

WORLDS: POSSIBLE & IMPOSSIBLE

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L&M Seminar
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§1 What are worlds  for?

§2 Start with worlds,
or somewhere else?

§3 Kinds of impossibility

§4 Logically deviant  worlds?

§1 What are worlds for?

* formal Semantics – Model Theory

{1} What are worlds for?

* Formal Semantics – Model Theory

... which can help us combine languages & logics
(interpret L_1 & L_2 on the same set of worlds)

{1} What are worlds for?

* Formal Semantics – Model Theory

- * ... which can help us combine languages & logics
- * ... ? which ~~can~~ aid with understanding & intuition

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- * formal Semantics – Model Theory
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- * ... ↗ which can aid with understanding & intuition
- * This might provide constraints on the design of a logic

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- * ... ? which can aid with understanding & intuition
- * This might provide constraints on the design of a logic
- * ... especially if we have some independent handle on what worlds ‘there are’

{1} What are worlds for?

* Formal Semantics – Model Theory

- * ... which can help us combine languages & logics
- * ... ↗ which can aid with understanding & intuition
- * This might provide constraints on the design of a logic
- * ... especially if we have some independent handle on what worlds ‘there are’
- * Which leads to a great deal of metaphysical Speculation.

{1} What are worlds for?

§2 Start with worlds, or somewhere else?

... possible worlds, in the sense of possible states of affairs are not *really* individuals (just as numbers are not *really* individuals).

To say that a state of affairs obtains is just to say that something is the case; to say that something is a possible state of affairs is just to say that something could be the case; and to say that something is the case 'in' a possible state of affairs is just to say that the thing in question would necessarily be the case if that state of affairs obtained, i.e. if something else were the case ...

We understand 'truth in states of affairs' because we understand 'necessarily'; not *vice versa*.

— "Worlds, Times and Selves"
(1969)



The fundamental connection:

It is necessary that p if

in every possible
world we have p

§2 Start with worlds, or somewhere else?

This is reminiscent of

It is true that p if

there is some fact that
makes p true

§2 Start with worlds, or somewhere else?

It is true that p if
Order of
priority?

there is some fact that
makes p true

§2 Start with worlds, or somewhere else?

It is true that p if

iff

p ↗ there is some fact that makes p true

Order of
priority?

§2 Start with worlds, or somewhere else?

The fundamental connection:

