Assertions, Denials Questions, Answers & the Common Ground

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To better understand the speech acts of *assertion* and *denial*, their relationship to *other* speech acts, and connections between these speech acts and logical notions, including the classical sequent calculus.

I want to revisit some themes
(and revise some of the claims)
in my 2005 paper "Multiple Conclusions."

The behaviour of two kinds of speech acts:

polar (yes/no) *questions*,

and *justification requests*.

Assertion and Denial

Polar Questions

Positions and Rules

Justification Requests

ASSERTION AND DENIAL

$$X \succ Y$$

Don't *assert* each member of X
and *deny* each member of Y.

Defining Rules for Logical Concepts

This allows for a uniform, modular
system of rules of logical vocabulary.

$$\frac{X, A, B \succ Y}{X, A \land B \succ Y}\,{\land}Df \qquad \frac{X \succ A, B, Y}{X \succ A \lor B, Y}\,{\lor}Df \qquad \frac{X \succ A, Y}{X, \neg A \succ Y}\,{\neg}Df \qquad \frac{X, A \succ B, Y}{X \succ A \to B, Y}\,{\to}Df$$

$$\frac{X \succ A(n), Y}{X \succ \forall x A(x), Y}\,{\forall}Df \qquad \frac{X, A(n) \succ Y}{X, \exists x A(x) \succ Y}\,{\exists}Df \qquad \frac{X, Fa \succ Fb, Y \quad X, Fb \succ Fa, Y}{X \succ a = b, Y}\,{=}Df$$

Terms & conditions: the singular term n (in $\forall/\exists$*Df*) and the predicate F (in $=$*Df*)
do not appear below the line in those rules.

Structural Rules

$$X, A \succ A, Y \; {\scriptstyle Id} \qquad \frac{X \succ A, Y \quad X, A \succ Y}{X \succ Y}\, {\scriptstyle Cut}$$

These rules constrain assertion and denial *as such*.

… I was wading into a pre-existing literature about assertion. A *very large* literature.

It is fruitful to think of assertion as an act governed by *norms*.

For me: *Production Norms*

Aim to say what is *true*!

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Aim to say what is *true*!

Only say what you *know*!

Aim to say what is *true*!

Only say what you *know*!

Be prepared to *back it up* when requested!

The hearer is entitled to re-assert.

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You can refer back to the asserter
to *vouch for* the assertion.

To assert is to bid for the content asserted
to be added to the COMMON GROUND,
the body of information that
we (together) take for granted.

Stalnaker on Common Ground

To presuppose something is to take it for granted, or at least to act as if one takes it for granted, as background information as common ground among the participants in the conversation. What is most distinctive about this propositional attitude is that it is a social or public attitude: one presupposes that ϕ *only if one presupposes that others presuppose it as well.*

— "Common Ground" *L&P* (2002)

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This does not help distinguish *denial* from *retraction*, or from other speech acts.

Let's address this issue…

… by examining polar questions,
and their answers,
in the light of our background
interest in assertion and its norms.

POLAR QUESTIONS

This is a distinct speech act
with its own norms.

Is it the case that p?

This is a distinct speech act
with its own norms.

It raises an *issue*.

There are two ways to settle the issue

Yes

Yes *No*

If I say *yes* and you say *no*
to some polar question p?,
then we DISAGREE.

That is, we take *different* positions on p.

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then we DISAGREE.

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There is no *shared* position
incorporating both of our answers.

Other responses, like

Other responses don't settle the issue

Other responses, like

maybe

Other responses, like

maybe · *I don't know*

Other responses, like

maybe · *I don't know* · *I think so*

Other responses, like

maybe · *I don't know* · *I think so*

are acceptable responses to p?,

Other responses, like

maybe · *I don't know* · *I think so*

are acceptable responses to p?,

but they don't answer the question.

They don't settle the issue of p.

Settling answers are assertions

A *yes* or a *no* to p? counts as an assertion.

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A *yes* or a *no* to p? counts as an assertion.

