PY4601 Paradoxes

Recent Work on the Liar Paradox

Greg Restall

University of
St Andrews

March & April 2023

WEEK 10: Kripke's fixed point construction

WEEK 11: What it might mean

TODAY'S PLAN

FORMALISING one paradoxical argument

LOCATING different positions on this map

FORMALISING a different paradoxical argument

Noticing the PARALLELS

Kripke's MODEL
- why it's needed
- $\{0, n, 1\}$ & refinement
- stages & the fixed point

THE UPSHOT

TODAY'S PLAN

FORMALISING one paradoxical argument

LOCATING different positions on this map

FORMALISING a different paradoxical argument

Noticing the PARALLELS

Kripke's MODEL
- why it's needed
- $\{0, n, 1\}$ & refinement
- stages & the fixed point

THE UPSHOT

(λ) λ is not true

$$
\begin{prooftree}
\AxiomC{$\lambda = \langle \neg T\lambda \rangle$}
\AxiomC{$[T\lambda]^1$}
\RightLabel{$=E$}
\BinaryInfC{$T\langle \neg T\lambda \rangle$}
\RightLabel{TE}
\UnaryInfC{$\neg T\lambda$}
\AxiomC{$[T\lambda]^1$}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\RightLabel{$\neg I^1$}
\UnaryInfC{$\neg T\lambda$}
\AxiomC{$\lambda = \langle \neg T\lambda \rangle$}
\AxiomC{$\lambda = \langle \neg T\lambda \rangle$}
\AxiomC{$[T\lambda]^2$}
\RightLabel{$=E$}
\BinaryInfC{$T\langle \neg T\lambda \rangle$}
\RightLabel{TE}
\UnaryInfC{$\neg T\lambda$}
\AxiomC{$[T\lambda]^2$}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\RightLabel{$\neg I^2$}
\UnaryInfC{$\neg T\lambda$}
\RightLabel{TI}
\UnaryInfC{$T\langle \neg T\lambda \rangle$}
\RightLabel{$=E$}
\BinaryInfC{$T\lambda$}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\end{prooftree}
$$

$$
\begin{prooftree}
\AxiomC{$\neg A$}
\AxiomC{A}
\RightLabel{$\neg E$}
\BinaryInfC{$\bot$}
\end{prooftree}
\qquad
\begin{prooftree}
\AxiomC{$[A]^i$}
\noLine
\UnaryInfC{$\vdots$}
\noLine
\UnaryInfC{$\bot$}
\RightLabel{$\neg I^i$}
\UnaryInfC{$\neg A$}
\end{prooftree}
\qquad
\begin{prooftree}
\AxiomC{A}
\RightLabel{TI}
\UnaryInfC{$T\langle A \rangle$}
\end{prooftree}
\quad
\begin{prooftree}
\AxiomC{$T\langle A \rangle$}
\RightLabel{TE}
\UnaryInfC{A}
\end{prooftree}
\qquad
\begin{prooftree}
\AxiomC{$a = b$}
\AxiomC{Fa}
\RightLabel{$=E$}
\BinaryInfC{Fb}
\end{prooftree}
$$

TODAY'S PLAN

FORMALISING one paradoxical argument

Locating different positions on this map

FORMALISING a different paradoxical argument

Noticing the PARALLELS

Kripke's MODEL
- why it's needed
- $\{0, n, 1\}$ & refinement
- stages & the fixed point

THE UPSHOT

LEVEL SOLUTIONS

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{\cancel{\lambda = \langle \neg T\lambda \rangle} \qquad [T\lambda]^1}{T\langle \neg T\lambda \rangle}\,{=}E
      }{\neg T\lambda}\,TE
      \qquad [T\lambda]^1
    }{\bot}\,\neg E
  }{\neg T\lambda}\,\neg I^1
  \qquad
  \dfrac{
    \lambda = \langle \neg T\lambda \rangle
    \qquad
    \dfrac{
      \dfrac{
        \dfrac{
          \dfrac{
            \dfrac{\cancel{\lambda = \langle \neg T\lambda \rangle} \qquad [T\lambda]^2}{T\langle \neg T\lambda \rangle}\,{=}E
          }{\neg T\lambda}\,TE
          \qquad [T\lambda]^2
        }{\bot}\,\neg E
      }{\neg T\lambda}\,\neg I^2
    }{T\langle \neg T\lambda \rangle}\,TI
  }{T\lambda}\,{=}E
}{\bot}\,\neg E
$$

