

# Proof Identity, Invariants and Hyperintensionality

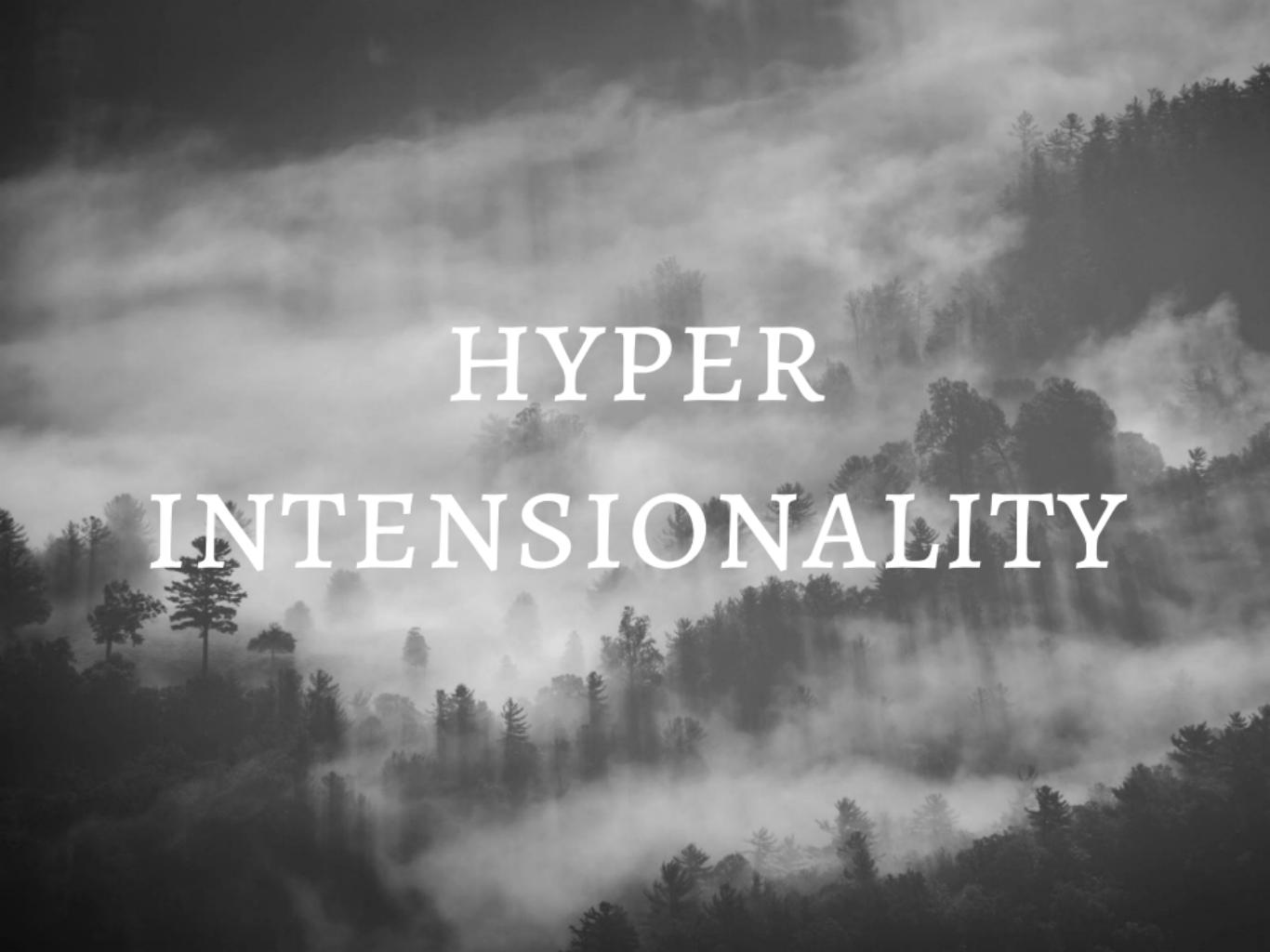
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THE UNIVERSITY OF  
MELBOURNE

DIP COLLOQUIUM · 7 MARCH 2017

Hyperintensionality  
Models and Proofs  
Truthmakers  
Subject Matter  
Proof Invariants  
Where we've got, and where to from here



# HYPER INTENSIONALITY

# Extensionality, Intensionality and Hyperintensionality

- ▶ EXTENSIONAL: agree on the same truth value or referent —  $\wedge, \vee, \neg, \forall, \exists, =$ .
- ▶ INTENSIONAL: agree on the same truth value or referent in every possible world —  $\Box, \Diamond, F, P, \rightarrow$ .
- ▶ HYPERINTENSIONAL: more finely grained still. *proves that, makes true, because.*
  - Distinguish between logically equivalent propositions.  $p \vee \neg p$  and  $\top$ ;  $p \vee (p \wedge q)$  and  $p$ , etc.
- ▶ I want to explore the difference between some *model theoretic* and *proof theoretic* approaches to hyperintensionality.

