Why *logic* matters for *philosophy* & why *philosophy* matters for *logic*

Greg Restall · University of St Andrews · November 2025

1. Orientation
2. Why *logic* matters for *philosophy*
3. Why *philosophy* matters for *logic*

1. **Orientation**

2. **Why *logic* matters for *philosophy***

3. **Why *philosophy* matters for *logic***

What is *philosophy?*

Wilfrid Sellars

1912 – 1989

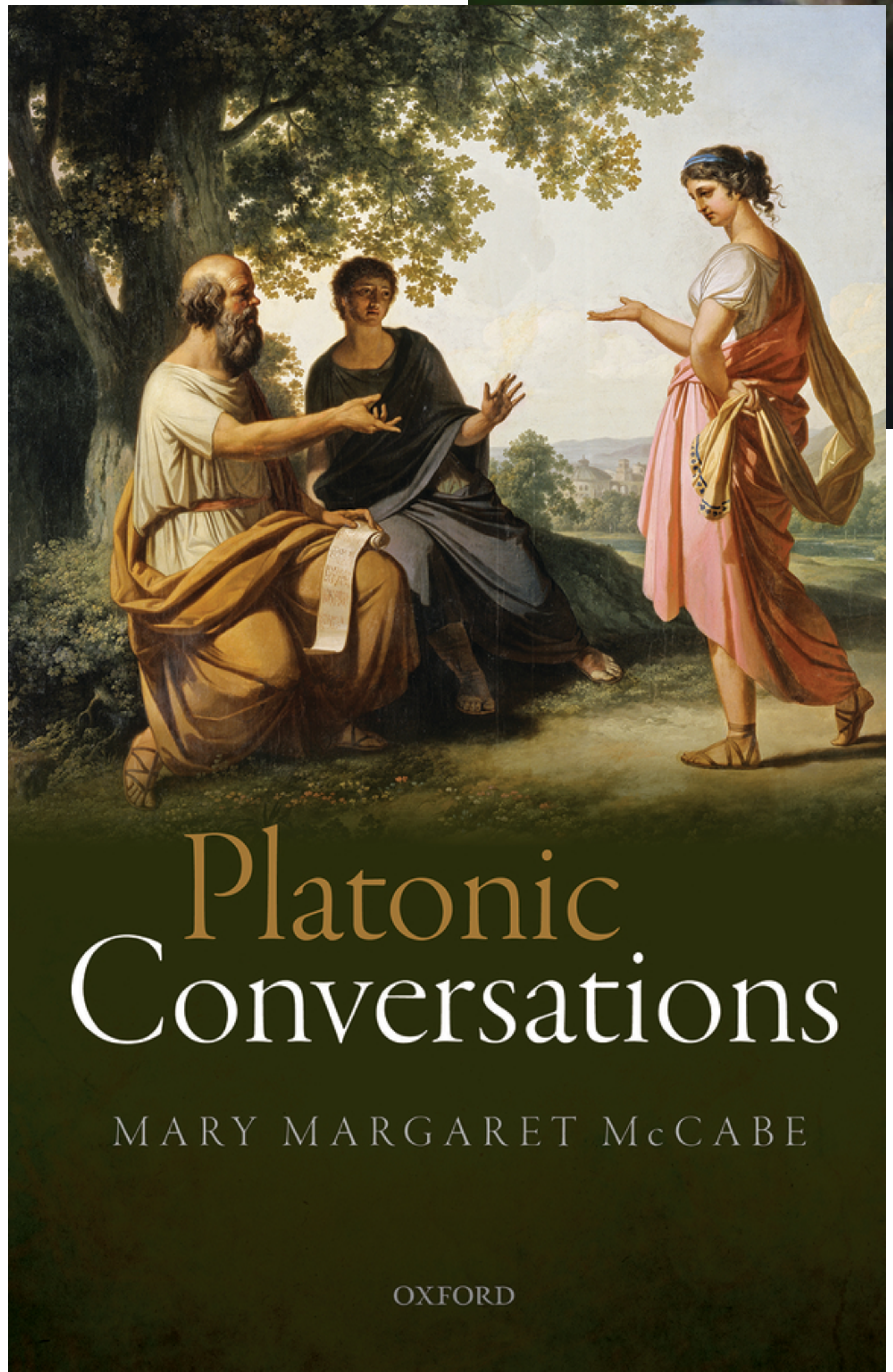
The aim of philosophy,
  abstractly formulated,
is to understand how things
  in the broadest possible sense of the term
hang together
  in the broadest possible sense of the term.

"Philosophy and the Scientific Image of Man" (1962)

Mary Margaret McCabe

For me, philosophy is best done by *conversation* …
Philosophical conversation is hard, for it
demands a kind of openness, thoughtfulness,
and courtesy that is difficult to achieve …
it allows one to try ideas without fear of ridicule
or dismissal and at the right speed …

Platonic Conversations (2015)

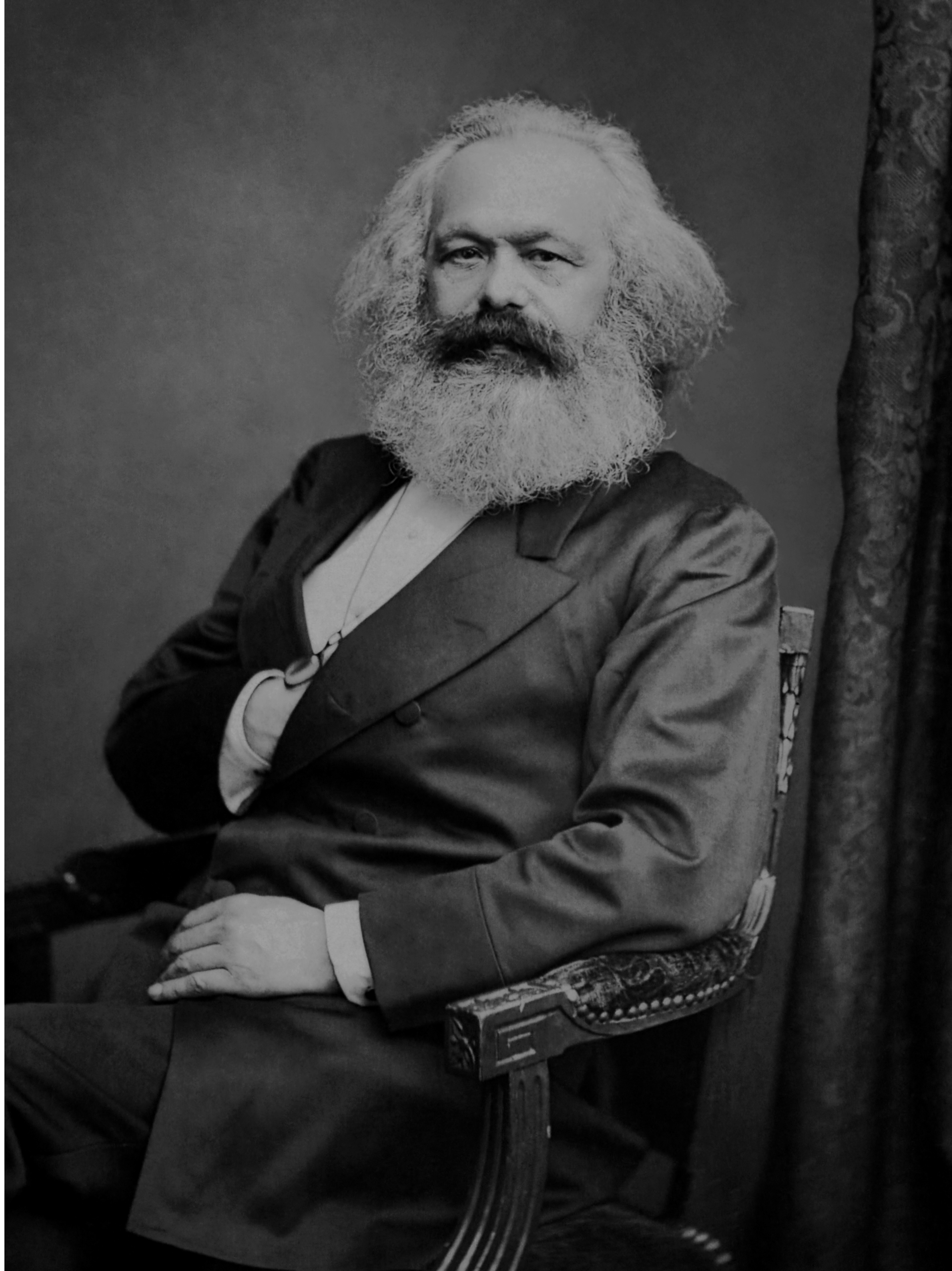


Karl Marx

1818 – 1883

Philosophers have hitherto only *interpreted* the world in various ways; the point is to *change* it.

"Theses on Feuerbach" (1845)



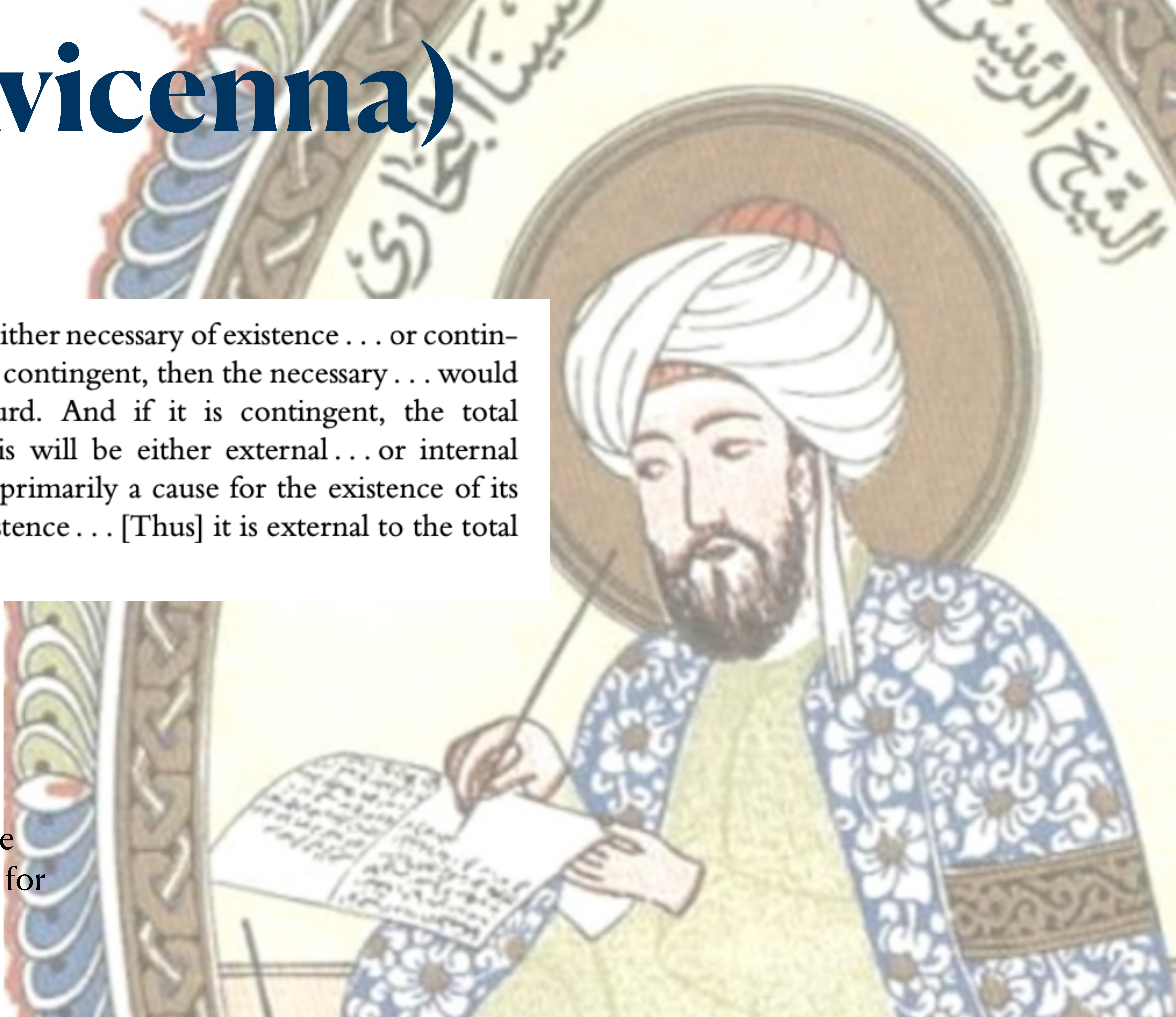
Philosophy is a long-running **conversation**, reaching across times and cultures, in which we try to **make sense of things**. In taking part in this conversation, we **change ourselves** and our **world**.

