Existence, Definedness

and the semantics of possibility and necessity

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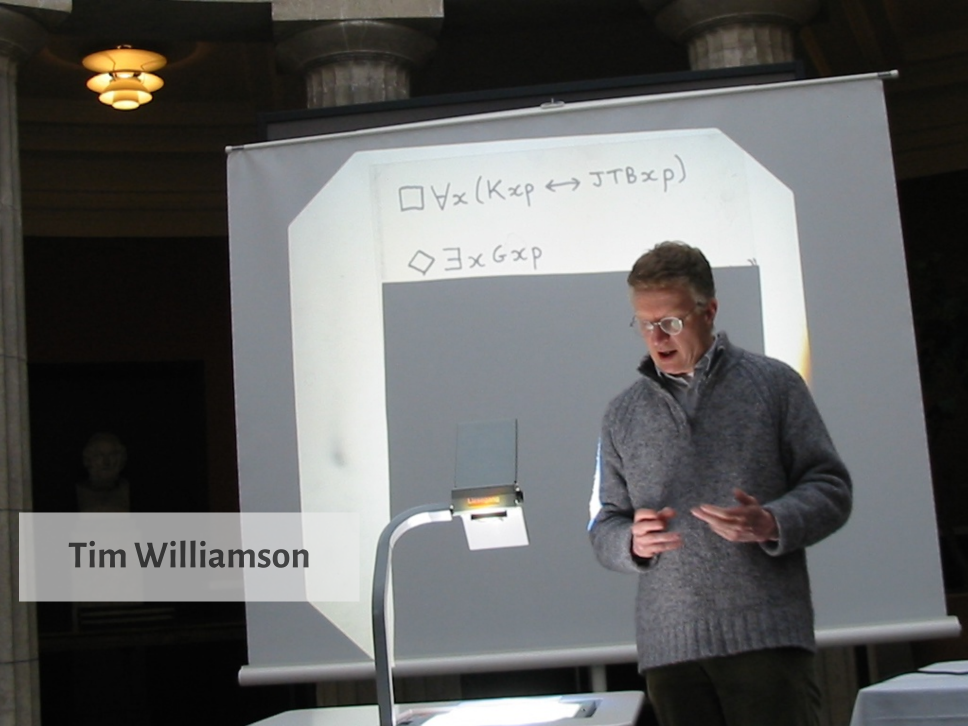
AWPL 2016 · TPLC 2016 · OCTOBER 2016

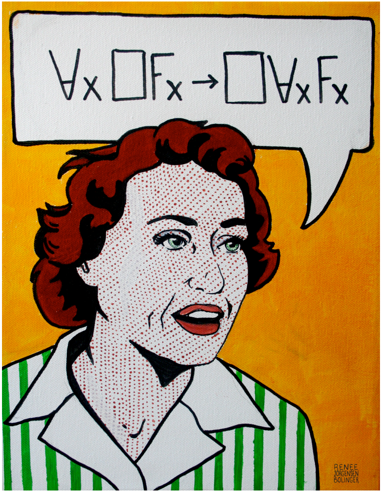
… is to discuss Professor Williamson's treatment of *necessitism* and *contingentism*, in the light of a hypersequent semantics for quantified modal logic.



Tim Williamson

MODAL MODEL THEORY AND THE BARCAN FORMULA

Ruth Barcan and the Barcan Formula



Portrait by *Renee Jorgensen Bolinger*

It's *very* easy to prove the Barcan Formula

Just add $(\forall L)$, $(\forall R)$, $(\Box L)$ & $(\Box R)$ and stir.

Barcan Derivations

$$
\dfrac{\dfrac{\dfrac{\dfrac{Fa \succ Fa}{\Box Fa \succ \quad | \quad \succ Fa}\,[\Box L]}{\forall x \Box Fx \succ \quad | \quad \succ Fa}\,[\forall L]}{\forall x \Box Fx \succ \quad | \quad \succ \forall x Fx}\,[\forall R]}{\forall x \Box Fx \succ \Box \forall x Fx}\,[\Box R]
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\color{red}{Fa \succ Fa}}{\Box Fa \succ \quad | \quad \succ Fa}[\Box L]}{\forall x \Box Fx \succ \quad | \quad \succ Fa}[\forall L]}{\forall x \Box Fx \succ \quad | \quad \succ \forall x Fx}[\forall R]}{\forall x \Box Fx \succ \Box \forall x Fx}[\Box R]
$$

Barcan Derivations

$$
\dfrac{\dfrac{\dfrac{\dfrac{Fa \succ Fa}{\Box Fa \succ \ \mid\ \succ Fa}[\Box L]}{\forall x \Box Fx \succ \ \mid\ \succ Fa}[\forall L]}{\forall x \Box Fx \succ \ \mid\ \succ \forall x Fx}[\forall R]}{\forall x \Box Fx \succ \Box \forall x Fx}[\Box R]
$$

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$$\dfrac{\dfrac{\dfrac{\dfrac{Fa \succ Fa}{\Box Fa \succ \quad \mid \quad \succ Fa}\,[\Box L]}{\forall x \Box Fx \succ \quad \mid \quad \succ Fa}\,[\forall L]}{\forall x \Box Fx \succ \quad \mid \quad \succ \forall x Fx}\,[\forall R]}{\forall x \Box Fx \succ \Box \forall x Fx}\,[\Box R]$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{Fa \succ Fa}{\Box Fa \succ \quad \mid \quad \succ Fa}\,[\Box L]}{\forall x \Box Fx \succ \quad \mid \quad \succ Fa}\,[\forall L]}{\forall x \Box Fx \succ \quad \mid \quad \succ \forall x Fx}\,[\forall R]}{\forall x \Box Fx \succ \Box \forall x Fx}\,[\Box R]
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$$\dfrac{\dfrac{\dfrac{\dfrac{Fa \succ Fa}{\Box Fa \succ \ \mid\ \succ Fa}[\Box L]}{\forall x \Box Fx \succ \ \mid\ \succ Fa}[\forall L]}{\forall x \Box Fx \succ \ \mid\ \succ \forall x Fx}[\forall R]}{\forall x \Box Fx \succ \Box \forall x Fx}[\Box R]$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{Fa \succ Fa}{\Box Fa \succ \quad \mid \quad \succ Fa}\,[\Box L]}{\forall x \Box Fx \succ \quad \mid \quad \succ Fa}\,[\forall L]}{\forall x \Box Fx \succ \quad \mid \quad \succ \forall x Fx}\,[\forall R]}{\forall x \Box Fx \succ \Box \forall x Fx}\,[\Box R]
$$