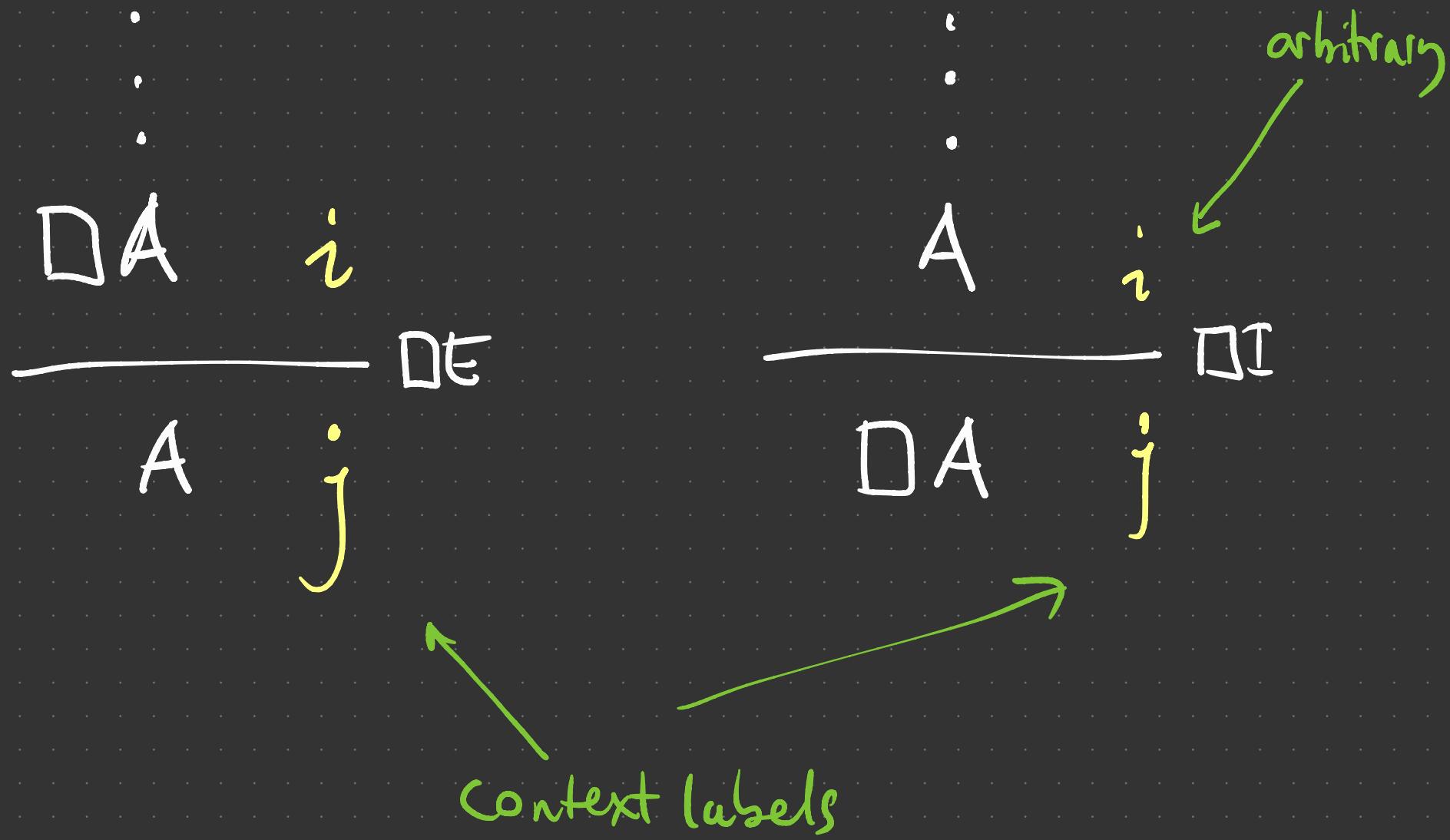
It is necessary that p if

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???

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§2 Start with worlds, or somewhere else?



§2 Start with worlds, or somewhere else?

EXAMPLE PROOF

$$\frac{\frac{D(p \wedge Dq) \ 1}{Dp \ 1} \quad \frac{D(p \wedge \Box q) \ 1}{Dq \ 1}}{\frac{p \ 2 \qquad \qquad \qquad q \ 2}{p \wedge q \ 2}} \quad \frac{}{D(p \wedge q) \ 1}$$

Here we transit
through a different
context in the
proof

§2 Start with worlds, or somewhere else?

These are rules for use, ...



... they coordinate \square judgements with context shifts in reasoning.

§2 Start with worlds, or somewhere else?

What is a world?

P j

asserting P under the scope of
some alternate Supposition
(marked with j))

taking P to hold in some
alternate Circumstance j.

§2 Start with worlds, or somewhere else?

Why are worlds complete?

$$\frac{\neg(\neg(p \vee \neg p))_2 \quad [p]_2 \supseteq}{p \vee \neg p}_2 \quad \text{VI}$$

$$\frac{\perp}{\neg I^1}$$

$$\frac{\neg p_2}{\neg I}$$

$$\frac{\neg(\neg(p \vee \neg p))_2 \quad p \vee \neg p_2}{\perp}_2 \quad \text{IE}$$

$$\frac{\perp}{\neg I^2}$$

$$\frac{\neg\neg(p \vee \neg p)_2}{\text{DNE}}$$

$$p \vee \neg p_2$$

§2 Start with worlds, or somewhere else?