The background image shows a vast mountain range with deep green forests covering the lower slopes and rocky peaks rising in the distance. A winding river or road cuts through the valley floor. The sky is a clear blue with scattered white and grey clouds.

# MODELS AND PROOFS

# Two Traditions in Logic and Semantics

**MODEL THEORY** (1) Define *models* and give recursive conditions explaining when a claim is true in a model.

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**PROOF THEORY** (1) Define *proof rules* giving recursive conditions explaining what follows from what. (2) Define *consequence* as the existence of *some* proof from premises to conclusion. (3) An *invalid* argument is one that has *no* proof.

# Two More Tools for Understanding Aboutness

MODEL THEORY

*Situations as Truthmakers*

PROOF THEORY

*Proof Invariants*

# TRUTHMAKERS

Case 1:  $p \vee \neg p$

$p \vee \neg p$

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$$p \vee \neg p$$

$s \Vdash p \vee \neg p$  iff  $s \Vdash p$  or  $s \Vdash \neg p$

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$$p \vee \neg p$$

$s \Vdash p \vee \neg p$  iff  $s \Vdash p$  or  $s \Vdash \neg p$

Many situations are silent on whether  $p$  or not.

(*Making  $p \vee \neg p$  true* is different from *making  $q \vee \neg q$  true*.)

Case 2:  $p \vee (p \wedge q)$

$$p \vee (p \wedge q)$$

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$$p \vee (p \wedge q)$$

$s \Vdash p \vee (p \wedge q)$  iff  $s \Vdash p$  or  $s \Vdash p \wedge q$ ,  
iff  $s \Vdash p$  or ( $s \Vdash p$  and  $s \Vdash q$ ),  
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iff  $s \Vdash p$  or ( $s \Vdash p$  and  $s \Vdash q$ ),  
iff  $s \Vdash p$ .

Making  $p \vee (p \wedge q)$  true just is making  $p$  true.

Case 3:  $(p \vee \neg p) \vee (q \vee \neg q)$  and  $(p \vee \neg p) \wedge (q \vee \neg q)$

$$(p \vee \neg p) \vee (q \vee \neg q)$$

Either make  $p$  true, or make  $p$  false,  
or make  $q$  true, or make  $q$  false.

Case 3:  $(p \vee \neg p) \vee (q \vee \neg q)$  and  $(p \vee \neg p) \wedge (q \vee \neg q)$

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$$(p \vee \neg p) \wedge (q \vee \neg q)$$

Either make  $p$  and  $q$  true, or make  $p$  true and  $q$  false,  
or make  $p$  false and  $q$  true, or make  $p$  and  $q$  false.

A photograph showing a vast, dense grid of green stadium seats, likely made of plastic or metal, arranged in rows. The perspective is from a low angle, looking up at the rows of seats that recede into the distance. The seats are a uniform green color with dark green armrests and backrests. Some small red objects, possibly flags or markers, are visible on top of some of the seats.

SUBJECT MATTER

## Positive Subject Matter

“The [positive] subject matter of  $S$   
is the relation  $m$  such that  
worlds are  $m$ -dissimilar iff  
 $S$  is differently true in them.”

## Positive Subject Matter

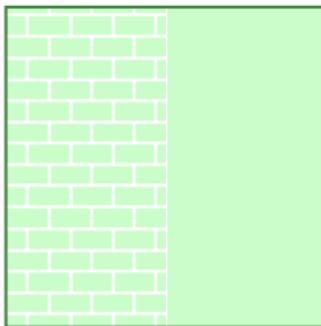
“The [positive] subject matter of  $S$   
is the relation  $m$  such that  
worlds are  $m$ -dissimilar iff  
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This is: how  $S$  is true

# Picturing Positive Subject Matter



# Picturing Positive Subject Matter



## Negative Subject Matter

The negative subject matter of  $S$   
is the relation  $m'$  such that  
worlds are  $m'$ -dissimilar iff  
 $S$  is differently false in them.”

