Justification Requests Inference, and Definitions

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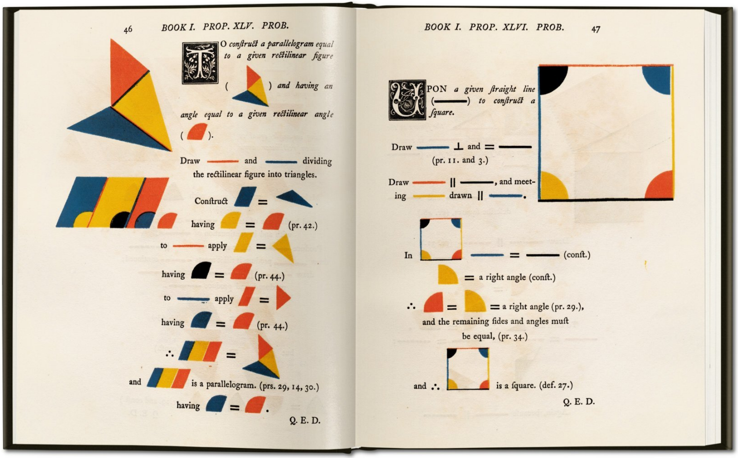
Deduction is compelling



The First Six Books of The Elements of Euclid, Oliver Byrne, 1847.

What is the compulsion?

"That beautiful First Proposition of Euclid!" the Tortoise murmured dreamily. "You admire Euclid?"

"Passionately! So far, at least, as one *can* admire a treatise that wo'n't be published for some centuries to come!"

"Well, now, let's take a little bit of the argument in that First Proposition—just *two* steps, and the conclusion drawn from them. Kindly enter them in your note-book. And in order to refer to them conveniently, let's call them *A*, *B*, and *Z*:—

(*A*) Things that are equal to the same are equal to each other.

(*B*) The two sides of this Triangle are things that are equal to the same.

(*Z*) The two sides of this Triangle are equal to each other.

Readers of Euclid will grant, I suppose, that *Z* follows logically from *A* and *B*, so that any one who accepts *A* and *B* as true, *must* accept *Z* as true?"

"Undoubtedly! The youngest child in a High School—as soon as High Schools are invented, which will not be till some two thousand years later—will grant *that*."

"And if some reader had *not* yet accepted *A* and *B* as true, he might still accept the *sequence* as a *valid* one, I suppose?"

"What the Tortoise Said to Achilles", Lewis Carroll, *Mind* **4** 1895, 278–280.

What is the compulsion?

"No doubt such a reader might exist. He might say 'I accept as true the Hypothetical Proposition that, *if* A and B be true, Z must be true; but, I *don't* accept A and B as true.' Such a reader would do wisely in abandoning Euclid, and taking to football."

"And might there not *also* be some reader who would say 'I accept A and B as true, but I *don't* accept the Hypothetical'?"

"Certainly there might. *He*, also, had better take to football."

"And *neither* of these readers," the Tortoise continued, "is *as yet* under any logical necessity to accept Z as true?"

"Quite so," Achilles assented.

"Well, now, I want you to consider *me* as a reader of the *second* kind, and to force me, logically, to accept Z as true."

"A tortoise playing football would be—" Achilles was beginning

"—an anomaly, of course," the Tortoise hastily interrupted. "Don't wander from the point. Let's have Z first, and football afterwards!"

"I'm to force you to accept Z, am I?" Achilles said musingly. "And your present position is that you accept A and B, but you *don't* accept the Hypothetical—"

"Let's call it C," said the Tortoise.

"—but you *don't* accept

(C) If A and B are true, Z must be true."

"That is my present position," said the Tortoise.

"Then I must ask you to accept C."

"I'll do so," said the Tortoise, "as soon as you've entered it in that note-book of yours. What else have you got in it?"

"What the Tortoise Said to Achilles", Lewis Carroll, *Mind* **4** 1895, 278–280.

What is the compulsion?

"Plenty of blank leaves, I see!" the Tortoise cheerily remarked. "We shall need them *all*!" (Achilles shuddered.) "Now write as I dictate:—

(*A*) Things that are equal to the same are equal to each other.

(*B*) The two sides of this Triangle are things that are equal to the same.

(*C*) If *A* and *B* are true, *Z* must be true.

(*Z*) The two sides of this Triangle are equal to each other."

"You should call it *D*, not *Z*," said Achilles. "It comes *next* to the other three. If you accept *A* and *B* and *C*, you *must* accept *Z*."