Why are worlds complete?

$$\frac{[\neg(p \vee \neg p)]_2 \quad [p]^\dagger \vdash}{p \vee \neg p \vdash} \neg E$$

$$\frac{\perp}{\neg I^\dagger}$$

$$\frac{\neg p \vdash}{\neg I}$$

$$\frac{[\neg(p \vee \neg p)]_2 \quad p \vee \neg p \vdash}{\perp} \neg E$$

$$\frac{\perp}{\neg I^2}$$

$$\frac{\neg \neg(p \vee \neg p) \vdash}{DNE}$$

$$p \vee \neg p \vdash$$

This reasoning applies
in any
context if our
logical rules are
context agnostic

§2 Start with worlds, or somewhere else?

KINDS OF CONTEXT SHIFT

Subjunctive : Suppose Oswald hadn't shot JFK

Indicative : Suppose Oswald didn't shoot JFK

§2 Start with worlds, or somewhere else?

KINDS OF CONTEXT SHIFT

'METAPHYSICAL' necessity: \Box_M

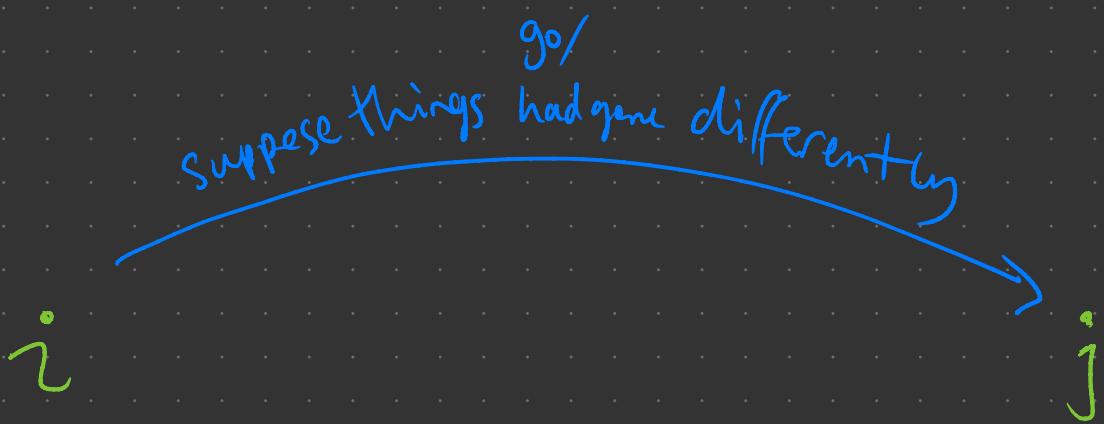
Subjunctive: Suppose Oswald hadn't shot JFK

Indicative: Suppose Oswald didn't shoot JFK

'Epistemic' necessity: \Box_E

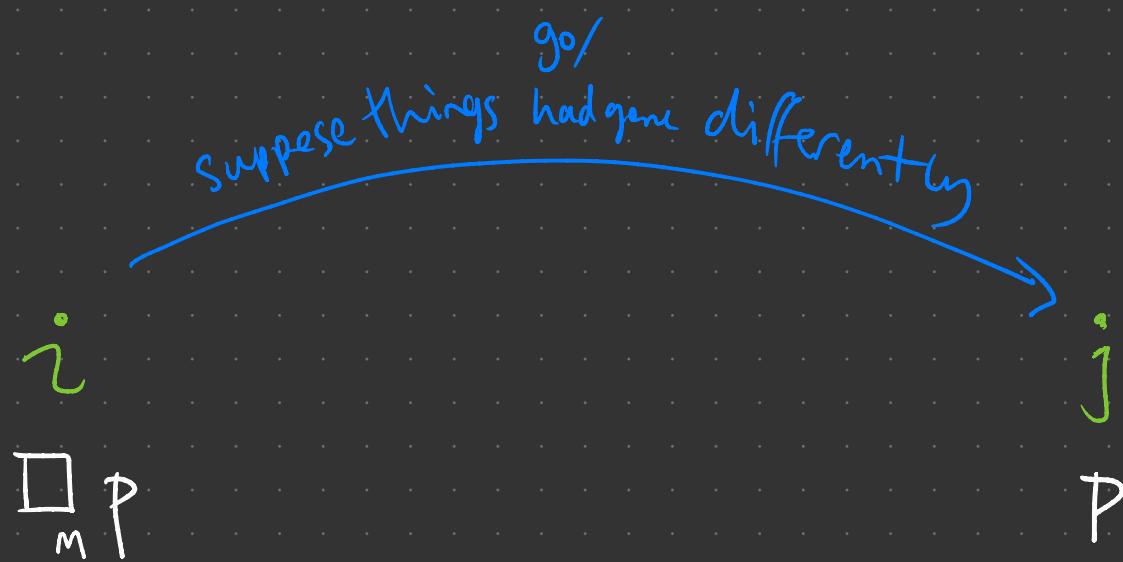
§2 Start with worlds, or somewhere else?

