

What Proofs and Truthmakers are About

Some reflections on Stephen Yablo's *Aboutness*

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MELBOURNE

ABOUTNESS WORKSHOP · MELBOURNE · 16 JULY 2016

Subject Matter

Models and Proofs

Truthmakers

Proof Invariants

Where we've got, and where to from here

A photograph showing a vast, dense grid of green stadium seats, likely made of plastic or metal, arranged in rows. The perspective is from a low angle, looking up at the rows of seats that recede into the distance. The seats are a uniform green color with dark green armrests and backrests. Some small red objects, possibly flags or markers, are visible on top of some of the seats.

SUBJECT MATTER

Positive Subject Matter

“The [positive] subject matter of S
is the relation m such that
worlds are m -dissimilar iff
 S is differently true in them.”

Positive Subject Matter

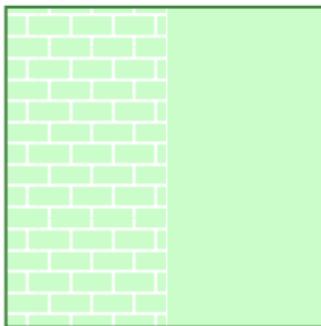
“The [positive] subject matter of S
is the relation m such that
worlds are m -dissimilar iff
 S is differently true in them.”

This is: how S is true

Picturing Positive Subject Matter



Picturing Positive Subject Matter



Negative Subject Matter

The negative subject matter of S
is the relation m' such that
worlds are m' -dissimilar iff
 S is differently false in them.”

Negative Subject Matter

The negative subject matter of S
is the relation m' such that
worlds are m' -dissimilar iff
 S is differently false in them.”

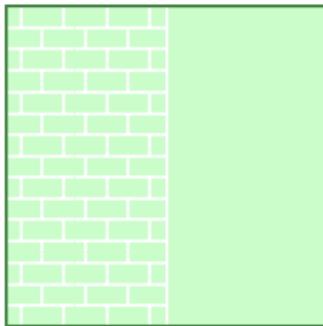
This is: how S is false

The Subject Matter of S

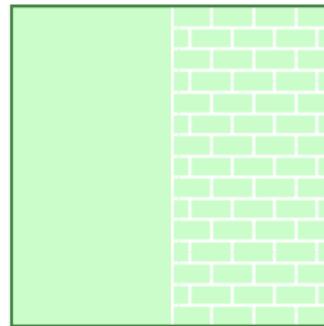
{how S is true, how S is false}

Aboutness, Chapter 2

Picturing Subject Matter

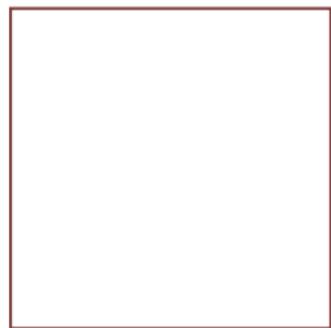
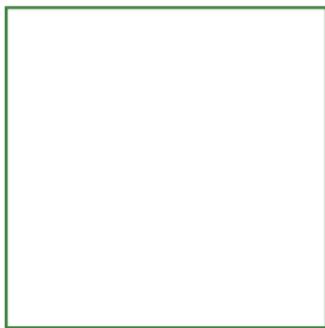


