What's so Special about Logic? Practices, Rules and Definitions

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LOGIC DAY / 1 NOVEMBER 2019 / MELBOURNE

My ***Aim***

To understand *logic* better…

My ***Aim***

To understand *logic* better…

… and to come to grips with *anti-exceptionalism*.

What logic *is*

Anti-exceptionalism

Quine

Practices

Rules

Definitions

WHAT LOGIC *IS*

PROOF THEORY

- Design and construction of different proof systems, proofs in those systems, and results *about* those proof systems.
- Axiomatic development of different theories. Translations between theories, reductions, embeddings …

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- Model constructions for different theories. Modelling different phenomena. Independence results.

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MODEL THEORY

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METATHEORY

- Soundness and completeness. Limitative Results.

External & Internal

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as mathematical structures to be *analysed*, and *adopting* them.

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There's a difference between comparing different logics, and *using* a logic, by using a given *proof* or taking a *model* to interpret a theory.

ANTI-EXCEPTIONALISM

What is anti-exceptionalism?

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CrossMark

Anti-exceptionalism about logic

Ole Thomassen Hjortland[1]

Published online: 9 June 2016

Abstract Logic isn't special. Its theories are continuous with science; its method continuous with scientific method. Logic isn't a priori, nor are its truths analytic truths. Logical theories are revisable, and if they are revised, they are revised on the same grounds as scientific theories. These are the tenets of anti-exceptionalism about logic. The position is most famously defended by Quine, but has more recent advocates in Maddy (Proc Address Am Philos Assoc 76:61–90, 2002), Priest (Doubt truth to be a liar, OUP, Oxford, 2006a, The metaphysics of logic, CUP, Cambridge, 2014, Log et Anal, 2016), Russell (Philos Stud 171:161–175, 2014, J Philos Log 0:1–11, 2015), and Williamson (Modal logic as metaphysics, Oxford University Press, Oxford, 2013b, The relevance of the liar, OUP, Oxford, 2015). Although

What is anti-exceptionalism?

- Logic isn't special.
- Logic's theories are continuous with science.
- Logic's methods are continuous with scientific method.
- Logic isn't *a priori*.
- Logic's truths are not analytic truths.
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Compare Arithmetic

- Arithmetic isn't special.
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$$
\dfrac{
  \dfrac{
    \dfrac{}{0''+0''=(0''+0')'}\,{\scriptstyle Q5}
    \qquad
    \dfrac{\dfrac{}{0''+0'=(0''+0)'}\,{\scriptstyle Q5}}{(0''+0')'=(0''+0)''}\,{\scriptstyle '=}
  }{0''+0''=(0''+0)''}\,{\scriptstyle =T}
  \qquad
  \dfrac{\dfrac{\dfrac{}{0''+0=0''}\,{\scriptstyle Q4}}{(0''+0)'=0'''}\,{\scriptstyle '=}}{(0''+0)''=0''''}\,{\scriptstyle '=}
}{0''+0''=0''''}\,{\scriptstyle =T}
$$

(Q4) $x+0=x$ (Q5) $x+y'=(x+y)'$ ($'=$) $x=y\,/\,x'=y'$ ($=T$) $x=y, y=z\,/\,x=z$

Here's a proof that $2 + 2 = 4$, in Robinson's Arithmetic

$$\dfrac{\dfrac{\dfrac{}{0''+0''=(0''+0')'}\,\text{Q5} \qquad \dfrac{\dfrac{}{0''+0'=(0''+0)'}\,\text{Q5}}{(0''+0')'=(0''+0)''}\,{'}{=}}{0''+0''=(0''+0)''}\,{=}T \qquad \dfrac{\dfrac{\dfrac{}{0''+0=0''}\,\text{Q4}}{(0''+0)'=0'''}\,{'}{=}}{(0''+0)''=0''''}\,{'}{=}}{0''+0''=0''''}\,{=}T$$

(Q4) $x + 0 = x$ (Q5) $x + y' = (x + y)'$ (${'}{=}$) $x = y \,/\, x' = y'$ (${=}T$) $x = y, y = z \,/\, x = z$

Is this derivation *a priori* or *a posteriori*?