## Negative Subject Matter

The negative subject matter of  $S$   
is the relation  $m'$  such that  
worlds are  $m'$ -dissimilar iff  
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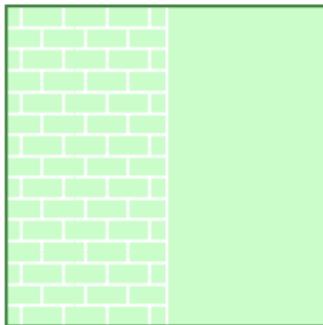
This is: how  $S$  is false

# The Subject Matter of S

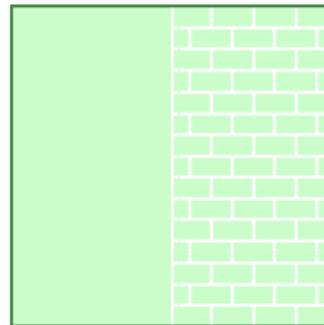
{how S is true, how S is false}

*Aboutness, Chapter 2*

# Picturing Subject Matter



how  $S$  is true



how  $S$  is false

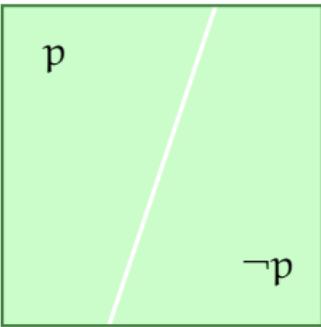
## The subject matter of $p \vee \neg p$



how  $p \vee \neg p$  is true

how  $p \vee \neg p$  is false

## The subject matter of $p \vee \neg p$

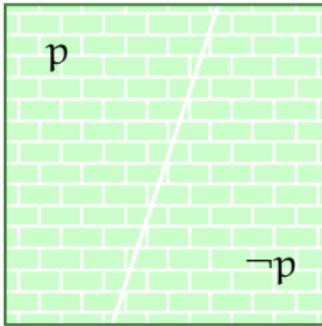


how  $p \vee \neg p$  is true

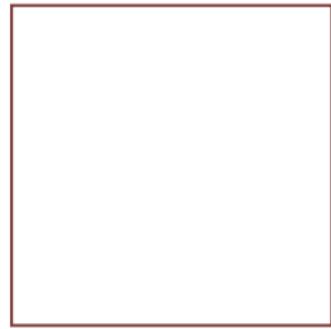


how  $p \vee \neg p$  is false

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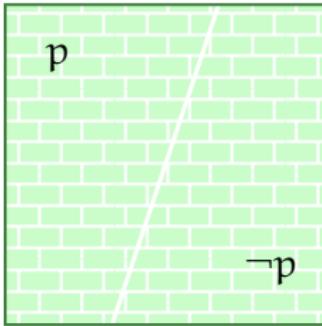


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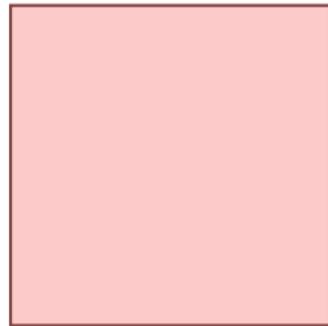


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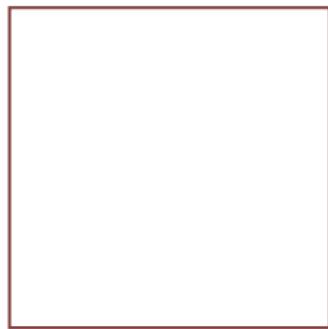
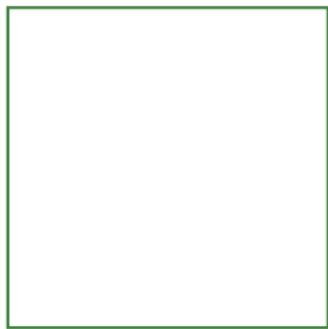


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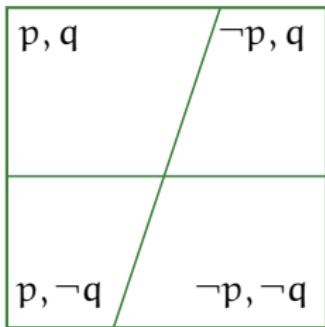
## The subject matter of $p \vee (p \wedge q)$



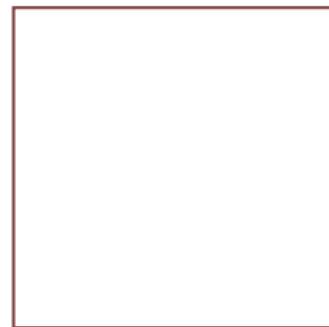
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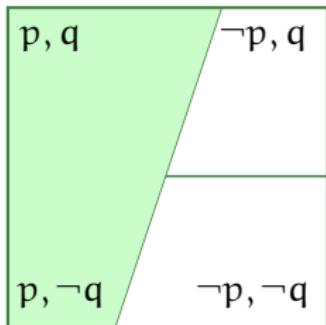


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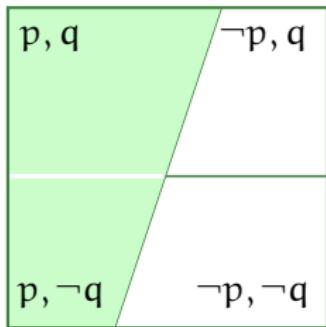