how S is true



how S is false

The subject matter of $p \vee \neg p$



how $p \vee \neg p$ is true

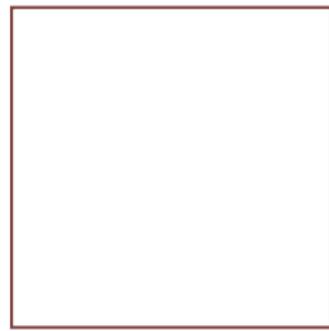
how $p \vee \neg p$ is false

The subject matter of $p \vee \neg p$

p

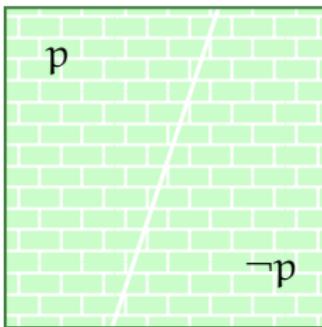
$\neg p$

how $p \vee \neg p$ is true

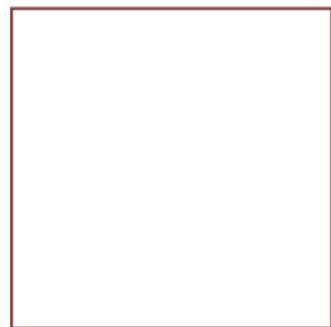


how $p \vee \neg p$ is false

The subject matter of $p \vee \neg p$

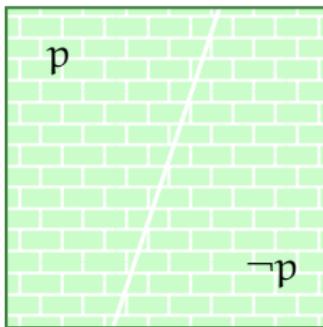


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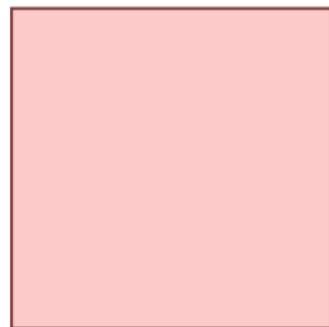


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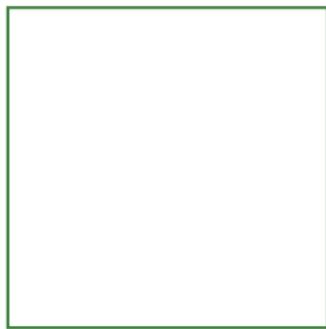


how $p \vee \neg p$ is true



how $p \vee \neg p$ is false

The subject matter of $p \vee (p \wedge q)$

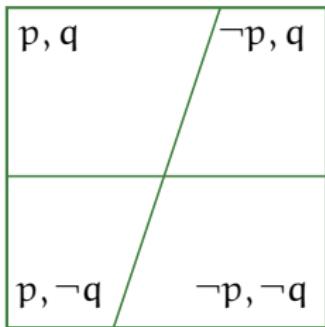


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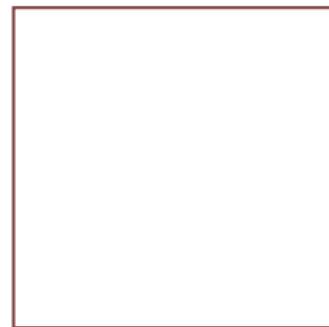


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The subject matter of $p \vee (p \wedge q)$

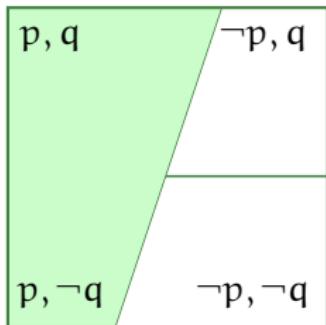


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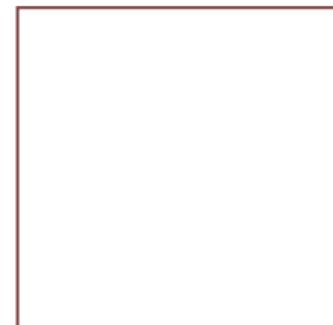


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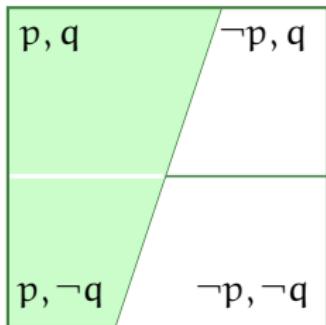


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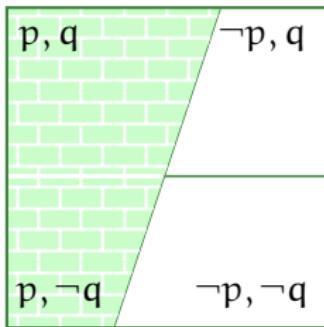


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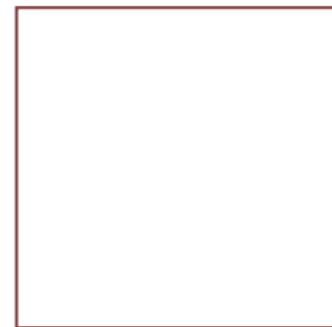