(Either answer is governed by all of the assertion norms we've seen.)

What does a "no" to p? assert?

Presumably $\neg p$.

What does a "no" to p? *assert?*

Presumably ¬p.

However, I prefer to think of
a *yes* to p? as ruling p *in*,
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and those with more limited expressive resources.

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(Nothing important hangs on this distinction.)

$$[X : Y]$$

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- We have ruled *in* everything in X, the POSITIVE COMMON GROUND.

$$[X : Y]$$

- We have ruled *in* everything in X, the POSITIVE COMMON GROUND.
- We have ruled *out* everything in Y, the NEGATIVE COMMON GROUND.

Denial and Retraction

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ELOISE: No, he is in the kitchen.

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WEAK DENIAL

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PARTIAL ANSWER

- To *strongly deny* p is to bid to add p to the *negative common ground*.

Strong and Weak Denial

- To *strongly deny* p is to bid to add p to the *negative common ground*.
- To *weakly deny* p is to *block* the addition of p to the *positive common ground*, or to bid for its *retraction* if it is already there.

Strong and Weak Denial, and the Common Ground

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- A strong denial of p is one way to settle the question p? — this is generally an appropriate response.

- A weak denial of p is not generally an appropriate response to the polar question p?, as the polar question does not place p in the positive common ground, and the question is inappropriate if p is already in the positive common ground, so there is no p to block or retract.

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- STRONG ASSERTION: add to the positive common ground.
- WEAK DENIAL: retract (or block) from the positive common ground.
- WEAK ASSERTION: retract (or block) from the negative common ground. — “Perhaps p.”

That's one way to understand the relationship between assertion and denial, and how to distinguish strong denial from other negative speech acts.

One Consequence

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ELOISE: No. The interior angles of *this* triangle add up to 180°. Can you prove the general case?

Eloise here seems to block from the common ground (weakly deny)
a logical consequence of claims in the common ground (the axioms of geometry),
for the same general reason as for other weak denials.

Any position $[X, A : A, Y]$
in which A has been
strongly asserted and
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If $X \not\succ Y$ then $[X : Y]$ is *available*.

A Word on Cut

$$\frac{X \succ A, Y \quad X, A \succ Y}{X \succ Y}\,\mathit{Cut}$$

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In any available position $[X : Y]$, if one way to settle $A?$ is *not* available, then the other way to settle it *is* available.

POSITIONS AND RULES

Defining Rules

$$\frac{X, A, B \succ Y}{X, A \land B \succ Y}\ \land Df \qquad \frac{X \succ A, B, Y}{X \succ A \lor B, Y}\ \lor Df$$

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These are kinds of *definitions*, showing how to treat assertions or denials of the *defined* concept in terms of the assertions or denials of their components.

Derivations

$$\cfrac{\cfrac{\neg p \succ \neg p}{\succ p, \neg p}\,\neg Df}{\succ p \vee \neg p}\,\vee Df$$

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$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{p, q \vee r \succ p \wedge q, q \vee r}{p, q \vee r \succ p \wedge q, r, q}\,\vee Df \quad \dfrac{p \wedge q, q \vee r \succ p \wedge q, r}{q, p, q \vee r \succ p \wedge q, r}\,\wedge Df}{p, q \vee r \succ p \wedge q, r}\,Cut}{p, q \vee r \succ (p \wedge q) \vee r}\,\vee Df}{p \wedge (q \vee r) \succ (p \wedge q) \vee r}\,\wedge Df
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- They don't have the same *shape* as proofs.
- (Where is the *conclusion* in $p \vee q \succ p, q$?)
- A endsequent $X \succ A$ doesn't tell you to *infer* A *from* X — it merely tells you to not assert all members of X and deny A.

Let's make this problem sharp

"Well, now, let's take a little bit of the argument in that First Proposition—just *two* steps, and the conclusion drawn from them. Kindly enter them in your note-book. And in order to refer to them conveniently, let's call them *A*, *B*, and *Z*:—

(*A*) Things that are equal to the same are equal to each other.