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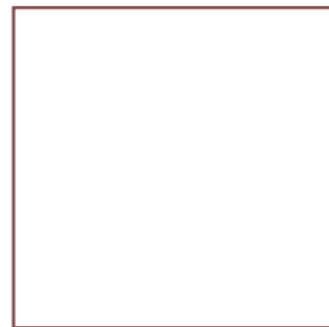


how  $p \vee (p \wedge q)$  is false

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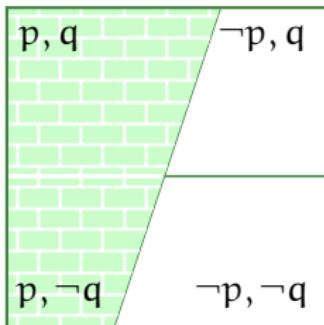


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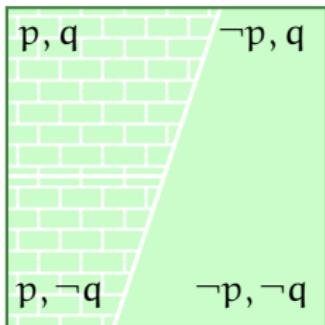


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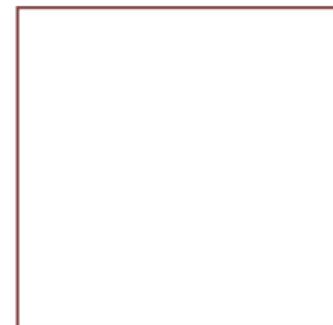


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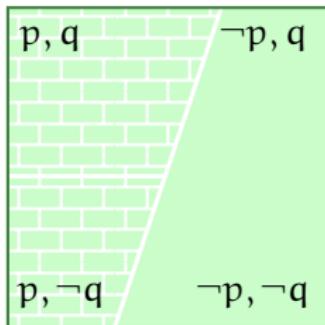


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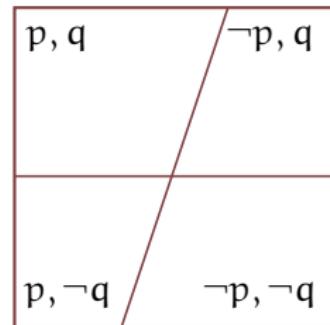


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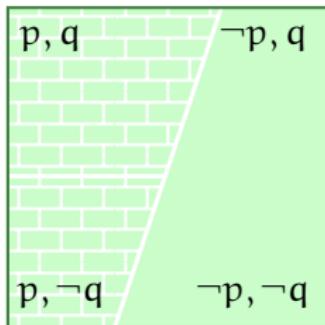


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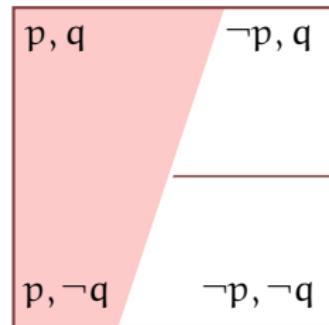


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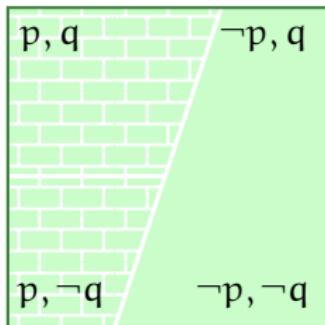


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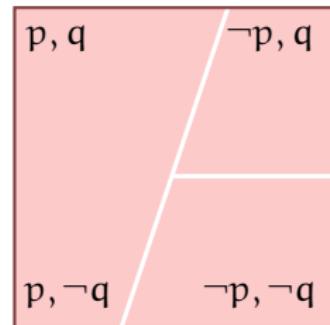


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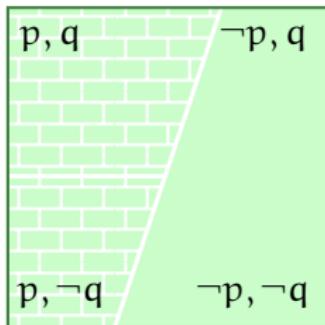