What is *logic*?

Ibn Sīnā (Avicenna)

[Persia, c950–1037CE]

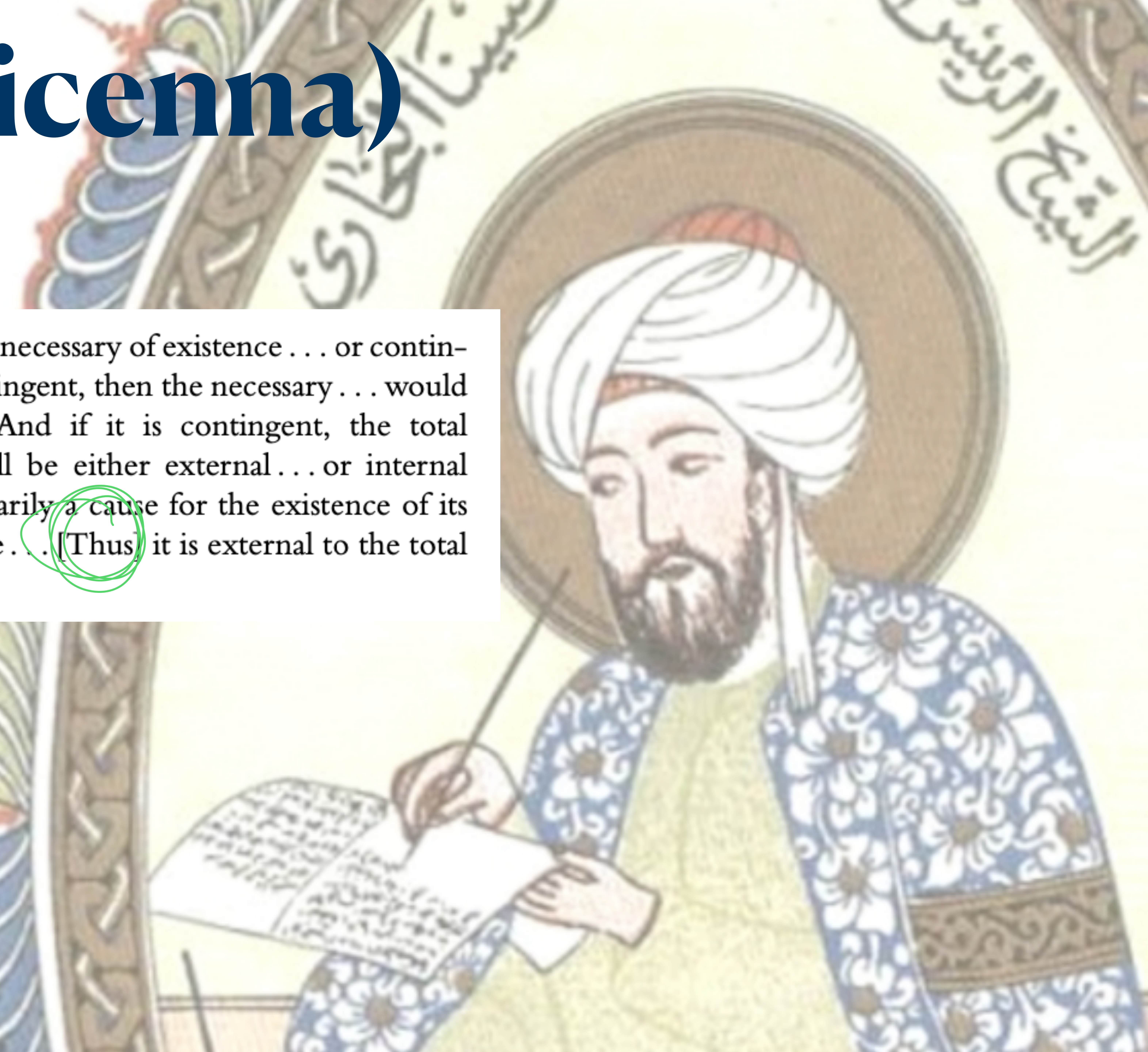
Their total . . . whether . . . finite or infinite, is either necessary of existence . . . or contingent. If it is necessary . . . but each of its units is contingent, then the necessary . . . would be composed of contingents, which is absurd. And if it is contingent, the total needs . . . something to bestow existence. This will be either external . . . or internal to it. If it is internal . . . a cause of the total is primarily a cause for the existence of its parts . . . and thus it will . . . cause [its own] existence . . . [Thus] it is external to the total and [so] not contingent.[17]



A section from *Al-Najāt*, translated by George Hourani, cited in Brian Leftow's "Arguments for God's Existence", *The Cambridge History of Medieval Philosophy*, CUP 2014.

Ibn Sīnā (Avicenna)

[Persia, c950–1037CE]



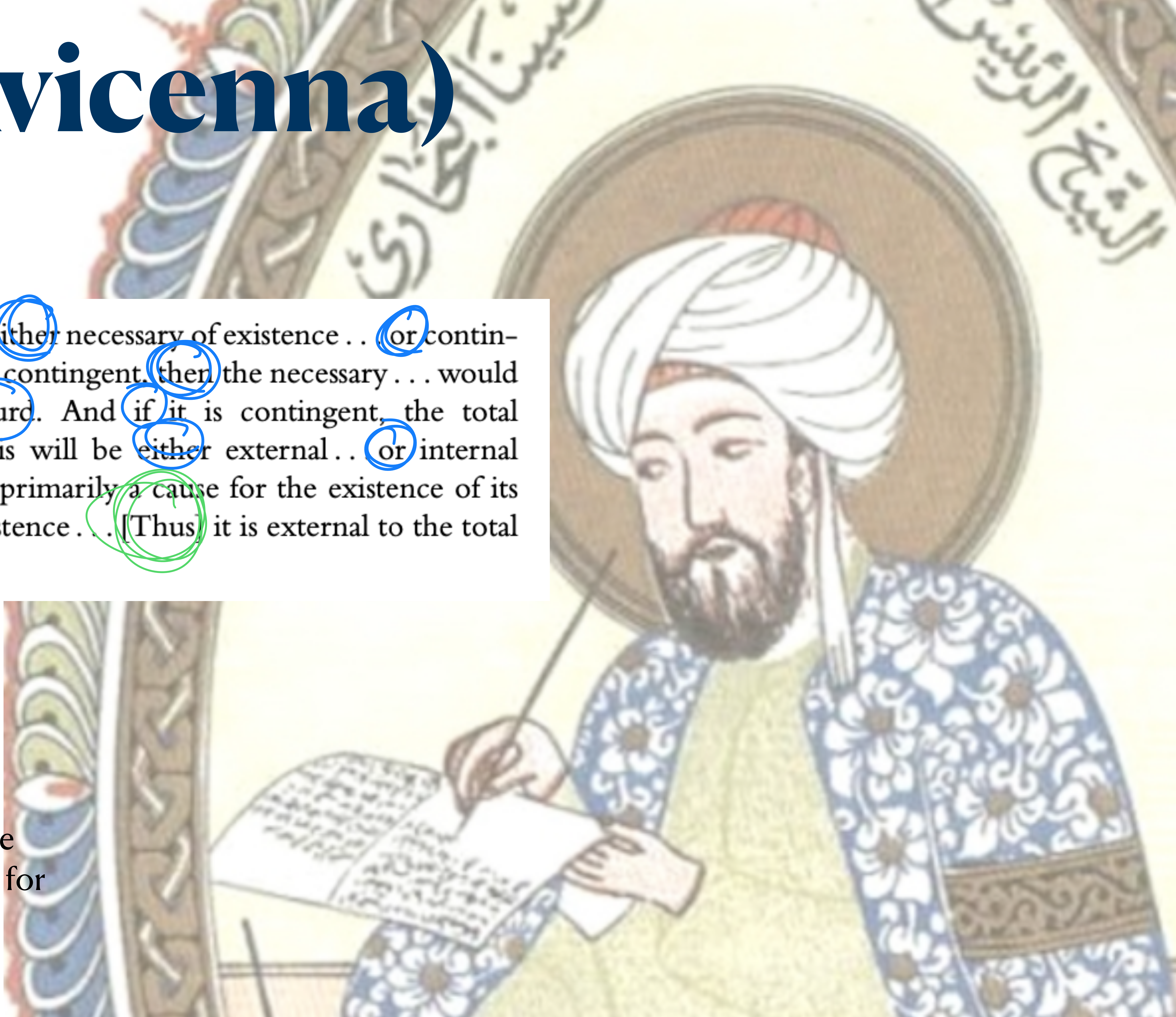
Their total . . . whether . . . finite or infinite, is either necessary of existence . . . or contingent. If it is necessary . . . but each of its units is contingent, then the necessary . . . would be composed of contingents, which is absurd. And if it is contingent, the total needs . . . something to bestow existence. This will be either external . . . or internal to it. If it is internal . . . a cause of the total is primarily a cause for the existence of its parts . . . and thus it will . . . cause [its own] existence . . . [Thus] it is external to the total and [so] not contingent.[17]

INFERENCE STEPS

A section from *Al-Najāt*, translated by George Hourani, cited in Brian Leftow's "Arguments for God's Existence", *The Cambridge History of Medieval Philosophy*, CUP 2014.

Ibn Sīnā (Avicenna)

[Persia, c950–1037CE]



Their total . . . whether . . . finite or infinite, is either necessary of existence . . . or contingent. If it is necessary . . . but each of its units is contingent, then the necessary . . . would be composed of contingents, which is absurd. And if it is contingent, the total needs . . . something to bestow existence. This will be either external . . . or internal to it. If it is internal . . . a cause of the total is primarily a cause for the existence of its parts . . . and thus it will . . . cause [its own] existence . . . [Thus] it is external to the total and [so] not contingent.[17]

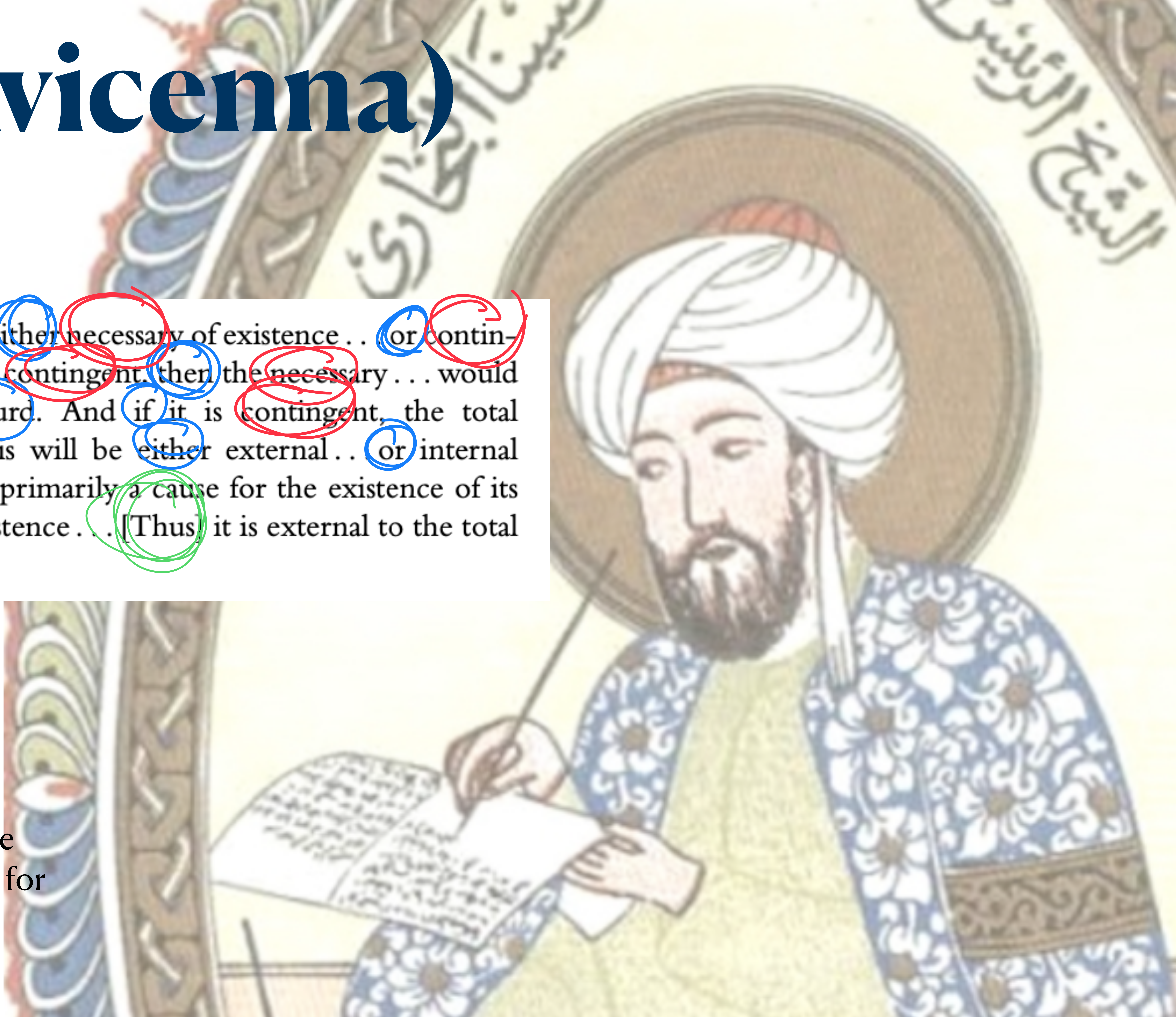
INFERENCE STEPS

LOGICAL PARTICLES

A section from *Al-Najāt,* translated by George Hourani, cited in Brian Leftow's "Arguments for God's Existence", *The Cambridge History of Medieval Philosophy,* CUP 2014.