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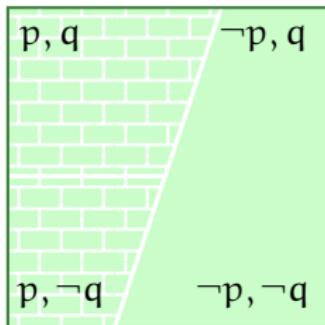


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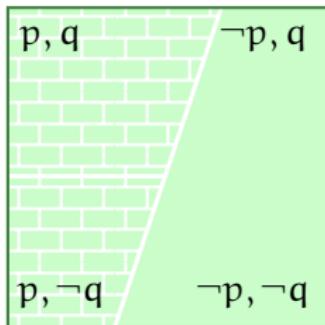


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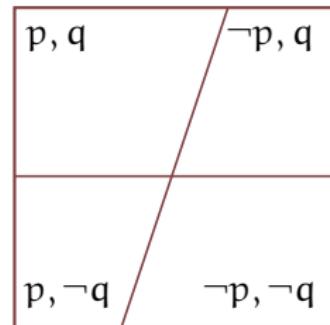


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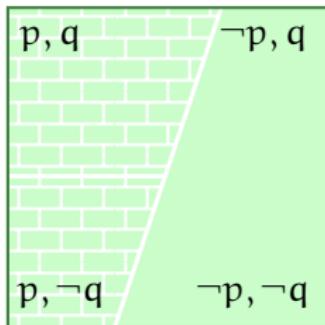


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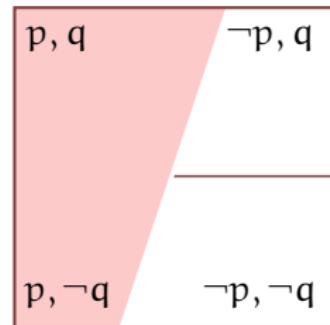


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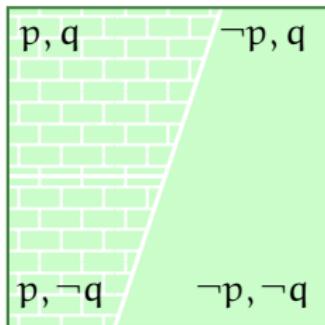


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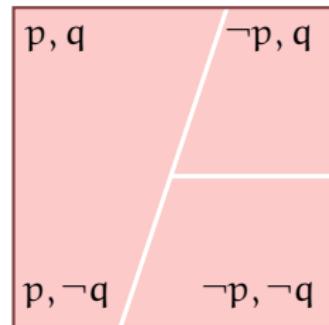


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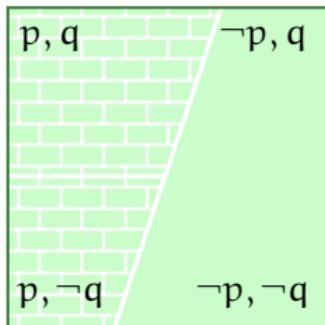


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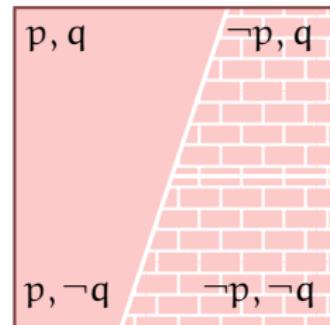


how $p \vee (p \wedge q)$ is false

The subject matter of $p \vee (p \wedge q)$



how $p \vee (p \wedge q)$ is true



how $p \vee (p \wedge q)$ is false

$$(p \vee \neg p) \vee (q \vee \neg q)$$

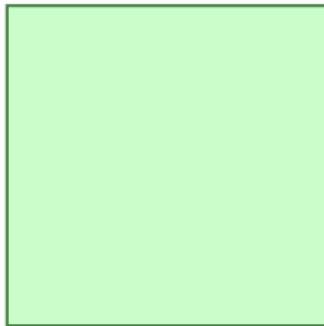


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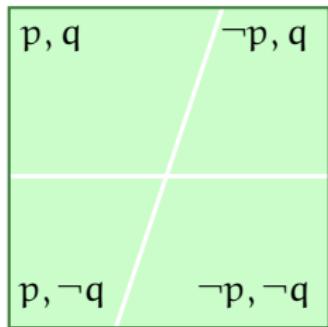