Barcan Derivations

$$\dfrac{\dfrac{\dfrac{\dfrac{Fa \succ Fa}{\Box Fa \succ \quad \mid \quad \succ Fa}[\Box L]}{\forall x \Box Fx \succ \quad \mid \quad \succ Fa}[\forall L]}{\forall x \Box Fx \succ \quad \mid \quad \succ \forall x Fx}[\forall R]}{\forall x \Box Fx \succ \Box \forall x Fx}[\Box R]$$

$$\dfrac{\dfrac{\dfrac{\dfrac{Fa \succ Fa}{Fa \succ \quad \mid \quad \succ \Diamond Fa}[\Diamond R]}{Fa \succ \quad \mid \quad \succ \exists x \Diamond Fx}[\exists R]}{\exists x Fx \succ \quad \mid \quad \succ \exists x \Diamond Fx}[\exists L]}{\Diamond \exists x Fx \succ \exists x \Diamond Fx}[\Diamond L]$$

Falsifying the Barcan Formula

w_1
$\Diamond(\exists x)Fx$

Falsifying the Barcan Formula

$$
\begin{array}{cc}
w_1 & \\
\hline
\Diamond(\exists x)Fx & (\exists x)Fx
\end{array}
$$

Falsifying the Barcan Formula

w_1	w_2
$\Diamond(\exists x)Fx$	$(\exists x)Fx$
	Fa

Falsifying the Barcan Formula

w_1	w_2
$\Diamond(\exists x)Fx$	$(\exists x)Fx$
	Fa
$\neg(\exists x)\Diamond Fx$	

Falsifying the Barcan Formula

w_1	w_2
$\Diamond(\exists x)Fx$	$(\exists x)Fx$
	Fa
$\neg(\exists x)\Diamond Fx$	
$\neg\Diamond Fb$	

Falsifying the Barcan Formula

w_1	w_2
$\Diamond(\exists x)Fx$	$(\exists x)Fx$
	Fa
$\neg(\exists x)\Diamond Fx$	
$\neg\Diamond Fb$	$\neg Fb$

Falsifying the Barcan Formula

w_1	w_2
$\Diamond(\exists x)Fx$	$(\exists x)Fx$
	Fa
$\neg(\exists x)\Diamond Fx$	
$\neg\Diamond Fb$	$\neg Fb$
$\neg\Diamond Fc$	

Falsifying the Barcan Formula

w_1	w_2
$\Diamond(\exists x)Fx$	$(\exists x)Fx$
	Fa
$\neg(\exists x)\Diamond Fx$	
$\neg\Diamond Fb$	$\neg Fb$
$\neg\Diamond Fc$	$\neg Fc$

Falsifying the Barcan Formula

$$
\begin{array}{cc}
w_1 & w_2 \\
\hline
\Diamond(\exists x)Fx & (\exists x)Fx \\
 & Fa \\
\neg(\exists x)\Diamond Fx & \\
\neg\Diamond Fb & \neg Fb \\
\neg\Diamond Fc & \neg Fc \\
\vdots & \vdots
\end{array}
$$

Falsifying the Barcan Formula

w_1	w_2
$\Diamond(\exists x)Fx$	$(\exists x)Fx$
	Fa
$\neg(\exists x)\Diamond Fx$	
$\neg\Diamond Fb$	$\neg Fb$
$\neg\Diamond Fc$	$\neg Fc$
$\vdots$	$\vdots$
but *not* a	

Falsifying the Barcan Formula

w_1	w_2
$\Diamond(\exists x)Fx$	$(\exists x)Fx$
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$\vdots$	$\vdots$
but *not* a	

The *domain of quantification* for $\exists$ must vary from world to world.

Questions about such models

- If w_1 is the actual world, what *are* those objects in the domains of other worlds, like w_2?
- Is there meant to be a single *intended model*? If so, what is it?
- If we collect together all the domains, we can *define* a necessitist quantifier.
- *Metaphysical Universality*

SEQUENTS & DEFINING RULES

$$\Gamma \succ \Delta$$

Don't assert each element of Γ
and deny each element of Δ.

Structural Rules

Identity: $A \succ A$

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Weakening: $$\frac{\Gamma \succ \Delta}{\Gamma, A \succ \Delta} \qquad \frac{\Gamma \succ \Delta}{\Gamma \succ A, \Delta}$$

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Contraction: $$\frac{\Gamma, A, A \succ \Delta}{\Gamma, A \succ \Delta} \qquad \frac{\Gamma \succ A, A, \Delta}{\Gamma \succ A, \Delta}$$

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Contraction: $\dfrac{\Gamma, A, A \succ \Delta}{\Gamma, A \succ \Delta}$ $\quad$ $\dfrac{\Gamma \succ A, A, \Delta}{\Gamma \succ A, \Delta}$

Cut: $\dfrac{\Gamma \succ A, \Delta \quad \Gamma, A \succ \Delta}{\Gamma \succ \Delta}$

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Cut: $$\frac{\Gamma \succ A, \Delta \quad \Gamma, A \succ \Delta}{\Gamma \succ \Delta}$$

Structural rules govern declarative sentences *as such*.

With Left/Right rules?