Ibn Sīnā (Avicenna)

[Persia, c950–1037CE]



Their total . . . whether . . . finite or infinite, is either necessary of existence . . . or contingent. If it is necessary . . . but each of its units is contingent, then the necessary . . . would be composed of contingents, which is absurd. And if it is contingent, the total needs . . . something to bestow existence. This will be either external . . . or internal to it. If it is internal . . . a cause of the total is primarily a cause for the existence of its parts . . . and thus it will . . . cause [its own] existence . . . [Thus] it is external to the total and [so] not contingent.[17]

INFERENCE STEPS
LOGICAL PARTICLES
MODAL CONCEPTS

A section from *Al-Najāt*, translated by George Hourani, cited in Brian Leftow's "Arguments for God's Existence", *The Cambridge History of Medieval Philosophy*, CUP 2014.

Ibn Sīnā (Avicenna)

[Persia, c950–1037CE]

Their total . . . whether . . . finite or infinite, is either necessary of existence . . . or contingent. If it is necessary . . . but each of its units is contingent, then the necessary . . . would be composed of contingents, which is absurd. And if it is contingent, the total needs . . . something to bestow existence. This will be either external . . . or internal to it. If it is internal . . . a cause of the total is primarily a cause for the existence of its parts . . . and thus it will . . . cause [its own] existence . . . [Thus] it is external to the total and [so] not contingent.[17]

INFERENCE STEPS
LOGICAL PARTICLES
MODAL CONCEPTS
MEREOLOGICAL CONCEPTS

A section from *Al-Najāt*, translated by George Hourani, cited in Brian Leftow's "Arguments for God's Existence", *The Cambridge History of Medieval Philosophy*, CUP 2014.

Ibn Sīnā (Avicenna)

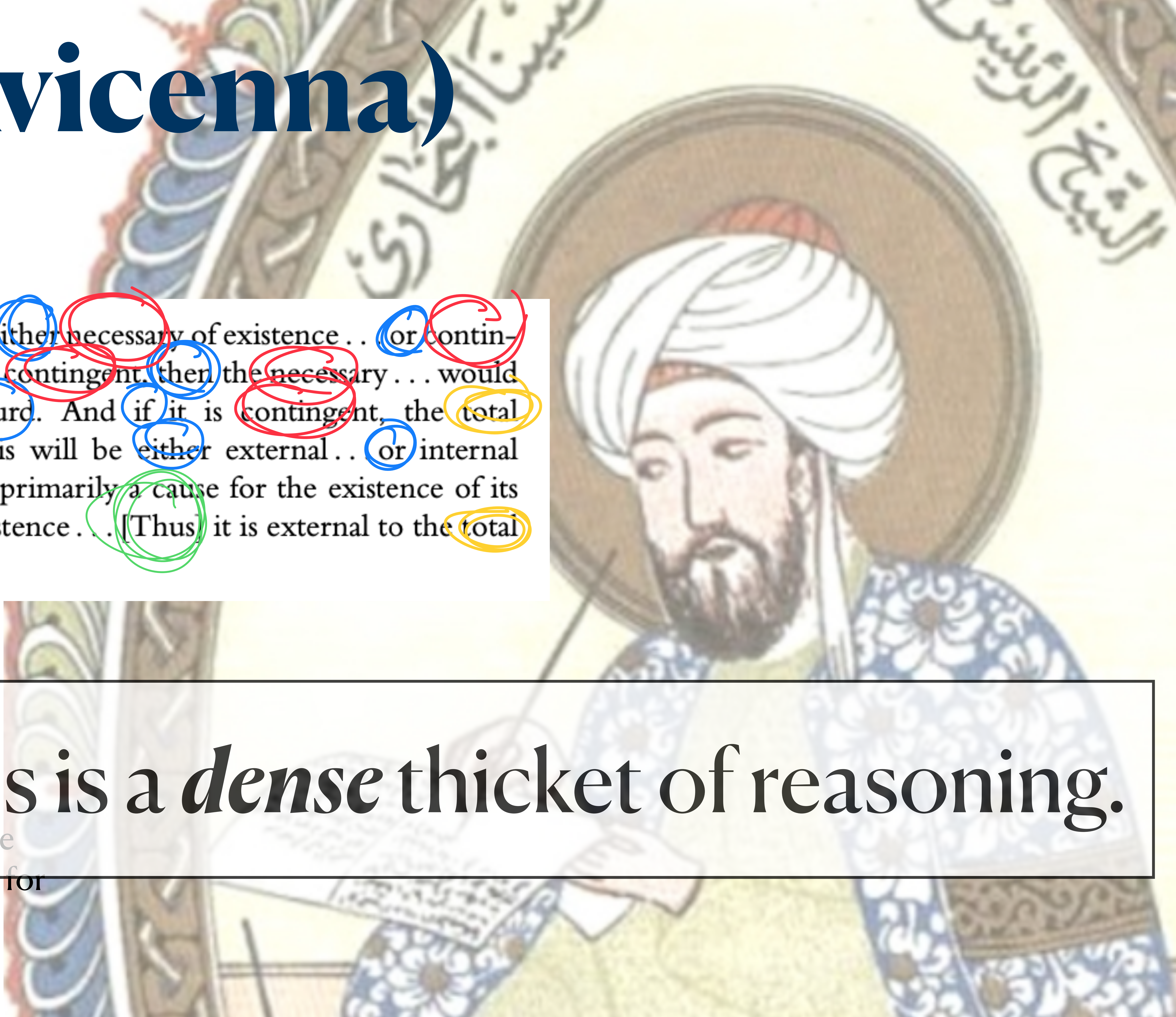
[Persia, c950–1037CE]

Their total . . . whether . . . finite or infinite, is either necessary of existence . . . or contingent. If it is necessary . . . but each of its units is contingent, then the necessary . . . would be composed of contingents, which is absurd. And if it is contingent, the total needs . . . something to bestow existence. This will be either external . . . or internal to it. If it is internal . . . a cause of the total is primarily a cause for the existence of its parts . . . and thus it will . . . cause [its own] existence . . . [Thus] it is external to the total and [so] not contingent.[17]

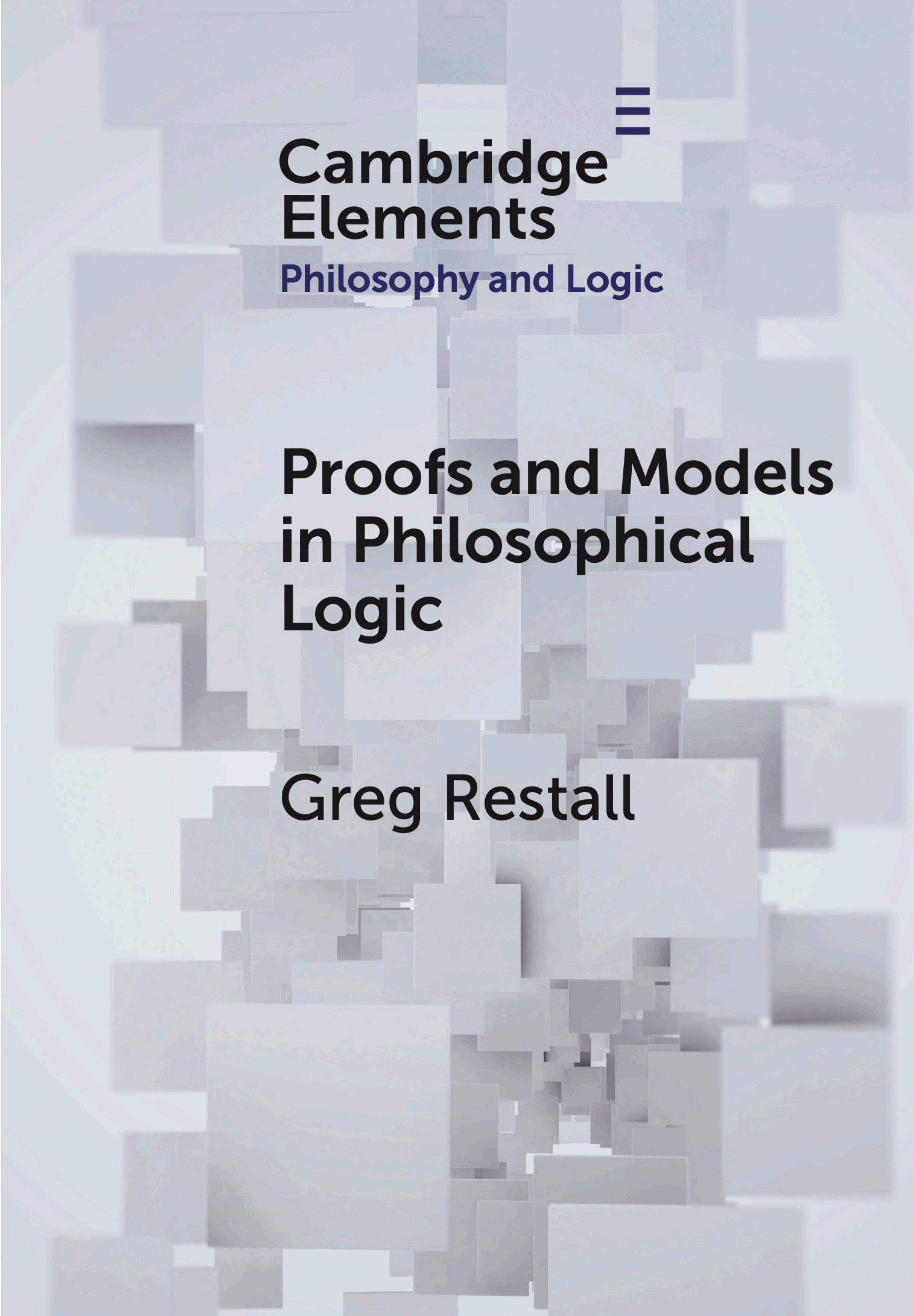
INFERENCE STEPS
LOGICAL PARTICLES
MODAL CONCEPTS
MEREOLOGICAL CONCEPTS

This is a *dense* thicket of reasoning.

A section from *Al-Najāt*, translated by George Hourani, cited in Brian Leftow's "Arguments for God's Existence", *The Cambridge History of Medieval Philosophy*, CUP 2014.



