PY4601 Paradoxes

Recent Work on the Liar Paradox

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WEEK 10: Kripke's fixed point construction

WEEK 11: What it might mean

TODAY'S PLAN

FORMALISING one paradoxical argument

LOCATING different positions on this map

FORMALISING a different paradoxical argument

Noticing the PARALLELS

Kripke's MODEL
- why it's needed
- $\{0, n, 1\}$ & refinement
- stages & the fixed point

THE UPSHOT

TODAY'S PLAN

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THE UPSHOT

(λ) $\qquad$ λ is not true

$$
\begin{prooftree}
\AxiomC{$\lambda = \langle \neg T\lambda \rangle$}
\AxiomC{$[T\lambda]^1$}
\RightLabel{$=E$}
\BinaryInfC{$T\langle \neg T\lambda \rangle$}
\RightLabel{TE}
\UnaryInfC{$\neg T\lambda$}
\AxiomC{$[T\lambda]^1$}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\RightLabel{$\neg I^1$}
\UnaryInfC{$\neg T\lambda$}
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\AxiomC{$\lambda = \langle \neg T\lambda \rangle$}
\AxiomC{$[T\lambda]^2$}
\RightLabel{$=E$}
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\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\RightLabel{$\neg I^2$}
\UnaryInfC{$\neg T\lambda$}
\RightLabel{TI}
\UnaryInfC{$T\langle \neg T\lambda \rangle$}
\RightLabel{$=E$}
\BinaryInfC{$T\lambda$}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\end{prooftree}
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\begin{prooftree}
\AxiomC{$\neg A$}
\AxiomC{A}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\end{prooftree}
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\begin{prooftree}
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\UnaryInfC{$\vdots$}
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\UnaryInfC{$\bot$}
\RightLabel{$\neg I^i$}
\UnaryInfC{$\neg A$}
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\qquad
\begin{prooftree}
\AxiomC{A}
\RightLabel{TI}
\UnaryInfC{$T\langle A \rangle$}
\end{prooftree}
\quad
\begin{prooftree}
\AxiomC{$T\langle A \rangle$}
\RightLabel{TE}
\UnaryInfC{A}
\end{prooftree}
\qquad
\begin{prooftree}
\AxiomC{$a = b$}
\AxiomC{Fa}
\RightLabel{$=E$}
\BinaryInfC{Fb}
\end{prooftree}
$$

TODAY'S PLAN

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THE UPSHOT

LEVEL SOLUTIONS

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\AxiomC{$\cancel{\lambda = \langle \neg T\lambda \rangle}$}
\AxiomC{$[T\lambda]^1$}
\RightLabel{$=E$}
\BinaryInfC{$T\langle \neg T\lambda \rangle$}
\RightLabel{TE}
\UnaryInfC{$\neg T\lambda$}
\AxiomC{$[T\lambda]^1$}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\RightLabel{$\neg I^1$}
\UnaryInfC{$\neg T\lambda$}
\AxiomC{$\lambda = \langle \neg T\lambda \rangle$}
\AxiomC{$\cancel{\lambda = \langle \neg T\lambda \rangle}$}
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\RightLabel{TE}
\UnaryInfC{$\neg T\lambda$}
\AxiomC{$[T\lambda]^2$}
\RightLabel{$\neg E$}
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\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\end{prooftree}
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\end{prooftree}
\qquad
\begin{prooftree}
\AxiomC{$[A]^i$}
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\RightLabel{$\neg I^i$}
\UnaryInfC{$\neg A$}
\end{prooftree}
\qquad
\begin{prooftree}
\AxiomC{A}
\RightLabel{TI}
\UnaryInfC{$T\langle A \rangle$}
\end{prooftree}
\quad
\begin{prooftree}
\AxiomC{$T\langle A \rangle$}
\RightLabel{TE}
\UnaryInfC{A}
\end{prooftree}
\qquad
\begin{prooftree}
\AxiomC{$a = b$}
\AxiomC{Fa}
\RightLabel{$=E$}
\BinaryInfC{Fb}
\end{prooftree}
$$

NO-PROPOSITION VIEWS

$$
\dfrac{
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\dfrac{\cancel{\lambda = \langle \neg T\lambda \rangle} \qquad [T\lambda]^1}{T\langle \neg T\lambda \rangle}\,{=}E
}{\neg T\lambda}\,TE
\qquad [T\lambda]^1
}{\bot}\,\neg E
}{\neg T\lambda}\,\neg I^1
\qquad
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\lambda = \langle \neg T\lambda \rangle
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\dfrac{
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\dfrac{
\dfrac{\cancel{\lambda = \langle \neg T\lambda \rangle} \qquad [T\lambda]^2}{T\langle \neg T\lambda \rangle}\,{=}E
}{\neg T\lambda}\,TE
\qquad [T\lambda]^2
}{\bot}\,\neg E
}{\neg T\lambda}\,\neg I^2
}{T\langle \neg T\lambda \rangle}\,TI
}{T\lambda}\,{=}E
}{\bot}\,\neg E
$$

$$
\dfrac{\neg A \qquad A}{\bot}\,\neg E
\qquad\qquad
\dfrac{\begin{matrix}[A]^i \\ \vdots \\ \bot\end{matrix}}{\neg A}\,\neg I^i
\qquad\qquad
\dfrac{A}{T\langle A \rangle}\,TI
\qquad
\dfrac{T\langle A \rangle}{A}\,TE
\qquad\qquad
\dfrac{a = b \qquad Fa}{Fb}\,{=}E
$$

$$
\AxiomC{$\lambda = \langle \neg T\lambda \rangle$}
\AxiomC{$[T\lambda]^1$}
\RightLabel{$=E$}
\BinaryInfC{$T\langle \neg T\lambda \rangle$}
\RightLabel{TE}
\UnaryInfC{$\neg T\lambda$}
\AxiomC{$[T\lambda]^1$}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\RightLabel{$\cancel{\neg I^1}$}
\UnaryInfC{$\neg T\lambda$}
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\AxiomC{$\lambda = \langle \neg T\lambda \rangle$}
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\RightLabel{TE}
\UnaryInfC{$\neg T\lambda$}
\AxiomC{$[T\lambda]^2$}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\RightLabel{$\cancel{\neg I^2}$}
\UnaryInfC{$\neg T\lambda$}
\RightLabel{TI}
\UnaryInfC{$T\langle \neg T\lambda \rangle$}
\RightLabel{$=E$}
\BinaryInfC{$T\lambda$}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\DisplayProof
$$

GAP VIEWS

$$
\AxiomC{$\neg A$}
\AxiomC{A}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\DisplayProof
\qquad
\AxiomC{$[A]^i$}
\noLine
\UnaryInfC{$\vdots$}
\noLine
\UnaryInfC{$\bot$}
\RightLabel{$\neg I^i$}
\UnaryInfC{$\neg A$}
\DisplayProof
\qquad
\AxiomC{A}
\RightLabel{TI}
\UnaryInfC{$T\langle A \rangle$}
\DisplayProof
\quad
\AxiomC{$T\langle A \rangle$}
\RightLabel{TE}
\UnaryInfC{A}
\DisplayProof
\qquad
\AxiomC{$a=b$}
\AxiomC{Fa}
\RightLabel{$=E$}
\BinaryInfC{Fb}
\DisplayProof
$$

GAP VIEWS

GLUT VIEWS

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^1}{T\langle \neg T\lambda \rangle}\,{\scriptstyle =E}}{\neg T\lambda}\,{\scriptstyle TE} \qquad [T\lambda]^1}{\bot}\,{\scriptstyle \cancel{\neg E}}}{\neg T\lambda}\,{\scriptstyle \neg I^1} \qquad \dfrac{\lambda = \langle \neg T\lambda \rangle \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^2}{T\langle \neg T\lambda \rangle}\,{\scriptstyle =E}}{\neg T\lambda}\,{\scriptstyle TE} \qquad [T\lambda]^2}{\bot}\,{\scriptstyle \cancel{\neg E}}}{\neg T\lambda}\,{\scriptstyle \neg I^2}}{T\langle \neg T\lambda \rangle}\,{\scriptstyle TI}}{T\lambda}\,{\scriptstyle =E}}{\bot}\,{\scriptstyle \cancel{\neg E}}$$

$$\cancel{\dfrac{\neg A \qquad A}{\bot}}\,{\scriptstyle \neg E}$$

GLUT VIEWS

$$\dfrac{\begin{array}{c}[A]^i \\ \vdots \\ \bot\end{array}}{\neg A}\,{\scriptstyle \neg I^i}$$

$$\dfrac{A}{T\langle A \rangle}\,{\scriptstyle TI} \qquad \dfrac{T\langle A \rangle}{A}\,{\scriptstyle TE}$$

$$\dfrac{a = b \qquad Fa}{Fb}\,{\scriptstyle =E}$$

$$
\dfrac{
\dfrac{\dfrac{\dfrac{\dfrac{\lambda=\langle\neg T\lambda\rangle \qquad [T\lambda]^1}{T\langle\neg T\lambda\rangle}\,{\scriptstyle =E}}{\neg T\lambda}\,{\scriptstyle \cancel{TE}} \qquad [T\lambda]^1}{\bot}\,{\scriptstyle \neg E}}{\neg T\lambda}\,{\scriptstyle \neg I^1}
\qquad
\dfrac{\lambda=\langle\neg T\lambda\rangle \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\lambda=\langle\neg T\lambda\rangle \qquad [T\lambda]^2}{T\langle\neg T\lambda\rangle}\,{\scriptstyle =E}}{\neg T\lambda}\,{\scriptstyle \cancel{TE}} \qquad [T\lambda]^2}{\bot}\,{\scriptstyle \neg E}}{\neg T\lambda}\,{\scriptstyle \neg I^2}}{T\langle\neg T\lambda\rangle}\,{\scriptstyle \cancel{TI}}}{T\lambda}\,{\scriptstyle =E}
}{\bot}\,{\scriptstyle \neg E}
$$

REVISING TRUTH

$$
\dfrac{\neg A \qquad A}{\bot}\,{\scriptstyle \neg E}
\qquad\qquad
\dfrac{\begin{matrix}[A]^i\\ \vdots\\ \bot\end{matrix}}{\neg A}\,{\scriptstyle \neg I^i}
\qquad\qquad
\cancel{\dfrac{A}{T\langle A\rangle}\,{\scriptstyle TI} \qquad \dfrac{T\langle A\rangle}{A}\,{\scriptstyle TE}}
\qquad\qquad
\dfrac{a=b \qquad Fa}{Fb}\,{\scriptstyle =E}
$$

REVISING TRUTH

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^1}{T\langle \neg T\lambda \rangle}\,{=}E\,???
}{\neg T\lambda}\,TE
\qquad [T\lambda]^1
}{\bot}\,\neg E
}{\neg T\lambda}\,\neg I^1
\qquad
\dfrac{
\lambda = \langle \neg T\lambda \rangle
\qquad
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^2}{T\langle \neg T\lambda \rangle}\,{=}E\,???
}{\neg T\lambda}\,TE
\qquad [T\lambda]^2
}{\bot}\,\neg E
}{\neg T\lambda}\,\neg I^2
}{T\langle \neg T\lambda \rangle}\,TI
}{T\lambda}\,{=}E\,???
}{\bot}\,\neg E
$$

$$
\dfrac{\neg A \qquad A}{\bot}\,\neg E
\qquad\qquad
\dfrac{\begin{matrix}[A]^i \\ \vdots \\ \bot\end{matrix}}{\neg A}\,\neg I^i
\qquad\qquad
\dfrac{A}{T\langle A \rangle}\,TI
\quad
\dfrac{T\langle A \rangle}{A}\,TE
\qquad\qquad
\overset{???}{\dfrac{a = b \qquad Fa}{Fb}}\,{=}E
$$

TODAY'S PLAN

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THE UPSHOT

t is A

$$\frac{A(t)}{t \in \{x : A(x)\}}\ \in\mathrm{I} \qquad\qquad \frac{t \in \{x : A(x)\}}{A(t)}\ \in\mathrm{E}$$

t has the property A.

Consider $\{x : x \in x \to p\}$,

for a given statement p.

Does $\{x : x \in x \to p\} \in \{x : x \in x \to p\}$?

Consider $\overset{c}{\{x : x \in x \to p\}}$,

for a given statement p.

Does $\{x : x \in x \to p\} \in \{x : x \in x \to p\}$?

