

# *What's so Special about Logic? Practices, Rules and Definitions*

*Greg Restall*



ALICE AMBROSE LAZEROWITZ – THOMAS TYMOCKO MEMORIAL LOGIC LECTURE

4 DECEMBER 2019 / SMITH COLLEGE

## *My Aim*

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To understand *logic* better...

## *My Aim*

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To understand *logic* better...

... and especially, to understand  
what makes logic *distinctive*.

# Alice Ambrose Lazerowitz

## A CONTROVERSY IN THE LOGIC OF MATHEMATICS\*

THE impetus of the attack on Cantor's theory of transfinities carried assailants and defenders into an examination of the foundations of mathematics, and thence into a questioning of logic as a substructure for mathematics. Is logic basic, as Frege and Peano assumed, and are its laws final? Brouwer has answered this question with a flat no. Scorning any grounding of mathematics in logic, he has given primacy to mathematical intuition and forced logic to play second fiddle. This divergence of view among workers in foundations of mathematics will constitute the subject-matter of the present discussion.

The issue between Brouwer's intuitionist school and its opponents was sharply defined with the declaration by the latter of the unlimited validity of the law of excluded middle while the former protested an *ignorabimus* with reference to it. The considerations which governed the stand taken were certain conceptions of the nature of mathematics and of the relation of logic to it. Now on this point a threefold rather than twofold cleavage came to pass, each camp proclaiming its own remedy for the mathematical difficulty. Two of the three schools agree on the legitimacy of the processes at issue: unrestricted definition of the questionable entities, *e.g.*, of the set of all real numbers, and the extension of the law of excluded middle to reasoning on infinite sets. But these same schools have different answers to Brouwer's criticisms. At least I

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THE FOUR-COLOR PROBLEM AND ITS  
PHILOSOPHICAL SIGNIFICANCE \*

THE old four-color problem was a problem of mathematics for over a century. Mathematicians appear to have solved it to their satisfaction, but their solution raises a problem for philosophy which we might call the *new four-color problem*.

The old four-color problem was whether every map on the plane or sphere can be colored with no more than four colors in such a way that neighboring regions are never colored alike. This problem is so simple to state that even a child can understand it. Nevertheless, the four-color problem resisted attempts by mathematicians for more than one hundred years. From very early on it was proved that five colors suffice to color a map, but no map was ever found that required more than four colors. In fact some mathematicians thought that four colors were not sufficient and were working on methods to produce a counterexample when Kenneth Appel and Wolfgang Haken, assisted by John Koch, published a proof that four colors suffice.<sup>†</sup> Their proof has been accepted by most mathemati-

## *My Plan*

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What logic is  
Anti-exceptionalism  
Quine  
Practices  
Rules  
Definitions

WHAT LOGIC *IS*

### PROOF THEORY

- ▶ Design and construction of different proof systems, proofs in those systems, and results *about* those proof systems.
- ▶ Axiomatic development of different theories. Translations between theories, reductions, embeddings ...

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## METATHEORY

- ▶ Soundness and completeness. Limitative Results.

## *External & Internal*

There's a difference between treating proofs and models as mathematical structures to be *analysed*, and *adopting* them.

### *External & Internal*

There's a difference between treating proofs and models as mathematical structures to be *analysed*, and *adopting* them.

There's a difference between comparing different logics, and *using* some logic, by using a given *proof* to justify a conclusion, or taking a *model* to interpret a theory.

# ANTI- EXCEPTIONALISM

# *What is anti-exceptionalism?*

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DOI 10.1007/s11098-016-0701-8



## **Anti-exceptionalism about logic**

**Ole Thomassen Hjortland<sup>1</sup>**

Published online: 9 June 2016  
© Springer Science+Business Media Dordrecht 2016

**Abstract** Logic isn't special. Its theories are continuous with science; its method continuous with scientific method. Logic isn't a priori, nor are its truths analytic truths. Logical theories are revisable, and if they are revised, they are revised on the same grounds as scientific theories. These are the tenets of anti-exceptionalism about logic. The position is most famously defended by Quine, but has more recent advocates in Maddy (*Proc Address Am Philos Assoc* 76:61–90, 2002), Priest (Doubt truth to be a liar, OUP, Oxford, 2006a, *The metaphysics of logic*, CUP, Cambridge, 2014, *Log et Anal*, 2016), Russell (*Philos Stud* 171:161–175, 2014, *J Philos Log* 0:1–11, 2015), and Williamson (Modal logic as metaphysics, Oxford University Press, Oxford, 2013b, *The relevance of the liar*, OUP, Oxford, 2015). Although