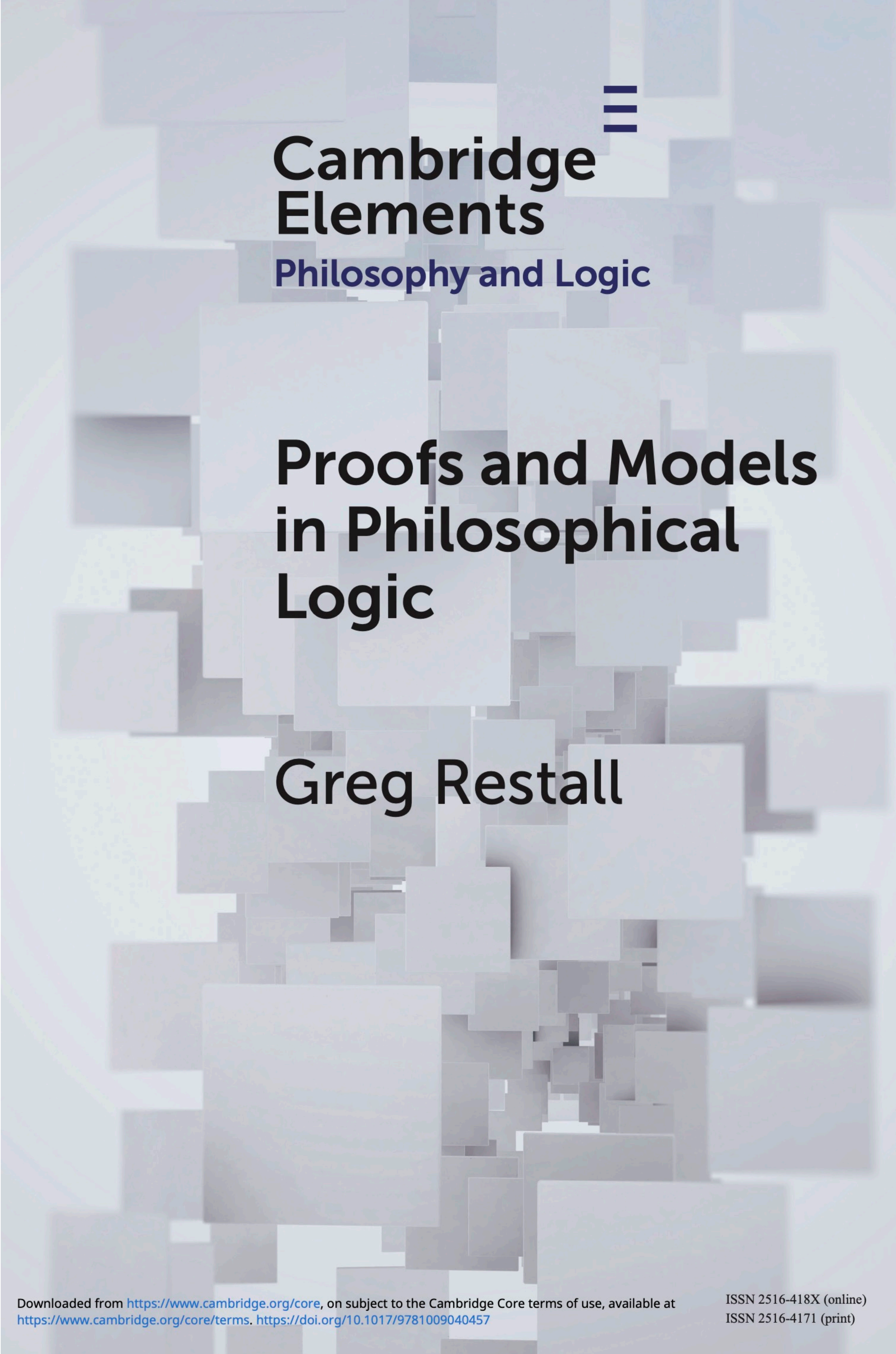
Logic is the *science* & *art* of **reasoning**.



ISSN 2516-418X (online)
ISSN 2516-4171 (print)

Logic is the *science* & *art* of **reasoning**.

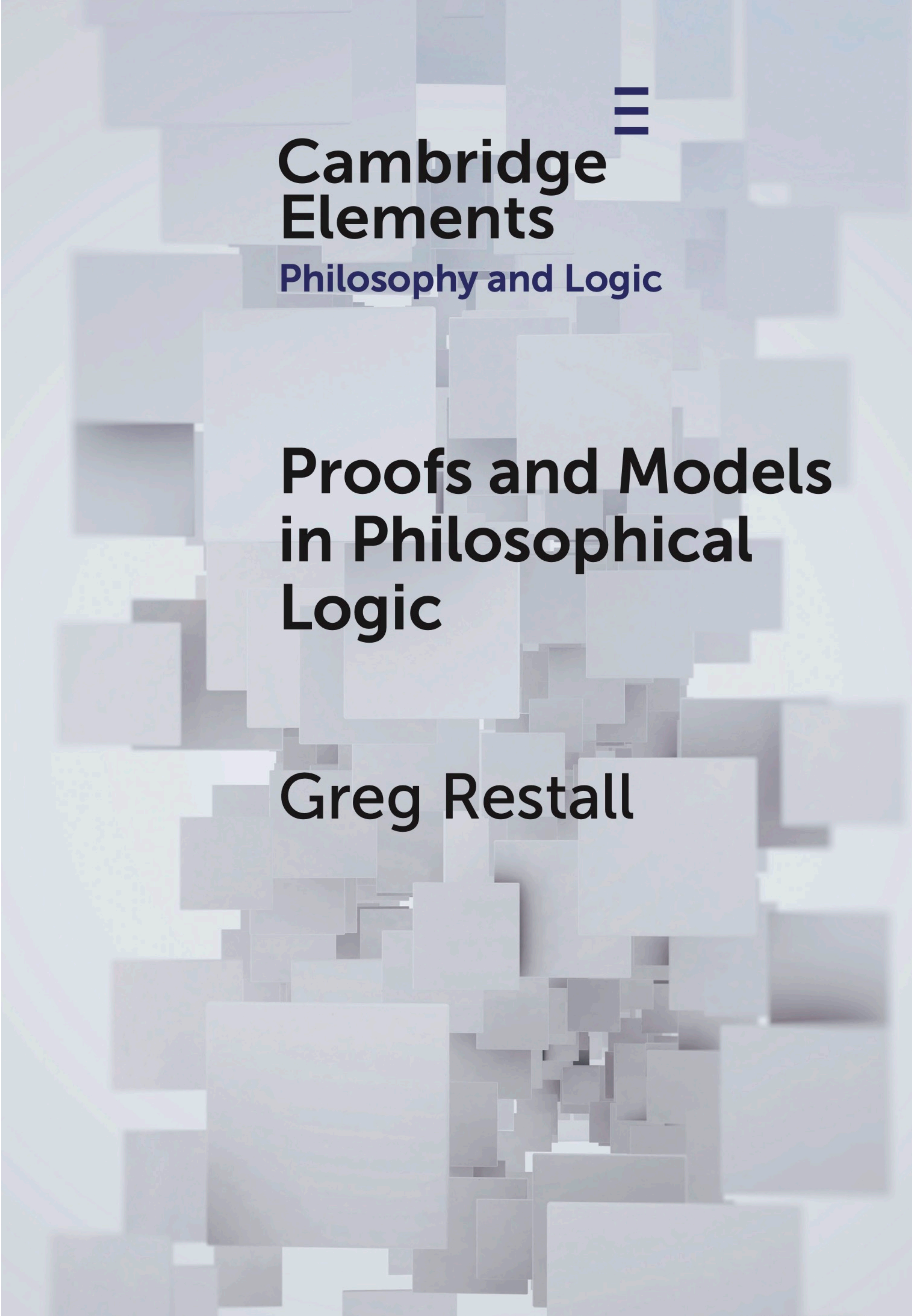
Proofs: analyse reasoning steps into basic components.



Logic is the *science* & *art* of **reasoning**.

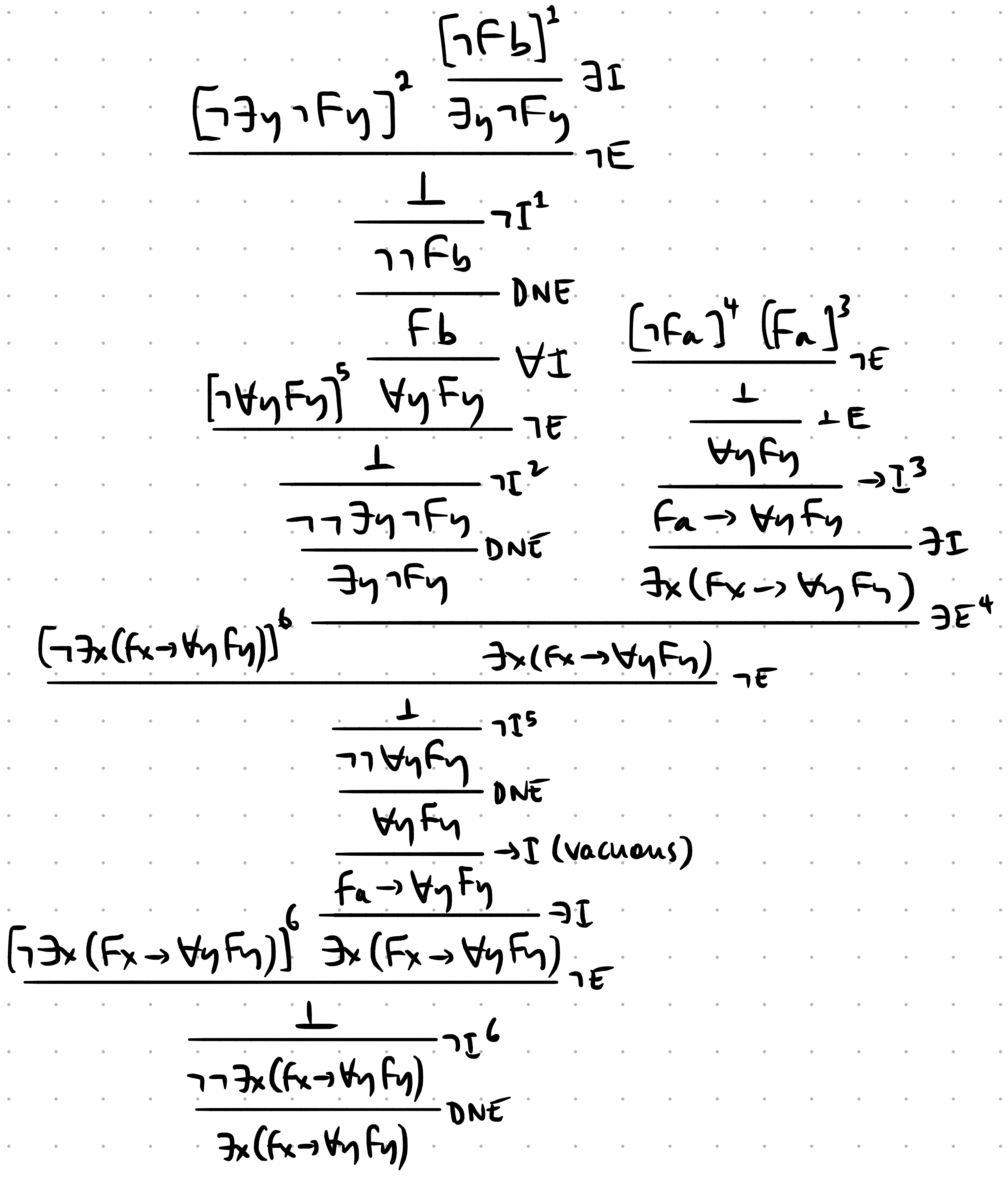
Proofs: analyse reasoning steps into basic components.

Models: represent *possibilities*, the space of claims we can make and how they hang together.



ISSN 2516-418X (online)
ISSN 2516-4171 (print)

Logic has become rather *formal*



Definition 55 (Double Indexed Models) *A double indexed model is given by a pair $\langle W, V\rangle$ consisting of a non-empty set W of worlds and a valuation function V assigning a truth value from $\{0,1\}$ to each pair consisting of an atom and a world. Given this data, we can define a function assigning a truth value to each formula and a pair of worlds, like this:*

- $V(p,w,v)=1$ *iff* $V(p,w)=1$.
- $V(A\wedge B,w,v)=1$ *iff* $V(A,w,v)=1$ *and* $V(B,w,v)=1$.
- $V(A\vee B,w,v)=1$ *iff* $V(A,w,v)=1$ *or* $V(B,w,v)=1$.
- $V(A\rightarrow B,w,v)=1$ *iff* $V(A,w,v)=0$ *or* $V(B,w,v)=1$.
- $V(\neg A,w,v)=1$ *iff* $V(A,w,v)=0$.
- $V(\bot,w,v)=1$ *never.*
- $V(\Box A,w,v)=1$ *iff* $V(A,u,v)=1$ *for each world* $u\in W$.
- $V(\Diamond A,w,v)=1$ *iff* $V(A,u,v)=1$ *for some world* $u\in W$.
- $V(\mathbb{A}A,w,v)=1$ *iff* $V(A,v,v)=1$.