$c \in c$?

$$\frac{c \in c}{c \in c \to p}\,{\in}E \qquad \frac{c \in c \to p}{c \in c}\,{\in}I$$

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{[C\in C]^1}{C\in C\to P}\,{\scriptstyle \in E}
      \qquad [C\in C]^1
    }{P}\,{\scriptstyle \to E}
  }{C\in C\to P}\,{\scriptstyle \to I^1}
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{[C\in C]^2}{C\in C\to P}\,{\scriptstyle \in E}
        \qquad [C\in C]^2
      }{P}\,{\scriptstyle \to E}
    }{C\in C\to P}\,{\scriptstyle \to I^2}
  }{C\in C}\,{\scriptstyle \in I}
}{P}\,{\scriptstyle \to E}
$$

C is $\{x : x\in x\to p\}$

$$
\dfrac{A\to B \qquad A}{B}\,{\scriptstyle \to E}
\qquad
\dfrac{\begin{matrix}[A]^i\\ \vdots \\ B\end{matrix}}{A\to B}\,{\scriptstyle \to I^i}
\qquad
\dfrac{A(t)}{t\in\{x:A(x)\}}\,{\scriptstyle \in I}
\qquad
\dfrac{t\in\{x:A(x)\}}{A(t)}\,{\scriptstyle \in E}
$$

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$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{[\cancel{c \in c}]^1}{c \in c \to p}\;{\in}E
\qquad [\cancel{c \in c}]^1
}{p}\;{\to}E
}{c \in c \to p}\;{\to}I^1
\qquad
\dfrac{
\dfrac{
\dfrac{
\dfrac{[\cancel{c \in c}]^2}{c \in c \to p}\;{\in}E
\qquad [\cancel{c \in c}]^2
}{p}\;{\to}E
}{c \in c \to p}\;{\to}I^2
}{c \in c}\;{\in}I
}{p}\;{\to}E
$$

Level & no proposition views

---

c is $\cancel{\{x : x \in x \to p\}}$

$$
\dfrac{A \to B \qquad A}{B}\;{\to}E
\qquad
\dfrac{\begin{matrix}[A]^i \\ \vdots \\ B\end{matrix}}{A \to B}\;{\to}I^i
\qquad
\dfrac{A(t)}{t \in \{x : A(x)\}}\;{\in}I
\qquad
\dfrac{t \in \{x : A(x)\}}{A(t)}\;{\in}E
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{[c \in c]^1}{c \in c \to p}\,{\scriptstyle \in E} \qquad [c \in c]^1}{p}\,{\scriptstyle \to E}}{c \in c \to p}\,{\scriptstyle \cancel{\to I^1}} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[c \in c]^2}{c \in c \to p}\,{\scriptstyle \in E} \qquad [c \in c]^2}{p}\,{\scriptstyle \to E}}{c \in c \to p}\,{\scriptstyle \cancel{\to I^2}}}{c \in c}\,{\scriptstyle \in I}}{}}{p}\,{\scriptstyle \to E}
$$

'GAP' views?

---

c is $\{x : x \in x \to p\}$

$$
\dfrac{A \to B \qquad A}{B}\,{\scriptstyle \to E}
\qquad
\cancel{\dfrac{\begin{array}{c}[A]^i \\ \vdots \\ B\end{array}}{A \to B}\,{\scriptstyle \to I^i}}
\qquad
\dfrac{A(t)}{t \in \{x : A(x)\}}\,{\scriptstyle \in I}
\qquad
\dfrac{t \in \{x : A(x)\}}{A(t)}\,{\scriptstyle \in E}
$$

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{[C\in C]^1}{C\in C\to p}\,{\in}E
      \qquad [C\in C]^1
    }{p}\,\cancel{\to E}
  }{C\in C\to p}\,{\to}I^1
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{[C\in C]^2}{C\in C\to p}\,{\in}E
        \qquad [C\in C]^2
      }{p}\,\cancel{\to E}
    }{C\in C\to p}\,{\to}I^2
  }{C\in C}\,{\in}I
}{p}\,{\to}E
$$

'GLUT' VIEWS?

C is $\{x : x\in x \to p\}$

$$
\dfrac{A\to B \qquad A}{\cancel{B}}\,\cancel{\to E}
\qquad
\dfrac{\begin{matrix}[A]^i\\ \vdots \\ B\end{matrix}}{A\to B}\,{\to}I^i
\qquad
\dfrac{A(t)}{t\in\{x : A(x)\}}\,{\in}I
\qquad
\dfrac{t\in\{x : A(x)\}}{A(t)}\,{\in}E
$$

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{[c\in c]^1}{c\in c\to p}\,\cancel{\in E} \qquad [c\in c]^1
}{p}\,{\to}E
}{c\in c\to p}\,{\to}I^1
\qquad
\dfrac{
\dfrac{
\dfrac{
\dfrac{[c\in c]^2}{c\in c\to p}\,\cancel{\in E} \qquad [c\in c]^2
}{p}\,{\to}E
}{c\in c\to p}\,{\to}I^2
}{c\in c}\,\cancel{\in I}
}{p}\,{\to}E
$$

Revision about property ascription

C is $\{x : x\in x \to p\}$

$$
\dfrac{A\to B \qquad A}{B}\,{\to}E
\qquad\qquad
\dfrac{\begin{matrix}[A]^i\\ \vdots \\ B\end{matrix}}{A\to B}\,{\to}I^i
\qquad\qquad
\cancel{\dfrac{A(t)}{t\in\{x : A(x)\}}\,\in I}
\qquad\qquad
\cancel{\dfrac{t\in\{x : A(x)\}}{A(t)}\,\in E}
$$

Does your diagnosis of the liar paradox generalise to Curry's paradox?

Should it?

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THE UPSHOT

$m(A \wedge B) = 1$ iff $m(A) = 1$ & $m(B) = 1$

$m(T\langle A \rangle) = \ldots\ldots ?$

$$m(A \wedge B) = 1 \text{ iff } m(A) = 1 \;\&\; m(B) = 1$$

$$m(T\langle A \rangle) = m(A) \;?$$

$$m(A \wedge B) = 1 \text{ iff } \begin{array}{l} m(A) = 1 \ \& \\ m(B) = 1 \end{array}$$

$$m(T\langle A \rangle) = m(A) \ ?$$

$$\tau = \langle T\tau \rangle : \quad \begin{aligned} m(T\tau) &= m(T\langle T\tau \rangle) \\ &= m(T\tau) \\ &= m(T\langle T\tau \rangle) \\ &= \cdots \end{aligned}$$

$$m(A \wedge B) = 1 \text{ iff } m(A) = 1 \;\&\; m(B) = 1$$

$$m(T\langle A \rangle) = m(A) \;?$$

$$\lambda = \langle \neg T \tau \rangle : \quad m(T\lambda) = m(T\langle \neg T\lambda \rangle)$$
$$= m(\neg T\lambda)$$
$$= m(\neg T\langle \neg T\lambda \rangle)$$
$$= \dots$$

$$m(A \wedge B) = 1 \text{ iff } \begin{array}{l} m(A) = 1 \ \& \\ m(B) = 1 \end{array}$$

$$m(T\langle A \rangle) = m(A) \ ?$$

In the presence of self reference, these rules do not assign values of a complex expression in terms of the values of <u>simpler</u> expressions.

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Noticing the PARALLELS

why it's needed

Kripke's MODEL – $\{0, n, 1\}$ & refinement

stages & the fixed point

THE UPSHOT

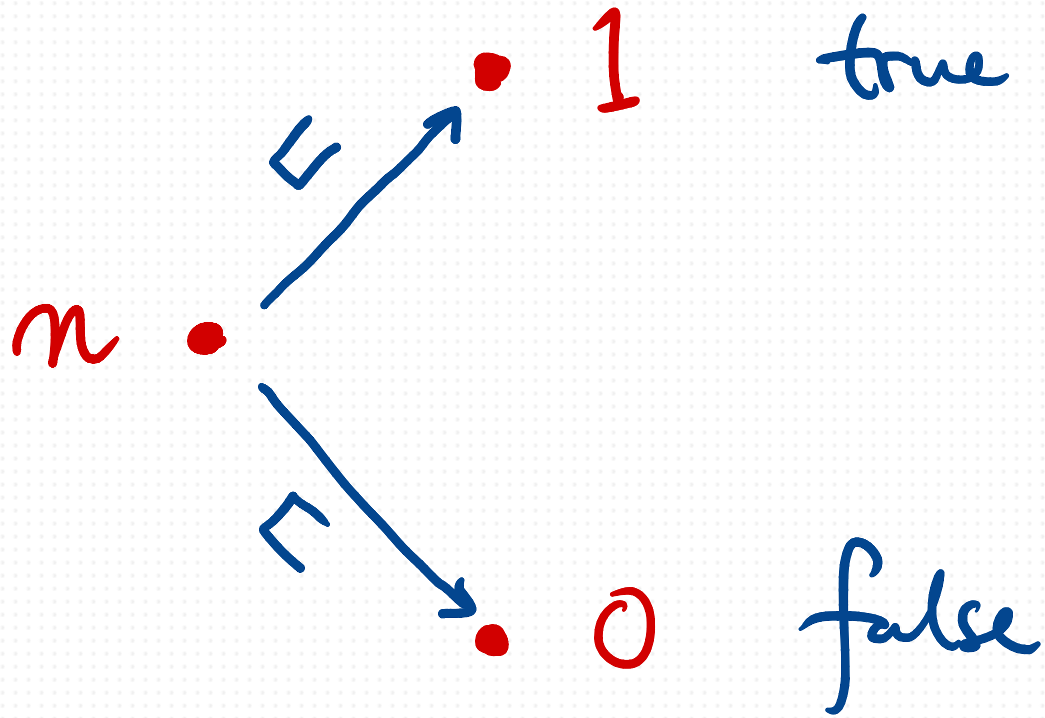
- 1 true

- 0 false

$\bullet\ 1$ true

$n\ \bullet$

$\bullet\ 0$ false



$x \sqsubseteq y$ iff $x \sqsubset y$ or $x = y$

$$m(\bot) = 0 \qquad m(\neg A) = 1 \text{ iff } m(A) = 0$$
$$= 0 \text{ iff } m(A) = 1$$

$$m(A \wedge B) = 1 \text{ iff } m(A) = 1 \;\&\; m(B) = 1$$
$$= 0 \text{ iff } m(A) = 0 \text{ or } m(B) = 0$$

$$m(A \vee B) = 1 \text{ iff } m(A) = 1 \text{ or } m(B) = 1$$
$$= 0 \text{ iff } m(A) = 0 \;\&\; m(B) = 0$$

$$m(A \to B) = 1 \text{ iff } m(A) = 0 \text{ or } m(B) = 1$$
$$= 0 \text{ iff } m(A) = 1 \;\&\; m(B) = 0$$

A	$\neg A$
0	1
n	n
1	0

$\wedge$	0	n	1
0	0	0	0
n	0	n	n
1	0	n	1

$\vee$	0	n	1
0	0	n	1
n	n	n	1
1	1	1	1

$\rightarrow$	0	n	1
0	1	1	1
n	n	n	1
1	0	n	1

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THE UPSHOT

If $m_1(A) \sqsubseteq m_2(A)$ then $m_1(\neg A) \sqsubseteq m_2(\neg A)$

& if $m_1(B) \sqsubseteq m_2(B)$ then $m_1(A \wedge B) \sqsubseteq m_2(A \wedge B)$

$m_1(A \vee B) \sqsubseteq m_2(A \vee B)$

$m_1(A \to B) \sqsubseteq m_2(A \to B)$

eg.

	m_1		m_2
A:	n	$\sqsubseteq$	1
B:	n	$\sqsubseteq$	0
$A \to B$:	n	$\sqsubseteq$	0

	m_1		m_2
A:	n	$\sqsubseteq$	1
B:	n	$\sqsubseteq$	1
$A \to B$:	n	$\sqsubseteq$	1

Adding T to a formal language.