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \wedge B \succ \Delta}\,[\wedge L] \qquad \frac{\Gamma \succ A, \Delta \quad \Gamma \succ B, \Delta}{\Gamma \succ A \wedge B, \Delta}\,[\wedge R]$$

With Left/Right rules?

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \wedge B \succ \Delta}\,[\wedge L] \qquad \frac{\Gamma \succ A, \Delta \quad \Gamma \succ B, \Delta}{\Gamma \succ A \wedge B, \Delta}\,[\wedge R]$$

$$\frac{\Gamma, B \succ \Delta}{\Gamma, A \mathit{tonk} B \succ \Delta}\,[\mathit{tonk}L] \qquad \frac{\Gamma \succ A, \Delta}{\Gamma \succ A \mathit{tonk} B, \Delta}\,[\mathit{tonk}R]$$

Use $\succ_{\mathcal{L}}$ to *define* $\succ_{\mathcal{L}'}$.

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Desideratum #1: $\succ_{\mathcal{L}'}$ is *conservative*: $(\succ_{\mathcal{L}'})|_{\mathcal{L}}$ is $\succ_{\mathcal{L}}$.

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Desideratum #1: $\succ_{\mathcal{L}'}$ is *conservative*: $(\succ_{\mathcal{L}'})|_{\mathcal{L}}$ is $\succ_{\mathcal{L}}$.

Desideratum #2: Concepts are defined *uniquely*.

A Defining Rule

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \land B \succ \Delta}\ [\land Df]$$

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Fully specifies norms governing conjunctions on the *left* in terms of simpler vocabulary.

$$\frac{\Gamma, A, B \succ \Delta}{\Gamma, A \wedge B \succ \Delta}\ [\wedge Df]$$

Fully specifies norms governing conjunctions on the *left* in terms of simpler vocabulary.

Identity and *Cut* determine the behaviour of conjunctions on the *right*.

$$
\dfrac{\Gamma \succ A, \Delta \qquad \dfrac{\Gamma \succ B, \Delta \qquad \dfrac{\dfrac{}{A \wedge B \succ A \wedge B}\,[Id]}{A, B \succ A \wedge B}\,[\wedge Df]}{\Gamma, A \succ A \wedge B, \Delta}\,[Cut]}{\Gamma \succ A \wedge B, \Delta}\,[Cut]
$$

From [$\wedge Df$] to [$\wedge L/R$]

$$
\dfrac{\Gamma \succ A, \Delta \qquad \dfrac{\Gamma \succ B, \Delta \qquad \dfrac{\dfrac{}{A \wedge B \succ A \wedge B}\,[Id]}{A, B \succ A \wedge B}\,[\wedge Df]}{\Gamma, A \succ A \wedge B, \Delta}\,[Cut]}{\Gamma \succ A \wedge B, \Delta}\,[Cut]
$$

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$$

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$$\dfrac{\Gamma \succ A, \Delta \qquad \dfrac{\Gamma \succ B, \Delta \qquad \dfrac{\dfrac{}{A \wedge B \succ A \wedge B}[Id]}{A, B \succ A \wedge B}[\wedge Df]}{\Gamma, A \succ A \wedge B, \Delta}[Cut]}{\Gamma \succ A \wedge B, \Delta}[Cut]$$

$$\dfrac{\Gamma \succ A, \Delta \quad \Gamma \succ B, \Delta}{\Gamma \succ A \wedge B, \Delta}[\wedge R]$$

And Back

$$\dfrac{\dfrac{A \succ A \quad B \succ B}{A, B \succ A \land B}[\land R] \quad \Gamma, A \land B \succ \Delta}{\Gamma, A, B \succ \Delta}[Cut]$$

This works for more than the classical logical constants

I want to see how this works for modal operators, and examine their interaction with the quantifiers.

Explaining *why* the modal operators have the logical properties they exhibit is an open question.

… possible worlds, in the sense of possible states of affairs are not *really* individuals (just as numbers are not *really* individuals).

To say that a state of affairs obtains is just to say that something is the case; to say that something is a possible state of affairs is just to say that something could be the case; and to say that something is the case 'in' a possible state of affairs is just to say that the thing in question would necessarily be the case if that state of affairs obtained, i.e. if something else were the case …

We understand 'truth in states of affairs' because we understand 'necessarily'; not *vice versa*.
— "Worlds, Times and Selves" (1969)

I agree with Prior…

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- (Why does possibility distribute of disjunction, necessity over disjunction? Why do the modalities *work* like normal modal logics?)
- If modality is *primitive* we have no explanation.
- If modality is governed by the rules introduced here, then we can see *why* possible worlds are useful, and model the behaviour of modal concepts.

HYPERSEQUENTS, POSSIBILITY AND NECESSITY

Modal Reasoning involves *Shifts*

Suppose it's necessary that p and necessary that q.

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Suppose it's necessary that p and necessary that q.
Is it necessary that both p and q?

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Since it's necessary that p, here we have p.

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Consider any way it could go:

Since it's necessary that p, here we have p.

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So, we have both p and q.

Modal Reasoning involves *Shifts*

Suppose it's necessary that p and necessary that q.
Is it necessary that both p and q?

Could we *avoid* p and q?

Consider any way it could go:

Since it's necessary that p, here we have p.

Since it's necessary that q, here we have q.

So, we have both p and q.

So, no matter how things go, we have p and q.

Modal Reasoning involves *Shifts*

Suppose it's necessary that p and necessary that q.
Is it necessary that both p and q?

Could we *avoid* p and q?

Consider any way it could go:

Since it's necessary that p, here we have p.

Since it's necessary that q, here we have q.

So, we have both p and q.

So, no matter how things go, we have p and q.

So the conjunction p and q is necessary.