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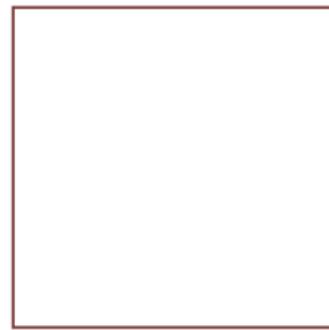
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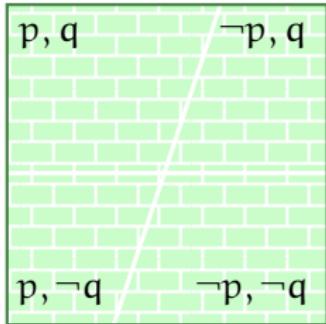


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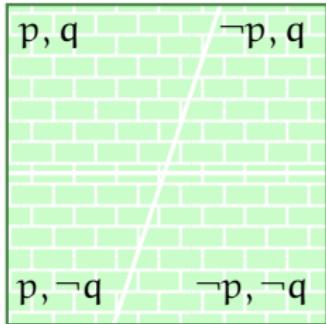


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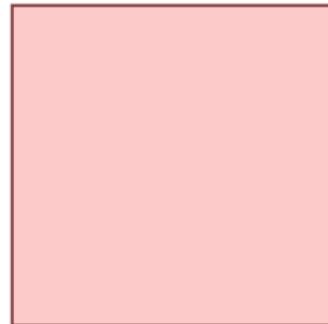


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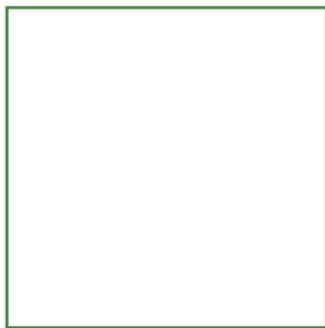


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$$(p \vee \neg p) \wedge (q \vee \neg q)$$



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how $(p \vee \neg p) \wedge (q \vee \neg q)$ is false

$$(p \vee \neg p) \wedge (q \vee \neg q)$$

p, q

$\neg p, q$

$p, \neg q$

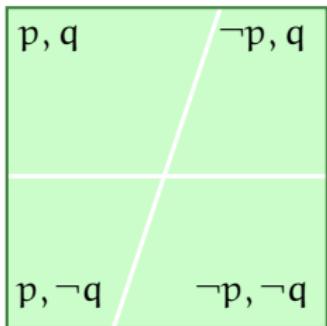
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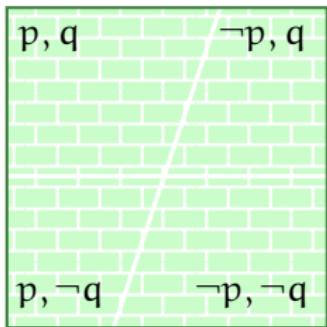


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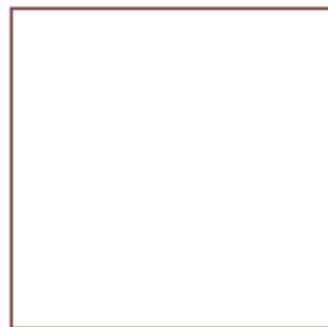


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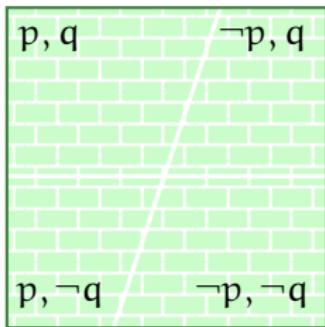


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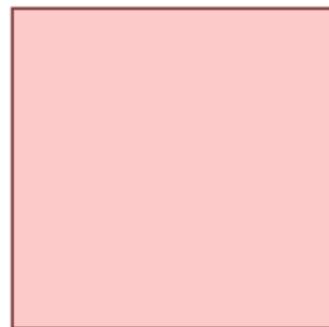


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how $(p \vee \neg p) \wedge (q \vee \neg q)$ is true

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The background image shows a vast mountain range with deep green forests covering the lower slopes and rocky peaks rising in the distance. A winding river or road cuts through the valley floor. The sky is a clear blue with scattered white and grey clouds.

MODELS AND PROOFS

Two Traditions in Logic and Semantics

MODEL THEORY (1) Define *models* and give recursive conditions explaining when a claim is true in a model.