(*B*) The two sides of this Triangle are things that are equal to the same.

(*Z*) The two sides of this Triangle are equal to each other.

Readers of Euclid will grant, I suppose, that *Z* follows logically from *A* and *B*, so that any one who accepts *A* and *B* as true, *must* accept *Z* as true?"

"Undoubtedly! The youngest child in a High School—as soon as High Schools are invented, which will not be till some two thousand years later—will grant *that*."

"And if some reader had *not* yet accepted *A* and *B* as true, he might still accept the *sequence* as a *valid* one, I suppose?"

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The Tortoise never asserts A and A $\rightarrow$ Z while denying Z,
but he doesn't accept A and A $\rightarrow$ Z *as a reason* for Z.

JUSTIFICATION REQUESTS

What is a justification request?

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ELOISE: *Really*?

ABELARD: I saw him there five minutes ago.

ELOISE: OK.

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ABELARD: Astralabe is in the kitchen.
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ELOISE: Yes, but he was in the study two minutes ago.

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That's a JUSTIFICATION REQUEST.

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This reason is another assertion [or denial] which must be *granted*, (added to the common ground) in order for the request to be met.

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A justification request for a strong assertion [or strong denial] is an attempt to block the addition to the common ground, until a *reason* is given.

This reason is another assertion [or denial] which must be *granted*, (added to the common ground) in order for the request to be met.

Granting the given reason is *necessary* but not *sufficient* for satisfying the justification request.

Definitions and Justification Requests

ACHILLES So … this is an *equilateral* triangle.

TORTOISE I'm sorry, I don't follow, my heroic friend. I've not heard that word before: what does '*equilateral*' mean?

ACHILLES Oh, that's easy to explain. '*Equilateral*' means having sides of the same length. An *equilateral* triangle is a triangle with all three sides the same length.

TORTOISE OK. That sounds good. You may continue with your reasoning.

ACHILLES Well, as I was saying, the sides of this triangle are all one cubit in length, so it is an equilateral triangle.

TORTOISE Perhaps you will forgive me, Achilles, but I still don't follow. I grant to you that the sides of this triangle all have the same length. I fail to see, however, that it *follows* that it is an equilateral triangle. Could you explain why it is?

If I accept the definition $A =_{\mathrm{df}} B$,
then I should accept granting A as meeting
a justification request for the assertion of B
and ruling out A as meeting a justification
request for B's denial and *vice versa*.

A failure to accept this is a sign
that I have not mastered the definition.

What goes for a definition of the form $A =_{\mathrm{df}} B$ can also go for *defining rules*:

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It is a mistake to grant A and grant B
and to look for something more to discharge
a justification request for an assertion of $A \land B$,
if you take $\land Df$ as a *definition*.

Justification Requests and Defining Rules

$$\frac{X, A \succ B, Y}{X \succ A \to B, Y} \to\!Df$$

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It is a mistake to rule A in and rule B out and to look for something more to discharge a justification request for a denial of $A \to B$ if you accept $\to\!Df$ as a definition.

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Consider this *focussed* derivation:

$$\frac{A \to Z \succ \boxed{A \to Z}}{A \to Z, A \succ \boxed{Z}}\ {\to}Df$$

- Read the *premise* as telling us that in a position in which $A \to Z$ is already ruled in, we have an answer to the justification request for $A \to Z$'s assertion.

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- Read the *premise* as telling us that in a position in which $A \to Z$ is already ruled in, we have an answer to the justification request for $A \to Z$'s assertion.
- Then applying $\to$*Df* we see why we have an answer to the request concerning Z's assertion, in a context in which $A \to Z$ and A have both been ruled in. (In granting $A \to Z$ and A we have settled Z positively. Its denial is ruled out, since to assert A and deny Z amounts to denying $A \to Z$.)

SLOGAN: A derivation of $X \succ \boxed{A}, Y$ shows us how to meet a justification request for the assertion of A in any available position extending $[X : Y]$.