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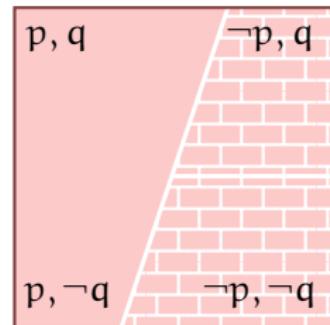


how  $p \vee (p \wedge q)$  is false

# The subject matter of $p \vee (p \wedge q)$



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how  $p \vee (p \wedge q)$  is false

$$(p \vee \neg p) \vee (q \vee \neg q)$$

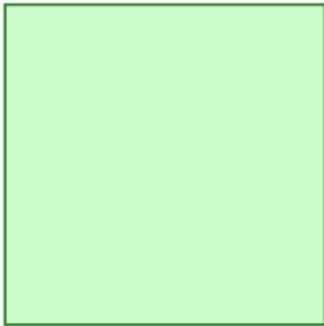


how  $(p \vee \neg p) \vee (q \vee \neg q)$  is true



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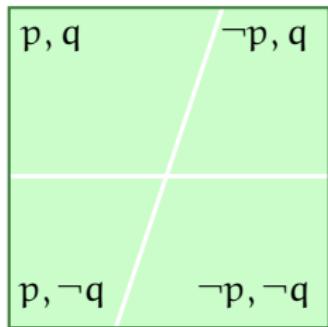


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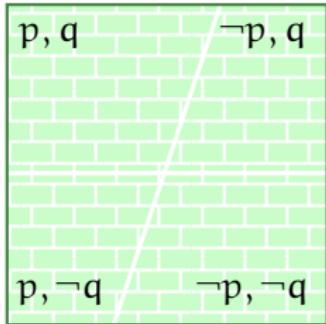


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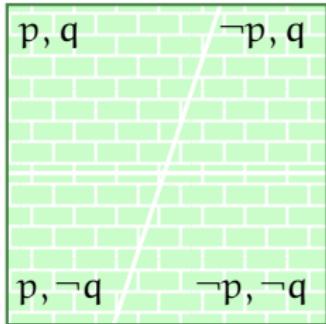


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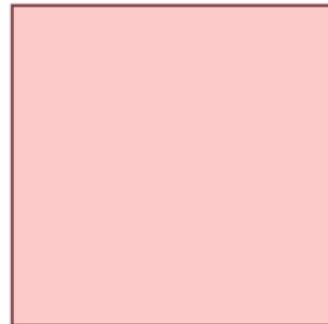


$$(p \vee \neg p) \vee (q \vee \neg q)$$

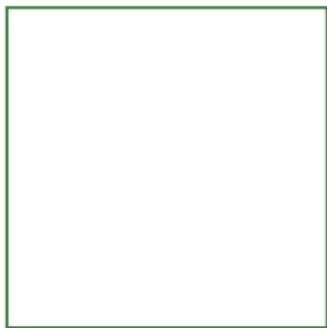


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$$(p \vee \neg p) \wedge (q \vee \neg q)$$



how  $(p \vee \neg p) \wedge (q \vee \neg q)$  is true



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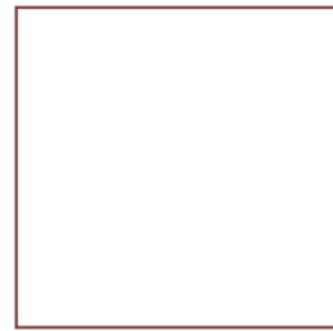
$$(p \vee \neg p) \wedge (q \vee \neg q)$$

$p, q$

$\neg p, q$

$p, \neg q$

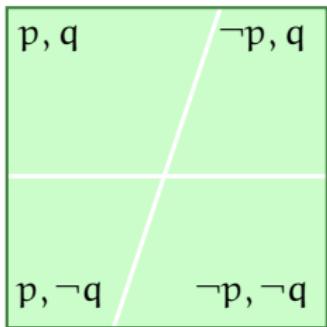
$\neg p, \neg q$



how  $(p \vee \neg p) \wedge (q \vee \neg q)$  is true

how  $(p \vee \neg p) \wedge (q \vee \neg q)$  is false

$$(p \vee \neg p) \wedge (q \vee \neg q)$$

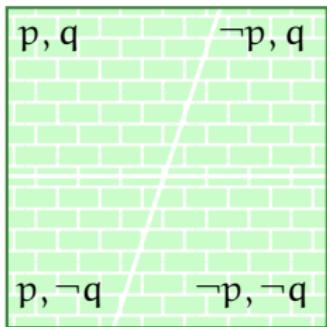


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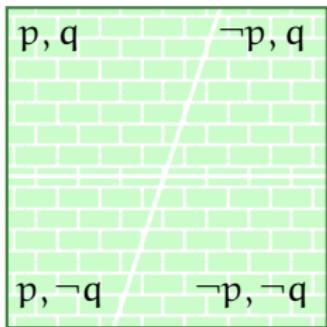