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Two Tools for Understanding Aboutness

MODEL THEORY

Situations as Truthmakers

PROOF THEORY

Proof Invariants

TRUTHMAKERS

Case 1: $p \vee \neg p$

$p \vee \neg p$

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$$p \vee \neg p$$

$s \Vdash p \vee \neg p$ iff $s \Vdash p$ or $s \Vdash \neg p$

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$$p \vee \neg p$$

$s \Vdash p \vee \neg p$ iff $s \Vdash p$ or $s \Vdash \neg p$

Many situations are silent on whether p or not.

(*Making $p \vee \neg p$ true* is different from *making $q \vee \neg q$ true*.)

Case 2: $p \vee (p \wedge q)$

$p \vee (p \wedge q)$

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$$p \vee (p \wedge q)$$

$s \Vdash p \vee (p \wedge q)$ iff $s \Vdash p$ or $s \Vdash p \wedge q$,
iff $s \Vdash p$ or ($s \Vdash p$ and $s \Vdash q$),
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$$p \vee (p \wedge q)$$

$s \Vdash p \vee (p \wedge q)$ iff $s \Vdash p$ or $s \Vdash p \wedge q$,
iff $s \Vdash p$ or ($s \Vdash p$ and $s \Vdash q$),
iff $s \Vdash p$.

Making $p \vee (p \wedge q)$ true just is making p true.

Case 3: $(p \vee \neg p) \vee (q \vee \neg q)$ and $(p \vee \neg p) \wedge (q \vee \neg q)$

$$(p \vee \neg p) \vee (q \vee \neg q)$$

Either make p true, or make p false,
or make q true, or make q false.

Case 3: $(p \vee \neg p) \vee (q \vee \neg q)$ and $(p \vee \neg p) \wedge (q \vee \neg q)$

$$(p \vee \neg p) \vee (q \vee \neg q)$$

Either make p true, or make p false,
or make q true, or make q false.

$$(p \vee \neg p) \wedge (q \vee \neg q)$$

Either make p and q true, or make p true and q false,
or make p false and q true, or make p and q false.

PROOF INVARIANTS

When is π_1 the same proof as π_2 ?

$$\frac{p \wedge q}{\frac{p}{p \vee q}} \wedge E$$

$$\frac{p \succ p}{\frac{p \succ p \vee q}{p \wedge q \succ p \vee q}} \vee R$$

When is π_1 the same proof as π_2 ?

$$\frac{\frac{p \wedge q}{p} \wedge E}{p \vee q} \vee I$$

$$\frac{\frac{p \succ p}{p \succ p \vee q} \vee R}{p \wedge q \succ p \vee q} \wedge L$$

When is π_1 the same proof as π_2 ?

$$\frac{p \wedge q}{\frac{p}{p \vee q}} \wedge E$$

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$$\frac{\frac{q \succ q}{q \succ p \vee q} \vee R}{p \wedge q \succ p \vee q} \wedge L$$

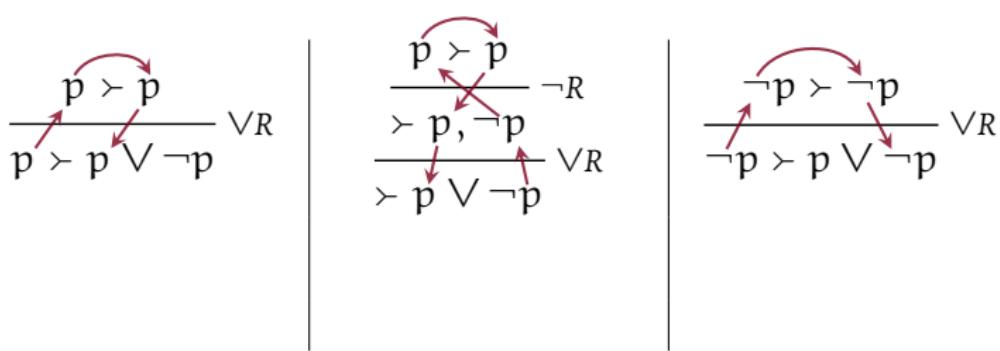
Proofs, proofinvariants and truth conditions

SLOGAN 1: A proof of C from P shows how C can obtain in P-circumstances.