SUBJUNCTIVE



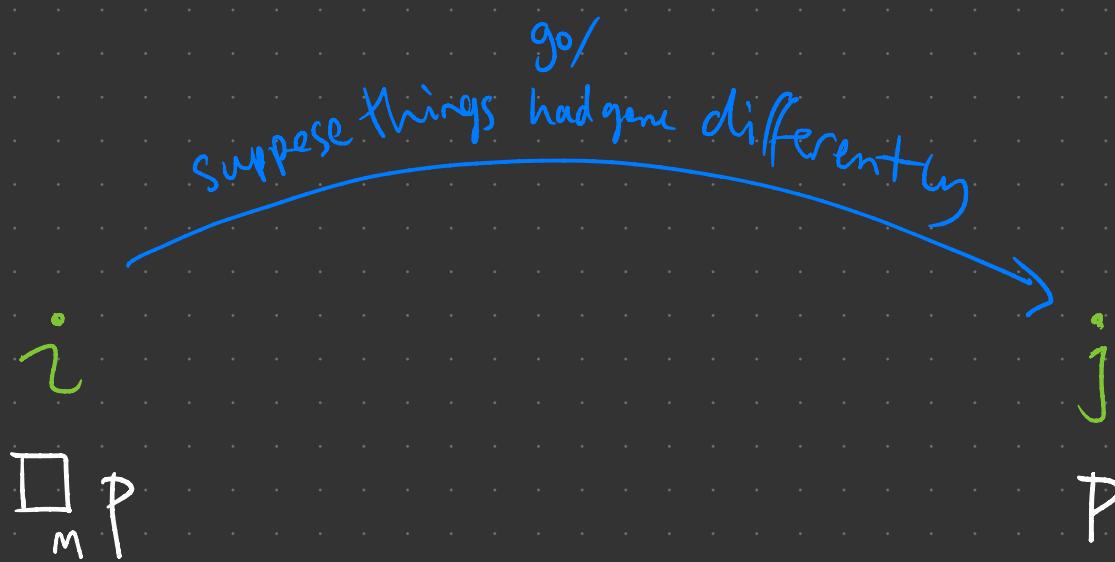
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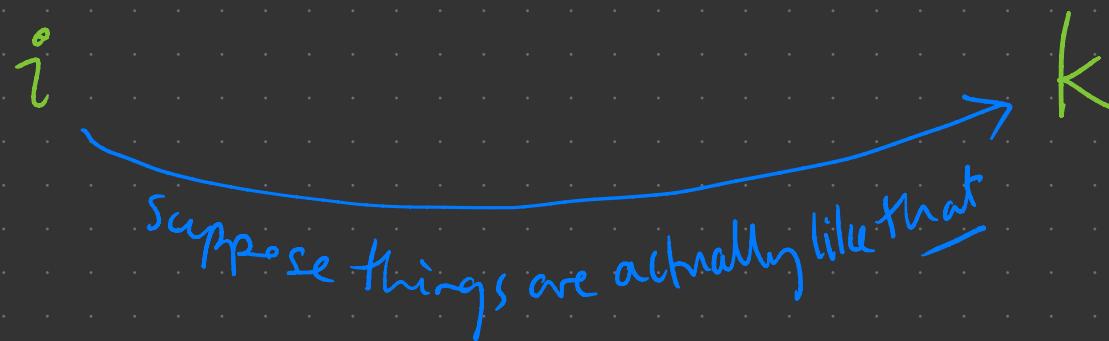