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A derivation of $X, \boxed{A} \succ Y$ shows us how to meet a justification request for the denial of A in any available position extending $[X : Y]$.

Focussed Structural Rules

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$$\frac{X \succ \boxed{A}, Y \quad X, A \succ \boxed{B}, Y}{X \succ \boxed{B}, Y}\,Cut \qquad \frac{X \succ \boxed{A}, Y \quad X, A, \boxed{B} \succ Y}{X, \boxed{B} \succ Y}\,Cut$$

$$\frac{X, A, \boxed{A} \succ Y}{X, \boxed{A} \succ Y}\,W \qquad \frac{X \succ A, \boxed{A}, Y}{X \succ \boxed{A}, Y}\,W$$

Swap

$$\dfrac{\dfrac{X \succ \boxed{A}, B, Y \quad X, A \succ A, \boxed{B}, Y}{X \succ A, B, \boxed{B}, Y}\,\textit{Cut}}{X \succ A, \boxed{B}, Y}\,\textit{W}$$

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$$\dfrac{\dfrac{X \succ \boxed{A}, B, Y \quad X, A \succ A, \boxed{B}, Y}{X \succ A, B, \boxed{B}, Y}\,\textit{Cut}}{X \succ A, \boxed{B}, Y}\,\textit{W}$$

$$\dfrac{X \succ \boxed{A}, B, Y}{X \succ A, \boxed{B}, Y}\,\textit{Swap}$$

Focussed Defining Rules

$$\frac{X, A, B \succ Y}{X, A \wedge B \succ Y}\ {\wedge}Df \qquad \frac{X, A, B \succ Y}{X, A \wedge B \succ Y}\ {\wedge}Df$$

$$\frac{X \succ A, B, Y}{X \succ A \vee B, Y}\ {\vee}Df \qquad \frac{X \succ A, B, Y}{X \succ A \vee B, Y}\ {\vee}Df$$

$$\frac{X \succ A, Y}{X, \neg A \succ Y}\ {\neg}Df \qquad \frac{X, A \succ B, Y}{X \succ A \to B, Y}\ {\to}Df \qquad \frac{X, A \succ B, Y}{X \succ A \to B, Y}\ {\to}Df$$

Proof and Supposition

$$\frac{X, A \succ \boxed{B}, Y}{X \succ \boxed{A \to B}, Y}{\to}Df \qquad \frac{X, \boxed{A} \succ B, Y}{X \succ \boxed{A \to B}, Y}{\to}Df$$

To prove $A \to B$, *rule* A *in* (suppose it) and prove B.

Or, *rule* B *out* (suppose it), and refute A.

A Focussed Derivation

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{p \succ p, q}{\succ p, p \to q}\,{\to}\mathit{Df}
\qquad
\dfrac{(p \to q) \to p \succ (p \to q) \to p}{(p \to q) \to p, p \to q \succ p}\,{\to}\mathit{Df}
}{(p \to q) \to p \succ p, p}\,\mathit{Cut}
}{(p \to q) \to p \succ p}\,\mathit{W}
}{\succ ((p \to q) \to p) \to p}\,{\to}\mathit{Df}
$$

This can be represented as a *dialogue*,
meeting a justification request for
an assertion of $((p \to q) \to p) \to p$.
(See the handout for an example.)

Answers!

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- The making of an *inference* is a (possibly preemptive) answer to a justification request.
- A derivation of a sequent $X \succ A, Y$ [$X, A \succ Y$] can be transformed into a *procedure* for meeting a justification request for an assertion of A [denial of A] in any available position, appealing only what is granted in [$X : Y$], and to the defining rules used in that derivation.

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- Derivations are one way we can *grasp* complex bounds and *enforce* them.
- The *negative* view of the bounds is seen in the clash between assertion and denial, and the *positive* view is found in the answers we can give to justification requests.
- Adopting *defining rules* is one way to be *very* precise about the norms governing the concepts so defined, combining *safety*, *univocity* and *expressive power*.

THANK YOU!