Exposing the Structure of that Deduction

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{\Box p, \Box q \succ \Box p}{\Box p, \Box q \succ \;\mid\; \succ p}\,[\Box Df]
\qquad
\dfrac{\Box p, \Box q \succ \Box q}{\Box p, \Box q \succ \;\mid\; \succ q}\,[\Box Df]
}{\Box p, \Box q \succ \;\mid\; \succ p \land q}\,[\land R]
}{\Box p, \Box q \succ \Box(p \land q)}\,[\Box Df]
}{\Box p \land \Box q \succ \Box(p \land q)}\,[\land Df]
$$

$$
\cfrac{
\cfrac{
\cfrac{\Box p, \Box q \succ \Box p}{\Box p, \Box q \succ \;\mid\; \succ p}\,[\Box Df]
\qquad
\cfrac{\Box p, \Box q \succ \Box q}{\Box p, \Box q \succ \;\mid\; \succ q}\,[\Box Df]
}{\Box p, \Box q \succ \;\mid\; \succ p \land q}\,[\land R]
}{
\cfrac{\Box p, \Box q \succ \Box(p \land q)}{\color{red}{\Box p \land \Box q} \succ \Box(p \land q)}\,[\land Df]
}\,[\Box Df]
$$

Exposing the Structure of that Deduction

$$\dfrac{\dfrac{\dfrac{\Box p, \Box q \succ \Box p}{\Box p, \Box q \succ \;\mid\; \succ p}[\Box Df] \quad \dfrac{\Box p, \Box q \succ \Box q}{\Box p, \Box q \succ \;\mid\; \succ q}[\Box Df]}{\dfrac{\Box p, \Box q \succ \;\mid\; \succ p \land q}{\dfrac{\color{red}{\Box p, \Box q} \succ \Box(p \land q)}{\Box p \land \Box q \succ \Box(p \land q)}[\land Df]}[\Box Df]}[\land R]$$

Exposing the Structure of that Deduction

$$
\dfrac{\dfrac{\dfrac{\Box p, \Box q \succ \Box p}{\color{red}{\Box p, \Box q \succ \ \mid\ \succ p}}[\Box Df] \qquad \dfrac{\Box p, \Box q \succ \Box q}{\Box p, \Box q \succ \ \mid\ \succ q}[\Box Df]}{\dfrac{\Box p, \Box q \succ \ \mid\ \succ p \land q}{\Box p, \Box q \succ \Box(p \land q)}[\Box Df]}[\land R]}{\Box p \land \Box q \succ \Box(p \land q)}[\land Df]
$$

Exposing the Structure of that Deduction

$$
\dfrac{\dfrac{\Box p, \Box q \succ \Box p}{\Box p, \Box q \succ \ \mid \ \succ p}[\Box Df] \qquad \dfrac{\Box p, \Box q \succ \Box q}{\color{red}{\Box p, \Box q \succ \ \mid \ \succ q}}[\Box Df]}{\dfrac{\dfrac{\Box p, \Box q \succ \ \mid \ \succ p \land q}{\Box p, \Box q \succ \Box(p \land q)}[\Box Df]}{\Box p \land \Box q \succ \Box(p \land q)}[\land Df]}[\land R]
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\Box p, \Box q \succ \Box p}{\Box p, \Box q \succ \ \mid\ \succ p}\,[\Box Df] \qquad \dfrac{\Box p, \Box q \succ \Box q}{\Box p, \Box q \succ \ \mid\ \succ q}\,[\Box Df]}{\Box p, \Box q \succ \ \mid\ \succ {\color{red} p \land q}}\,[\land R]}{\Box p, \Box q \succ \Box(p \land q)}\,[\Box Df]}{\Box p \land \Box q \succ \Box(p \land q)}\,[\land Df]
$$

Exposing the Structure of that Deduction

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{\Box p, \Box q \succ \Box p}{\Box p, \Box q \succ \;\mid\; \succ p}\,[\Box Df]
      \qquad
      \dfrac{\Box p, \Box q \succ \Box q}{\Box p, \Box q \succ \;\mid\; \succ q}\,[\Box Df]
    }{\Box p, \Box q \succ \;\mid\; \succ p \wedge q}\,[\wedge R]
  }{\Box p, \Box q \succ \Box(p \wedge q)}\,[\Box Df]
}{\Box p \wedge \Box q \succ \Box(p \wedge q)}\,[\wedge Df]
$$

Exposing the Structure of that Deduction

$$
\cfrac{
  \cfrac{\Box p, \Box q \succ \Box p}{\Box p, \Box q \succ \quad | \quad \succ p}\,[\Box Df]
  \qquad
  \cfrac{\Box p, \Box q \succ \Box q}{\Box p, \Box q \succ \quad | \quad \succ q}\,[\Box Df]
}{
  \cfrac{
    \cfrac{\Box p, \Box q \succ \quad | \quad \succ p \wedge q}{\Box p, \Box q \succ \Box(p \wedge q)}\,[\Box Df]
  }{
    \color{red}{\Box p \wedge \Box q \succ \Box(p \wedge q)}
  }\,[\wedge Df]
}\,[\wedge R]
$$

$$\Box p, \Box q \succ \quad | \quad \succ p \wedge q$$

Don't assert $\Box p$ and $\Box q$ in one 'zone'
and deny $p \wedge q$ in another.

$$\Gamma \succ \Delta \mid \Gamma' \succ \Delta'$$

Don't assert each member of Γ
and deny each member of Δ in one 'zone'
and assert each member of Γ'
and deny each member of Δ' in another.