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SUBJUNCTIVE

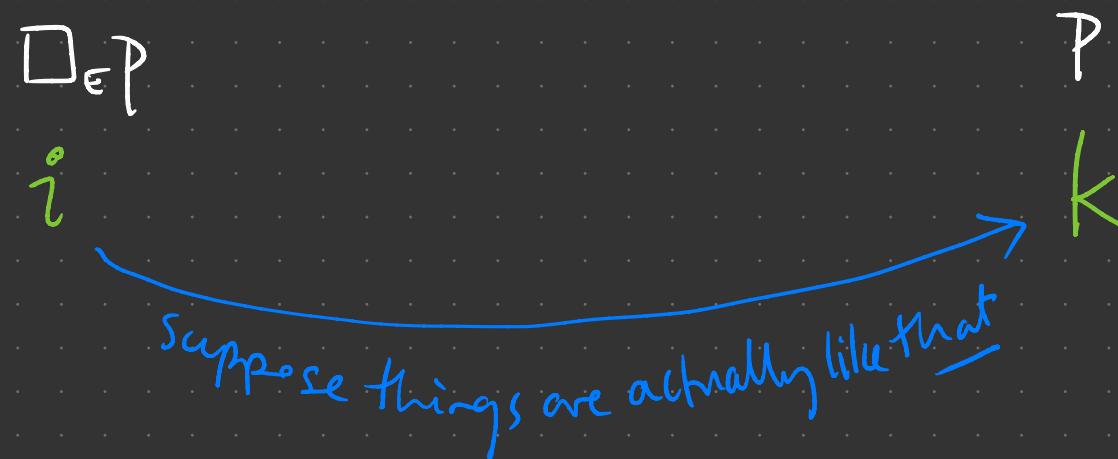
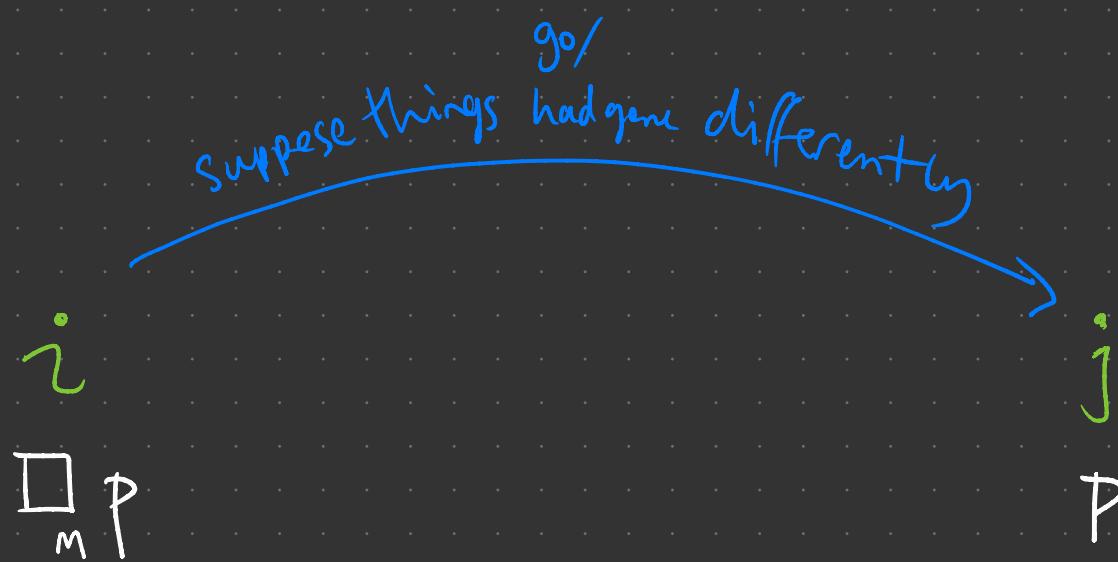


INDICATIVE



§2 Start with worlds, or somewhere else?