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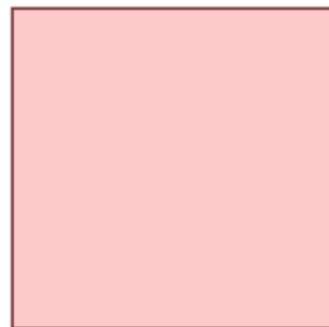


$$(p \vee \neg p) \wedge (q \vee \neg q)$$



how  $(p \vee \neg p) \wedge (q \vee \neg q)$  is true

how  $(p \vee \neg p) \wedge (q \vee \neg q)$  is false





# PROOF INVARIANTS

## When is $\pi_1$ the same proof as $\pi_2$ ?

$$\frac{p \wedge q}{\frac{p}{p \vee q}} \wedge E$$

$$\frac{p \succ p}{\frac{p \succ p \vee q}{p \wedge q \succ p \vee q}} \vee R$$

## When is $\pi_1$ the same proof as $\pi_2$ ?

$$\frac{\frac{p \wedge q}{p} \wedge E}{p \vee q} \vee I$$

$$\frac{p \succ p}{\frac{p \succ p \vee q}{p \wedge q \succ p \vee q}} \vee R \quad \wedge L$$

## When is $\pi_1$ the same proof as $\pi_2$ ?

$$\frac{p \wedge q}{\frac{p}{\frac{p \vee q}{p \wedge q}} \vee I} \wedge E$$

$$\frac{\frac{p \succ p}{p \succ p \vee q} \vee R}{p \wedge q \succ p \vee q} \wedge L$$

## When is $\pi_1$ the same proof as $\pi_2$ ?

$$\frac{\frac{p \wedge q}{p} \wedge E}{p \vee q} \vee I$$

$$\frac{\frac{p \succ p}{p \succ p \vee q} \vee R}{p \wedge q \succ p \vee q} \wedge L$$

## When is $\pi_1$ the same proof as $\pi_2$ ?

$$\frac{\begin{array}{c} p \wedge q \\ \hline p \end{array}}{p \vee q} \wedge E$$

$$\frac{\begin{array}{c} p \succ p \\ \hline p \succ p \vee q \end{array}}{p \wedge q \succ p \vee q} \wedge L$$

$$\frac{\begin{array}{c} p \wedge q \\ \hline q \end{array}}{p \vee q} \wedge E$$

$$\frac{\begin{array}{c} q \succ q \\ \hline q \succ p \vee q \end{array}}{p \wedge q \succ p \vee q} \wedge L$$

## When is $\pi_1$ the same proof as $\pi_2$ ?

$$\frac{\begin{array}{c} p \wedge q \\ \hline p \end{array}}{p \vee q} \wedge E$$

$$\frac{\begin{array}{c} p \succ p \\ \hline p \succ p \vee q \end{array}}{p \wedge q \succ p \vee q} \wedge L$$

$$\frac{\begin{array}{c} p \wedge q \\ \hline q \end{array}}{p \vee q} \wedge E$$

$$\frac{\begin{array}{c} q \succ q \\ \hline q \succ p \vee q \end{array}}{p \wedge q \succ p \vee q} \wedge L$$

## When is $\pi_1$ the same proof as $\pi_2$ ?

$$\frac{p \wedge q}{\frac{p}{p \vee q}} \wedge E$$

$$\frac{\frac{p \succ p}{p \succ p \vee q} \vee R}{p \wedge q \succ p \vee q} \wedge L$$

$$\frac{p \wedge q}{\frac{q}{p \vee q}} \wedge E$$

$$\frac{\frac{\frac{q \succ q}{q \succ p \vee q} \vee R}{p \wedge q \succ p \vee q} \wedge L}{p \wedge q \succ p \vee q} \wedge L$$

## When is $\pi_1$ the same proof as $\pi_2$ ?

$$\frac{p \wedge q}{\frac{p}{p \vee q} \vee I} \wedge E$$

$$\frac{\frac{p \succ p}{p \succ p \vee q} \vee R}{p \wedge q \succ p \vee q} \wedge L$$

$$\frac{p \wedge q}{\frac{q}{p \vee q} \vee I} \wedge E$$

$$\frac{\frac{q \succ q}{q \succ p \vee q} \vee R}{p \wedge q \succ p \vee q} \wedge L$$

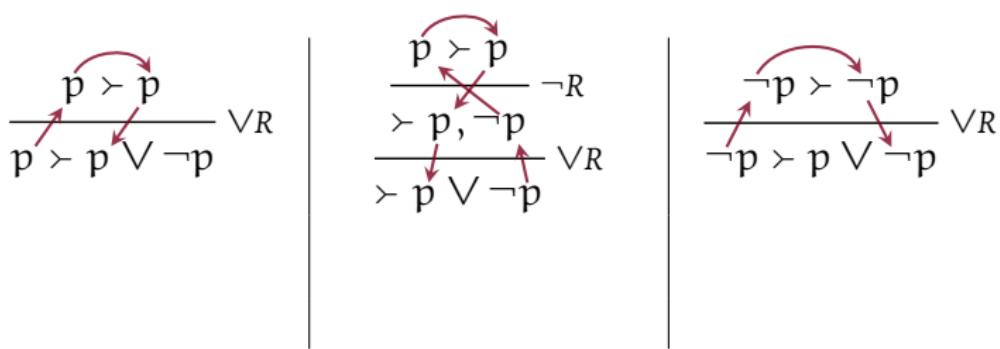
## *Proofs, proofinvariants and truth conditions*

SLOGAN 1: A proof of C from P shows how C can obtain in P-circumstances.