## *What is anti-exceptionalism?*

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- ▷ Logic's theories are continuous with science.
- ▷ Logic's methods are continuous with scientific method.
- ▷ Logic isn't *a priori*.
- ▷ Logic's truths are not analytic truths.
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## *Compare Arithmetic*

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Here's a proof that  $2 + 2 = 4$ , in Robinson's Arithmetic

$$\frac{\frac{0'' + 0'' = (0'' + 0')'}{Q5} \quad \frac{\overline{0'' + 0' = (0'' + 0)'} \stackrel{Q5}{=} \overline{0'' + 0 = 0''} \stackrel{Q4}{=}}{\overline{(0'' + 0')' = (0'' + 0)''} \stackrel{'=}{=} \overline{(0'' + 0)' = 0'''} \stackrel{'=}{=}} \stackrel{=T}{=} \frac{\overline{0'' + 0'' = (0'' + 0)''} \stackrel{'=}{=}}{\overline{0'' + 0'' = 0''''} \stackrel{'=}{=}}$$

(Q4)  $x + 0 = x$  (Q5)  $x + y' = (x + y)'$  ( $' =$ )  $x = y / x' = y'$  ( $=T$ )  $x = y, y = z / x = z$

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$$(Q4) x + 0 = x \quad (Q5) x + y' = (x + y)' \quad ('=) x = y / x' = y' \quad (=T) x = y, y = z / x = z$$

Is this derivation *a priori* or *a posteriori*?

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$$0'' + 0'' = (0'' + 0)''$$
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$$(Q4) x + 0 = x \quad (Q5) x + y' = (x + y)' \quad ('=) x = y / x' = y' \quad (=T) x = y, y = z / x = z$$

Is this derivation *a priori* or *a posteriori*?

If some evidence were needed to supplement  
the argument, where would we add it?

## *External Questions and Internal Questions*

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*What does this proof do?*

$$\frac{\frac{\frac{[r \rightarrow p]^2 \quad [r]^1}{p} \rightarrow E}{[p \rightarrow q]^3} \rightarrow E}{\frac{q}{r \rightarrow q} \rightarrow I^1} \rightarrow I^2}{(r \rightarrow p) \rightarrow (r \rightarrow q)} \rightarrow I^3$$
$$\frac{(p \rightarrow q) \rightarrow ((r \rightarrow p) \rightarrow (r \rightarrow q))}{(p \rightarrow q) \rightarrow ((r \rightarrow p) \rightarrow (r \rightarrow q))} \rightarrow I^3$$

## *What does this proof do?*

$$\frac{\frac{\frac{[p \rightarrow q]^3}{\frac{[r \rightarrow p]^2 \quad [r]^1}{p} \rightarrow E}}{\frac{q}{r \rightarrow q} \rightarrow I^1} \rightarrow E}{(r \rightarrow p) \rightarrow (r \rightarrow q) \rightarrow I^2} \rightarrow I^3}{(p \rightarrow q) \rightarrow ((r \rightarrow p) \rightarrow (r \rightarrow q)) \rightarrow I^3}$$

Is the conclusion *analytic*?

## *What does this proof do?*

$$\frac{\frac{\frac{[r \rightarrow p]^2 \quad [r]^1}{p} \rightarrow E}{[p \rightarrow q]^3} \rightarrow E}{\frac{q}{r \rightarrow q} \rightarrow I^1} \rightarrow I^2}{(r \rightarrow p) \rightarrow (r \rightarrow q)} \rightarrow I^3$$
$$(p \rightarrow q) \rightarrow ((r \rightarrow p) \rightarrow (r \rightarrow q)) \rightarrow I^3$$

Is the conclusion *analytic*? ¶ Is the conclusion *a priori*?

