge q)}\;\vee R$$

$$\dfrac{\dfrac{p \succ p \qquad q \succ q}{p\,,\, q \succ p \wedge q}\;\wedge R}{p\,,\, q \succ p \vee (p \wedge q)}\;\vee R$$

Holds *when* p

Case 2: $p \lor (p \land q)$

$$\dfrac{p, q \succ p}{p, q \succ p \lor (p \land q)}\ \lor R$$

$$\dfrac{\dfrac{p \succ p \qquad q \succ q}{p, q \succ p \land q}\ \land R}{p, q \succ p \lor (p \land q)}\ \lor R$$

Holds *when* p

Case 2: $p \vee (p \wedge q)$

$$\dfrac{p, q \succ p}{p, q \succ p \vee (p \wedge q)} \vee R$$

Holds *when* p

$$\dfrac{\dfrac{p \succ p \qquad q \succ q}{p, q \succ p \wedge q} \wedge R}{p, q \succ p \vee (p \wedge q)} \vee R$$

Case 2: $p \vee (p \wedge q)$

$$\dfrac{p, q \succ p}{p, q \succ p \vee (p \wedge q)}\ \vee R$$

Holds *when* p

$$\dfrac{\dfrac{p \succ p \qquad q \succ q}{p, q \succ p \wedge q}\ \wedge R}{p, q \succ p \vee (p \wedge q)}\ \vee R$$

Holds *when* p *and* q

Case 3a: $(p \lor \neg p) \lor (q \lor \neg q)$

$$\cfrac{\cfrac{p \succ p}{p \succ p \lor \neg p}\ {\lor}R}{p \succ (p \lor \neg p) \lor (q \lor \neg q)}\ {\lor}R \qquad \cfrac{\cfrac{\neg p \succ \neg p}{\neg p \succ p \lor \neg p}\ {\lor}R}{\neg p \succ (p \lor \neg p) \lor (q \lor \neg q)}\ {\lor}R$$

$$\cfrac{\cfrac{\cfrac{p \succ p}{\succ p, \neg p}\ \neg R}{\succ p \lor \neg p}\ {\lor}R}{\succ (p \lor \neg p) \lor (q \lor \neg q)}\ {\lor}R$$

$$\dfrac{\dfrac{p \succ p}{p \succ p \lor \neg p}\,{\lor R}}{p \succ (p \lor \neg p) \lor (q \lor \neg q)}\,{\lor R} \qquad \dfrac{\dfrac{\neg p \succ \neg p}{\neg p \succ p \lor \neg p}\,{\lor R}}{\neg p \succ (p \lor \neg p) \lor (q \lor \neg q)}\,{\lor R}$$

$$\dfrac{\dfrac{\dfrac{p \succ p}{\succ p, \neg p}\,{\neg R}}{\succ p \lor \neg p}\,{\lor R}}{\succ (p \lor \neg p) \lor (q \lor \neg q)}\,{\lor R}$$

These conditions *don't involve* q.

Case 3a: $(p \lor \neg p) \lor (q \lor \neg q)$

$$\dfrac{\dfrac{q \succ q}{q \succ q \lor \neg q}\ {\lor R}}{q \succ (p \lor \neg p) \lor (q \lor \neg q)}\ {\lor R} \qquad \dfrac{\dfrac{\neg q \succ \neg q}{\neg q \succ q \lor \neg q}\ {\lor R}}{\neg q \succ (p \lor \neg p) \lor (q \lor \neg q)}\ {\lor R}$$

$$\dfrac{\dfrac{\dfrac{q \succ q}{\succ q, \neg q}\ {\neg R}}{\succ q \lor \neg q}\ {\lor R}}{\succ (p \lor \neg p) \lor (q \lor \neg q)}\ {\lor R}$$

Case 3a: $(p \vee \neg p) \vee (q \vee \neg q)$

$$\dfrac{\dfrac{q \succ q}{q \succ q \vee \neg q}\,{\vee}R}{q \succ (p \vee \neg p) \vee (q \vee \neg q)}\,{\vee}R \qquad \dfrac{\dfrac{\neg q \succ \neg q}{\neg q \succ q \vee \neg q}\,{\vee}R}{\neg q \succ (p \vee \neg p) \vee (q \vee \neg q)}\,{\vee}R$$

$$\dfrac{\dfrac{\dfrac{q \succ q}{\succ q, \neg q}\,\neg R}{\succ q \vee \neg q}\,{\vee}R}{\succ (p \vee \neg p) \vee (q \vee \neg q)}\,{\vee}R$$

These conditions *don't involve* p.

Case 3b: $(p \lor \neg p) \land (q \lor \neg q)$

$$\dfrac{\dfrac{q \succ q}{q \succ q \lor \neg q}\,{\lor}R \qquad \dfrac{\neg p \succ \neg p}{\neg p \succ p \lor \neg p}\,{\lor}R}{\neg p, q \succ (p \lor \neg p) \land (q \lor \neg q)}\,{\land}R \qquad\qquad \dfrac{\dfrac{\dfrac{p \succ p}{\succ p, \neg p}\,{\neg}R}{\succ p \lor \neg p}\,{\lor}R \qquad \dfrac{\dfrac{q \succ q}{\succ q, \neg q}\,{\neg}R}{\succ q \lor \neg q}\,{\lor}R}{\succ (p \lor \neg p) \land (q \lor \neg q)}\,{\land}R$$

These conditions *always involve* both p and q.


WHERE WE'VE GOT, AND WHERE TO FROM HERE

Scorecard

	SUBJECT MATTER	SITUATIONS	INVARIANTS
$p \vee \neg p$ / $\top$	*different*	*different*	*different*
$p \vee (p \wedge q)$ / p	*different*	*same*	*different*
$(p \vee \neg p) \vee (q \vee \neg q)$ / $(p \vee \neg p) \wedge (q \vee \neg q)$	*same*	*different*	*different*

Places to Go

- In what can proofs and proof invariants do interesting work in articulating *subject matter*?
- Can invariants be used to define a notion of *logical subtraction*? (See Stephen Yablo's *Aboutness* for details.)
- Extend proof invariants beyond propositional logic.

THANK YOU!

http://consequently.org/presentation/2017/proof-identity-invariants-and-hyperintensionality

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