Defining Rules for $\Box$ and $\Diamond$

$$\frac{\Gamma \succ \Delta \mid \ \succ A \mid \mathcal{H}}{\Gamma \succ \Box A, \Delta \mid \mathcal{H}}\ [\Box Df] \qquad \frac{\Gamma \succ \Delta \mid A \succ \ \mid \mathcal{H}}{\Gamma, \Diamond A \succ \Delta \mid \mathcal{H}}\ [\Diamond Df]$$

QUANTIFICATION, DEFINEDNESS & THE BARCAN FORMULA

The Standard Quantifier Rules

$$\frac{\Gamma \succ A(n), \Delta}{\Gamma \succ (\forall x)A(x), \Delta}\,[\forall Df] \qquad \frac{\Gamma, A(n) \succ \Delta}{\Gamma, (\exists x)A(x) \succ \Delta}\,[\exists Df]$$

Deriving the Barcan Formula

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(\forall x)\Box Fx \succ (\forall x)\Box Fx}{(\forall x)\Box Fx \succ \Box Fn}[\forall Df]}{(\forall x)\Box Fx \succ \quad | \quad \succ Fn}[\Box Df]}{(\forall x)\Box Fx \succ \quad | \quad \succ (\forall x)Fx}[\forall Df]}{(\forall x)\Box Fx \succ \Box(\forall x)Fx}[\Box Df]}{\succ (\forall x)\Box Fx \supset \Box(\forall x)Fx}[\supset Df]
$$

Where the derivation breaks down

$$
\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\color{red}{(\forall x)\Box Fx \succ (\forall x)\Box Fx}}{(\forall x)\Box Fx \succ \Box Fn}\,[\forall \mathit{Df}]}{(\forall x)\Box Fx \succ \quad \mid \quad \succ Fn}\,[\Box \mathit{Df}]}{(\forall x)\Box Fx \succ \quad \mid \quad \succ (\forall x)Fx}\,[\forall \mathit{Df}]}{(\forall x)\Box Fx \succ \Box(\forall x)Fx}\,[\Box \mathit{Df}]}{\succ (\forall x)\Box Fx \supset \Box(\forall x)Fx}\,[\supset \mathit{Df}]
$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(\forall x)\Box Fx \succ (\forall x)\Box Fx}{\color{red}{(\forall x)\Box Fx \succ \Box Fn}}\,[\forall Df]}{(\forall x)\Box Fx \succ \quad \mid \quad \succ Fn}\,[\Box Df]}{(\forall x)\Box Fx \succ \quad \mid \quad \succ (\forall x)Fx}\,[\forall Df]}{(\forall x)\Box Fx \succ \Box(\forall x)Fx}\,[\Box Df]}{\succ (\forall x)\Box Fx \supset \Box(\forall x)Fx}\,[\supset Df]$$

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\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{(\forall x)\Box Fx \succ (\forall x)\Box Fx}{(\forall x)\Box Fx \succ \Box Fn}\,[\forall Df]}{\color{red}{(\forall x)\Box Fx \succ \quad | \quad \succ Fn}}\,[\Box Df]}{(\forall x)\Box Fx \succ \quad | \quad \succ (\forall x)Fx}\,[\forall Df]}{(\forall x)\Box Fx \succ \Box(\forall x)Fx}\,[\Box Df]}{\succ (\forall x)\Box Fx \supset \Box(\forall x)Fx}\,[\supset Df]
$$

Pro and *Con* attitudes to Terms

To rule a term *in* is to take it as suitable
to substitute into a quantifier,
i.e., to take the term to *denote*.

To rule a term *out* is to take it as unsuitable
to substitute into a quantifier,
i.e., to take the term to *not denote*.

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To rule a term *out* is to take it as unsuitable
to substitute into a quantifier,
i.e., to take the term to *not denote*.

We add terms to the LHS and RHS of sequents $\Gamma \succ \Delta$.

Structural Rules remain as before

Identity: $X \succ X$

Weakening: $\dfrac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma, X \succ \Delta]} \qquad \dfrac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ X, \Delta]}$

Contraction: $\dfrac{\mathcal{H}[\Gamma, X, X \succ \Delta]}{\mathcal{H}[\Gamma, X \succ \Delta]} \qquad \dfrac{\mathcal{H}[\Gamma \succ X, X, \Delta]}{\mathcal{H}[\Gamma \succ X, \Delta]}$

Cut: $\dfrac{\mathcal{H}[\Gamma \succ X, \Delta] \quad \mathcal{H}[\Gamma, X \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta]}$

Here X is either a sentence or a term.

...and there are some more

$$\textit{Ext. Weak.:}\quad \frac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta \mid \Gamma' \succ \Delta']} \quad \frac{\mathcal{H}[\Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta \parallel \Gamma' \succ \Delta']}$$

$$\textit{Ext. Contr.:}\quad \frac{\mathcal{H}[\Gamma \succ \Delta \mid \Gamma \succ \Delta]}{\mathcal{H}[\Gamma \succ \Delta]} \quad \frac{\mathcal{H}[\mathcal{S} \parallel \mathcal{S}]}{\mathcal{H}[\mathcal{S}]}$$

Quantifier Rules, allowing for non-denoting terms

$$\frac{\mathcal{H}[\Gamma, n \succ A(n), \Delta]}{\mathcal{H}[\Gamma \succ (\forall x)A(x), \Delta]}\ [\forall Df] \qquad \frac{\mathcal{H}[\Gamma, n, A(n) \succ \Delta]}{\mathcal{H}[\Gamma, (\exists x)A(x) \succ \Delta]}\ [\exists Df]$$

Now you *can't* derive the Barcan Formula

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Now you *can't* derive the Barcan Formula

$$(\forall x)\Box Fx \succ \Box(\forall x)Fx \;\mid\; {\color{red}b} \succ {\color{red}Fb}, (\forall x)Fx$$

Now you *can't* derive the Barcan Formula

$$(\forall x)\Box Fx \succ b, Fb, \Box(\forall x)Fx \mid b \succ Fb, (\forall x)Fx$$

Now you *can't* derive the Barcan Formula

$$a, (\forall x)\Box Fx \succ b, Fb, \Box(\forall x)Fx \mid a, b \succ Fb, (\forall x)Fx$$

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$$a, \Box Fa, (\forall x)\Box Fx \succ b, Fb, \Box(\forall x)Fx \mid a, b \succ Fb, (\forall x)Fx$$

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This hypersequent is underivable…

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This hypersequent is underivable...