## *What does this proof do?*

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$$(p \rightarrow q) \rightarrow ((r \rightarrow p) \rightarrow (r \rightarrow q))$$

Is the conclusion *analytic*? ¶ Is the conclusion *a priori*? ¶ Is the proof *special*?

QUINE

## Anti-exceptionalism about logic

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**Abstract** Logic isn't special. Its theories are continuous with science; its method continuous with scientific method. Logic isn't a priori, nor are its truths analytic truths. Logical theories are revisable, and if they are revised, they are revised on the same grounds as scientific theories. These are the tenets of anti-exceptionalism about logic. The position is most famously defended by Quine, but has more recent advocates in Maddy (Proc Address Am Philos Assoc 76:61–90, 2002), Priest (Doubt truth to be a liar, OUP, Oxford, 2006a, The metaphysics of logic, CUP, Cambridge, 2014, Log et Anal, 2016), Russell (Philos Stud 171:161–175, 2014, J Philos Log 0:1–11, 2015), and Williamson (Modal logic as metaphysics, Oxford University Press, Oxford, 2013b). The relevance of the liar (OUP, Oxford, 2015). Although

# *... had its limits. (Word and Object)*

## §13 Translating Logical Connectives

In §§7 through 11 we accounted for radical translation of occasion sentences, by approximate identification of stimulus meanings. Now there is also a decidedly different domain that lends itself directly to radical translation: that of *truth functions* such as negation, logical conjunction, and alternation. For this purpose the sentences put to the native for assent or dissent may be occasion sentences and standing sentences indifferently. Those that are occasion sentences will have to be accompanied by a prompting stimulation, if assent or dissent is to be elicited; the standing sentences, on the other hand, can be put without props. Now by reference to assent and dissent we can state semantic criteria for truth functions; i.e., criteria for determining whether a given native idiom is to be construed as expressing the truth function in question. The semantic criterion of negation is that it turns any short sentence to which one will assent into a sentence from which one will dissent, and vice versa. That of conjunction is that it produces compounds to which (so long as the component sentences are short) one is prepared to assent always and only when one is prepared to assent to each component. That of alternation is similar with assent changed twice to dissent.

## *Constraints*

---

Why, then, is a conjunction true when both conjuncts are true?

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Why is a disjunction false when both disjuncts are false?

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For the Quine of *Word and Object*, inferences like these *are a priori* valid.

## *Constraints*

Why, then, is a conjunction true when both conjuncts are true?

Why is a disjunction false when both disjuncts are false?

For the Quine of *Word and Object*, inferences like these *are a priori* valid.

(Not *a priori* in the sense that they are unrevisable,  
but in the sense that if the terms have the meanings  
we have postulated, we do not need to appeal to evidence  
to ground the validity of the inference.)

## *The constitutive and relativized a priori*

---

*... the concept of the relativized a priori, as originally formulated within the tradition of logical empiricism, was explicitly intended to prise apart two meanings that were discerned within the original Kantian conception: necessary and unrevisable, true for all time, on the one hand, and “constitutive of the concept of the object of [scientific] knowledge,” on the other.*

— Michael Friedman, *The Dynamics of Reason* (2002)

## *Constraints*

---

What does “*and*”, in this sense, *mean*?

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What does “*and*”, in this sense, *mean*?

What does “*or*”, in this sense, *mean*?

## *Constraints*

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What does “*or*”, in this sense, *mean*?

For the Quine of *Word and Object*, it is not a bridge too far to say that principles governing these particles are *definitionally analytic*.

## *Anti-anti-exceptionalism—internally*

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# PRACTICES

For the Quine of *Word and Object*,  
you locate the logical connectives  
by identifying their interaction  
with *assent* and *dissent*.

## *Quine's Criteria for Negation,*

---

*The semantic criterion of negation is that it turns any short sentence to which one will assent into a sentence from which one will dissent, and vice versa.*

## *Quine's Criteria for Negation, Conjunction*

*The semantic criterion of negation is that it turns any short sentence to which one will assent into a sentence from which one will dissent, and vice versa. That of conjunction is that it produces compounds to which (so long as the component sentences are short) one is prepared to assent always and only when one is prepared to assent to each component.*

## *Quine's Criteria for Negation, Conjunction and Disjunction*

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These criteria are not enough to generate truth functional logic, unless supplemented.