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$$
\dfrac{
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  \qquad
  \dfrac{\dfrac{}{0'' + 0' = (0'' + 0)'}\,{\scriptstyle Q5}}{(0'' + 0')' = (0'' + 0)''}\,{\scriptstyle '=}
}{0'' + 0'' = (0'' + 0)''}\,{\scriptstyle =T}
\qquad
\dfrac{}{}
\!\!\!\!
\dfrac{
  0'' + 0'' = (0'' + 0)''
  \qquad
  \dfrac{\dfrac{\dfrac{}{0'' + 0 = 0''}\,{\scriptstyle Q4}}{(0'' + 0)' = 0'''}\,{\scriptstyle '=}}{(0'' + 0)'' = 0''''}\,{\scriptstyle '=}
}{0'' + 0'' = 0''''}\,{\scriptstyle =T}
$$

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If some evidence were needed to supplement the argument, where would we add it?

External Questions and Internal Questions

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What does this proof do?

$$
\dfrac{[p \to q]^3 \qquad \dfrac{[r \to p]^2 \qquad [r]^1}{p}{\to}E}{\dfrac{\dfrac{\dfrac{q}{r \to q}{\to}I^1}{(r \to p) \to (r \to q)}{\to}I^2}{(p \to q) \to ((r \to p) \to (r \to q))}{\to}I^3}{\to}E
$$

What does this proof do?

$$
\begin{prooftree}
\AxiomC{$[p \to q]^3$}
\AxiomC{$[r \to p]^2$}
\AxiomC{$[r]^1$}
\RightLabel{$\to E$}
\BinaryInfC{p}
\RightLabel{$\to E$}
\BinaryInfC{q}
\RightLabel{$\to I^1$}
\UnaryInfC{$r \to q$}
\RightLabel{$\to I^2$}
\UnaryInfC{$(r \to p) \to (r \to q)$}
\RightLabel{$\to I^3$}
\UnaryInfC{$(p \to q) \to ((r \to p) \to (r \to q))$}
\end{prooftree}
$$

Is the conclusion *analytic*?

What does this proof do?

$$\dfrac{\dfrac{\dfrac{[p \to q]^3 \qquad \dfrac{[r \to p]^2 \qquad [r]^1}{p}{\scriptstyle \to E}}{q}{\scriptstyle \to E}}{\dfrac{r \to q}{(r \to p) \to (r \to q)}{\scriptstyle \to I^2}}{\scriptstyle \to I^1}}{(p \to q) \to ((r \to p) \to (r \to q))}{\scriptstyle \to I^3}$$

Is the conclusion *analytic*? ¶ Is the conclusion *a priori*?

What does this proof do?

$$
\dfrac{\dfrac{\dfrac{\dfrac{[p \to q]^3 \qquad \dfrac{[r \to p]^2 \qquad [r]^1}{p}\,{\to}E}{q}\,{\to}E}{r \to q}\,{\to}I^1}{(r \to p) \to (r \to q)}\,{\to}I^2}{(p \to q) \to ((r \to p) \to (r \to q))}\,{\to}I^3
$$

Is the conclusion *analytic*? ¶ Is the conclusion *a priori*? ¶ Is the proof *special*?

QUINE

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§13 Translating Logical Connectives

In §§7 through 11 we accounted for radical translation of occasion sentences, by approximate identification of stimulus meanings. Now there is also a decidedly different domain that lends itself directly to radical translation: that of *truth functions* such as negation, logical conjunction, and alternation. For this purpose the sentences put to the native for assent or dissent may be occasion sentences and standing sentences indifferently. Those that are occasion sentences will have to be accompanied by a prompting stimulation, if assent or dissent is to be elicited; the standing sentences, on the other hand, can be put without props. Now by reference to assent and dissent we can state semantic criteria for truth functions; i.e., criteria for determining whether a given native idiom is to be construed as expressing the truth function in question. The semantic criterion of negation is that it turns any short sentence to which one will assent into a sentence from which one will dissent, and vice versa. That of conjunction is that it produces compounds to which (so long as the component sentences are short) one is prepared to assent always and only when one is prepared to assent to each component. That of alternation is similar with assent changed twice to dissent.

Why, then, is a conjunction true when both conjuncts are true?

Constraints

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Why is a disjunction false when both disjuncts are false?

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For the Quine of *Word and Object*, inferences like these *are a priori* valid.

Why, then, is a conjunction true when both conjuncts are true?

Why is a disjunction false when both disjuncts are false?

For the Quine of *Word and Object*, inferences like these *are a priori* valid.