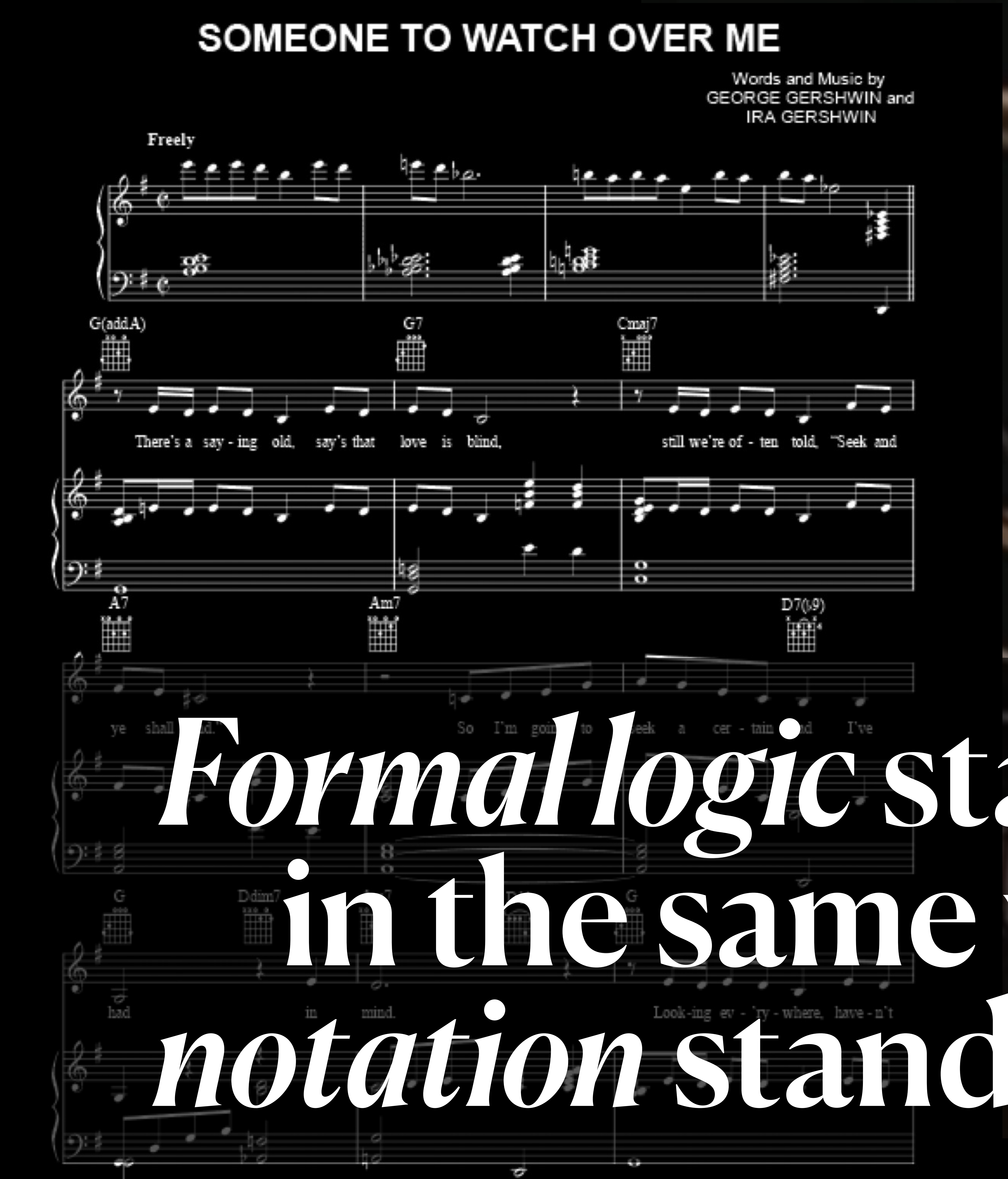
We understand $V(A,w,v)=1$ as saying that the formula A is true at *the world w, given that the world v is actual.*

Double indexed models can do everything that actuality models can do, without forcing the choice of an actual world. *Any* world can be treated as actual.

Definition 56 (Double indexed counterexamples, validity) *A double indexed model $\langle W, V\rangle$ is a* counterexample *to an argument $X\succ A$ iff there are worlds $w,v\in W$ such that $V(X,w,v)=1$ and $V(A,w,v)=0$.*

An argument $X\succ A$ is double indexed valid *iff there are no counterexamples to the argument. Or, equivalently, for all models $\langle W,V\rangle$, for all $w,v\in W$, if $V(X,w,v)=1$ then $V(A,w,v)=1$. If the argument $X\succ A$ is double indexed valid, then we write $X\vDash_{\mathsf{DI}} A$.*

These are examples from the last few of weeks' content in *PY2010 Intermediate Logic*.



Formal logic stands to *reasoning* in the same way that music *notation* stands to *performance*.

Rózsa Péter

1905 – 1977

The writing down of a *formula* is an expression of our joy that we can answer all these questions by means of one argument.

Playing with Infinity (1957)

What *matters*?

x* *matters to* *y (where ***x*** and ***y*** are activities)

when doing ***y*** well involves doing ***x***.

x matters to y (where ***x*** and ***y*** are activities)

when doing ***y*** well involves doing ***x***.

So, I'll try to show why doing *philosophy* well involves doing *logic*, and why doing *logic* well involves doing *philosophy*.

1. Orientation

2. **Why *logic* matters for *philosophy***

3. Why *philosophy* matters for *logic*

Logic is *not* philosophy's ***cop***.

Philosophy is a long-running **conversation**, reaching across times and cultures, in which we try to **make sense of things**. In taking part in this conversation, we **change ourselves** and our **world**.

Attention to what ***we*** mean and what ***others*** mean is a part of being a good conversation partner.

Philosophy is a long-running **conversation**, reaching across times and cultures, in which we try to **make sense of things**. In taking part in this conversation, we **change ourselves** and our **world**.

Attention to what ***we*** mean and what ***others*** mean is a part of being a good conversation partner.

Noticing what ***follows*** from what we say, what is a reason ***for*** and a reason ***against***, is part of sense making.

Philosophy is a long-running **conversation**, reaching across times and cultures, in which we try to **make sense of things**. In taking part in this conversation, we **change ourselves** and our **world**.

Attention to what ***we*** mean and what ***others*** mean is a part of being a good conversation partner.

Noticing what ***follows*** from what we say, what is a reason ***for*** and a reason ***against***, is part of sense making.

Philosophy is a long-running **conversation**,

reach ... try to

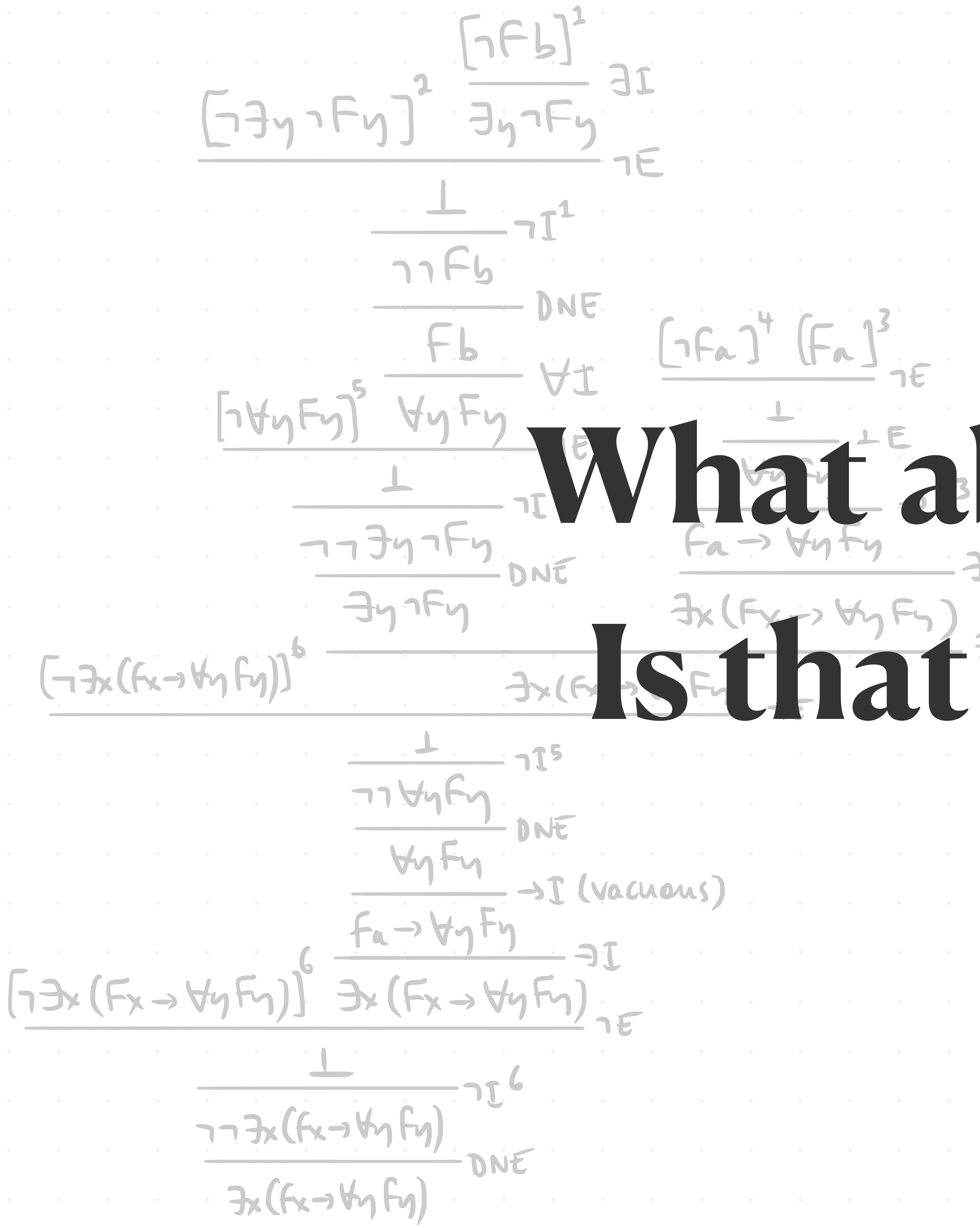
make ...

When we're trying to make sense in areas where ***concepts are contested*** and where our judgements diverge, reflection *on* our reasoning is crucial.

conversation, we **change ourselves** and our **world**.

Engaging in philosophy well involves *good reasoning*: in that sense, logic matters for philosophy.

Logic has become rather *formal*



Definition 55 (Double Indexed Models) *A double indexed model is given by a pair $\langle W, V\rangle$ consisting of a non-empty set W of worlds and a valuation function V assigning a truth value from $\{0,1\}$ to each pair consisting of an atom and a world. Given this data, we can define a function assigning a truth value to each formula and a* pair *of worlds, like this:*

- $V(p,w,v)=1$ *iff* $V(p,w)=1$.
- $V(A\wedge B,w,v)=1$ *iff* $V(A,w,v)=1$ *and* $V(B,w,v)=1$.
- $V(A\vee B,w,v)=1$ *iff* $V(A,w,v)=1$ *or* $V(B,w,v)=1$.
- $V(A\to B,w,v)=1$ *iff* $V(A,w,v)=0$ *or* $V(B,w,v)=1$.
- $V(\neg A,w,v)=1$ *iff* $V(A,w,v)=0$.
- [illegible]
- $V(\Box A,w,v)=1$ *iff* $V(A,u,v)=1$ *for each world* $u\in W$.
- $V(\Diamond A,w,v)=1$ *iff* $V(A,u,v)=1$ *for some world* $u\in W$.
- $V(@A,w,v)=1$ *iff* $V(A,v,v)=1$.

We understand $V(A,w,v)=1$ as saying that the formula A is true at the world w, given that the world v is actual.

Double indexed models can do everything that actuality models can do, without forcing the choice of an actual world. *Any* world can be treated as actual.

Definition 56 (Double indexed counterexamples, validity) *A double indexed model $\langle W,V\rangle$ is a* counterexample *to an argument $X \succ A$ iff there are worlds $w,v\in W$ such that $V(X,w,v)=1$ and $V(A,w,v)=0$.*

An argument $X\succ A$ is double indexed valid *iff there are no counterexamples to the argument. Or, equivalently, for all models $\langle W,V\rangle$, for all $w,v\in W$, if $V(X,w,v)=1$ then $V(A,w,v)=1$. If the argument $X\succ A$ is double indexed valid, then we write $X\models_{\mathsf{DI}} A$.*

What about *formal* logic?
Is that *truly* important?