## *Illustrating the issue*

---

Suppose I dissent from  $p \vee \neg p$ .

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Suppose I dissent from  $p \vee \neg p$ .

So, I dissent from  $p$  and dissent from  $\neg p$ .

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*Counterexample:*  $(p \vee \neg p) \wedge (q \vee \neg q)$ .

(Quine gives no conditions concerning  
when to *dissent* from a conjunction.)

## *Quine's Project*

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Quine's project in *Word and Object*  
involved radical translation, stimulus meaning  
and occasion sentences, and much besides.

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logic is constitutively *a priori*.

## *Quine's Project*

Quine's project in *Word and Object* involved radical translation, stimulus meaning and occasion sentences, and much besides.

It does arrive at a radical holism, but one in which a certain amount of logic is constitutively *a priori*.

I will not be adopting Quine's project, but I am interested in exploring how logical concepts can be *defined* and how this explains how logic is *special*.

## *Quine's Criteria for Negation, Conjunction and Disjunction*

*The semantic criterion of negation is that it turns any short sentence to which one will assent into a sentence from which one will dissent, and vice versa. That of conjunction is that it produces compounds to which (so long as the component sentences are short) one is prepared to assent always and only when one is prepared to assent to each component. That of alternation is similar with assent changed twice to dissent.*

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A more important question: How could we tell that we have located such items in someone's vocabulary?

Are these criteria descriptions of behaviour, or are they better understood in terms of ...

# RULES

*We can bind ourselves by adopting a rule*

---

Instead of just looking for an item in our vocabulary with the desired behaviour, we could *define* one.

*We can bind ourselves by adopting a rule*

---

Instead of just looking for an item in our vocabulary with the desired behaviour, we could *define* one.

We can adopt a RULE: “use ‘ $\wedge$ ’ like *this* ...”

## *'Rules' a la Quine*

DEFINIENDUM	DEFINIENS
$+ \neg A$	$\neg A$
$+ A \wedge B$	$+ A, + B$
$- A \vee B$	$- A, - B$

## *'Rules' a la Quine*

DEFINIENDUM	DEFINIENS
$+ \neg A$	$\neg A$
$+ A \wedge B$	$+ A, + B$
$- A \vee B$	$- A, - B$
$- A \rightarrow B$	$+ A, - B$

## *'Rules' a la Quine*

DEFINIENDUM	DEFINIENS
$+ \neg A$	$\neg A$
$+ A \wedge B$	$+ A, + B$
$\neg A \vee B$	$\neg A, \neg B$
$\neg A \rightarrow B$	$+ A, \neg B$
$\neg \forall x A$	$\neg A[x/n] \text{ (n new)}$
$+ \exists x A$	$+ A[x/n] \text{ (n new)}$

## *'Rules' a la Quine*

DEFINIENDUM	DEFINIENS
$+ \neg A$	$\neg A$
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$+ \exists x A$	$+ A[x/n] \text{ (n new)}$
$- s = t$	$+ Fs, - Ft \text{ (F new)}$

## *'Rules' a la Quine*

DEFINIENDUM	DEFINIENS
$+ \neg A$	$\neg A$
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$+ \exists x A$	$+ A[x/n] \text{ (n new)}$
$- s = t$	$+ Fs, - Ft \text{ (F new)}$

To make sense of these, we need to say more about assertion and denial, assent and dissent.

# *This sounds familiar*

---

## MULTIPLE CONCLUSIONS

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VERSION 1.03

March 19, 2004

*Abstract:* I argue for the following four theses. (1) Denial is not to be analysed as the assertion of a negation. (2) Given the concepts of assertion and denial, we have the resources to analyse logical consequence as relating arguments with *multiple* premises and *multiple* conclusions. Gentzen's multiple conclusion calculus can be understood in a straightforward, motivated, non-question-begging way. (3) If a broadly anti-realist or inferentialist justification of a logical system works, it works just as well for *classical* logic as it does for *intuitionistic* logic. The special case for an anti-realist justification of intuitionistic logic over and above a justification of classical logic relies on an unjustified assumption about the shape of proofs. Finally, (4) this picture of logical consequence provides a relatively neutral shared vocabulary which can help us understand and adjudicate debates between proponents of classical and non-classical logics.