"And why *must* I?"

"Because it follows *logically* from them. If *A* and *B* and *C* are true, *Z must* be true. You don't dispute *that*, I imagine?"

"If *A* and *B* and *C* are true, *Z must* be true," the Tortoise thoughtfully repeated. "That's *another* Hypothetical, isn't it? And, if I failed to see its truth, I might accept *A* and *B* and *C*, and *still* not accept *Z*, mightn't I?"

"You might," the candid hero admitted; "though such obtuseness would certainly be phenomenal. Still, the event is *possible*. So I must ask you to grant *one* more Hypothetical."

"Very good. I'm quite willing to grant it, as soon as you've written it down. We will call it

(*D*) If *A* and *B* and *C* are true, *Z* must be true.

Have you entered that in your note-book?"

"I *have*!" Achilles joyfully exclaimed, as he ran the pencil into its sheath. "And at last we've got to the end of this ideal race-course! Now that you accept *A* and *B* and *C* and *D*, *of course* you accept *Z*."

"Do I?" said the Tortoise innocently. "Let's make that quite clear. I accept *A* and *B* and *C* and *D*. Suppose I *still* refused to accept *Z*?"

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Where (if anywhere) does the Tortoise go *wrong*?

What kind of error (if any) is involved in resisting valid deduction?

These are *public*, *communicative* acts.

I'll be looking at *speech acts*,
and the norms governing them
to shed light on our question.

The behaviour of two kinds of speech acts:

(1) *polar* (yes/no) *questions*,

and (2) *justification requests*.

My ***Plan***

Assertion and Denial

Polar Questions

Positions and Rules

Justification Requests

ASSERTION AND DENIAL

Multiple Conclusions (2005)

$$X \succ Y$$

Don't *assert* each member of X
and *deny* each member of Y.

An example derivation

$$
\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{(\neg q \land \neg r) \to p \succ (\neg q \land \neg r) \to p}{(\neg q \land \neg r) \to p, \neg q \land \neg r \succ p}\,{\to}Df}{(\neg q \land \neg r) \to p, \neg q, \neg r \succ p}\,{\land}Df}{(\neg q \land \neg r) \to p, \neg r \succ p, q}\,{\neg}Df}{(\neg q \land \neg r) \to p \succ p, q, r}\,{\neg}Df}{(\neg q \land \neg r) \to p, \neg p \succ q, r}\,{\neg}Df}{(\neg q \land \neg r) \to p, \neg p \succ q, \neg p \to r}\,{\to}Df}{(\neg q \land \neg r) \to p \succ \neg p \to q, \neg p \to r}\,{\to}Df}{(\neg q \land \neg r) \to p \succ (\neg p \to q) \lor (\neg p \to r)}\,{\lor}Df
$$

Structural Rules

$$X, A \succ A, Y \; \mathit{Id} \qquad \frac{X \succ A, Y \quad X, A \succ Y}{X \succ Y} \mathit{Cut}$$

These rules govern assertion and denial *as such*.

Defining Rules for Logical Concepts

$$\frac{X, A, B \succ Y}{X, A \land B \succ Y}\,\land Df \qquad \frac{X \succ A, B, Y}{X \succ A \lor B, Y}\,\lor Df \qquad \frac{X \succ A, Y}{X, \neg A \succ Y}\,\neg Df \qquad \frac{X, A \succ B, Y}{X \succ A \to B, Y}\,\to Df$$

$$\frac{X \succ A(n), Y}{X \succ \forall x A(x), Y}\,\forall Df \qquad \frac{X, A(n) \succ Y}{X, \exists x A(x) \succ Y}\,\exists Df \qquad \frac{X, Fa \succ Fb, Y \quad X, Fb \succ Fa, Y}{X \succ a = b, Y}\,{=}Df$$

Terms & conditions: the singular term n (in $\forall/\exists$*Df*) and the predicate F (in $=$*Df*) do not appear below the line in those rules.

These rules can be understood as *definitions* of the concepts they introduce (below the double line).

See (Scott 1974; Došen 1980, 1989; Restall 2019).

Let's get clearer on assertion and denial

It is fruitful to think of assertion as an act governed by *norms*.

*For **me**: Production Norms*

Aim to say what is *true*!

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Aim to say what is *true*!

Only say what you *know*!

*For **me**: Production Norms*

Aim to say what is *true*!

Only say what you *know*!

Be prepared to *back it up* when requested!