...and it's *fully refined*.

Positions

- A *finite* position is an underivable hypersequent.

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- An arbitrary position is a set (*indicative* alternatives) of sets (*subjunctive* alternatives) of pairs of sets of formulas or terms (*components*), where one component in each indicative alternative is marked with an @.

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  - If $\Box A$ is in the LHS of a component, A is in the LHS of every subjunctive alternative of that component.
  - If $\Box A$ is in the RHS of a component, A is in the RHS of some subjunctive alternative of that component.

Models

Fully refinied positions are examples of *models*,
with variable domains and the expected truth conditions for
the connectives, quantifiers and modal operators.

Soundness and Completeness

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- That fully refined position determines a model in which the hypersequent does not hold.
- So the models are adequate for the logic.
- And in the logic, the cut rule is admissible in the cut-free system.

THE STATUS OF CONTINGENTIST MODELS

The structure of modal concepts is explained
in terms of the rules for their use, *not* the models.

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Worlds (and their domains) are idealised positions.

There is no single intended model.

Given a language, any coherent position $\Gamma \succ \Delta$ can be canonically extended into a model.

QUANTIFIERS: WIDER AND STILL WIDER

Inner and Outer Quantification

'Outer' quantification is an issue for contingentism.
On most approaches to contingentism, it can be *defined*.

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On most approaches to contingentism, it can be *defined*.

This proof theoretical semantics is no different in that regard....

We have *Outer* Quantification

$$\frac{\mathcal{H}(n \succ\ \mid\ \Gamma \succ A(n), \Delta)}{\mathcal{H}(\Gamma \succ (\forall^{\Diamond} x)A(x), \Delta)}\ [\forall^{\Diamond} Df] \qquad \frac{\mathcal{H}(n \succ\ \mid\ \Gamma, A(n) \succ \Delta)}{\mathcal{H}(\Gamma, (\exists^{\Diamond} x)A(x) \succ \Delta)}\ [\exists^{\Diamond} Df]$$

for which the substituted term need be defined in *some* zone.

The Barcan Formula is Derivable

$$\dfrac{\dfrac{\dfrac{\dfrac{(\forall^{\Diamond}x)\Box A(x) \succ (\forall^{\Diamond}x)\Box A(x)}{n \succ \mid (\forall^{\Diamond}x)\Box A(x) \succ \Box A(n)}\,[\forall^{\Diamond}Df]}{n \succ \mid (\forall^{\Diamond}x)\Box A(x) \succ \mid \succ A(n)}\,[\Box Df]}{(\forall^{\Diamond}x)\Box A(x) \succ \mid \succ (\forall^{\Diamond}x)A(x)}\,[\forall^{\Diamond}Df]}{(\forall^{\Diamond}x)\Box A(x) \succ \Box(\forall^{\Diamond}x)A(x)}\,[\Box Df]$$

But we also have *Way Out* Quantification

$$\frac{\mathcal{H}(\Gamma \succ A(n), \Delta)}{\mathcal{H}(\Gamma \succ (\Pi x)A(x), \Delta)}\ [\Pi Df] \qquad \frac{\mathcal{H}(\Gamma, A(n) \succ \Delta)}{\mathcal{H}(\Gamma, (\Sigma x)A(x) \succ \Delta)}\ [\Sigma Df]$$

for which the term need not be defined *anywhere*.

What about undefined terms?

For a restriction to completely free logic undermines the application of scientific method by permitting one to hold on to a universal generalization after one of its instances has been refuted: one denies Ga *but still asserts* $\forall x Gx$ *by also denying* $\exists y\ a = y$*, still retaining the constant* a *in the language. We assume that the formal languages under consideration in this chapter are well designed in the relevant sense, so that metaphysical universality implies truth. For our present aim is neither to model natural languages, for example in their use of fictional and mythological names…nor to stick to what is knowable a priori in some sense, which might exclude whether some names refer. Rather, our business is to clarify the structure of metaphysical universality in a broadly scientific spirit. Non-referring uses of 'Pegasus' have no more place in such an enquiry than they have in physics or zoology. Of course, the term 'phlogiston' did occur in scientific language, but if it failed to refer (rather than referring to an empty kind) then its presence in any scientific theory was a defect in that theory. Consequently, we should not distort our formal language by allowing for such a term.* MLM, *pages 131–132.*

Are these *scientific* terms?

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We should not distort our formal language by *banning* these terms.

SOLOMON FEFERMAN*

DEFINEDNESS

ABSTRACT. Questions of definedness are ubiquitous in mathematics. Informally, these involve reasoning about expressions which may or may not have a value. This paper surveys work on logics in which such reasoning can be carried out directly, especially in computational contexts. It begins with a general logic of "partial terms", continues with partial combinatory and lambda calculi, and concludes with an expressively rich theory of partial functions and polymorphic types, where termination of functional programs can be established in a natural way.

Erkenntnis 43: 295–320, 1995.

t is defined

$$t\downarrow$$

t is defined

$$\mathsf{t}{\downarrow}$$

$$\frac{\Gamma, \mathsf{t} \succ \Delta \mid \mathcal{H}}{\Gamma, \mathsf{t}{\downarrow} \succ \Delta \mid \mathcal{H}}\,[{\downarrow}Df]$$

Which is the really *universal* quantifier?