$\mathcal{M}_0$

interpret the non T sentences however you like in $\{0, n, 1\}$.

assign Tx the value n

We treat T-sentences as undetermined at stage 0, and we progressively refine them over stages.

$$m_0(\text{non T atom}) = m_1(\text{non-T atom})$$

so, $m_0(p) \sqsubseteq m_1(p)$

$$m_0(A) = m_1(T\langle A \rangle)$$

so, $\underset{= n}{m_0(T\langle A \rangle)} \sqsubseteq m_1(T\langle A \rangle)$

so, $m_0 \sqsubseteq m_1$

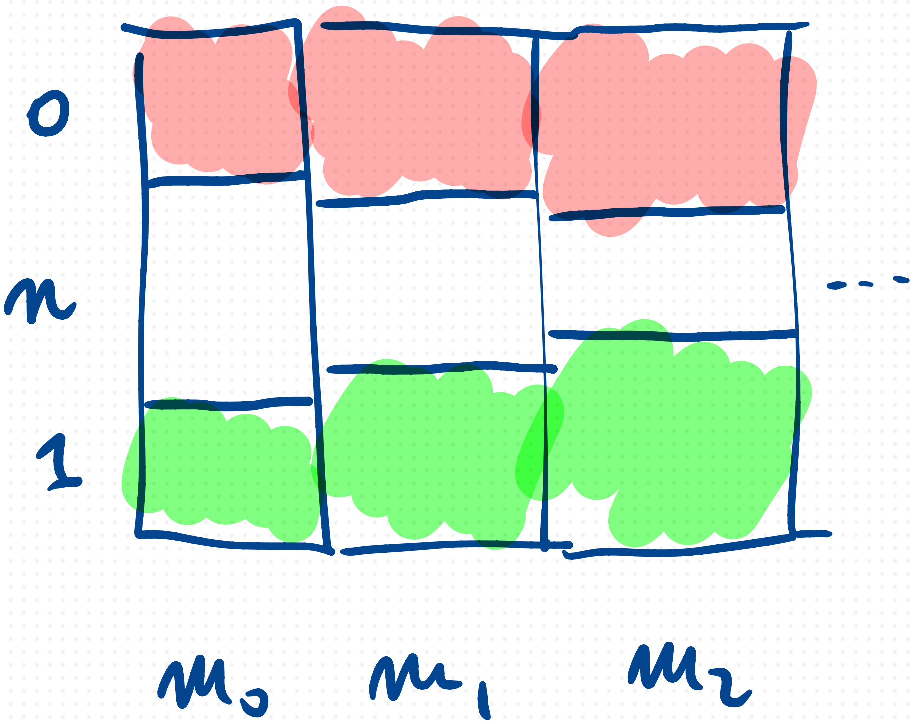
$$m_i(\text{non T atom}) = m_{i+1}(\text{non-T atom})$$

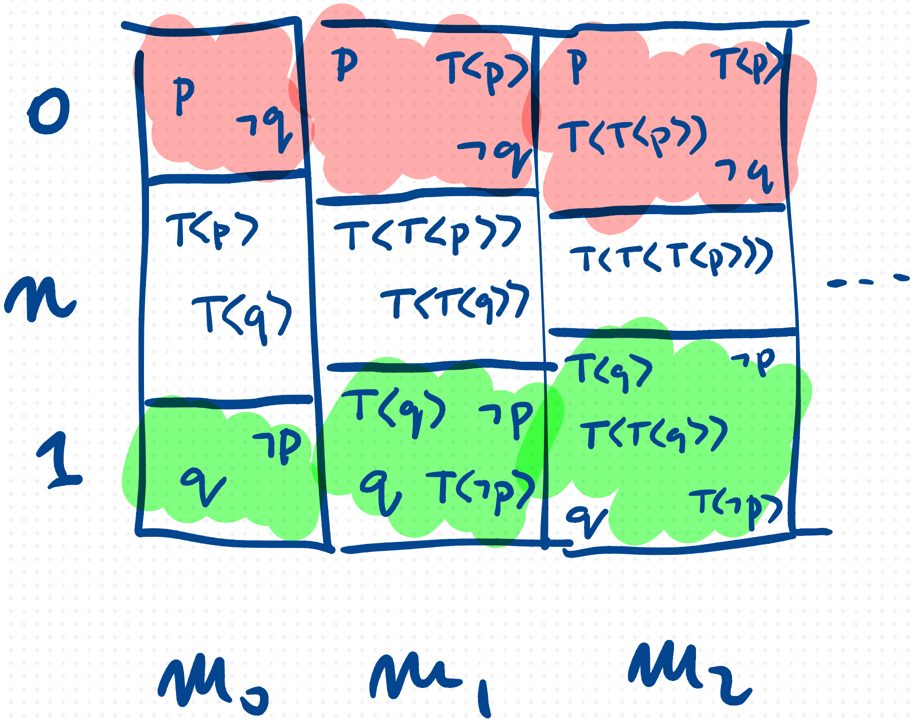
so, $m_i(p) \sqsubseteq m_{i+1}(p)$

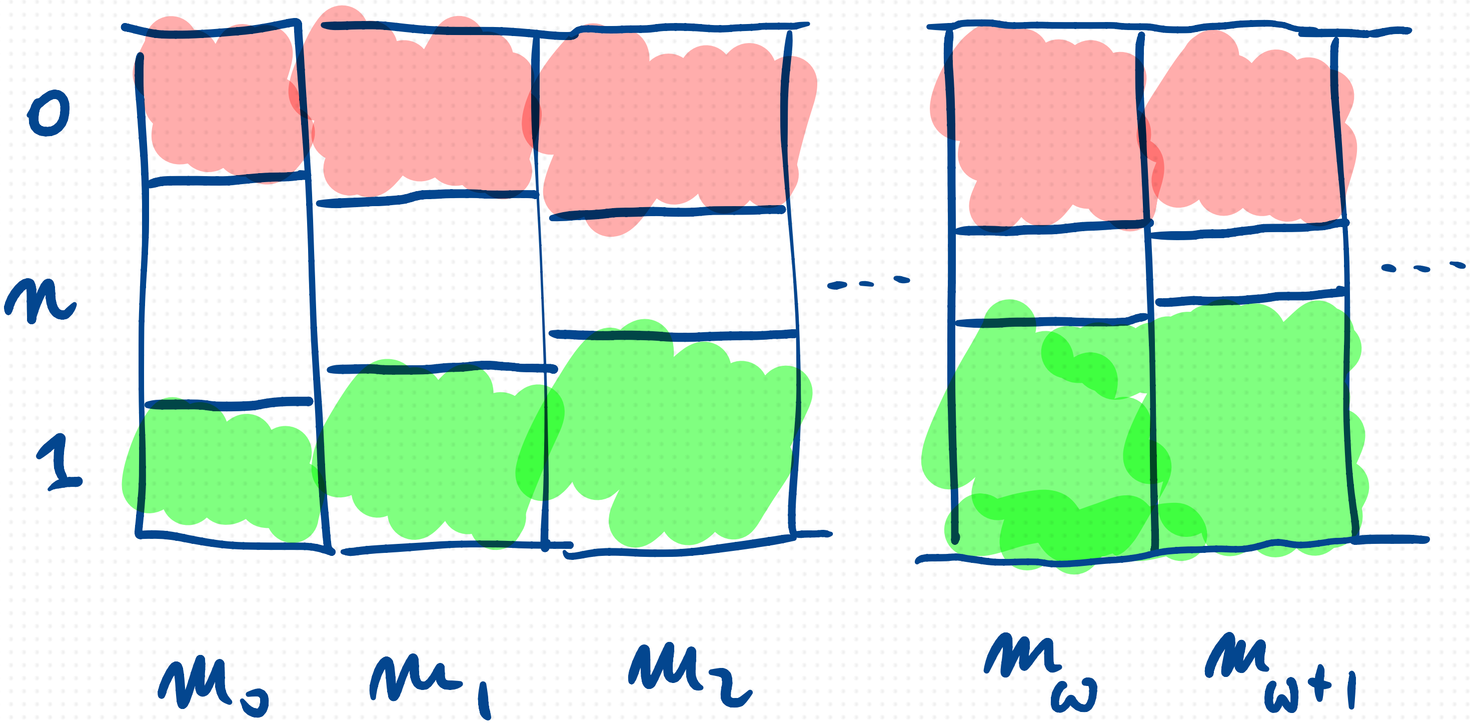
$$m_i(A) = m_{i+1}(T\langle A\rangle)$$

so, $\underset{m_{i-1}(A)}{\overset{m_i(T\langle A\rangle)}{\parallel}} \sqsubseteq \underset{m_i(A)}{\overset{m_{i+1}(T\langle A\rangle)}{\parallel}}$

so, $m_i \sqsubseteq m_{i+1}$


0
n
1
m_0
m_1
m_2
...

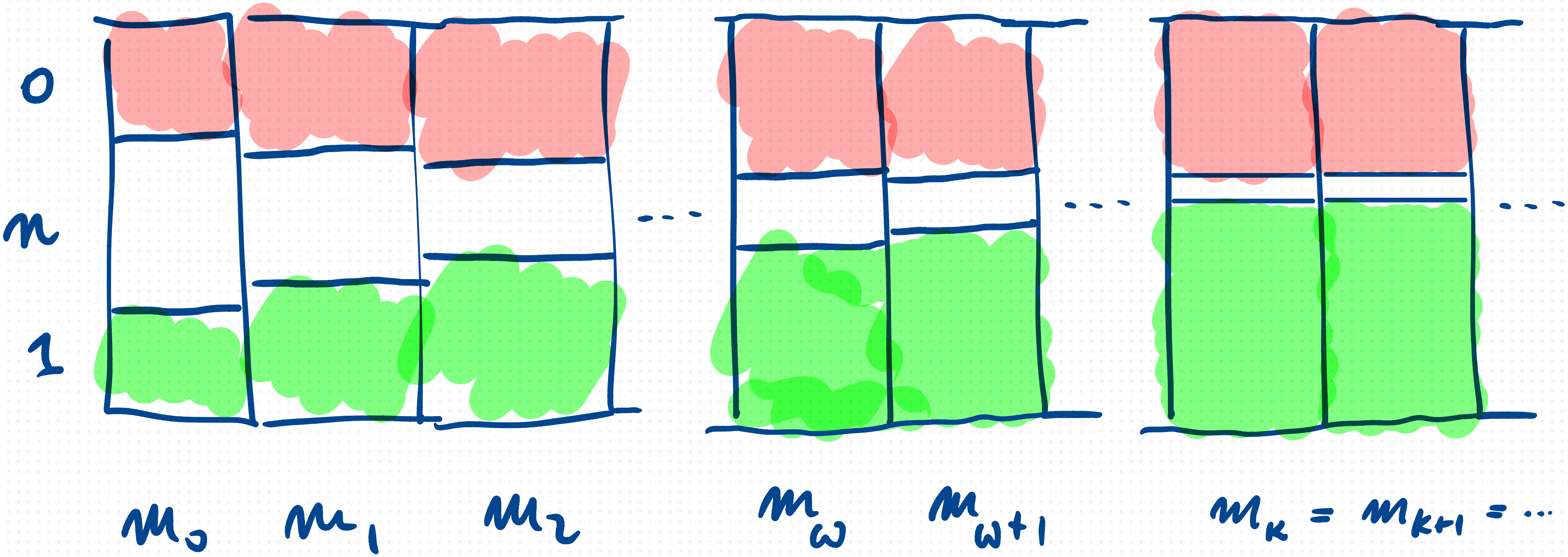

0
n
1
p
¬q
T⟨p⟩
T⟨q⟩
q
¬p
p
T⟨p⟩
¬q
T⟨T⟨p⟩⟩
T⟨T⟨q⟩⟩
T⟨q⟩
¬p
q
T⟨¬p⟩
p
T⟨p⟩
T⟨T⟨p⟩⟩
¬q
T⟨T⟨T⟨p⟩⟩⟩
T⟨q⟩
¬p
T⟨T⟨q⟩⟩
q
T⟨¬p⟩
...
m0
m1
m2



0
n
1
m_0
m_1
m_2
m_ω
$m_{\omega+1}$

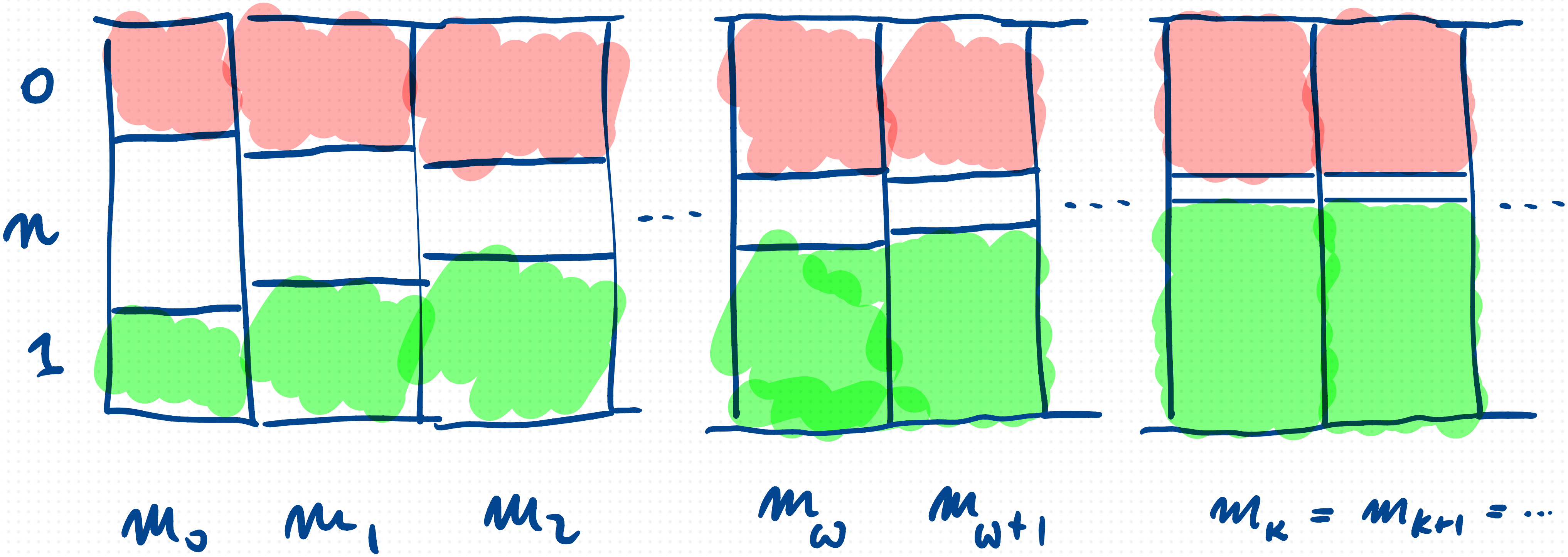
(!)