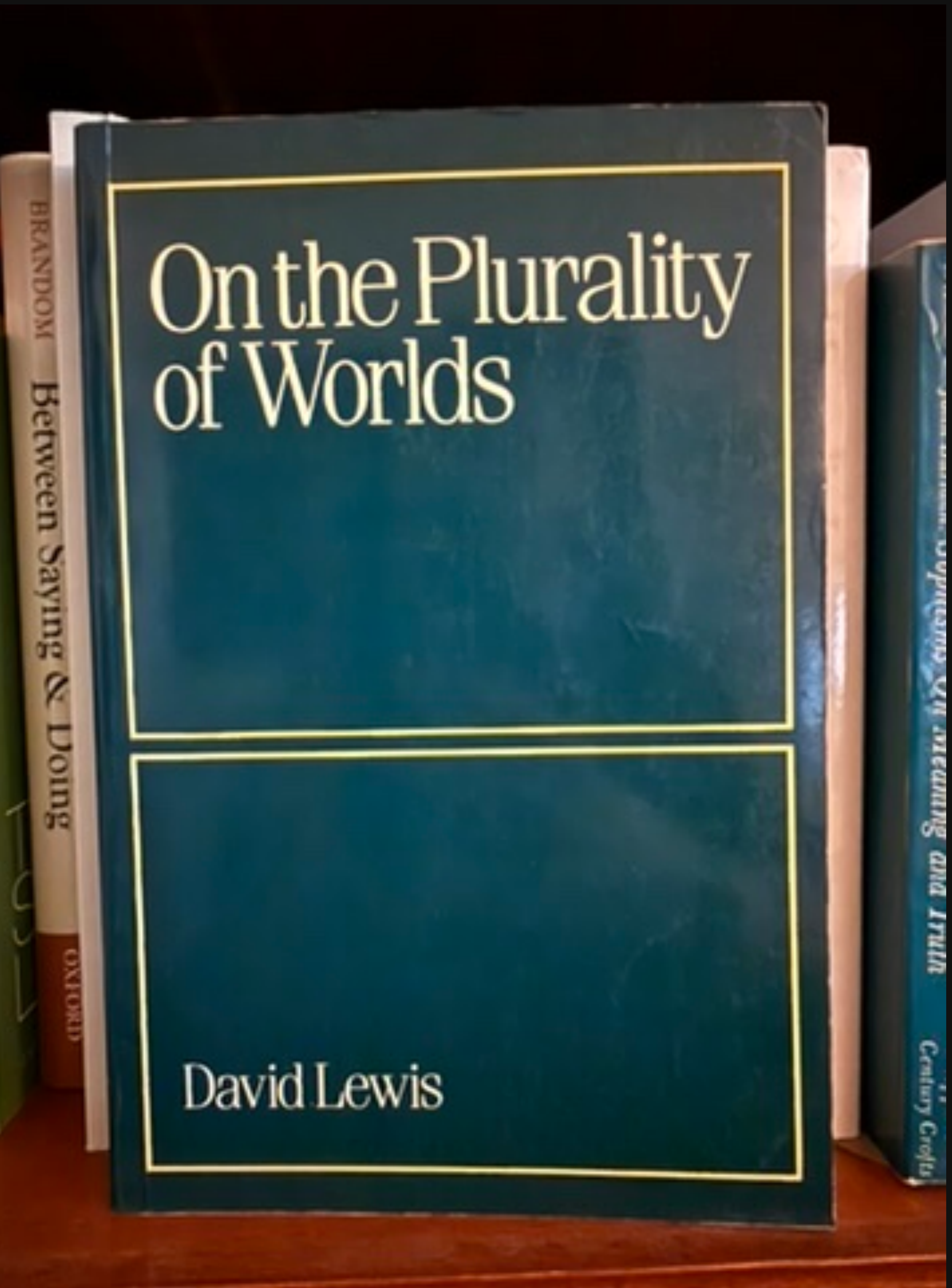
These are examples from the last few of weeks' content in *PY2010 Intermediate Logic*.

The more *general* your philosophical perspective or position, the more *abstract* and *formal* your reasoning will become.

A case study: the philosophy of **necessity** and **contingency**.