SLOGAN 2: Different *proofinvariants* give different truth conditions.

$$\frac{\frac{p \wedge q}{\frac{p}{\frac{p \vee q}{\vee I}} \wedge E}{\wedge E} \quad \frac{p \wedge q}{\frac{q}{\frac{p \vee q}{\vee I}} \wedge E}$$

Case 1: $p \vee \neg p$



Case 1: $p \vee \neg p$

$$\frac{p \succ p}{p \succ p \vee \neg p} \vee R$$

$$\frac{\begin{array}{c} p \succ p \\ \hline \succ p, \neg p \end{array}}{\succ p \vee \neg p} \vee R$$

$$\frac{\begin{array}{c} \neg p \succ \neg p \\ \hline \neg p \succ p \vee \neg p \end{array}}{\neg p \succ p \vee \neg p} \vee R$$

Holds when p

Case 1: $p \vee \neg p$

$$\frac{p \succ p}{p \succ p \vee \neg p} \vee R$$

Holds when p

$$\frac{\begin{array}{c} p \succ p \\ \hline \succ p, \neg p \end{array}}{\succ p \vee \neg p} \vee R$$

$$\frac{\neg p \succ \neg p}{\neg p \succ p \vee \neg p} \vee R$$

Holds when $\neg p$

Case 1: $p \vee \neg p$

$$\frac{p \succ p}{p \succ p \vee \neg p} \text{VR}$$

Holds when p

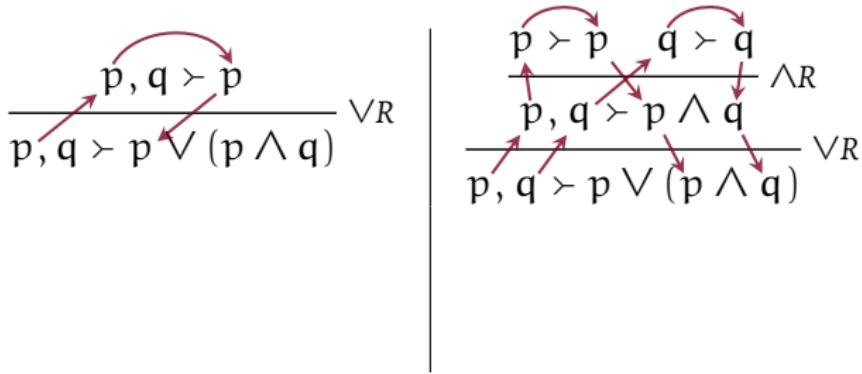
$$\frac{\begin{array}{c} p \succ p \\ \hline \succ p, \neg p \end{array}}{\succ p \vee \neg p} \text{VR}$$

Holds independently

$$\frac{\neg p \succ \neg p}{\neg p \succ p \vee \neg p} \text{VR}$$

Holds when $\neg p$

Case 2: $p \vee (p \wedge q)$



Case 2: $p \vee (p \wedge q)$

$$\frac{p, q \succ p}{p, q \succ p \vee (p \wedge q)} \vee R$$

$$\frac{\begin{array}{c} p \succ p \\ \hline p, q \succ p \wedge q \end{array}}{\begin{array}{c} p \succ p \wedge q \\ \hline p, q \succ p \vee (p \wedge q) \end{array}} \wedge R$$

Holds when p

Case 2: $p \vee (p \wedge q)$

$$\frac{p, q \succ p}{p, q \succ p \vee (p \wedge q)} \vee R$$

$$\frac{\begin{array}{c} p \succ p \\ q \succ q \end{array}}{p, q \succ p \wedge q} \wedge R$$
$$\frac{}{p, q \succ p \vee (p \wedge q)} \vee R$$

Holds when p

Case 2: $p \vee (p \wedge q)$

$$\frac{p, q \succ p}{p, q \succ p \vee (p \wedge q)} \vee R$$

$$\frac{\begin{array}{c} p \succ p \\ q \succ q \end{array}}{p, q \succ p \wedge q} \wedge R$$
$$\frac{p, q \succ p \wedge q}{p, q \succ p \vee (p \wedge q)} \vee R$$

Holds when p

Case 2: $p \vee (p \wedge q)$

$$\frac{p, q \succ p}{p, q \succ p \vee (p \wedge q)} \vee R$$

Holds when p

$$\frac{\begin{array}{c} p \succ p \\ q \succ q \end{array}}{\frac{p, q \succ p \wedge q}{p, q \succ p \vee (p \wedge q)}} \wedge R$$