We eventually reach a fixed-point.



(!)

We eventually reach a fixed-point.

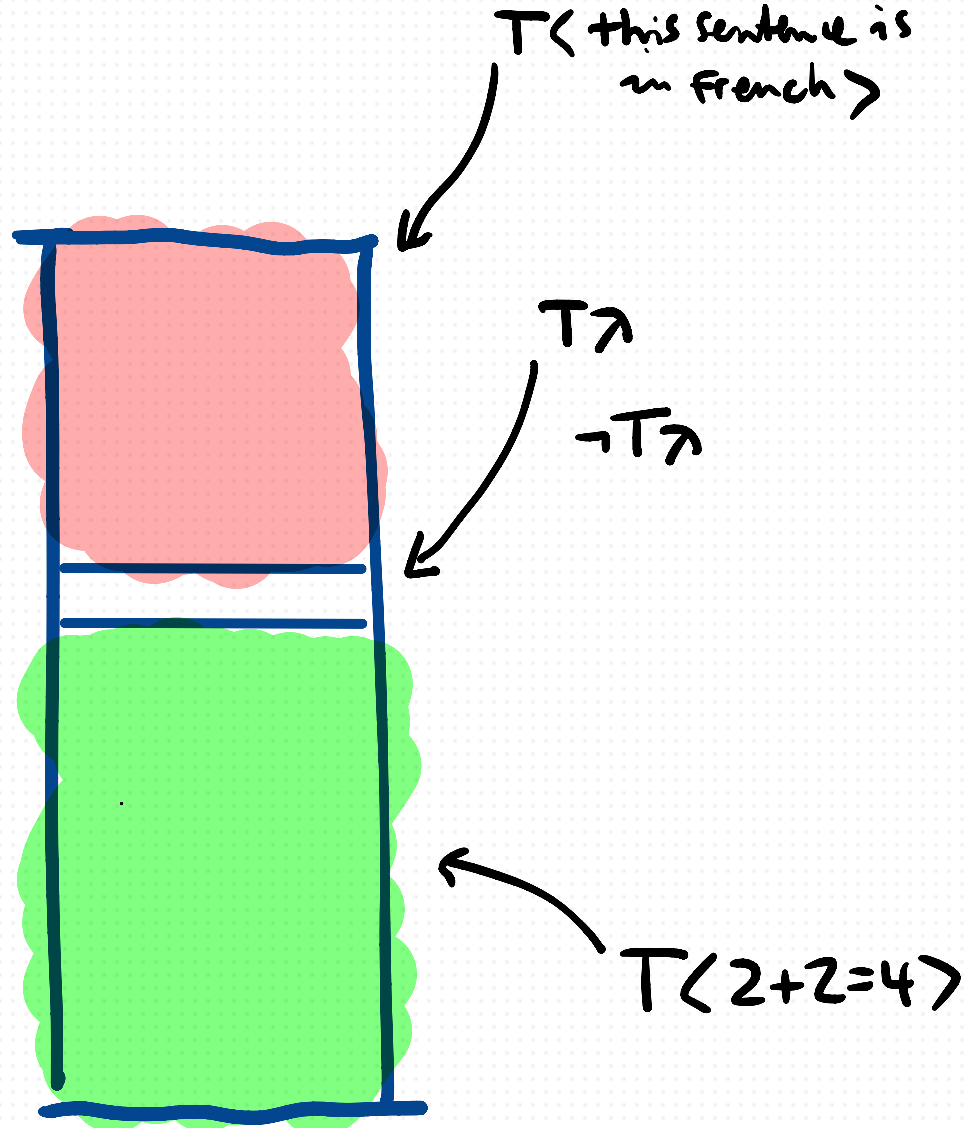


$$m_\kappa(A) = m_{\kappa+1}(T\langle A\rangle) = m_\kappa(T\langle A\rangle)$$

by definition

mk is a fixed point

$\lambda = \langle \neg T\lambda \rangle$



TODAY'S PLAN

FORMALISING one paradoxical argument

LOCATING different positions on this map

FORMALISING a different paradoxical argument

Noticing the PARALLELS

Kripke's MODEL
- why it's needed
- $\{0, n, 1\}$ & refinement
- stages & the fixed point

THE UPSHOT

NOT a 'SOLUTION' to the LIAR PARADOX

The naive reading $\left\{\begin{array}{ll} 0 & \text{TRUE} \\ n & \text{NEITHER} \\ 1 & \text{FALSE} \end{array}\right\}$ seems

susceptible to a revenge paradox.

λ is neither true nor false

$\neg T\lambda$ $\qquad\qquad$ $\neg\neg T\lambda$

BOTH n.

MORE THAN JUST THE LIAR PARADOX

- It's a **MODEL**: it gives a safety guarantee for the language it interprets.

- Nothing special about **negation**. Refinement is doing all the work ~ and it applies equally to → and the Curry paradox, as to the Liar

- Nothing special about **truth**. This technique applies equally well to property ascription and any other circular definitions.

NEXT WEEK

What can these models mean?

Can we be neutralist about these paradoxes?

Thank You!

https://consequently.org/

TODAY'S PLAN

THE FIXED POINT CONSTRUCTION

INTERPRETING $\{0, n, 1\}$

- TRUTH-VALUE GAPS
- TRUTH-VALUE GLUTS
- STRICT / TOLERANT ASSERTION

NEUTRALISM?

$$
\begin{prooftree}
\AxiomC{$\lambda = \langle \neg T\lambda \rangle$}
\AxiomC{$[T\lambda]^1$}
\RightLabel{$=E$}
\BinaryInfC{$T\langle \neg T\lambda \rangle$}
\RightLabel{TE}
\UnaryInfC{$\neg T\lambda$}
\AxiomC{$[T\lambda]^1$}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\RightLabel{$\neg I^1$}
\UnaryInfC{$\neg T\lambda$}
\AxiomC{$\lambda = \langle \neg T\lambda \rangle$}
\AxiomC{$\lambda = \langle \neg T\lambda \rangle$}
\AxiomC{$[T\lambda]^2$}
\RightLabel{$=E$}
\BinaryInfC{$T\langle \neg T\lambda \rangle$}
\RightLabel{TE}
\UnaryInfC{$\neg T\lambda$}
\AxiomC{$[T\lambda]^2$}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\RightLabel{$\neg I^2$}
\UnaryInfC{$\neg T\lambda$}
\RightLabel{TI}
\UnaryInfC{$T\langle \neg T\lambda \rangle$}
\RightLabel{$=E$}
\BinaryInfC{$T\lambda$}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\end{prooftree}
$$

$$
\begin{prooftree}
\AxiomC{$\neg A$}
\AxiomC{A}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\end{prooftree}
\qquad
\begin{prooftree}
\AxiomC{$[A]^i$}
\noLine
\UnaryInfC{$\vdots$}
\noLine
\UnaryInfC{$\bot$}
\RightLabel{$\neg I^i$}
\UnaryInfC{$\neg A$}
\end{prooftree}
\qquad
\begin{prooftree}
\AxiomC{A}
\RightLabel{TI}
\UnaryInfC{$T\langle A \rangle$}
\end{prooftree}
\quad
\begin{prooftree}
\AxiomC{$T\langle A \rangle$}
\RightLabel{TE}
\UnaryInfC{A}
\end{prooftree}
\qquad
\begin{prooftree}
\AxiomC{$a = b$}
\AxiomC{Fa}
\RightLabel{$=E$}
\BinaryInfC{Fb}
\end{prooftree}
$$

TODAY'S PLAN

THE FIXED POINT CONSTRUCTION

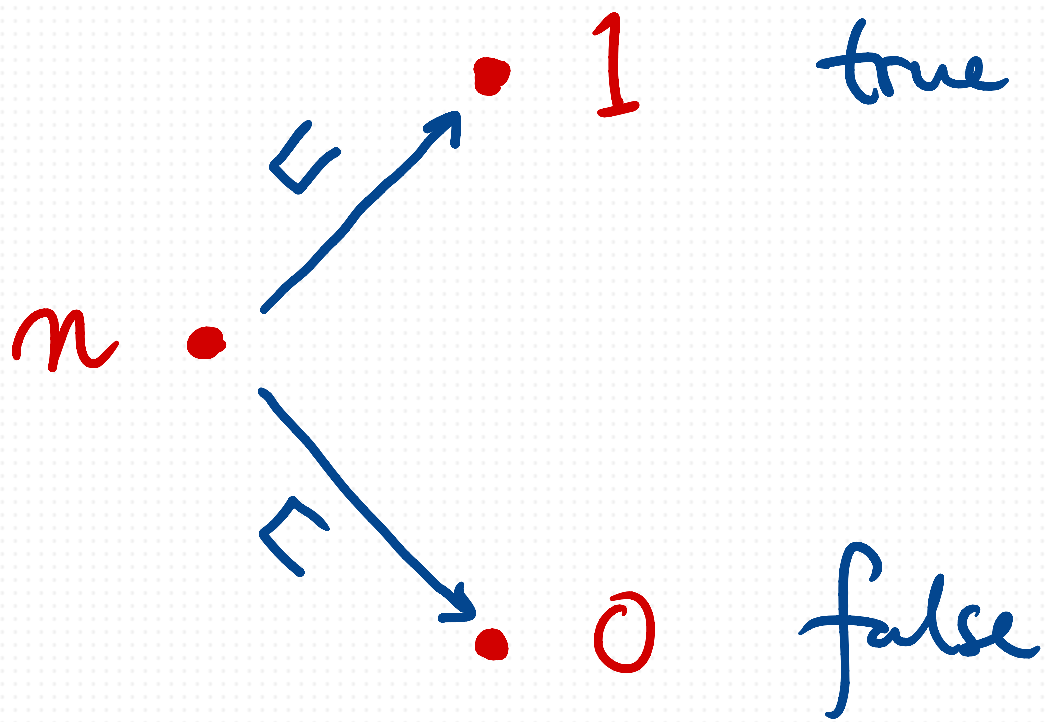
INTERPRETING $\{0, n, 1\}$

TRUTH-VALUE GAPS

TRUTH-VALUE GLUTS

STRICT / TOLERANT ASSERTION

NEUTRALISM?



$$x \sqsubseteq y \text{ iff } x \sqsubset y \text{ or } x = y$$

$m(\bot) = 0$

$$m(\neg A) = 1 \text{ iff } m(A) = 0$$
$$= 0 \text{ iff } m(A) = 1$$

$$m(A \wedge B) = 1 \text{ iff } m(A) = 1 \ \& \ m(B) = 1$$
$$= 0 \text{ iff } m(A) = 0 \text{ or } m(B) = 0$$

$$m(A \vee B) = 1 \text{ iff } m(A) = 1 \text{ or } m(B) = 1$$
$$= 0 \text{ iff } m(A) = 0 \ \& \ m(B) = 0$$

$$m(A \to B) = 1 \text{ iff } m(A) = 0 \text{ or } m(B) = 1$$
$$= 0 \text{ iff } m(A) = 1 \ \& \ m(B) = 0$$

Adding T to a formal language.