The hearer is entitled to re-assert.

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You can refer back to the asserter
to *vouch for* the assertion.

*For **us**: Our Common Ground*

To assert is to bid for the content asserted
to be added to the COMMON GROUND,
the body of information that
we (together) take for granted.

Stalnaker on Common Ground

To presuppose something is to take it for granted, or at least to act as if one takes it for granted, as background information as common ground among the participants in the conversation. What is most distinctive about this propositional attitude is that it is a social or public attitude: *one presupposes that* ϕ *only if one presupposes that others presuppose it as well.*

— "Common Ground" *L&P* (2002)

What is the relationship between Assertion and Denial?

In "Multiple Conclusions", I said little beyond the claim that assertion and denial clash

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This does not help distinguish *denial* from *retraction*, or from other speech acts.

Let's address this issue…

… by examining polar questions,
and their answers,
in the light of our background
interest in assertion and its norms.

POLAR QUESTIONS

This is a distinct speech act
with its own norms.

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It raises an *issue*.

*There are two ways to **settle** the issue*

Yes

*There are two ways to **settle** the issue*

Yes *No*

The two ways ***clash***

If I say *yes* and you say *no*
to some polar question p?,
then we DISAGREE.

That is, we take *different* positions on p.

The two ways ***clash***

If I say *yes* and you say *no*
to some polar question p?,
then we DISAGREE.

That is, we take *different* positions on p.

There is no *shared* position
incorporating both of our answers.

Other responses, like

Other responses, like

maybe

Other responses, like

maybe · *I don’t know*

Other responses don't settle the issue

Other responses, like

maybe · *I don't know* · *I think so*

Other responses, like

maybe · *I don't know* · *I think so*

are acceptable responses to p?,

Other responses, like

maybe · *I don't know* · *I think so*

are acceptable responses to p?,

but they don't settle the issue.

A *yes* or a *no* to p? counts as an assertion.

Settling answers are assertions

A *yes* or a *no* to p? counts as an assertion.

(Either answer is governed by all of the assertion norms we've seen.)

What does a "no" to p? ***assert***?

Presumably $\neg p$.

*What does a "no" to p? **assert**?*

Presumably ¬p.

However, I prefer to think of
a *yes* to p? as ruling p *in*,
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This way, we can distinguish practices
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(Nothing important *here* hangs on this distinction.)

$$[X : Y]$$

a pair of sets of sentences

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- We have ruled *in* everything in X, the POSITIVE COMMON GROUND.

Common Ground

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- We have ruled *in* everything in X, the POSITIVE COMMON GROUND.
- We have ruled *out* everything in Y, the NEGATIVE COMMON GROUND.

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- We have ruled *in* everything in X, the POSITIVE COMMON GROUND.
- We have ruled *out* everything in Y, the NEGATIVE COMMON GROUND.

Think of this as part of the *conversational scoreboard*.
There are also our public *individual* commitments,
the questions under discussion, and much more.

Denial and Retraction

ABELARD: Astralabe is in the study.

ELOISE: No, he is in the kitchen.

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ABELARD: Is Astralabe in the study?

ELOISE: *No, he is *either* in the kitchen *or* the study.

INAPPROPRIATE

ELOISE: Maybe. He's *either* in the kitchen *or* the study.

Denial and Retraction

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ELOISE: No, he is in the kitchen.

ABELARD: Is Astralabe in the study?

ELOISE: No, he is in the kitchen.

STRONG DENIAL

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PARTIAL ANSWER

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WEAK DENIAL

ABELARD: Is Astralabe in the study?

ELOISE: No, he is in the kitchen.

STRONG DENIAL

ABELARD: Is Astralabe in the study?

ELOISE: *No, he is *either* in the kitchen *or* the study.

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PARTIAL ANSWER

Strong and Weak Denial

- To *strongly deny* p is to bid to add p to the *negative common ground*.

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- To *strongly deny* p is to bid to add p to the *negative common ground*.
- To *weakly deny* p is to *block* the addition of p to the *positive common ground*, or to bid for its *retraction* if it is already there.

Strong and Weak Denial, and the Common Ground

- Strong *or* weak denials of p are appropriate responses to an assertion of p, because the assertion bids to add p to the positive common ground.

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Strong and Weak Denial, and the Common Ground

- Strong *or* weak denials of p are appropriate responses to an assertion of p, because the assertion bids to add p to the positive common ground.

- A strong denial of p is one way to settle $p?$ — negatively.