SLOGAN 2: Different *proofinvariants* give different truth conditions.

$$\frac{\frac{p \wedge q}{p} \wedge E}{p \vee q} \vee I \qquad \frac{\frac{p \wedge q}{q} \wedge E}{p \vee q} \vee I$$

## Case 1: $p \vee \neg p$



## Case 1: $p \vee \neg p$

$$\frac{p \succ p}{p \succ p \vee \neg p} \text{ VR}$$

$$\frac{\frac{\frac{p \succ p}{\cancel{\succ p, \neg p}} \neg R}{\succ p \vee \neg p} \text{ VR}}{\succ p \vee \neg p}$$

$$\frac{\frac{\frac{\neg p \succ \neg p}{\cancel{\neg p \succ p \vee \neg p}} \neg R}{\neg p \succ p \vee \neg p} \text{ VR}}{\neg p \succ p \vee \neg p}$$

Holds when  $p$

## Case 1: $p \vee \neg p$

$$\frac{p \succ p}{p \succ p \vee \neg p} \text{ VR}$$

Holds when  $p$

$$\frac{\begin{array}{c} p \succ p \\ \hline \succ p, \neg p \end{array}}{\succ p \vee \neg p} \text{ VR}$$

$$\frac{\neg p \succ \neg p}{\neg p \succ p \vee \neg p} \text{ VR}$$

Holds when  $\neg p$

## Case 1: $p \vee \neg p$

$$\frac{p \succ p}{p \succ p \vee \neg p} \text{VR}$$

Holds when  $p$

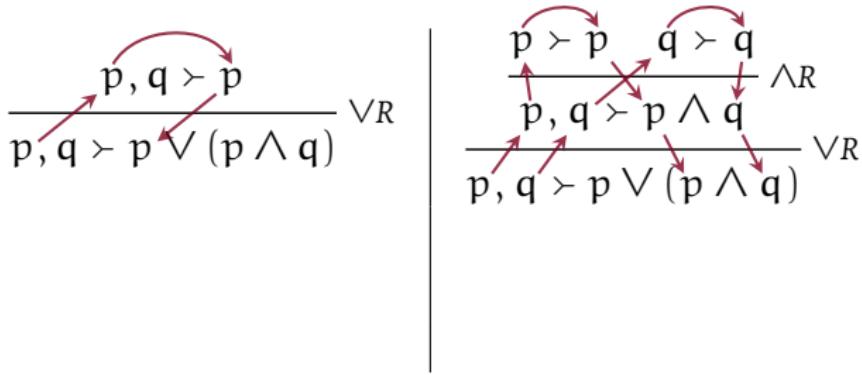
$$\frac{\begin{array}{c} p \succ p \\ \hline \succ p, \neg p \end{array}}{\succ p \vee \neg p} \text{VR}$$

Holds independently

$$\frac{\neg p \succ \neg p}{\neg p \succ p \vee \neg p} \text{VR}$$

Holds when  $\neg p$

## Case 2: $p \vee (p \wedge q)$



## Case 2: $p \vee (p \wedge q)$

$$\frac{p, q \succ p}{p, q \succ p \vee (p \wedge q)} \vee R$$

$$\frac{\begin{array}{c} p \succ p \\ \hline p, q \succ p \wedge q \end{array}}{\begin{array}{c} p \succ p \wedge q \\ \hline p, q \succ p \vee (p \wedge q) \end{array}} \wedge R \quad \vee R$$

Holds when p

## Case 2: $p \vee (p \wedge q)$

$$\frac{p, q \succ p}{p, q \succ p \vee (p \wedge q)} \vee R$$

$$\frac{\begin{array}{c} p \succ p \\ q \succ q \end{array}}{p, q \succ p \wedge q} \wedge R$$
$$\frac{}{p, q \succ p \vee (p \wedge q)} \vee R$$

Holds when p

## Case 2: $p \vee (p \wedge q)$

$$\frac{p, q \succ p}{p, q \succ p \vee (p \wedge q)} \vee R$$

*Holds when p*

$$\frac{\begin{array}{c} p \succ p \\ q \succ q \end{array}}{p, q \succ p \wedge q} \wedge R$$
$$\frac{p, q \succ p \wedge q}{p, q \succ p \vee (p \wedge q)} \vee R$$