$(\forall x)$ is a restricted $(\forall^{\Diamond} x)$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Gamma \succ (\forall x)A(x), \Delta \mid \mathcal{H}}{\Gamma, n \succ A(n), \Delta \mid \mathcal{H}}\,[\forall Df]}{n \succ \ \mid \Gamma, n \succ A(n), \Delta \mid \mathcal{H}}\,[K,W]}{n \succ \ \mid \Gamma, n\!\downarrow\, \succ A(n), \Delta \mid \mathcal{H}}\,[\downarrow Df]}{n \succ \ \mid \Gamma \succ n\!\downarrow\, \supset A(n), \Delta \mid \mathcal{H}}\,[\supset Df]}{\Gamma \succ (\forall^{\Diamond} x)(x\!\downarrow\, \supset A(x)), \Delta \mid \mathcal{H}}\,[\forall^{\Diamond} Df]$$

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$(\forall^{\Diamond}x)$ is a restricted (Πx)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Gamma \succ (\forall^{\Diamond}x)A(x), \Delta \mid \mathcal{H}}{n \succ \mid \Gamma \succ A(n), \Delta \mid \mathcal{H}}[\forall^{\Diamond}Df]}{n{\downarrow} \succ \mid \Gamma \succ A(n), \Delta \mid \mathcal{H}}[{\downarrow}Df]}{\Gamma, \Diamond n{\downarrow} \succ A(n), \Delta \mid \mathcal{H}}[\Diamond Df]}{\Gamma \succ \Diamond n{\downarrow} \supset A(n), \Delta \mid \mathcal{H}}[{\supset}Df]}{\Gamma \succ (\Pi x)(\Diamond x{\downarrow} \supset A(x)), \Delta \mid \mathcal{H}}[\Pi Df]$$

EPISTEMIC MODALS

Two Kinds of Zone Shift

INDICATIVE: suppose I'm wrong and that. . .

SUBJUNCTIVE: suppose things go differently.
or *had gone* differently.

The New York Times.

LATE CITY EDITION

KENNEDY IS KILLED BY SNIPER AS HE RIDES IN CAR IN DALLAS; JOHNSON SWORN IN ON PLANE

Gov. Connally Shot; Mrs. Kennedy Safe

Why America Weeps


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- Suppose Oswald *hadn't* shot JFK.

Philosophers' Imprint

volume 7, no. 4
may 2007

STEREOSCOPIC VISION:

Persons, Freedom, and Two Spaces of Material Inference

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Georgetown University

W. Heath White
University of North Carolina at Wilmington

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<www.philosophersimprint.org/007004/>

Freedom, oh freedom, well that's just some people talkin'.
— The Eagles

What is a person, as opposed to a non-person? One might begin to address the question by appealing to a second distinction: between *agents*, characterized by the ability to act freely and intentionally, and mere patients, caught up in events but in no sense authors of the happenings involving them. An alternative way to address the question appeals to a third distinction: between *subjects*—bearers of rights and responsibilities, commitments and entitlements, makers of claims, thinkers of thoughts, issuers of orders, and posers of questions—and mere objects, graspable or evaluable by subjects but not themselves graspers or evaluators.

We take it as a methodological point of departure that these three distinctions are largely coextensive, indeed coextensive in conceptually central cases. Granted, these distinctions can come apart. One might think that 'person' applies to anything that is worthy of a distinctive sort of moral respect and think this applicable to some fetuses or the deeply infirm elderly. Even if the particular respect due such beings is importantly different from "what we owe each other", such respect could still be thought to be of the kind distinctively due people, and think this even while holding that such people lack agentive or subjective capacity. Similarly, one might think dogs or various severely impaired humans to be attenuated subjects but not agents.

Without taking any particular stand on such examples, our methodological hypothesis is that such cases, if they exist, are understood as persons (agents, subjects) essentially by reference to paradigm cases and, indeed, to a single paradigm within which person/non-person, subject/object, and agent/patient are conceptually connected.[1] Stated

1. For one detailed development of this sort of paradigm-riff structure, and a defense of the possibility of concepts essentially governed by such a structure, see Lance and Little (2004). Discussions with Hilda Lindeman have helped

We are *social* creatures, who *act* on the basis of *views*

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- We do *many different and strange things* in our messy zone-shifting practices, but we can isolate a particular convention or practice, idealise it, to see what we could do following *those* rules.

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- We do *many different and strange things* in our messy zone-shifting practices, but we can isolate a particular convention or practice, idealise it, to see what we could do following *those* rules.
  - (*Analogies*: $\forall x$ from first order logic and natural language's 'all.' Frictionless planes. etc.)

Example Subjunctive Shifts

Oswald *did* shoot JFK, but suppose he *hadn't*? How would history have gone differently then?

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$$[\mathrm{oSk}\ :\ \ ]_{@}\ \mid\ [@\mathrm{oSk}\ :\ \mathrm{oSk}]$$

We open up a zone for consideration, in which we deny oSk, while keeping track of the initial zone where we assert it.

(And if we like, we can assert @oSk in the zone under the counterfactual supposition.)

Disagreement and Indicative Shifting

I think that Oswald shot JFK, but you don't.

I consider what it would mean for you to be right.

If you're right, Oswald *actually didn't* shoot JFK.

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Epistemic alternatives interact differently with actuality.

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Epistemic alternatives interact differently with actuality.

$$[\mathrm{oSk} : \ ]_{@} \Longrightarrow [\ : \mathrm{oSk}]_{@}$$

I think that Hesperus is Phosphorous, but I recognise that *you don't*. I don't take *you* to be *inconsistent* or misusing names.

Indicative Shifting

I think that Hesperus is Phosphorous, but I recognise that *you don't*. I don't take *you* to be *inconsistent* or misusing names.

We *don't* have this:

$$a = b \succ \quad \Longrightarrow \quad Fa \succ Fb$$

It's coherent for you to assert Fa and deny Fb even if I take it that $a = b$, and it's coherent for me to consider an alternative in which $a \neq b$ even if I don't agree.

Idealised Indicative Shifts

- Let's take any zone found by indicatively shifting from here (or from anywhere from here, etc.) to be an alternative indicative context from any *other* context indicatively shifted from here.

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- This is as *liberal* as possible about what counts as an *alternative* from any alternative zone.

- This gives us a motivation for a richer family of hypersequents.