- A weak denial of p is not generally an appropriate response to the $p?$, as the question does not place p in the positive common ground. The question would be inapt if p were already in the positive common ground, so there is no p to block or retract.

- STRONG DENIAL: add to the negative common ground.

Strong and Weak Denials, and Strong and Weak ***Assertions***

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Strong and Weak Denials, and Strong and Weak ***Assertions***

- STRONG DENIAL: add to the negative common ground.
- STRONG ASSERTION: add to the positive common ground.
- WEAK DENIAL: retract (or block) from the positive common ground.
- WEAK ASSERTION: retract (or block) from the negative common ground. — "Perhaps p."

That's one way to understand the relationship between assertion and denial, and how to distinguish strong denial from other negative speech acts.

One Consequence

The common ground
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ELOISE: No. The interior angles of *this* triangle add up to 180°. Can you prove the general case?

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ELOISE: No. The interior angles of *this* triangle add up to 180°. Can you prove the general case?

Eloise blocks from the common ground (weakly denies)
a *logical consequence* of the common ground (the axioms of geometry),
for the same kind of reason we accept for other weak denials.

(This would be impossible if the common ground was simply a set of worlds.)

If $X \succ Y$ is derivable,
then it's out of bounds
to *strongly assert* each member of X
and *strongly deny* each member of Y.

Logical Consequence and Strong or Weak Denial

If $X \succ Y$ is derivable,
then it's out of bounds
to *strongly assert* each member of X
and *strongly deny* each member of Y.

But this example shows that
it *need not* be out of bounds to
strongly assert each member of X
and *weakly deny* each member of Y.

POSITIONS AND RULES

Defining Rules

$$\frac{X, A, B \succ Y}{X, A \land B \succ Y}\ {\land}Df \qquad \frac{X \succ A, B, Y}{X \succ A \lor B, Y}\ {\lor}Df$$

$$\frac{X \succ A, Y}{X, \neg A \succ Y}\ {\neg}Df \qquad \frac{X, A \succ B, Y}{X \succ A \to B, Y}\ {\to}Df$$

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These are kinds of *definitions*, showing how to treat assertions or denials of the *defined* concept in terms of the assertions or denials of their components.

Derivations

$$\dfrac{\dfrac{\neg p \succ \neg p}{\succ p, \neg p}\,\neg Df}{\succ p \lor \neg p}\,\lor Df$$

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$$\dfrac{\dfrac{\neg p \succ \neg p}{\succ p, \neg p}\,\neg Df}{\succ p \lor \neg p}\,\lor Df \qquad \dfrac{\dfrac{p \succ p}{p, \neg p \succ}\,\neg Df}{p \land \neg p \succ}\,\land Df$$

Derivations

$$\dfrac{\dfrac{\neg p \succ \neg p}{\succ p, \neg p}\,\neg Df}{\succ p \vee \neg p}\,\vee Df \qquad \dfrac{\dfrac{p \succ p}{p, \neg p \succ}\,\neg Df}{p \wedge \neg p \succ}\,\wedge Df$$

$$\dfrac{\dfrac{\dfrac{\dfrac{p, q \vee r \succ p \wedge q, q \vee r}{p, q \vee r \succ p \wedge q, r, q}\,\vee Df \qquad \dfrac{p \wedge q, q \vee r \succ p \wedge q, r}{q, p, q \vee r \succ p \wedge q, r}\,\wedge Df}{p, q \vee r \succ p \wedge q, r}\,Cut}{p, q \vee r \succ (p \wedge q) \vee r}\,\vee Df}{p \wedge (q \vee r) \succ (p \wedge q) \vee r}\,\wedge Df$$

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Sequent Derivations aren't exactly ***Proofs***

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Sequent Derivations aren't exactly ***Proofs***

- They don't have the same *shape* as proofs.
- (Where is the *conclusion* in $p \vee q \succ p, q$?)
- A endsequent $X \succ A$ doesn't tell you to *infer* A *from* X — it merely tells you to not assert all members of X and deny A.

The Tortoise doesn't violate this norm

"Well, now, let's take a little bit of the argument in that First Proposition—just *two* steps, and the conclusion drawn from them. Kindly enter them in your note-book. And in order to refer to them conveniently, let's call them *A*, *B*, and *Z*:—

(*A*) Things that are equal to the same are equal to each other.

(*B*) The two sides of this Triangle are things that are equal to the same.

(*Z*) The two sides of this Triangle are equal to each other.