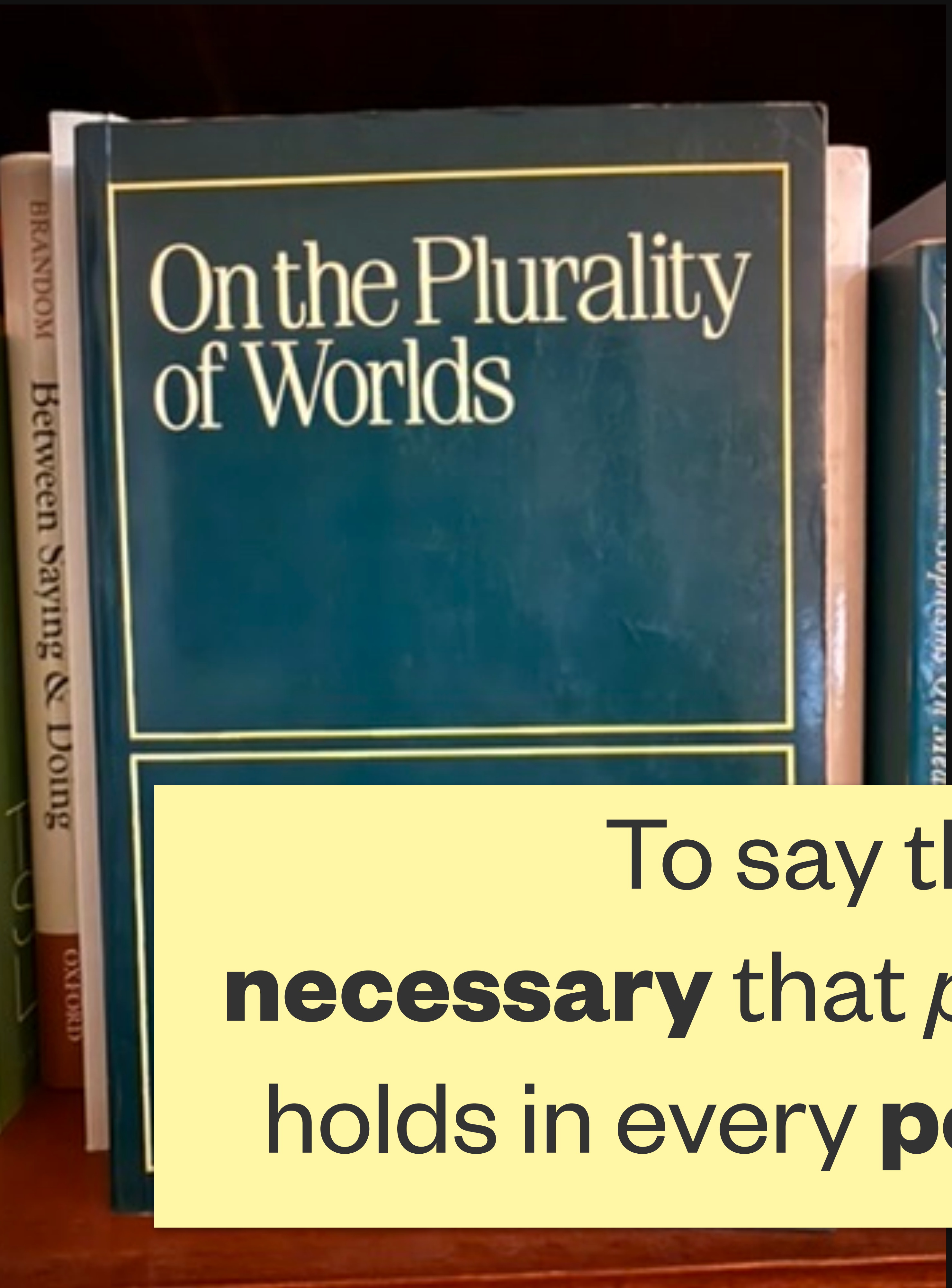
David Lewis

1941 – 2001

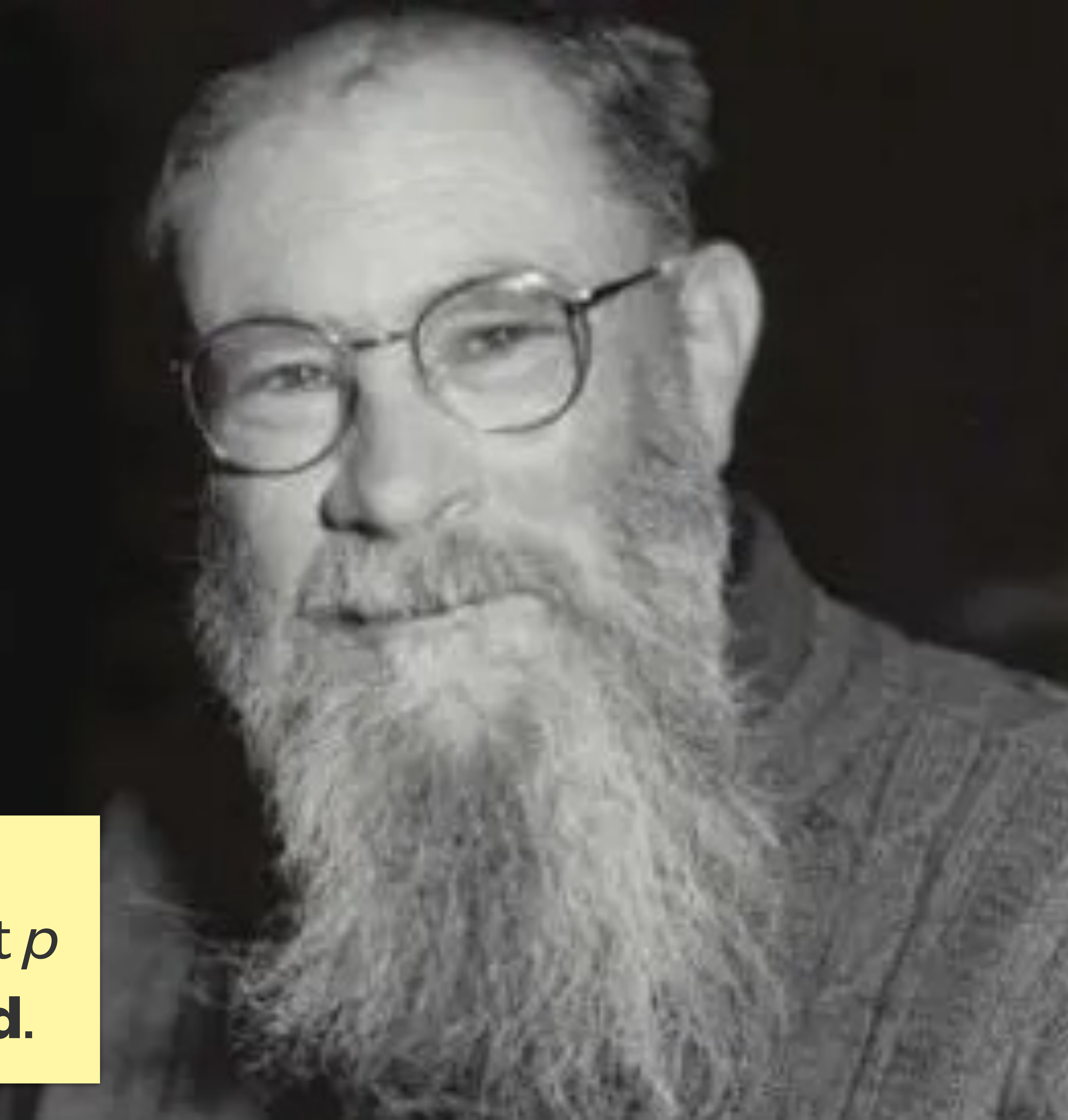


David Lewis

1941 – 2001

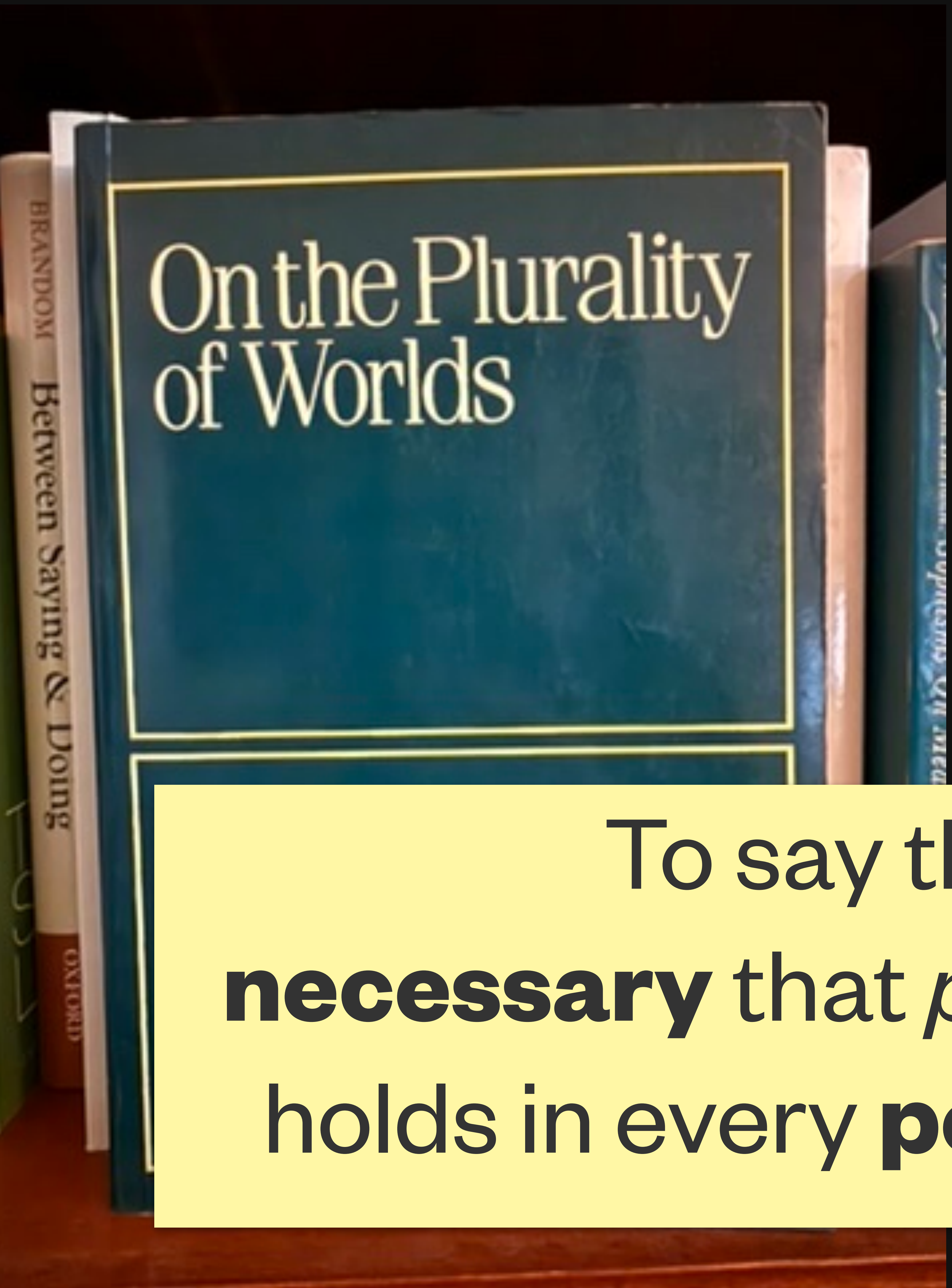


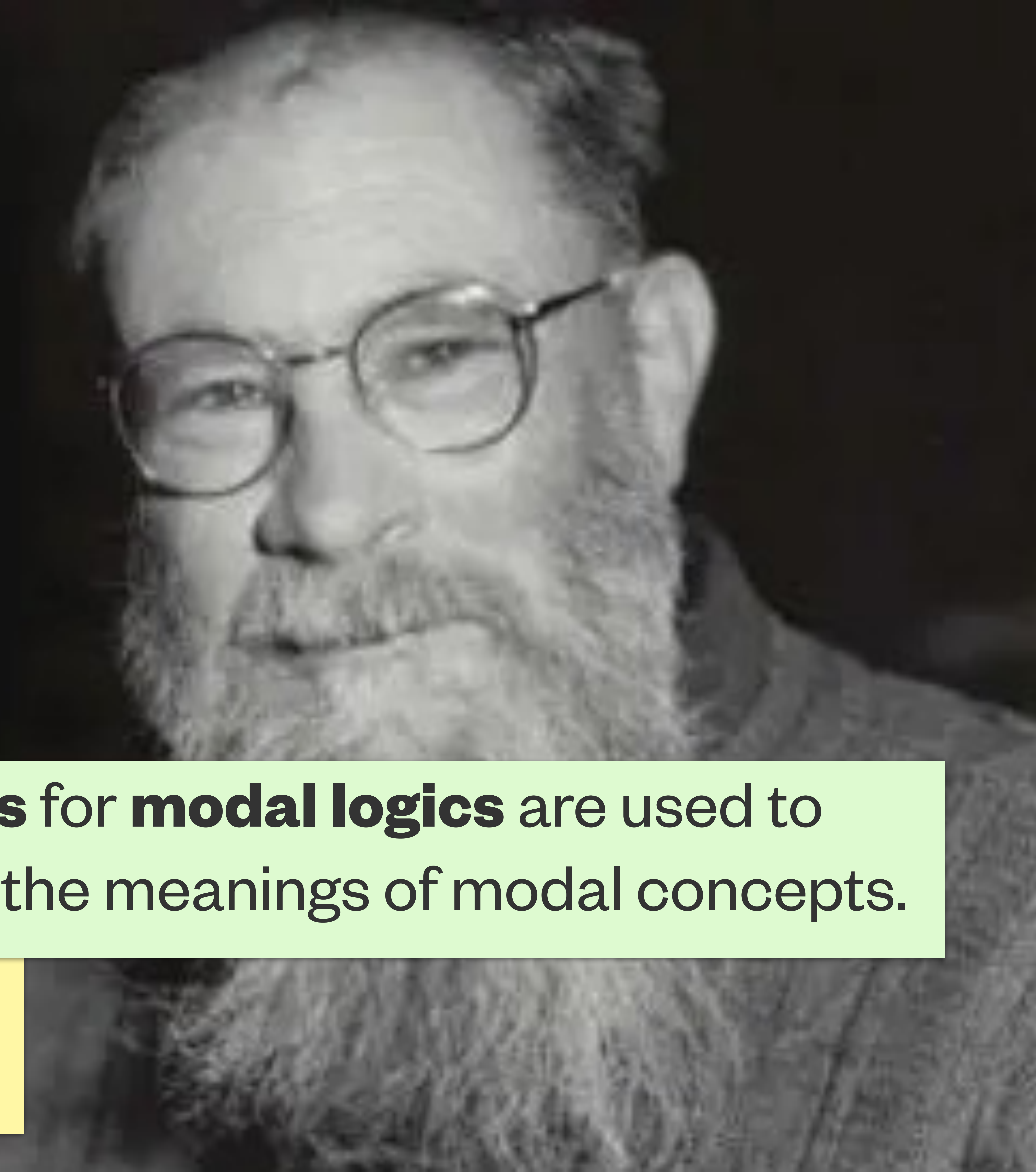
To say that it is **necessary** that *p* is to say that *p* holds in every **possible world**.



David Lewis

1941–2001





Models for **modal logics** are used to articulate the meanings of modal concepts.

To say that it
necessary that *p* is to say that *p* holds in every **possible world**.

Amie Thomasson

3

A Neopragmatist Approach to Modality

Amie L. Thomasson

Modality has long presented a range of philosophical problems and puzzles. For example, are there (really) modal properties, modal facts, or possible worlds? If there are modal properties, how could they be related to non-modal properties or relations? If there are modal facts, properties, or possible worlds, how could we come to know about them, given that modal features of the world seem not to be empirically detectable, and that possible worlds seem to be, in principle, causally disconnected from us?

I will argue that we can make a better approach to the problems of modality if we start a step back. That is, I will suggest that we should begin not with metaphysical questions about what modal properties or possible worlds *are* (or whether there are any), or with epistemological questions about how we could know them. Instead, I will argue, we should begin by asking what *functions* modal discourse serves, and what *rules* it follows. This, of course, is to take a broadly neopragmatist approach to the problems of modality.[1] For as neopragmatists such as David MacArthur and Huw Price have put it, "pragmatism begins with questions about the functions and genealogy of certain *linguistic* items…It begins with linguistic behaviour, and asks broadly anthropological questions: How are we to understand the roles and functions of the behaviour in question, in the lives of the creatures concerned?" (2007: 95). One overarching theme is that a better understanding of the language in question can lead us to reevaluate many traditional philosophical problems. But the proof is in the pudding. Here I will aim to demonstrate the plausibility and usefulness of the neopragmatist approach with some modal pudding.

Annals of Pure and Applied Logic 163 (2012) 1611–1623

Contents lists available at SciVerse ScienceDirect

Annals of Pure and Applied Logic

journal homepage: www.elsevier.com/locate/apal

A cut-free sequent system for two-dimensional modal logic, and why it matters

Greg Restall

Philosophy Department, The University of Melbourne, Parkville 3010, Australia

A R T I C L E   I N F O

Article history:
Available online 26 December 2011

MSC:
03A05
03B45
03F05

Keywords:
Modal logic
Hypersequent
Completeness
Semantics

A B S T R A C T

The two-dimensional modal logic of Davies and Humberstone (1980) [3] is an important aid to our understanding the relationship between *actuality*, *necessity* and *a priori knowability*. I show how a cut-free hypersequent calculus for 2D modal logic not only captures the logic precisely, but may be used to address issues in the epistemology and metaphysics of our modal concepts. I will explain how the use of our concepts motivates the inference rules of the sequent calculus, and then show that the completeness of the calculus for Davies–Humberstone models explains why those concepts have the structure described by those models. The result is yet another application of the completeness theorem.

1. Introduction

The 'two-dimensional modal logic' of Davies and Humberstone [3] is an important aid to our understanding the relationship between *actuality*, *necessity* and *a priori knowability*. It is widely used in philosophical discussions of these notions, but it is by no means uncontroversial [2,4,5,7,13]. Models for the logic are well understood. It is a standard modal logic, but instead of evaluating statements at worlds, we double index, and evaluate at pairs of worlds. A holds at $\langle w, v\rangle$ iff from the perspective of w as the actual world, A would have held had v obtained. Then $\Box A$ holds at $\langle w, v\rangle$ if A holds at $\langle w, v'\rangle$ for each different world v' (A is necessary if it holds at every alternative world), $@A$ holds at $\langle w, v\rangle$ iff A holds at $\langle w, w\rangle$ (A is actually the case if it holds back at the actual world), and $\mathsf{APK}A$ holds at $\langle w, v\rangle$ iff A holds at $\langle w', w'\rangle$ for each world w' (if A holds in every circumstance considered as actual, it holds, however things could actually be). In these models, we can consider another world as a subjunctive alternative (had things gone differently, that would have been the case) or as an indicative alternative (we might be wrong and that might actually be the case). $\Box$ is the modal logic corresponding to subjunctive alternatives and APK is the modal logic corresponding to indicative alternatives.

The two notions of necessity fall apart rather radically, as you can see with the presence of the actuality operator @. In any model, $p \equiv @p$ is true at each pair $\langle w, w\rangle$. Suppose p is true at $\langle w, w\rangle$, but that p is false at a subjunctive alternative

3

A Neopragmatist Approach to Modality

Amie L. Thomasson

Annals of Pure and Applied Logic 163 (2012) 1611–1623

Contents lists available at SciVerse ScienceDirect

Annals of Pure and Applied Logic

journal homepage: www.elsevier.com/locate/apal

ELSEVIER

Those areas of philosophy that involve ***generality*** and ***abstraction*** and the ***limits of our thought and talk*** can fruitfully involve techniques from formal logic.

neopragmatist approach to the problems of modality.[1] For as neopragmatists such as David MacArthur and Huw Price have put it, "pragmatism begins with questions about the functions and genealogy of certain *linguistic* items…It begins with linguistic behaviour, and asks broadly anthropological questions: How are we to understand the roles and functions of the behaviour in question, in the lives of the creatures concerned?" (2007: 95). One overarching theme is that a better understanding of the language in question can lead us to reevaluate many traditional philosophical problems. But the proof is in the pudding. Here I will aim to demonstrate the plausibility and usefulness of the neopragmatist approach with some modal pudding.

1. Introduction

The 'two-dimensional modal logic' of Davies and Humberstone [3] is an important aid to our understanding the relationship between *actuality*, *necessity* and *a priori knowability*. It is widely used in philosophical discussions of these notions, but it is by no means uncontroversial [2,4,5,7,13]. Models for the logic are well understood. It is a standard modal logic, but instead of evaluating statements at worlds, we double index, and evaluate at pairs of worlds. A holds at $\langle w, v\rangle$ iff from the perspective of w as the actual world, A would have held had v obtained. Then $\Box A$ holds at $\langle w, v\rangle$ if A holds at $\langle w, v'\rangle$ for each different world v' (A is necessary if it holds at every alternative world), $@A$ holds at $\langle w, v\rangle$ iff A holds at $\langle w, w\rangle$ (A is actually the case if it holds back at the actual world), and $\mathsf{APK}A$ holds at $\langle w, v\rangle$ iff A holds at $\langle w', w'\rangle$ for each world w' (if A holds in every circumstance considered as actual, it holds, however things could actually be). In these models, we can consider another world as a subjunctive alternative (had things gone differently, that would have been the case) or as an indicative alternative (we might be wrong and that might actually be the case). $\Box$ is the modal logic corresponding to subjunctive alternatives and APK is the modal logic corresponding to indicative alternatives.

The two notions of necessity fall apart rather radically, as you can see with the presence of the actuality operator @. In any model, $p \equiv @p$ is true at each pair $\langle w, w\rangle$. Suppose p is true at $\langle w, w\rangle$, but that p is false at a subjunctive alternative

1. Orientation

2. Why *logic* matters for *philosophy*

3. Why *philosophy* matters for *logic*

The proof system was ***designed*** to represent patterns in what you and I do when talking about different things that *could* or *must* be.