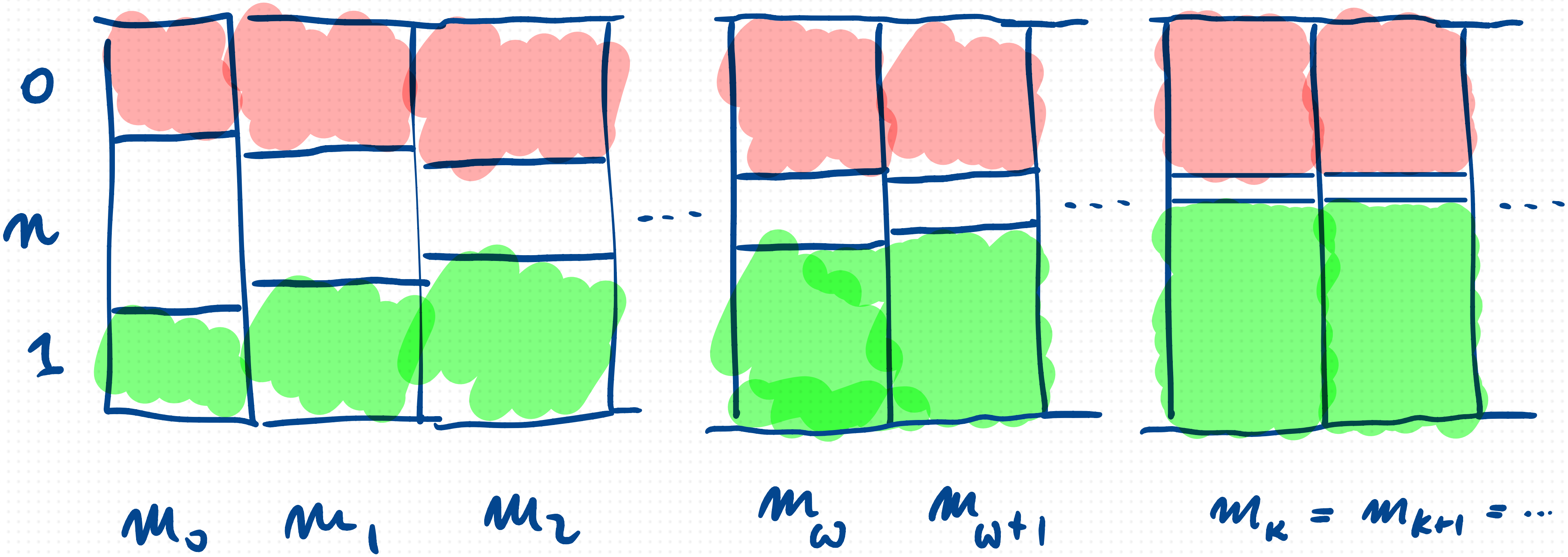
$\mathcal{M}_0$

- interpret the non T sentences however you like in $\{0, n, 1\}$.
- assign Tx the value n

We treat T-sentences as undetermined at stage 0, and we progressively refine them over stages.

(!)

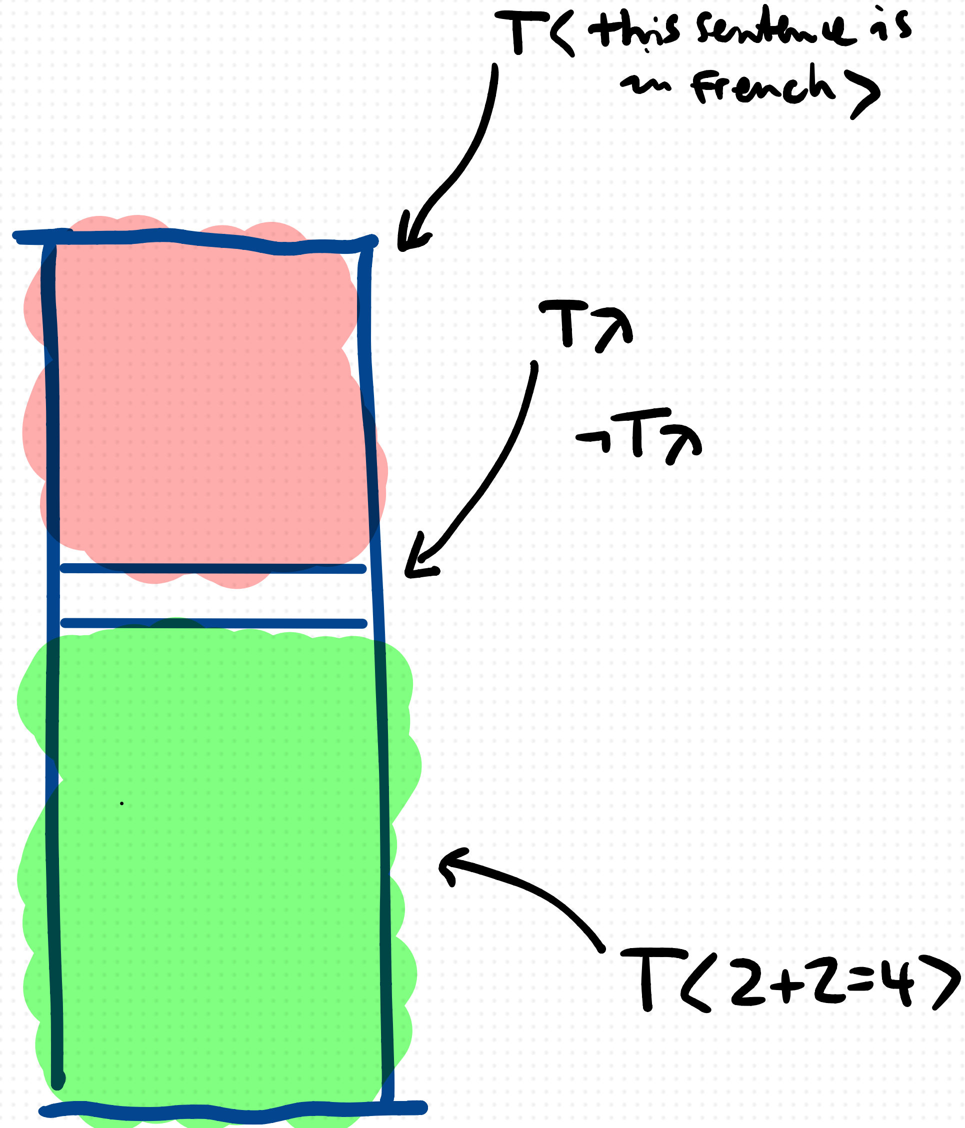
We eventually reach a fixed-point.



$$m_\kappa(A) = m_{\kappa+1}(T\langle A\rangle) = m_\kappa(T\langle A\rangle)$$

by definition $\qquad$ m_κ is a fixed point

$\lambda = \langle \neg T\lambda \rangle$



What does this mean?

TODAY'S PLAN

THE FIXED POINT CONSTRUCTION

INTERPRETING $\{0, n, 1\}$

TRUTH-VALUE GAPS

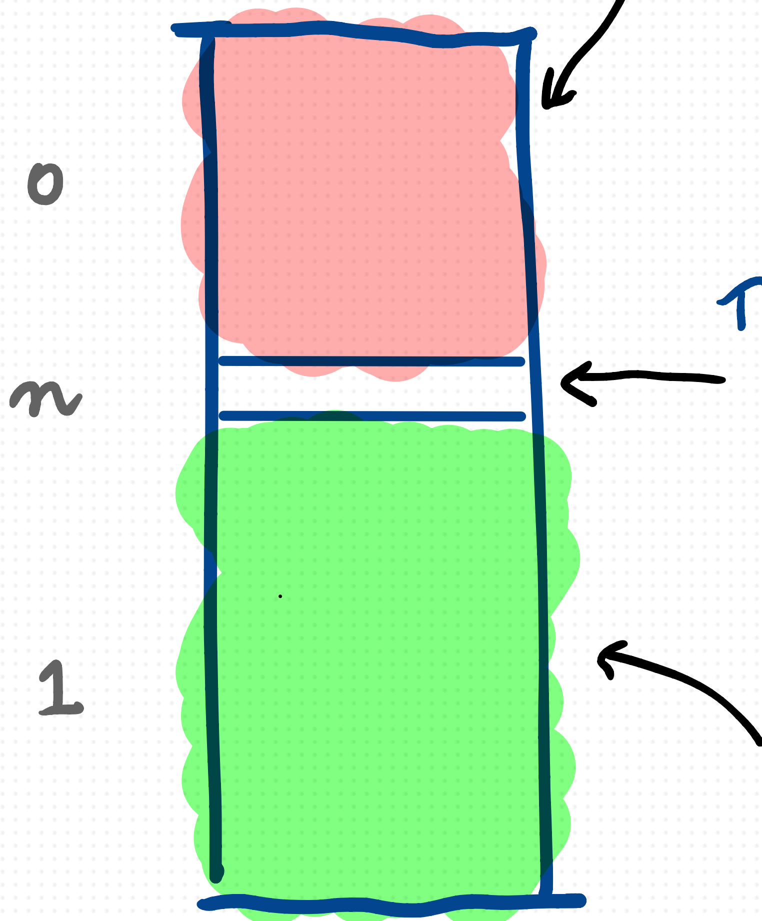
TRUTH-VALUE GLUTS

STRICT / TOLERANT ASSERTION

NEUTRALISM?

TRUTH-VALUE GAPS

According to this model...



These sentences are FALSE

These sentences are not TRUE & are not FALSE

These sentences are TRUE

$m(\bot) = 0$

$m(\neg A) = 1$ iff $m(A) = 0$
$ = 0$ iff $m(A) = 1$

$m(A \wedge B) = 1$ iff $m(A) = 1$ & $m(B) = 1$
$ = 0$ iff $m(A) = 0$ or $m(B) = 0$

$m(A \vee B) = 1$ iff $m(A) = 1$ or $m(B) = 1$
$ = 0$ iff $m(A) = 0$ & $m(B) = 0$

$m(A \to B) = 1$ iff $m(A) = 0$ or $m(B) = 1$
$ = 0$ iff $m(A) = 1$ & $m(B) = 0$

What is LOGICAL VALIDITY on this view?

An argument from X to A is valid iff for every model m, if $m(P) = 1$ for each $P \in X$, then $m(A) = 1$ too.

Validity is preservation of TRUTH.

VALID

VALID

NOT!

NOT VALID!

$$
\dfrac{
\dfrac{\dfrac{\dfrac{\dfrac{\lambda = \langle \neg T\lambda\rangle \quad [T\lambda]^1}{T\langle\neg T\lambda\rangle}\,{\scriptstyle =E}}{\neg T\lambda}\,{\scriptstyle TE} \quad [T\lambda]^1}{\bot}\,{\scriptstyle \neg E}}{\neg T\lambda}\,\cancel{\scriptstyle \neg I^1}
\qquad
\dfrac{\lambda = \langle\neg T\lambda\rangle \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\lambda = \langle\neg T\lambda\rangle \quad [T\lambda]^2}{T\langle\neg T\lambda\rangle}\,{\scriptstyle =E}}{\neg T\lambda}\,{\scriptstyle TE} \quad [T\lambda]^2}{\bot}\,{\scriptstyle \neg E}}{\neg T\lambda}\,{\scriptstyle \neg I^2}}{T\langle\neg T\lambda\rangle}\,{\scriptstyle TI}}{T\lambda}\,{\scriptstyle =E}
}{\bot}\,{\scriptstyle \neg E}
$$

VALID

$$\dfrac{\neg A \quad A}{\bot}\,{\scriptstyle \neg E}$$

NOT VALID

$$\cancel{\dfrac{\begin{matrix}[A]^1 \\ \vdots \\ \bot\end{matrix}}{\neg A}\,{\scriptstyle \neg I^1}}$$

VALID

$$\dfrac{A}{T\langle A\rangle}\,{\scriptstyle TI} \qquad \dfrac{T\langle A\rangle}{A}\,{\scriptstyle TE}$$

VALID

$$\dfrac{a=b \quad Fa}{Fb}\,{\scriptstyle =E}$$

- Sentences that are neither true nor false are still meaningful. (The meaning of 'T' forces $T\lambda$ to be assigned the value n.)