(Not *a priori* in the sense that they are unrevisable,
but in the sense that *if* the terms have the meanings
we have postulated, we do not need to appeal to evidence
to ground the validity of the inference.)

The constitutive and relativized a priori

... the concept of the relativized a priori, as originally formulated within the tradition of logical empiricism, was explicitly intended to prise apart two meanings that were discerned within the original Kantian conception: necessary and unrevisable, true for all time, on the one hand, and "constitutive of the concept of the object of [*scientific*] *knowledge," on the other.*

— Michael Friedman, *The Dynamics of Reason* (2002)

Constraints

What does "*and*", in this sense, *mean*?

What does “*and*”, in this sense, *mean*?

What does “*or*”, in this sense, *mean*?

Constraints

What does "*and*", in this sense, *mean*?

What does "*or*", in this sense, *mean*?

For the Quine of *Word and Object*, it is not a bridge too far to say that principles governing these particles are *definitionally analytic*.

Anti-*anti-exceptionalism—internally*

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PRACTICES

Assent and Dissent

For the Quine of *Word and Object*, you locate the logical connectives by identifying their interaction with *assent* and *dissent*.

MULTIPLE CONCLUSIONS

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March 19, 2004

Abstract: I argue for the following four theses. (1) Denial is not to be analysed as the assertion of a negation. (2) Given the concepts of assertion and denial, we have the resources to analyse logical consequence as relating arguments with *multiple* premises and *multiple* conclusions. Gentzen's multiple conclusion calculus can be understood in a straightforward, motivated, non-question-begging way. (3) If a broadly anti-realist or inferentialist justification of a logical system works, it works just as well for *classical* logic as it does for *intuitionistic* logic. The special case for an anti-realist justification of intuitionistic logic over and above a justification of classical logic relies on an unjustified assumption about the shape of proofs. Finally, (4) this picture of logical consequence provides a relatively neutral shared vocabulary which can help us understand and adjudicate debates between proponents of classical and non-classical logics.

* * *

Quine's Criteria for Negation,

The semantic criterion of negation is that it turns any short sentence to which one will assent into a sentence from which one will dissent, and vice versa.

Quine's Criteria for Negation, Conjunction

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These criteria are not enough to generate truth functional logic, unless supplemented.

Suppose I dissent from $p \vee \neg p$.

Illustrating the issue

Suppose I dissent from $p \vee \neg p$.

So, I dissent from p and dissent from $\neg p$.

So, I assent to p.

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Conjecture: I cannot dissent from any truth-functional tautology.

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Counterexample: $(p \vee \neg p) \wedge (q \vee \neg q)$.

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Counterexample: $(p \vee \neg p) \wedge (q \vee \neg q)$.

(Quine gives no conditions concerning
when to *dissent* from a conjunction.)

Quine's Project

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Quine's Project

Quine's project in *Word and Object* involved radical translation, stimulus meaning and occasion sentences, and much besides.

It does arrive at a radical holism, but one in which a certain amount of logic is constitutively *a priori*.

I will not be adopting Quine's project.

The semantic criterion of negation is that it turns any short sentence to which one will assent into a sentence from which one will dissent, and vice versa. That of conjunction is that it produces compounds to which (so long as the component sentences are short) one is prepared to assent always and only when one is prepared to assent to each component. That of alternation is similar with assent changed twice to dissent.

A more important question: How could we tell that we have located such items in someone's vocabulary?

RULES

We can bind ourselves by adopting a rule

Instead of just looking for an item in our vocabulary with the desired behaviour, we could *define* one.

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Instead of just looking for an item in our vocabulary with the desired behaviour, we could *define* one.

We can adopt a RULE: "use '$\wedge$' like *this* ..."

‘Rules’ a la Quine

DEFINIENDUM	DEFINIENS
$+\neg A$	$-A$
$+A \wedge B$	$+A, +B$
$-A \vee B$	$-A, -B$

'Rules' a la Quine

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$+\neg A$	$-A$
$+A \wedge B$	$+A, +B$
$-A \vee B$	$-A, -B$
$-A \to B$	$+A, -B$
$-\forall x A$	$-A[x/n]$ (n new)
$+\exists x A$	$+A[x/n]$ (n new)

‘Rules’ a la Quine

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$+\neg A$	$-A$
$+A \wedge B$	$+A, +B$
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$+\exists x A$	$+A[x/n]$ (n new)
$-s = t$	$+Fs, -Ft$ (F new)

‘Rules’ a la Quine

DEFINIENDUM	DEFINIENS
$+\neg A$	$-A$
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$-s = t$	$+Fs, -Ft$ (F new)

To make sense of these, we need to say more about assertion and denial, assent and dissent.