Two Dimensional Hypersequents

$$
\begin{array}{ccccccccc}
X_1^1 \succ_{@} Y_1^1 & | & X_2^1 \succ Y_2^1 & | & \cdots & | & X_{m_1}^1 \succ Y_{m_1}^1 & \| \\
X_1^2 \succ_{@} Y_1^2 & | & X_2^2 \succ Y_2^2 & | & \cdots & | & X_{m_2}^2 \succ Y_{m_2}^2 & \| \\
\vdots & & \vdots & & & & \vdots & \\
X_1^n \succ_{@} Y_1^n & | & X_2^n \succ Y_2^n & | & \cdots & | & X_{m_n}^n \succ Y_{m_n}^n &
\end{array}
$$

Two Dimensional Hypersequents

$$
\begin{array}{ccccccccc}
X_1^1 \succ_@ Y_1^1 & \mid & X_2^1 \succ Y_2^1 & \mid & \cdots & \mid & X_{m_1}^1 \succ Y_{m_1}^1 & \| \\
X_1^2 \succ_@ Y_1^2 & \mid & X_2^2 \succ Y_2^2 & \mid & \cdots & \mid & X_{m_2}^2 \succ Y_{m_2}^2 & \| \\
\vdots & & \vdots & & & & \vdots & \\
X_1^n \succ_@ Y_1^n & \mid & X_2^n \succ Y_2^n & \mid & \cdots & \mid & X_{m_n}^n \succ Y_{m_n}^n &
\end{array}
$$

Think of these as *scorecards*, keeping track of assertions and denials.

Notation

$$\mathcal{H}[\Gamma \succ \Delta]$$

Notation

$$\mathcal{H}[\Gamma' \succ \Delta']$$

Notation

$$\mathcal{H}[\Gamma' \succ \Delta']$$

$$\mathcal{H}[\Gamma \succ \Delta \;\mid\; \Gamma' \succ \Delta']$$

Notation

$$\mathcal{H}[\Gamma' \succ \Delta']$$

$$\mathcal{H}[\Gamma \succ \Delta \mid \Gamma' \succ \Delta']$$

$$\mathcal{H}[\Gamma \succ \Delta \parallel \Gamma' \succ \Delta']$$

Defining Rule for $\Box$

$$\frac{\mathcal{H}[\Gamma \succ \Delta \mid \ \succ A]}{\mathcal{H}[\Gamma \succ \Box A, \Delta]}\ [\Box Df]$$

Defining Rule for @

$$\begin{array}{c}\mathcal{H}[\Gamma, A \succ_{@} \Delta \mid \Gamma' \succ \Delta'] \\ \hline\hline \mathcal{H}[\Gamma \succ_{@} \Delta \mid \Gamma', @A \succ \Delta'] \end{array}\ [@Df]$$

Defining Rule for [e]

$$\frac{\mathcal{H}[\Gamma \succ \Delta \;\;\|\;\; \succ_{@} A]}{\mathcal{H}[\Gamma \succ [\mathsf{e}]A, \Delta]}\;[[\mathsf{e}]Df]$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\succ \quad \mid \; [\mathrm{e}]A \succ [\mathrm{e}]A}{\succ \quad \mid \; [\mathrm{e}]A \succ \quad \| \quad \succ_{@} A}\;[[\mathrm{e}]Df]}{\succ \quad \mid \; [\mathrm{e}]A \succ \quad \| \quad \succ_{@} @A}\;[@Df]}{\succ \quad \mid \; [\mathrm{e}]A \succ [\mathrm{e}]@A}\;[[\mathrm{e}]Df]}{\succ \quad \mid \quad \succ [\mathrm{e}]A \supset [\mathrm{e}]@A}\;[\supset Df]}{\succ \Box([\mathrm{e}]A \supset [\mathrm{e}]@A)}\;[\Box Df]
$$

Epistemic Barcan Formula

$$(\forall x)[e]Fx \succ [e](\forall x)Fx$$

Epistemic Barcan Formula

$$(\forall x)[e]Fx \succ [e](\forall x)Fx$$

$$\langle e\rangle(\exists x)Fx \succ (\exists x)\langle e\rangle Fx$$

Suppose $\langle e \rangle (\exists x)(\exists y)(Mx \;\&\; Ey \;\&\; x \neq y)$.

Suppose $\langle e \rangle (\exists x)(\exists y)(Mx \;\&\; Ey \;\&\; x \neq y)$.

Do we have $(\exists x)\langle e \rangle((\exists y)(Mx \;\&\; Ey \;\&\; x \neq y)$?

Morning Star and Evening Star

Suppose $\langle e \rangle (\exists x)(\exists y)(Mx \,\&\, Ey \,\&\, x \neq y)$.

Do we have $(\exists x)\langle e \rangle ((\exists y)(Mx \,\&\, Ey \,\&\, x \neq y)$?

And $(\exists x)(\exists y)\langle e \rangle (Mx \,\&\, Ey \,\&\, x \neq y)$?

Suppose $\langle e\rangle(\exists x)(\exists y)(Mx \mathbin{\&} Ey \mathbin{\&} x \neq y)$.

Do we have $(\exists x)\langle e\rangle((\exists y)(Mx \mathbin{\&} Ey \mathbin{\&} x \neq y)$?

And $(\exists x)(\exists y)\langle e\rangle(Mx \mathbin{\&} Ey \mathbin{\&} x \neq y)$?

What could such x and y be?

CONCLUDING THOUGHTS

I offer *compliments* and *complements*

- COMPLIMENTS: Williamson has shown us that the tools of modal logic are a fruitful setting for understanding modal metaphysics. We *need* the discipline of a formal theory to keep ourselves honest, and to help us understand exactly what theory it is we're adopting, and what its consequences *are*. *Modal Logic as Metaphysics* is a fruitful example of the best work in this tradition.

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- COMPLEMENTS: I've tried bring to bear the neglected half of logic—*proof theory*—to the task of modal semantics.

Thank you, Tim!

THANK YOU!

`http://consequently.org/presentation/2016/existence-definedness-awpl-tplc`

@consequently on Twitter