## Case 2: $p \vee (p \wedge q)$

$$\frac{p, q \succ p}{p, q \succ p \vee (p \wedge q)} \vee R$$

Holds when p

$$\frac{\begin{array}{c} p \succ p \\ q \succ q \end{array}}{p, q \succ p \wedge q} \wedge R$$
$$\frac{p, q \succ p \wedge q}{p, q \succ p \vee (p \wedge q)} \vee R$$

Holds when p and q

### Case 3a: $(p \vee \neg p) \vee (q \vee \neg q)$

$$\frac{\frac{\frac{p \succ p}{p \succ p \vee \neg p} \vee R}{p \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R \quad \frac{\neg p \succ \neg p}{\neg p \succ p \vee \neg p} \vee R}{\neg p \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R$$
$$\frac{\frac{\frac{p \succ p}{\succ p, \neg p} \neg R}{\succ p \vee \neg p} \vee R}{\succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R$$

### Case 3a: $(p \vee \neg p) \vee (q \vee \neg q)$

$$\frac{\frac{\frac{p \succ p}{p \succ p \vee \neg p} \vee R}{p \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R \quad \frac{\neg p \succ \neg p}{\neg p \succ p \vee \neg p} \vee R}{\neg p \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R$$
$$\frac{\frac{\frac{p \succ p}{\succ p, \neg p} \neg R}{\succ p \vee \neg p} \vee R}{\succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R$$

These conditions *don't involve q*.

### Case 3a: $(p \vee \neg p) \vee (q \vee \neg q)$

$$\frac{\frac{\frac{q \succ q}{q \succ q \vee \neg q} \vee R}{q \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R \quad \frac{\neg q \succ \neg q}{\neg q \succ q \vee \neg q} \vee R}{\neg q \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R$$
$$\frac{\frac{\frac{q \succ q}{\succ q, \neg q} \neg R}{\succ q \vee \neg q} \vee R}{\succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R$$

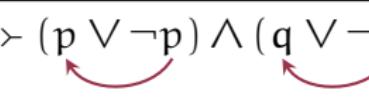
## Case 3a: $(p \vee \neg p) \vee (q \vee \neg q)$

$$\frac{\frac{\frac{q \succ q}{q \succ q \vee \neg q} \vee R}{q \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R \quad \frac{\neg q \succ \neg q}{\neg q \succ q \vee \neg q} \vee R}{\neg q \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R$$

$\frac{q \succ q}{\succ q, \neg q} \neg R$   
 $\frac{\succ q, \neg q}{\succ q \vee \neg q} \vee R$   
 $\frac{\succ q \vee \neg q}{\succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R$

These conditions *don't involve p*.

## Case 3b: $(p \vee \neg p) \wedge (q \vee \neg q)$

$$\frac{q \succ q}{q \succ q \vee \neg q} \vee R \quad \frac{\neg p \succ \neg p}{\neg p \succ p \vee \neg p} \vee R$$
$$\frac{}{\neg p, q \succ (p \vee \neg p) \wedge (q \vee \neg q)} \wedge R$$
  
$$\frac{p \succ p}{\succ p, \neg p} \neg R \quad \frac{q \succ q}{\succ q, \neg q} \neg R$$
$$\frac{}{\succ p \vee \neg p} \vee R \quad \frac{}{\succ q \vee \neg q} \vee R$$
$$\frac{}{\succ (p \vee \neg p) \wedge (q \vee \neg q)} \wedge R$$


These conditions *always involve both p and q.*

A scenic view of Bryce Canyon National Park, featuring a winding trail through a landscape of red rock hoodoos. The foreground shows a dirt path leading through the canyon, with several people walking along it. The middle ground is filled with the iconic hoodoo formations, and the background shows distant mountain ranges under a clear blue sky.

WHERE WE'VE GOT,  
AND WHERE TO  
FROM HERE

# Scorecard

SUBJECT MATTER	SITUATIONS	INVARIANTS
$p \vee \neg p / \top$	<i>different</i>	<i>different</i>
$p \vee (p \wedge q) / p$	<i>different</i>	<i>same</i>
$(p \vee \neg p) \vee (q \vee \neg q) /$ $(p \vee \neg p) \wedge (q \vee \neg q)$	<i>same</i>	<i>different</i>

## Places to Go

- ▶ In what can proofs and proof invariants do interesting work in articulating *subject matter*?
- ▶ Can invariants be used to define a notion of *logical subtraction*? (See Stephen Yablo's *Aboutness* for details.)
- ▶ Extend proof invariants beyond propositional logic.

# THANK YOU!

[http://consequently.org/presentation/2017/  
proof-identity-invariants-and-hyperintensionality](http://consequently.org/presentation/2017/proof-identity-invariants-and-hyperintensionality)

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