Holds when p and q

Case 3a: $(p \vee \neg p) \vee (q \vee \neg q)$

$$\frac{\frac{\frac{p \succ p}{p \succ p \vee \neg p} \vee R}{p \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R \quad \frac{\neg p \succ \neg p}{\neg p \succ p \vee \neg p} \vee R}{\neg p \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R$$
$$\frac{\frac{\frac{p \succ p}{\succ p, \neg p} \neg R}{\succ p \vee \neg p} \vee R}{\succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R$$

Case 3a: $(p \vee \neg p) \vee (q \vee \neg q)$

$$\frac{\frac{\frac{p \succ p}{p \succ p \vee \neg p} \vee R}{p \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R \quad \frac{\neg p \succ \neg p}{\neg p \succ p \vee \neg p} \vee R}{\neg p \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R$$
$$\frac{\frac{\frac{p \succ p}{\succ p, \neg p} \neg R}{\succ p \vee \neg p} \vee R}{\succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R$$

These conditions *don't involve q*.

Case 3a: $(p \vee \neg p) \vee (q \vee \neg q)$

$$\frac{\frac{\frac{q \succ q}{q \succ q \vee \neg q} \vee R}{q \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R \quad \frac{\neg q \succ \neg q}{\neg q \succ q \vee \neg q} \vee R}{\neg q \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R$$

$\frac{\frac{q \succ q}{\succ q, \neg q} \neg R}{\succ q \vee \neg q} \vee R$

$$\frac{\succ q \vee \neg q}{\succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R$$

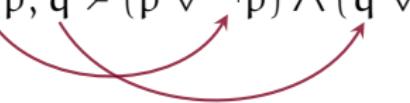
Case 3a: $(p \vee \neg p) \vee (q \vee \neg q)$

$$\frac{\frac{\frac{q \succ q}{q \succ q \vee \neg q} \vee R}{q \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R \quad \frac{\neg q \succ \neg q}{\neg q \succ q \vee \neg q} \vee R}{\neg q \succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R$$
$$\frac{\frac{\frac{q \succ q}{\succ q, \neg q} \neg R}{\succ q \vee \neg q} \vee R}{\succ (p \vee \neg p) \vee (q \vee \neg q)} \vee R}{\succ (p \vee \neg p) \vee (q \vee \neg q)}$$

These conditions *don't involve p*.

Case 3b: $(p \vee \neg p) \wedge (q \wedge \neg q)$

$$\frac{q \succ q}{q \succ q \vee \neg q} \vee R \quad \frac{\neg p \succ \neg p}{\neg p \succ p \vee \neg p} \vee R$$
$$\frac{}{\neg p, q \succ (p \vee \neg p) \wedge (q \vee \neg q)} \wedge R$$

$$\frac{p \succ p}{\succ p, \neg p} \neg R \quad \frac{q \succ q}{\succ q, \neg q} \neg R$$
$$\frac{}{\succ p \vee \neg p} \vee R \quad \frac{}{\succ q \vee \neg q} \vee R$$
$$\frac{}{\succ (p \vee \neg p) \wedge (q \vee \neg q)} \wedge R$$


These conditions *always involve both p and q.*

A scenic view of Bryce Canyon National Park, featuring a winding trail through a landscape of red rock hoodoos. The foreground shows a dirt path leading through the canyon, with several people walking along it. The middle ground is filled with the iconic hoodoo formations, and the background shows distant mountain ranges under a clear blue sky.

WHERE WE'VE GOT,
AND WHERE TO
FROM HERE

Scorecard

SUBJECT MATTER	SITUATIONS	INVARIANTS
$p \vee \neg p / \top$	<i>different</i>	<i>different</i>
$p \vee (p \wedge q) / p$	<i>different</i>	<i>same</i>
$(p \vee \neg p) \vee (q \vee \neg q) /$ $(p \vee \neg p) \wedge (q \vee \neg q)$	<i>same</i>	<i>different</i>

Places to Go

- ▶ In what can proofs and proof invariants do interesting work in articulating *subject matter*?
- ▶ Can invariants be used to define a notion of logical subtraction?
- ▶ Extend proof invariants beyond propositional logic.

THANK YOU!

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