G. Restall / Annals of Pure and Applied Logic 163 (2012) 1611–1623 1615

2.3. A priori knowability

Now consider *a priori* knowability. Here we move to consider two different ways we may suppose. For □ the different suppositions are different ways things might have been, had things gone differently. The different contexts are non-actual, not marked by @. On the other hand, when we consider what might be the case if we are wrong—if we consider *indicative* alternatives—we are considering different ways we may take the actual world to be.

> ". . . the ur-distinction, accounting for all the others, is between two practices of reasoning: indicative reasoning, which functions in the first instance to update our doxastic state in the face of new evidence, and subjunctive reasoning, the root context of which is deliberation about what to do and about the propriety of actual or potential actions. These different practices of reasoning are expressed in the differences between indicative and subjunctive conditionals, but in our view it is the conditionals that are to be understood in terms of the practices of reasoning rather than the other way around." — Mark Lance and W. Heath White [6]

Lance and White argue in this paper that these two abilities are deeply embedded in our nature as creatures who *act* on the basis of a *perspective*. We both correct our perspectives (hence the need for indicative update) and to reason about different plans (hence the need for subjunctive update). So, we have not only subjunctive alternatives, but we may also have indicative alternatives, which introduce into the discourse assertions with other zones taken as actual—they are alternative ways to take actual matters to be. The structure of a hypersequent is now more complex:

Definition 1 (*2D Hypersequents*)**.** A 2D hypersequent is a multiset of multisets of sequents. Each inner multiset has a single sequent marked with an @, its ACTUAL SEQUENT. The sequents in each inner multiset are SUBJUNCTIVE ALTERNATIVES to one another. Each of the actual sequents in hypersequent are the *indicative alternatives* to each and every sequent in the whole hypersequent. We use the following notation for a 2D hypersequent, marking off subjunctive alternatives with a single bar (|), indicative alternatives with a double bar (‖) and actual sequents with a subscripted @. The general structure is this.

$$\begin{array}{l} X_0^0 \vdash_@ Y_0^0 \;\mid\; X_1^0 \vdash Y_1^0 \;\mid\; \cdots \;\mid\; X_{n^0}^0 \vdash Y_{n^0}^0 \;\; \| \\ X_0^1 \vdash_@ Y_0^1 \;\mid\; X_1^1 \vdash Y_1^1 \;\mid\; \cdots \;\mid\; X_{n^1}^1 \vdash Y_{n^1}^1 \;\; \| \;\; \cdots \;\; \| \\ X_0^m \vdash_@ Y_0^m \;\mid\; X_1^m \vdash Y_1^m \;\mid\; \cdots \;\mid\; X_{n^m}^m \vdash Y_{n^m}^m \end{array}$$

in which each $X_0^i \vdash_@ Y_0^i$ is an indicative alternative of every sequent, and each $X_j^i \vdash Y_j^i$ is a subjunctive alternative of $X_k^i \vdash Y_k^i$.

The APK operator exploits indicative alternatives in just the same way as □ exploits subjunctive alternatives. Asserting APK p in one zone clashes with denying p in its indicative alternatives. The following sequent is derivable:

$$\mathsf{APK}\, p \vdash \;\; \| \;\; \vdash_@ p.$$

In fact, we will have the following sort of derivation:

$$\dfrac{\dfrac{\dfrac{\vdash \;\|\; p \vdash_@ p}{\vdash \;\|\; p \vdash_@ @p}[@R]}{\vdash \;\|\; \vdash_@ p \supset @p}[\supset R]}{\vdash \mathsf{APK}\,(p \supset @p)}[\mathsf{APK}\, R]$$

Now we have the resources to define the sequent system. We will use one more convention for notation. When we write

$$\mathcal{H}[X \vdash Y]$$

this is a 2D hypersequent, in which $X \vdash Y$ occurs as a particular component sequent. This component sequent may be marked as actual, it may not. When we write

$$\mathcal{H}[X' \vdash Y']$$

this is the 2D hypersequent which results from taking $\mathcal{H}[X \vdash Y]$ and replacing the component sequent $X \vdash Y$ with the sequent $X' \vdash Y'$. In addition, if the indicated $X \vdash Y$ was marked as actual in $\mathcal{H}[X \vdash Y]$, so is the indicated $X' \vdash Y'$ in

The proof system was ***designed*** to represent patterns in what you and I do when talking about different things that *could* or *must* be.

The *philosophy* came before the formal *logic.*

2.3. A priori knowability

Now consider *a priori* knowability. Here we move to consider two different ways we may suppose. For □ the different suppositions are different ways things might have been, had things gone differently. The different contexts are non-actual, not marked by @. On the other hand, when we consider what might be the case if we are wrong—if we consider *indicative* alternatives—we are considering different ways we may take the actual world to be.

> "... the ur-distinction, accounting for all the others, is between two practices of reasoning: indicative reasoning, which functions in the first instance to update our doxastic state in the face of new evidence, and subjunctive reasoning, the root context of which is deliberation about what to do and about the propriety of actual or potential actions. These different practices of reasoning are expressed in the differences between indicative and subjunctive conditionals, but in our view it is the conditionals that are to be understood in terms of the practices of reasoning rather than the other way around." — Mark Lance and W. Heath White [6]

Lance and White argue in this paper that these two abilities are deeply embedded in our nature as creatures who *act* on the basis of a *perspective*. We both correct our perspectives (hence the need for indicative update) and to reason about different plans (hence the need for subjunctive update). So, we have not only subjunctive alternatives, but we may also have indicative alternatives, which introduce into the discourse assertions with other zones taken as actual—they are alternative ways to take actual matters to be. The structure of a hypersequent is now more complex:

Definition 1 (*2D Hypersequents*). A 2D hypersequent is a multiset of multisets of sequents. Each inner multiset has a single sequent marked with an @, its ACTUAL SEQUENT. The sequents in each inner multiset are SUBJUNCTIVE ALTERNATIVES to one another. Each of the actual sequents in hypersequent are the *indicative alternatives* to each and every sequent in the whole hypersequent. We use the following notation for a 2D hypersequent, marking off subjunctive alternatives with a single bar (|), indicative alternatives with a double bar (‖) and actual sequents with a subscripted @. The general structure is this.

$$
\begin{array}{l}
X_0^0 \vdash_@ Y_0^0 \;\mid\; X_1^0 \vdash Y_1^0 \;\mid\; \cdots \;\mid\; X_{n^0}^0 \vdash Y_{n^0}^0 \;\;\| \\
X_0^1 \vdash_@ Y_0^1 \;\mid\; X_1^1 \vdash Y_1^1 \;\mid\; \cdots \;\mid\; X_{n^1}^1 \vdash Y_{n^1}^1 \;\;\| \;\cdots\; \| \\
X_0^m \vdash_@ Y_0^m \;\mid\; X_1^m \vdash Y_1^m \;\mid\; \cdots \;\mid\; X_{n^m}^m \vdash Y_{n^m}^m
\end{array}
$$

in which each $X_0^i \vdash_@ Y_0^i$ is an indicative alternative of every sequent, and each $X_j^i \vdash Y_j^i$ is a subjunctive alternative of $X_k^i \vdash Y_k^i$.

The APK operator exploits indicative alternatives in just the same way as □ exploits subjunctive alternatives. Asserting APK p in one zone clashes with denying p in its indicative alternatives. The following sequent is derivable:

$$\mathsf{APK}\, p \vdash \quad \| \quad \vdash_@ p.$$

In fact, we will have the following sort of derivation:

$$
\dfrac{\dfrac{\dfrac{\vdash \;\|\; p \vdash_@ p}{\vdash \;\|\; p \vdash_@ @p}\,[@R]}{\vdash \;\|\; \vdash_@ p \supset @p}\,[\supset R]}{\vdash \mathsf{APK}\,(p \supset @p)}\,[\mathsf{APK}\,R]
$$

Now we have the resources to define the sequent system. We will use one more convention for notation. When we write

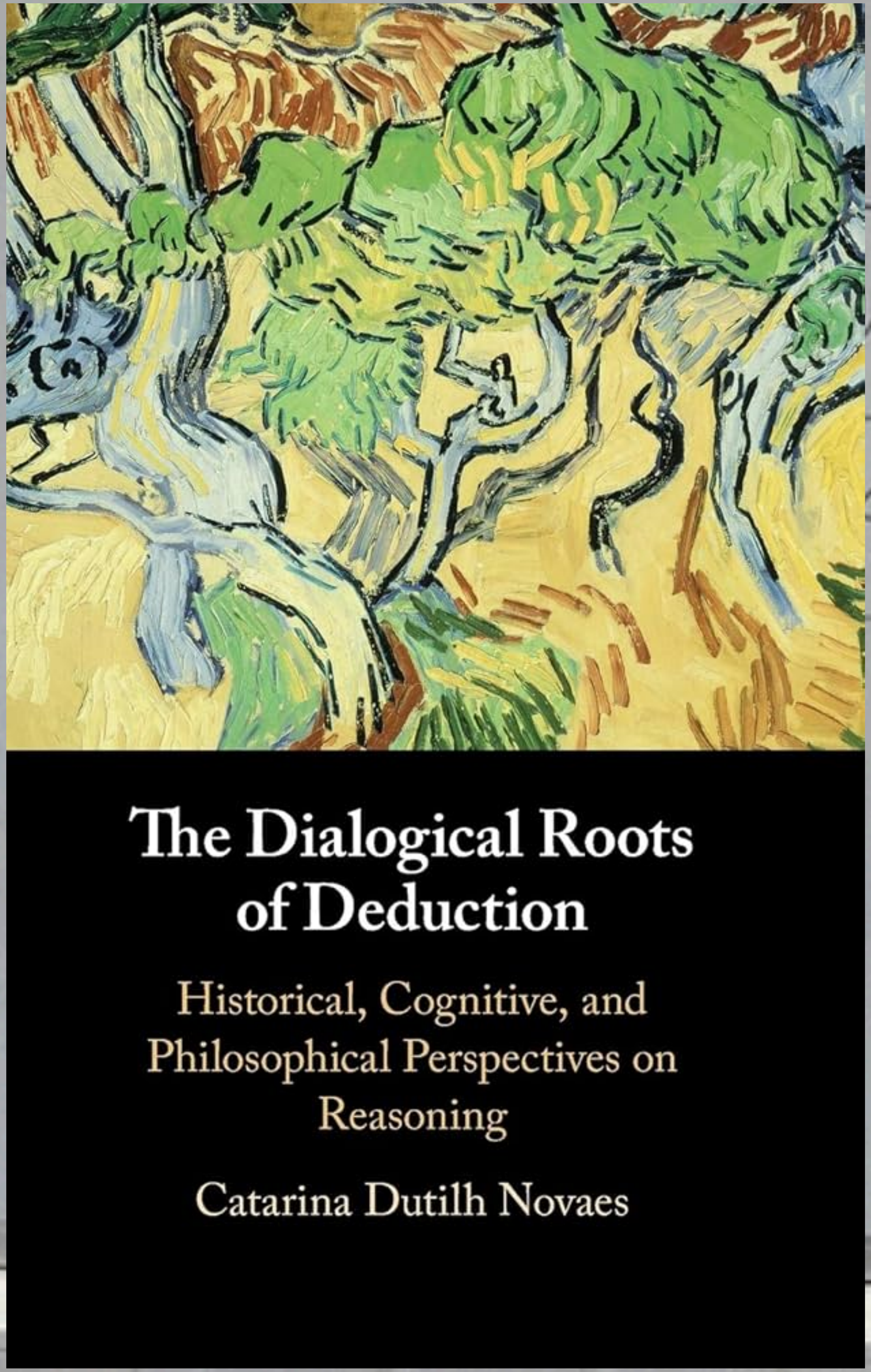
$$\mathcal{H}[X \vdash Y]$$

this is a 2D hypersequent, in which $X \vdash Y$ occurs as a particular component sequent. This component sequent may be marked as actual, it may not. When we write

$$\mathcal{H}[X' \vdash Y']$$

this is the 2D hypersequent which results from taking $\mathcal{H}[X \vdash Y]$ and replacing the component sequent $X \vdash Y$ with the sequent $X' \vdash Y'$. In addition, if the indicated $X \vdash Y$ was marked as actual in $\mathcal{H}[X \vdash Y]$, so is the indicated $X' \vdash Y'$ in

Catarina Dutilh Novaes




UNIVERSITY OF ST. ANDREWS
DEPARTMENT OF
MORAL
PHILOSOPHY
UNIVERSITY OF ST. ANDREWS
DEPARTMENT OF
LOGIC
AND
METAPHYSICS


UNIVERSITY OF ST. ANDREWS DEPARTMENT OF MORAL PHILOSOPHY
UNIVERSITY OF ST. ANDREWS DEPARTMENT OF LOGIC AND METAPHYSICS

Thanks