- ONE APPROACH: Sentences assigned n are interpreted as not expressing propositions (which must have determinate truth conditions, giving values 0 or 1.)
[So THE LOGIC OF PROPOSITIONS admits no GAPS]

- ANOTHER: These sentences do express propositions ~ they allow for gaps (a proposition is not just a set of worlds or anything like that).

REVENGE PROBLEMS

MODEL THEORY: $m(T\lambda) = n$

DIAGNOSIS: $T\lambda$ is neither true nor false.

IN THE LANGUAGE WE ARE MODELLING? $\neg(T\lambda \vee \neg T\lambda)$

This claim is also evaluated as n, not as TRUE.

Is there any way to represent the diagnosis in the language we are modelling?

TODAY'S PLAN

THE FIXED POINT CONSTRUCTION

INTERPRETING $\{0, n, 1\}$

TRUTH-VALUE GAPS

TRUTH-VALUE GLUTS

STRICT / TOLERANT

ASSERTION

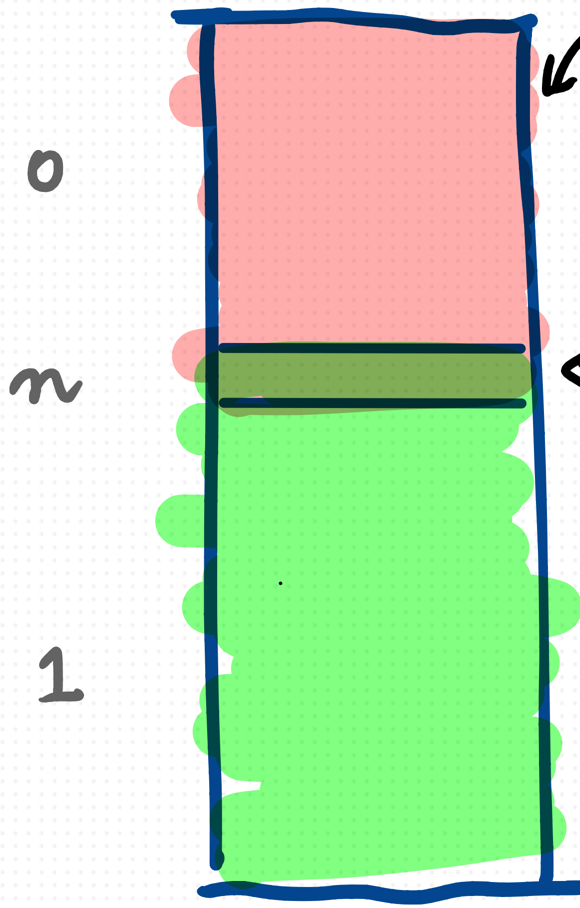
NEUTRALISM?

TRUTH-VALUE GLUTS

According to this model...

These sentences {0, n} are FALSE

These sentences {n} are both TRUE & FALSE



These sentences {n, 1} are TRUE

$$m(\bot) = 0 \qquad m(\neg A) = 1 \text{ iff } m(A) = 0$$
$$= 0 \text{ iff } m(A) = 1$$

$$m(A \wedge B) = 1 \text{ iff } m(A) = 1 \;\&\; m(B) = 1$$
$$= 0 \text{ iff } m(A) = 0 \text{ or } m(B) = 0$$

$$m(A \vee B) = 1 \text{ iff } m(A) = 1 \text{ or } m(B) = 1$$
$$= 0 \text{ iff } m(A) = 0 \;\&\; m(B) = 0$$

$$m(A \to B) = 1 \text{ iff } m(A) = 0 \text{ or } m(B) = 1$$
$$= 0 \text{ iff } m(A) = 1 \;\&\; m(B) = 0$$

What is LOGICAL VALIDITY on this view?

An argument from X to A is valid iff for every model m, if $m(P) \in \{n, 1\}$, for each $P \in X$, then $m(A) \in \{n, 1\}$ too.

Validity is preservation of TRUTH.

VALID

VALID

NOT VALID

NOT VALID

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\lambda = \langle \neg T\lambda \rangle \quad [T\lambda]^1}{T\langle \neg T\lambda \rangle}{\scriptstyle =E}}{\neg T\lambda}{\scriptstyle TE} \quad [T\lambda]^1}{\bot}{\scriptstyle \cancel{\neg E}}}{\neg T\lambda}{\scriptstyle \neg I^1} \quad \dfrac{\lambda = \langle \neg T\lambda \rangle \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\lambda = \langle \neg T\lambda \rangle \quad [T\lambda]^2}{T\langle \neg T\lambda \rangle}{\scriptstyle =E}}{\neg T\lambda}{\scriptstyle TE} \quad [T\lambda]^2}{\bot}{\scriptstyle \cancel{\neg E}}}{\neg T\lambda}{\scriptstyle \neg I^2}}{T\langle \neg T\lambda \rangle}{\scriptstyle TI}}{T\lambda}{\scriptstyle =E}}{\bot}{\scriptstyle \neg E}$$

NOT VALID

$$\cancel{\dfrac{\neg A \quad A}{\bot}{\scriptstyle \neg E}}$$

VALID

$$\dfrac{\begin{array}{c}[A]^i \\ \vdots \\ \bot\end{array}}{\neg A}{\scriptstyle \neg I^i}$$

VALID

$$\dfrac{A}{T\langle A \rangle}{\scriptstyle TI} \qquad \dfrac{T\langle A \rangle}{A}{\scriptstyle TE}$$

VALID

$$\dfrac{a = b \quad Fa}{Fb}{\scriptstyle =E}$$

$$m(A \vee \neg A) \in \{n, 1\}$$

A	A	$\vee$	$\neg A$
0	0	1	1
n	n	n	n
1	1	1	0

MODUS PONENS is INVALID for '$\to$'

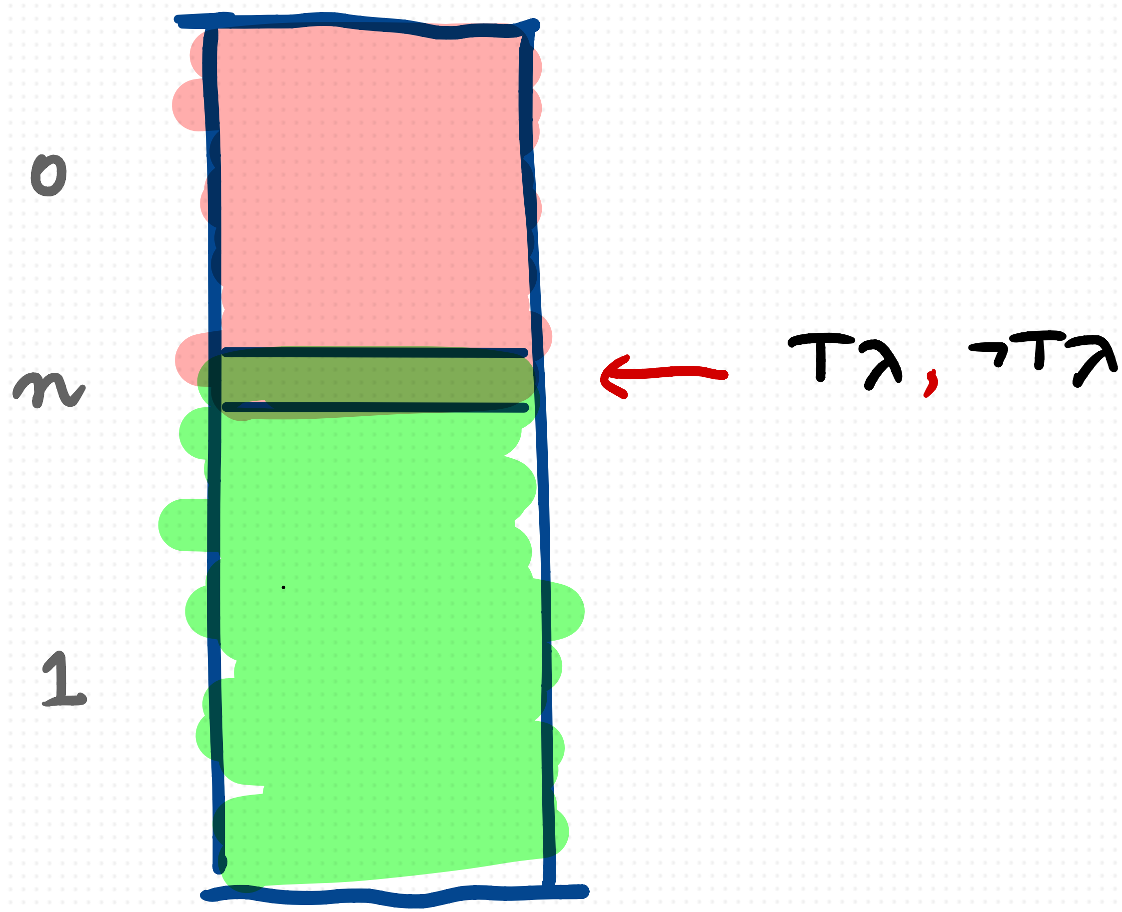
$$\underbrace{A \to B}_{n},\ \underbrace{A}_{n} \not\models \underbrace{B}_{0}$$

BOTH TRUE & FALSE

JUST FALSE

THE LIAR SENTENCE IS TRUE

(& its FALSE, too)



REVENGE PROBLEMS?

MODEL THEORY: $m(T\lambda) = n$

DIAGNOSIS: $T\lambda$ is both true and false.

IN THE LANGUAGE WE ARE MODELLING?

$$T\lambda \wedge \neg T\lambda$$

This claim is also evaluated as n, and so is TRUE.

THIS ALL SEEMS OK!

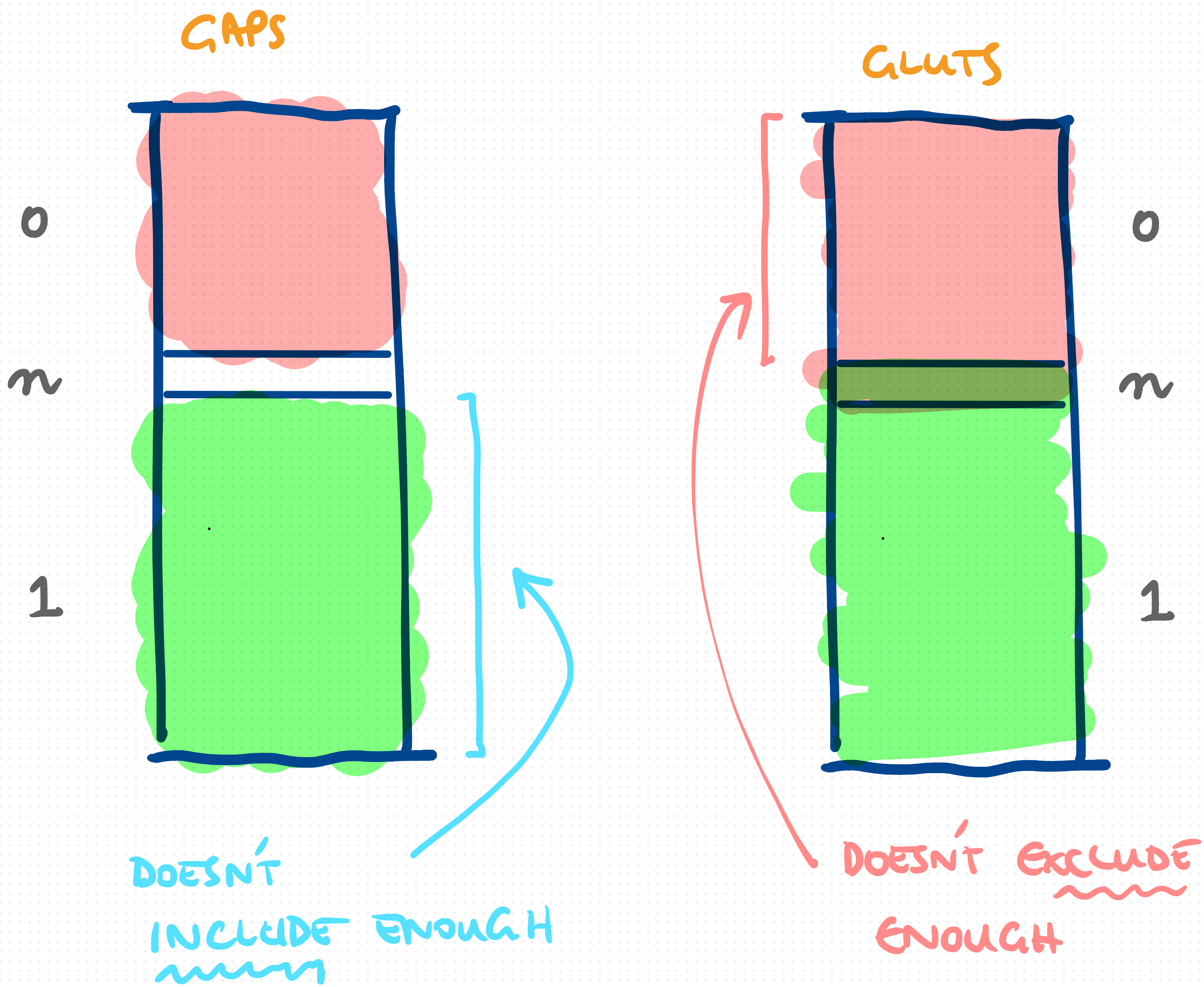
HOWEVER....