Readers of Euclid will grant, I suppose, that *Z* follows logically from *A* and *B*, so that any one who accepts *A* and *B* as true, *must* accept *Z* as true?"

"Undoubtedly! The youngest child in a High School—as soon as High Schools are invented, which will not be till some two thousand years later—will grant *that*."

"And if some reader had *not* yet accepted *A* and *B* as true, he might still accept the *sequence* as a *valid* one, I suppose?"

The Tortoise never asserts A, B and $(A \wedge B) \rightarrow Z$ while denying Z, but he doesn't accept A B and $(A \wedge B) \rightarrow Z$ *as a reason* for Z.

JUSTIFICATION REQUESTS

What is a justification request?

ABELARD: Astralabe is in the kitchen.

ELOISE: *Really*?

ABELARD: I saw him there five minutes ago.

ELOISE: OK.

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ELOISE: *Are you sure*? He's been in the study with me for the last half hour.

ABELARD: Astralabe is in the kitchen.
ELOISE: *Really*?
ABELARD: I saw him there five minutes ago.
ELOISE: Yes, but he was in the study two minutes ago.

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This reason is another assertion [or denial]
which must be *granted*, (added to the common ground)
in order for the request to be met.

What is a justification request?

A justification request for a strong assertion [or strong denial] is an attempt to block the addition to the common ground, until a *reason* is given.

This reason is another assertion [or denial] which must be *granted*, (added to the common ground) in order for the request to be met.

Granting the given reason is *necessary* but not *sufficient* for satisfying the justification request.

ACHILLES So … this is an *equilateral* triangle.

TORTOISE I'm sorry, I don't follow, my heroic friend. I've not heard that word before: what does '*equilateral*' mean?

ACHILLES Oh, that's easy to explain. '*Equilateral*' means having sides of the same length. An *equilateral* triangle is a triangle with all three sides the same length.

TORTOISE OK. That sounds good. You may continue with your reasoning.

ACHILLES Well, as I was saying, the sides of this triangle are all one cubit in length, so it is an equilateral triangle.

TORTOISE Perhaps you will forgive me, Achilles, but I still don't follow. I grant to you that the sides of this triangle all have the same length. I fail to see, however, that it *follows* that it is an equilateral triangle. Could you explain why it is?

If I accept the definition $A =_{df} B$,
then I should accept granting A as meeting
a justification request for the assertion of B
and ruling out A as meeting a justification
request for B's denial and *vice versa*.

A failure to accept this is a sign
that I have not mastered the definition.

What goes for a definition of the form $A =_{\mathrm{df}} B$ can also go for *defining rules*:

$$\frac{X, A, B \succ Y}{X, A \land B, \succ Y}\ {\land}Df$$

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It is a mistake to grant A and grant B
and to look for something more to discharge
a justification request for an assertion of $A \land B$,
if you take $\land Df$ as a *definition*.

Justification Requests and Defining Rules

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It is a mistake to rule A in and rule B out and to look for something more to discharge a justification request for a denial of $A \to B$ if you accept $\to$*Df* as a definition.

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Consider this *focussed* derivation:

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- Read the *premise* as telling us that in a position in which $A \to Z$ is already ruled in, we have an answer to the justification request for $A \to Z$'s assertion.

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- Then applying $\to Df$ we see why we have an answer to the request concerning Z's assertion, in a context in which $A \to Z$ and A have both been ruled in. (In granting $A \to Z$ and A we have settled Z positively. Its denial is ruled out, since to assert A and deny Z amounts to denying $A \to Z$.)

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(Note: it's the *derivation* that shows how to meet the justification request, not the mere validity of the sequent.)

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- That's permissible, of course, but if you resist *modus ponens* for a concept like our everyday "*if* . . . *then*. . ." raises the question of what you *do* take that phrase to mean.
- A derivation of a sequent $X \succ A, Y$ [$X, A \succ Y$] can be transformed into a *procedure* for meeting a justification request for an assertion of A [denial of A] in any available position, appealing only what is granted in [$X : Y$], and to the *defining rules* used in that derivation.

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- The bounds, by themselves, can transcend our grasp.
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- Derivations are one way we can *grasp* complex bounds and *enforce* them.
- The *negative* view of the bounds is seen in the clash between assertion and denial, and the *positive* view is found in the answers we can give to justification requests.
- Adopting *defining rules* is one way to be *very* precise about the norms governing the concepts so defined, combining *safety*, *univocity* and *expressive power*.

THANK YOU!

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