Positions

$$[X : Y]$$

$[X : Y]$

$[X, A : A, Y]$ is self-defeating.

Sequents: Unfocused and Focused

$$X \succ Y$$

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$$X \succ Y$$

$$X \succ \boxed{A}, Y \qquad X, \boxed{A} \succ Y$$

$$X, A \succ A, Y$$

Structural Rules: Identity

$$X, A \succ A, Y$$

$$X, A \succ \boxed{A}, Y$$

Structural Rules: Identity

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Structural Rules: Identity

$$X, A \succ A, Y$$

$$X, A \succ \boxed{A}, Y \qquad X, \boxed{A} \succ A, Y \qquad X, A \succ A, \boxed{B}, Y$$

Structural Rules: Cut

$$\frac{X \succ A, Y \quad X, A \succ Y}{X \succ Y}\, Cut$$

Structural Rules: Cut

$$\frac{X \succ A, Y \quad X, A \succ Y}{X \succ Y}\ \mathit{Cut}$$

$$\frac{X \succ \boxed{A}, Y \quad X, A \succ Y, \boxed{B}}{X \succ Y, \boxed{B}}\ \mathit{Cut}$$

DEFINITIONS

Defining Rules for Logical Concepts

$$\frac{X, A, B \succ Y}{X, A \land B \succ Y} \land Df \qquad \frac{X \succ A, B, Y}{X \succ A \lor B, Y} \lor Df \qquad \frac{X \succ A, Y}{X, \neg A \succ Y} \neg Df \qquad \frac{X, A \succ B, Y}{X \succ A \to B, Y} \to Df$$

$$\frac{X \succ A(n), Y}{X \succ \forall x A(x), Y} \forall Df \qquad \frac{X, A(n) \succ Y}{X, \exists x A(x) \succ Y} \exists Df \qquad \frac{X, Fa \succ Fb, Y}{X \succ a = b, Y} {=}Df$$

Terms & conditions: the singular term n (in $\forall/\exists$*Df*) and the predicate F (in $=$*Df*) do not appear below the line in those rules.

These rules can be understood as *definitions* of the concepts they introduce (below the double line).

See (Scott 1974; Došen 1980, 1989; Restall 2019).

$$
\begin{prooftree}
\AxiomC{$Fn \lor Gn \succ Fn \lor Gn, \exists x Gx$}
\RightLabel{$\lor Df$}
\UnaryInfC{$Fn \lor Gn \succ Fn, Gn, \exists x Gx$}
\RightLabel{$\forall Df$}
\UnaryInfC{$\forall x(Fx \lor Gx) \succ Fn, Gn, \exists x Gx$}
\AxiomC{$\forall x(Fx \lor Gx), \exists x Gx \succ Fn, \exists x Gx$}
\RightLabel{$\exists Df$}
\UnaryInfC{$\forall x(Fx \lor Gx), Gn \succ Fn, \exists x Gx$}
\RightLabel{Cut}
\BinaryInfC{$\forall x(Fx \lor Gx) \succ Fn, \exists x Gx$}
\RightLabel{$\forall Df$}
\UnaryInfC{$\forall x(Fx \lor Gx) \succ \forall x Fx, \exists x Gx$}
\RightLabel{$\lor Df$}
\UnaryInfC{$\forall x(Fx \lor Gx) \succ \forall x Fx \lor \exists x Gx$}
\end{prooftree}
$$

Definitions like these are Special

They are *safe* and *uniquely defining*.

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They introduce concepts governed by rules.

The concepts are well behaved.

The rules are constitutively *a priori*.

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The derivable formulas are *definitionally analytic*.

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Conservativity and *Unique Definability* are *very* special features of these logical concepts.

The rules are constitutively *a priori*.

The derivable formulas are *definitionally analytic*.

Conservativity and *Unique Definability* are *very* special features of these logical concepts.

In *this* way, logic is special.

THANK YOU!

http://consequently.org/presentation/2019/whats-so-special-about-logic-logicmelb