\* \* \*

# *Positions*

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[X : Y]

## *Positions*

---

[X : Y]

[X, A : A, Y] is self-defeating.

## *Sequents: Unfocused and Focused*

---

$$X \succ Y$$

## *Sequents: Unfocused and Focused*

$X \succ Y$

$X \succ [A], Y$        $X, [A] \succ Y$

## *Structural Rules: Identity*

---

$$X, A \succ A, Y$$

## *Structural Rules: Cut*

---

$$\frac{X \succ A, Y \quad X, A \succ Y}{X \succ Y} \textit{Cut}$$

# DEFINITIONS

# *Defining Rules for Logical Concepts*

$$\frac{X, A, B \succ Y}{X, A \wedge B \succ Y} \wedge Df \quad \frac{X \succ A, B, Y}{X \succ A \vee B, Y} \vee Df \quad \frac{X \succ A, Y}{X, \neg A \succ Y} \neg Df \quad \frac{X, A \succ B, Y}{X \succ A \rightarrow B, Y} \rightarrow Df$$

$$\frac{X \succ A(n), Y}{X \succ \forall x A(x), Y} \forall Df \quad \frac{X, A(n) \succ Y}{X, \exists x A(x) \succ Y} \exists Df \quad \frac{X, Fa \succ Fb, Y}{X \succ a = b, Y} = Df$$

*Terms & conditions:* the singular term  $n$  (in  $\forall/\exists Df$ ) and the predicate  $F$  (in  $=Df$ ) do not appear below the line in those rules.

These rules can be understood as *definitions* of the concepts they introduce (below the double line).

See (Scott 1974; Došen 1980, 1989; Restall 2019).

## *Adopting the Rules, Applying the Definitions*

$$\frac{\begin{array}{c} \text{Fn} \vee \text{Gn} \succ \text{Fn} \vee \text{Gn}, \exists x \text{Gx} \\ \hline \text{Fn} \vee \text{Gn} \succ \text{Fn}, \text{Gn}, \exists x \text{Gx} \end{array}}{\text{Fn} \vee \text{Gn} \succ \text{Fn}, \text{Gn}, \exists x \text{Gx}} \vee Df$$
$$\frac{\begin{array}{c} \forall x(\text{Fx} \vee \text{Gx}), \exists x \text{Gx} \succ \text{Fn}, \exists x \text{Gx} \\ \hline \forall x(\text{Fx} \vee \text{Gx}) \succ \text{Fn}, \text{Gn}, \exists x \text{Gx} \end{array}}{\forall x(\text{Fx} \vee \text{Gx}) \succ \text{Fn}, \text{Gn}, \exists x \text{Gx}} \forall Df$$
$$\frac{\begin{array}{c} \forall x(\text{Fx} \vee \text{Gx}) \succ \text{Fn}, \exists x \text{Gx} \\ \hline \forall x(\text{Fx} \vee \text{Gx}) \succ \forall x \text{Fx}, \exists x \text{Gx} \end{array}}{\forall x(\text{Fx} \vee \text{Gx}) \succ \forall x \text{Fx} \vee \exists x \text{Gx}} \forall Df$$
$$\frac{\begin{array}{c} \forall x(\text{Fx} \vee \text{Gx}), \exists x \text{Gx} \succ \text{Fn}, \exists x \text{Gx} \\ \hline \forall x(\text{Fx} \vee \text{Gx}), \text{Gn} \succ \text{Fn}, \exists x \text{Gx} \end{array}}{\forall x(\text{Fx} \vee \text{Gx}), \text{Gn} \succ \text{Fn}, \exists x \text{Gx}} \exists Df$$
$$\frac{}{\forall x(\text{Fx} \vee \text{Gx}) \succ \forall x \text{Fx} \vee \exists x \text{Gx}} Cut$$

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The derivable formulas are *definitionally analytic*.

The rules are *conservatively extending* and *uniquely defining* and these are *very special* features of these logical concepts.

In *this way*, logic is quite special.

# THANK YOU!

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