SUBJUNCTIVE



INDICATIVE

§2 Start with worlds, or somewhere else?

NOT MY QUESTIONS (HERE).

- What sort of objects are these worlds?
- What is the relationship between worlds & what exists^(..) in them?
- What logic(s) is(are) appropriate for non-modal
 $(\wedge, \vee, \neg, \rightarrow)$ proofs/models in general?

§2 Start with worlds, or somewhere else?

§3 Kinds of impossibility

Two PLASIBLE CLAIMS

1. What is necessary is necessarily so.
2. We can coherently disagree about what is necessary — & we do not know all necessary truths a priori.

1. What is necessary is necessarily so.

$$\frac{\Box_m P \ 1}{P \ 2} \Big) _m$$
$$\frac{\Box_m P \ 3}{\Box_m \Box_m P \ 1} \Big) _m$$

1. What is necessary is necessarily so.

$$\frac{\Box_m P \ 1}{P \ 2} \Big) _m^m$$
$$\frac{\Box_m P \ 3}{\Box_m \Box_m P \ 1}$$

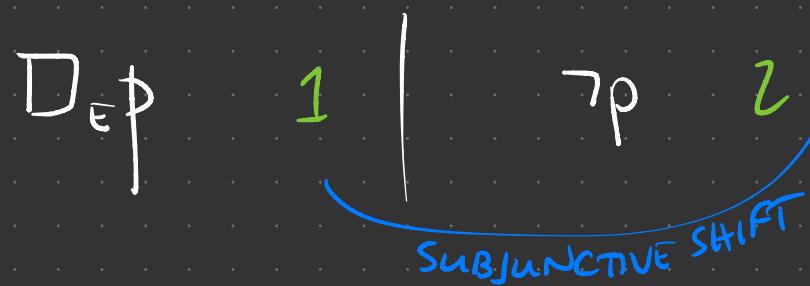
$$\frac{\Box_E P \ 1}{P \ 2} \Big) _E^E$$
$$\frac{\Box_E P \ 3}{\Box_E \Box_E P \ 1} \Big) _E^E$$

2. We can coherently disagree about necessities

- I grant $\Box_m P$, but suppose I'm wrong?



- I grant $\Box_e P$, but this could have been false.



§3 Kinds of impossibility

THE UPSHOT FOR WORLDS

- Some Epistemically Possible worlds
are metaphysically impossible.
- Some Metaphysically Possible worlds
are epistemically impossible.

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Hesperus ≠ Phosphorus ; Act_i, \neg Act_i ; D_mF_T, \neg D_mF_T
- Some Metaphysically Possible worlds
are epistemically impossible.

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are metaphysically impossible.
 $Hesperus \neq Phosphorus ; ACH, \neg ACT ; D_m^{\neg FR}, \neg D_m^{\neg FR}$

- Some Metaphysically Possible worlds
are epistemically impossible.

$P \wedge \neg CP$; perhaps other notions of indexicality

THE UPSHOT

It is utterly problematic
that there are impossible worlds

THE UPSHOT

It is utterly problematic
that there are impossible worlds
(if there are any worlds at all).

§4 Logically deviant worlds?

logically deviant

What about inconsistent worlds?

{4 logically ~~deviant~~ worlds?}

logically deviant

What about inconsistent worlds?

- restrictions (situations, Scenarios, etc.)
- expansions (gluts, clashes, etc.)

{4 logically ~~deviant~~ worlds?}

$$\frac{\frac{[\neg(p \vee \neg p)]_2 \quad [p]'_2}{p \vee \neg p \in \Gamma} \vee I}{\neg E}$$

$$\frac{\perp}{\neg p \in \Gamma} \neg I$$

$$\frac{[\neg(p \vee \neg p)]_2 \quad p \vee \neg p \in \Gamma}{\neg E}$$

$$\frac{\perp}{\neg\neg(p \vee \neg p) \in \Gamma^2} \neg\neg E$$

• Are inference rules
context agnostic?