In this framework, negation does not express exclusion.

If we want to rule out a claim, we cannot simply assert its negation.

This is the EXPRESSIBILITY problem for 'GLUT' approaches to the paradoxes.

THESE PROBLEMS ARE STRANGELY SYMMETRIC




Why don’t we have both?

PLEASE FILL IN THE MEQ!

2022/3-S2-PY4601-MEQ

TODAY'S PLAN

THE FIXED POINT CONSTRUCTION

INTERPRETING $\{0, n, 1\}$

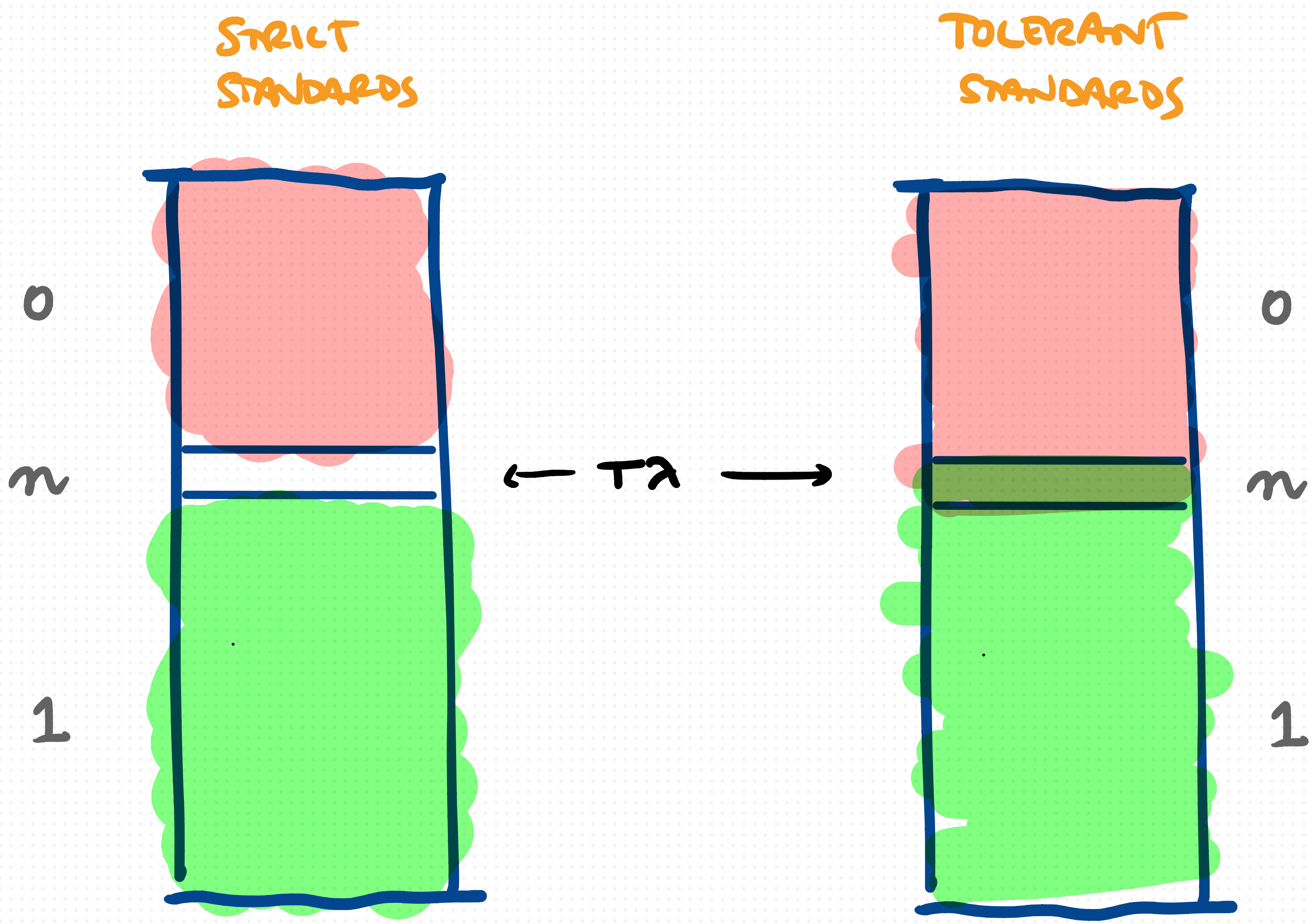
TRUTH-VALUE GAPS

TRUTH-VALUE GLUTS

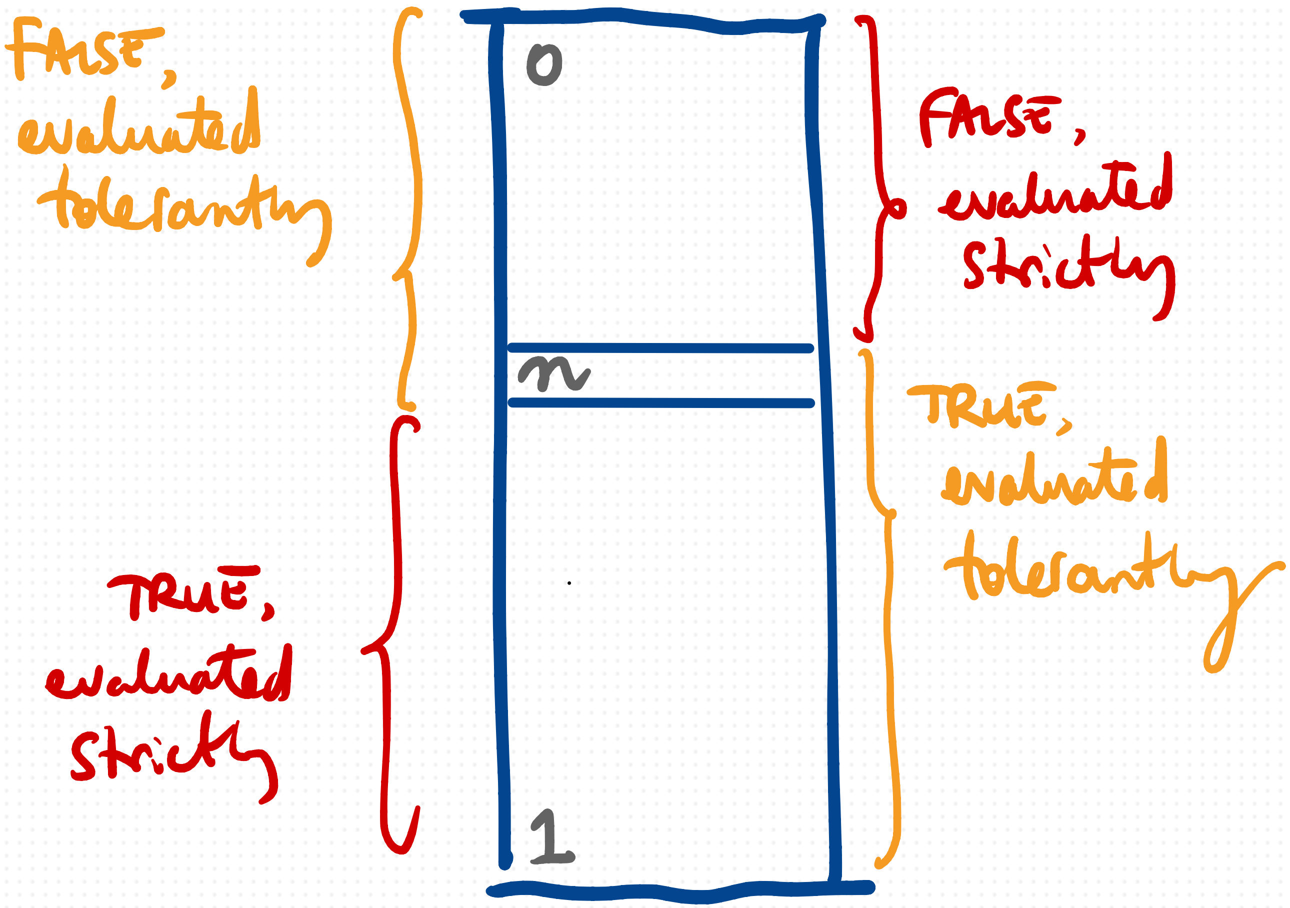
STRICT / TOLERANT ASSERTION

NEUTRALISM?

WE CAN BE STRICT OR TOLERANT



WE CAN BE STRICT OR TOLERANT



λ is true

fails, when evaluated strictly

succeeds, when evaluated tolerantly

THIS IS NOT SAYING THAT THERE ARE TWO KINDS OF TRUTH

REVISIONARY VIEWS OF TRUTH:

There are two truthlike concepts

T^{UP}: $A \to T^{UP}\langle A \rangle$

T_{DOWN}: $T_{DOWN}\langle A \rangle \to A$

STRICT/TOLERANT STANDARDS

There is one truth concept, for which $A \leftrightarrow T\langle A \rangle$,

But $T\langle A \rangle$ (and A) can be evaluated according to two different standards.

If there are two standards, what is validity?

$$A \overset{?}{\vDash} B$$

SS: If $m(A)=1$ then $m(B)=1$

TT: If $m(A)\in\{1,n\}$ then $m(B)\in\{1,n\}$

ST: If $m(A)=1$ then $m(B)\in\{1,n\}$

TS: If $m(A)\in\{1,n\}$ then $m(B)=1$

If there are two standards, what is validity?

$$A \overset{?}{\vDash} B$$

SS: If $m(A)=1$ then $m(B)=1$

Don't have $m(A)=1$ & $m(B)\in\{0,n\}$

TT: If $m(A)\in\{1,n\}$ then $m(B)\in\{1,n\}$

Don't have $m(A)\in\{1,n\}$ & $m(B)=0$

ST: If $m(A)=1$ then $m(B)\in\{1,n\}$

Don't have $m(A)=1$ & $m(B)=0$

TS: If $m(A)\in\{1,n\}$ then $m(B)=1$

Don't have $m(A)\in\{1,n\}$ & $m(B)\in\{0,n\}$

If there are two standards, what is validity?

$$A \overset{?}{\vDash} B$$

SS: If $m(A)=1$ then $m(B)=1$ → GAPS

Don't have $m(A)=1$ & $m(B)\in\{0,n\}$

TT: If $m(A)\in\{1,n\}$ then $m(B)\in\{1,n\}$ → GLUTS

Don't have $m(A)\in\{1,n\}$ & $m(B)=0$

ST: If $m(A)=1$ then $m(B)\in\{1,n\}$

Don't have $m(A)=1$ & $m(B)=0$

TS: If $m(A)\in\{1,n\}$ then $m(B)=1$

Don't have $m(A)\in\{1,n\}$ & $m(B)\in\{0,n\}$

If there are two standards, what is validity?

$$A \overset{?}{\vDash} B$$

SS: If $m(A)=1$ then $m(B)=1$ → GAPS

Don't have $m(A)=1$ & $m(B)\in\{0,n\}$

TT: If $m(A)\in\{1,n\}$ then $m(B)\in\{1,n\}$ → GLUTS

Don't have $m(A)\in\{1,n\}$ & $m(B)=0$

ST: If $m(A)=1$ then $m(B)\in\{1,n\}$

Don't have $m(A)=1$ & $m(B)=0$

$\boxed{\text{TS: If } m(A)\in\{1,n\} \text{ then } m(B)=1}$

ALMOST NOTHING IS VALID IN THIS SENSE!!