{4 logically deviant worlds?}

$$\frac{\frac{[\neg(p \vee \neg p)]_2 \quad [p]'_2}{p \vee \neg p _2} \vee I}{\neg E}$$

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- Are inference rules context agnostic?
— for some contexts, yes.

{4 Logically deviant worlds?}

$$\frac{\frac{[\neg(p \vee \neg p)]_2 \quad [P]'_2}{p \vee \neg p \in \Gamma} \vee I}{\neg E}$$

$$\frac{\perp}{\neg p \in \Gamma} \neg I$$

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- Are inference rules context agnostic?

— for some contexts, yes.

but for others?

{4 logically deviant worlds?}

$$\frac{\frac{[\neg(p \vee \neg p)]_2 \quad [P]'_2}{p \vee \neg p \in \Gamma} \vee I}{\neg E}$$

$$\frac{\perp}{\neg p \in \Gamma} \neg I$$

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$$\frac{\perp}{\neg\neg(p \vee \neg p) \in \Gamma^2} \neg\neg E$$

$$\frac{\neg\neg(p \vee \neg p) \in \Gamma^2}{p \vee \neg p \in \Gamma} DNE$$

• Are inference rules context agnostic?

— for some contexts, yes.

but for others?

~ keeping track of topic, fine-grained epistemic considerations, etc.

{4 Logically deviant worlds?}

$$\frac{P_i \quad Q_i}{P \wedge Q_i} \wedge I$$

$$\frac{P \wedge Q_i}{P_i} \wedge E$$

$$\frac{P \wedge Q_i}{Q_i} \wedge E$$

$$\frac{P_i}{P \vee Q_i} \vee I$$

$$\frac{Q_i}{P \vee Q_i} \vee I$$

$$\frac{\begin{array}{c} P \vee Q_i \\ \vdots \end{array}}{\begin{array}{c} C \\ \vdots \\ C \end{array}} \vee E$$

{4 logically deviant worlds?}

$$\frac{P \rightarrow q^i \quad P^i}{q^i} \rightarrow E$$

$$\frac{[P]^i \quad \vdots \quad q^i}{P \rightarrow q^i} \rightarrow I$$

{4 logically deviant worlds?}

$$\frac{\neg A_i \quad A_i}{\perp} \neg E$$

$$\frac{\vdash \perp}{\vdash A_i} \neg I$$

{4 logically deviant worlds?}

OPTIONS

- 1 Keep some context rules & restrict or modify others.
 - for topics, situations, etc.

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 - for topics, situations, etc.

eg

$$\frac{\neg A^i \quad A^j}{i \perp j} \neg E$$

$$\frac{[A]^i_j \quad i \perp j}{\neg I^i} \neg I$$

$$\neg A^i$$

{4 logically deviant worlds?}

OPTIONS

- 1 Keep some context rules & restrict or modify others.
 - for topics, situations, etc.
2. Restrict all the rules, for really fine-grained distinctions.

{4 Logically deviant worlds?

WHERE THIS LEADS

1. We have a uniform treatment of different kinds of context
— and so, different kinds of 'world'.

{4 Logically deviant worlds?}

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1. We have a uniform treatment of different kinds of context
 - and so, different kinds of 'world'
2. Logically 'transparent' contexts (such as those given by subjunctive & indicative shifts) give rise to worlds
 - both possible & impossible worlds.

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WHERE THIS LEADS

1. We have a uniform treatment of different kinds of context
 - and so, different kinds of ‘world’
2. Logically ‘transparent’ contexts (such as those given by subjunctive & indicative shifts) give rise to worlds
 - both possible & impossible worlds.
3. Logically ‘opaque’ contexts can give rise to logically restricted or deviant ‘worlds’.

{4 Logically ~~deviant~~ worlds?}

THANK You!