Don't have $m(A)\in\{1,n\}$ & $m(B)\in\{0,n\}$

If there are two standards, what is validity?

$$A \overset{?}{\vDash} B$$

GAPS

SS: If $m(A)=1$ then $m(B)=1$

Don't have $m(A)=1$ & $m(B)\in\{0,n\}$

GLUTS

TT: If $m(A)\in\{1,n\}$ then $m(B)\in\{1,n\}$

Don't have $m(A)\in\{1,n\}$ & $m(B)=0$

THIS IS INTERESTING!

ST: If $m(A)=1$ then $m(B)\in\{1,n\}$

Don't have $m(A)=1$ & $m(B)=0$

ALMOST NOTHING IS VALID IN THIS SENSE!!

TS: If $m(A)\in\{1,n\}$ then $m(B)=1$

Don't have $m(A)\in\{1,n\}$ & $m(B)\in\{0,n\}$

If there are two standards, what is validity?

$$A \vDash_{ST} B$$

IF THE PREMISES ARE STRICTLY TRUE, THE CONCLUSION IS AT LEAST TOLERANTLY TRUE.

THIS IS INTERESTING!

ST: If $m(A)=1$ then $m(B)\in\{1,n\}$

Don't have $m(A)=1$ & $m(B)=0$

If there are two standards, what is validity?

$$A \vDash_{ST} B$$

ST: If $m(A)=1$ then $m(B) \in \{1, n\}$

Don't have $m(A)=1$ & $m(B)=0$

IF THE PREMISES ARE STRICTLY TRUE, THE CONCLUSION IS AT LEAST TOLERANTLY TRUE.

$A \wedge \neg A \vDash_{ST} \perp$

$\top \vDash_{ST} A \vee \neg A$

If there are two standards, what is validity?

$$A \vDash_{ST} B$$

ST: If $m(A)=1$ then $m(B)\in\{1,n\}$

Don't have $m(A)=1$ & $m(B)=0$

IF THE PREMISES ARE STRICTLY TRUE, THE CONCLUSION IS AT LEAST TOLERANTLY TRUE.

$A \wedge \neg A \vDash_{ST} \perp$

$\top \vDash_{ST} A \vee \neg A$

In fact, if $X \vDash_{TV} A$ then $X \vDash_{ST} A$ too!

classical, two-valued logic

$$
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\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^1}{T\langle \neg T\lambda \rangle}{\scriptstyle =E}
}{\neg T\lambda}{\scriptstyle TE}
\qquad [T\lambda]^1
}{\bot}{\scriptstyle \neg E}
}{\neg T\lambda}{\scriptstyle \neg I^1}
\qquad
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\lambda = \langle \neg T\lambda \rangle
\qquad
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\dfrac{
\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^2}{T\langle \neg T\lambda \rangle}{\scriptstyle =E}
}{\neg T\lambda}{\scriptstyle TE}
\qquad [T\lambda]^2
}{\bot}{\scriptstyle \neg E}
}{\neg T\lambda}{\scriptstyle \neg I^2}
}{T\langle \neg T\lambda \rangle}{\scriptstyle TI}
}{T\lambda}{\scriptstyle =E}
}{\bot}{\scriptstyle \neg E}
$$

THESE MOVES ARE ST-VALID!

$$
\dfrac{\neg A \qquad A}{\bot}{\scriptstyle \neg E}
\qquad\qquad
\dfrac{\begin{matrix}[A]^i \\ \vdots \\ \bot\end{matrix}}{\neg A}{\scriptstyle \neg I^i}
\qquad\qquad
\dfrac{A}{T\langle A \rangle}{\scriptstyle TI}
\qquad
\dfrac{T\langle A \rangle}{A}{\scriptstyle TE}
\qquad\qquad
\dfrac{a=b \qquad Fa}{Fb}{\scriptstyle =E}
$$

$A, \neg A \models_{ST} \bot$ $\quad$ $X, A \models_{ST} \bot \Rightarrow X \models_{ST} \neg A$ $\quad$ $a=b, Fa \models_{ST} Fb$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^1}{T\langle \neg T\lambda \rangle}\,{\scriptstyle =E}}{\neg T\lambda}\,{\scriptstyle TE} \qquad [T\lambda]^1}{\bot}\,{\scriptstyle \neg E}}{\neg T\lambda}\,{\scriptstyle \neg I^1} \qquad \dfrac{\lambda = \langle \neg T\lambda \rangle \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^2}{T\langle \neg T\lambda \rangle}\,{\scriptstyle =E}}{\neg T\lambda}\,{\scriptstyle TE} \qquad [T\lambda]^2}{\bot}\,{\scriptstyle \neg E}}{\neg T\lambda}\,{\scriptstyle \neg I^2}}{T\langle \neg T\lambda \rangle}\,{\scriptstyle TI}}{T\lambda}\,{\scriptstyle =E}}{\bot}\,{\scriptstyle \neg E}$$

AND THESE HOLD IN ALL FIXED POINT MODELS!

THESE MOVES ARE ST-VALID!

$$\dfrac{\neg A \qquad A}{\bot}\,{\scriptstyle \neg E} \qquad\qquad \dfrac{\begin{matrix}[A]^i \\ \vdots \\ \bot\end{matrix}}{\neg A}\,{\scriptstyle \neg I^i} \qquad\qquad \dfrac{A}{T\langle A \rangle}\,{\scriptstyle TI} \qquad \dfrac{T\langle A \rangle}{A}\,{\scriptstyle TE} \qquad\qquad \dfrac{a=b \qquad Fa}{Fb}\,{\scriptstyle =E}$$

$A, \neg A \vDash_{ST} \bot$

$X, A \vDash_{ST} \bot \Rightarrow X \vDash_{ST} \neg A$

$A \vDash_{STT} T\langle A \rangle$

$T\langle A \rangle \vDash_{STT} A$

$a=b, Fa \vDash_{ST} Fb$

IN ANY FIXED POINT MODEL, the premises are strictly true

$$
\dfrac{
\dfrac{\dfrac{\dfrac{\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^1}{T\langle \neg T\lambda \rangle}{\scriptstyle =E}}{\neg T\lambda}{\scriptstyle TE} \qquad [T\lambda]^1}{\bot}{\scriptstyle \neg E}}{\neg T\lambda}{\scriptstyle \neg I^1}
\qquad
\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^2}{T\langle \neg T\lambda \rangle}{\scriptstyle =E}}{\neg T\lambda}{\scriptstyle TE} \qquad [T\lambda]^2}{\bot}{\scriptstyle \neg E}}{\neg T\lambda}{\scriptstyle \neg I^2}}{T\langle \neg T\lambda \rangle}{\scriptstyle TI}}{T\lambda}{\scriptstyle =E}
}{\bot}{\scriptstyle \neg E}
$$

& the conclusion is strictly false.

$$\dfrac{\neg A \qquad A}{\bot}{\scriptstyle \neg E} \qquad\qquad \dfrac{\begin{matrix}[A]^i \\ \vdots \\ \bot\end{matrix}}{\neg A}{\scriptstyle \neg I^i} \qquad\qquad \dfrac{A}{T\langle A \rangle}{\scriptstyle TI} \quad \dfrac{T\langle A \rangle}{A}{\scriptstyle TE} \qquad\qquad \dfrac{a=b \qquad Fa}{Fb}{\scriptstyle =E}$$

$$
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  \dfrac{
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      \dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^1}{T\langle \neg T\lambda \rangle}{\scriptstyle =E}
    }{\neg T\lambda}{\scriptstyle TE}
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  }{\dfrac{\bot}{\neg T\lambda}{\scriptstyle \neg I^1}}{\scriptstyle \neg E}
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    \lambda = \langle \neg T\lambda \rangle
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            \dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^2}{T\langle \neg T\lambda \rangle}{\scriptstyle =E}
          }{\neg T\lambda}{\scriptstyle TE}
          \qquad [T\lambda]^2
        }{\bot}{\scriptstyle \neg E}
      }{\neg T\lambda}{\scriptstyle \neg I^2}
    }{T\langle \neg T\lambda \rangle}{\scriptstyle TI}
  }{T\lambda}{\scriptstyle =E}
}{\bot}{\scriptstyle \neg E}
$$

So — WHERE IS THE ERROR IN THIS PROOF?

EVERY STEP IS VALID!

But the premises are strictly true & the conclusion is strictly false!

$$\dfrac{\neg A \qquad A}{\bot}\,{\scriptstyle \neg E} \qquad\qquad \dfrac{\begin{matrix}[A]^i \\ \vdots \\ \bot\end{matrix}}{\neg A}\,{\scriptstyle \neg I^i} \qquad\qquad \dfrac{A}{T\langle A \rangle}\,{\scriptstyle TI} \qquad \dfrac{T\langle A \rangle}{A}\,{\scriptstyle TE} \qquad\qquad \dfrac{a = b \qquad Fa}{Fb}\,{\scriptstyle =E}$$

THE ST-LOGIC OF FIXED POINT MODELS

$\lambda = \langle \neg T\lambda \rangle \vDash_{STT} \neg T\lambda$ $\qquad$ $\lambda = \langle \neg T\lambda \rangle \vDash_{STT} T\lambda$

$$\lambda = \langle \neg T\lambda \rangle \nvDash_{STT} \bot$$

$$T\lambda, \neg T\lambda \vDash_{STT} \bot$$

In general, STT consequence isn't transitive.

$$A \vDash_{STT} B, \quad B \vDash_{STT} C \not\Rightarrow A \vDash_{STT} C$$

STT-VALID

$$\dfrac{\dfrac{\dfrac{\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^1}{T\langle \neg T\lambda \rangle}\,{\scriptstyle =E}}{\neg T\lambda}\,{\scriptstyle TE} \qquad [T\lambda]^1}{\bot}\,{\scriptstyle \neg E}}{\neg T\lambda}\,{\scriptstyle \neg I^1}$$

STT-VALID

$$\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^2}{T\langle \neg T\lambda \rangle}\,{\scriptstyle =E}}{\neg T\lambda}\,{\scriptstyle TE} \qquad [T\lambda]^2}{\bot}\,{\scriptstyle \neg E}}{\neg T\lambda}\,{\scriptstyle \neg I^2}}{T\langle \neg T\lambda \rangle}\,{\scriptstyle TI}}{T\lambda}\,{\scriptstyle =E}$$

$$\dfrac{\neg T\lambda \qquad T\lambda}{\bot}\,{\scriptstyle \neg E}$$

STT-VALID

$$
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\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^1}{T\langle \neg T\lambda \rangle}{\scriptstyle =E}
}{\neg T\lambda}{\scriptstyle TE}
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\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^2}{T\langle \neg T\lambda \rangle}{\scriptstyle =E}
}{\neg T\lambda}{\scriptstyle TE}
\qquad [T\lambda]^2
}{\bot}{\scriptstyle \neg E}
}{\neg T\lambda}{\scriptstyle \neg I^2}
}{T\langle \neg T\lambda \rangle}{\scriptstyle TI}
}{T\lambda}{\scriptstyle =E}
}{\bot}{\scriptstyle \neg E}
$$

STT- INVALID

Although $A, \neg A \vDash_{STT} \bot$; $X \vDash_{STT} A$, $Y \vDash_{STT} \neg A$ $\not\Rightarrow$ $X, Y \vDash_{STT} \bot$

Speaking strictly, we reject the liar and its negation.

Speaking tolerantly, we accept the liar and its negation

(& so, we avoid the revenge problem for a gap-only approach)

$\neg A$ excludes A, speaking strictly

(& so, we avoid the expressibility problem for a glut-only approach)

Further Questions

- Logical Consequence is not transitive?

- Logical Consequence is plural & some consequence relations are not transitive?

TODAY'S PLAN

THE FIXED POINT CONSTRUCTION

INTERPRETING $\{0, n, 1\}$

- TRUTH-VALUE GAPS
- TRUTH-VALUE GLUTS
- STRICT/TOLERANT ASSERTION

NEUTRALISM?

NEUTRALISM?

Suppose A & B lead to a problematic conclusion Z.

Axis 1: Stay neutral as to which of A, B are false.

Axis 2: Reject Z using neutral theory.

Axis 3: Stay neutral as to whether — (a) only one of A, B are false & its feasible to find out which; (b) ... its infeasible to find out which; and (c) ... it is metaphysically impossible to find out ...

THIS IS LIKE NEUTRALISM, BUT DIFFERENT

Axis 1*: Incorporate components from GAP & GLUT approaches.

Axis 2*: Analyse the semantic paradoxes using standard account of fixed-point models.

Axis 3*: Find space for all three notions of validity SS-validity (gap-logic) TT-validity (glut-logic), and ST-validity (new, non-transitive logic)

Distinctive View: The very notion of what it is to accept & to reject a claim is itself at issue!

Any Questions?