$$
\dfrac{\neg A \qquad A}{\bot}\,\neg E
\qquad\qquad
\dfrac{\begin{matrix}[A]^i \\ \vdots \\ \bot\end{matrix}}{\neg A}\,\neg I^i
\qquad\qquad
\dfrac{A}{T\langle A \rangle}\,TI
\qquad
\dfrac{T\langle A \rangle}{A}\,TE
\qquad\qquad
\dfrac{a = b \qquad Fa}{Fb}\,{=}E
$$

NO-PROPOSITION VIEWS

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{\cancel{\lambda = \langle \neg T\lambda \rangle} \quad [T\lambda]^1}{T\langle \neg T\lambda \rangle}{\scriptstyle =E}
}{\neg T\lambda}{\scriptstyle TE} \quad [T\lambda]^1
}{\bot}{\scriptstyle \neg E}
}{\neg T\lambda}{\scriptstyle \neg I^1}
\qquad
\dfrac{\lambda = \langle \neg T\lambda \rangle \quad
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{\cancel{\lambda = \langle \neg T\lambda \rangle} \quad [T\lambda]^2}{T\langle \neg T\lambda \rangle}{\scriptstyle =E}
}{\neg T\lambda}{\scriptstyle TE} \quad [T\lambda]^2
}{\bot}{\scriptstyle \neg E}
}{\neg T\lambda}{\scriptstyle \neg I^2}
}{T\langle \neg T\lambda \rangle}{\scriptstyle TI}
}{T\lambda}{\scriptstyle =E}
}{\bot}{\scriptstyle \neg E}
$$

$$
\dfrac{\neg A \quad A}{\bot}\,{\scriptstyle \neg E}
\qquad
\dfrac{\begin{matrix}[A]^i \\ \vdots \\ \bot\end{matrix}}{\neg A}\,{\scriptstyle \neg I^i}
\qquad
\dfrac{A}{T\langle A \rangle}\,{\scriptstyle TI}
\quad
\dfrac{T\langle A \rangle}{A}\,{\scriptstyle TE}
\qquad
\dfrac{a = b \quad Fa}{Fb}\,{\scriptstyle =E}
$$

$$
\begin{prooftree}
\AxiomC{$\lambda = \langle \neg T\lambda \rangle$}
\AxiomC{$[T\lambda]^1$}
\RightLabel{$=E$}
\BinaryInfC{$T\langle \neg T\lambda \rangle$}
\RightLabel{TE}
\UnaryInfC{$\neg T\lambda$}
\AxiomC{$[T\lambda]^1$}
\RightLabel{$\neg E$}
\BinaryInfC{$\perp$}
\RightLabel{$\cancel{\neg I^1}$}
\UnaryInfC{$\neg T\lambda$}
\AxiomC{$\lambda = \langle \neg T\lambda \rangle$}
\AxiomC{$\lambda = \langle \neg T\lambda \rangle$}
\AxiomC{$[T\lambda]^2$}
\RightLabel{$=E$}
\BinaryInfC{$T\langle \neg T\lambda \rangle$}
\RightLabel{TE}
\UnaryInfC{$\neg T\lambda$}
\AxiomC{$[T\lambda]^2$}
\RightLabel{$\neg E$}
\BinaryInfC{$\perp$}
\RightLabel{$\cancel{\neg I^2}$}
\UnaryInfC{$\neg T\lambda$}
\RightLabel{TI}
\UnaryInfC{$T\langle \neg T\lambda \rangle$}
\RightLabel{$=E$}
\BinaryInfC{$T\lambda$}
\RightLabel{$\neg E$}
\BinaryInfC{$\perp$}
\end{prooftree}
$$

GAP VIEWS

$$
\begin{prooftree}
\AxiomC{$\neg A$}
\AxiomC{A}
\RightLabel{$\neg E$}
\BinaryInfC{$\perp$}
\end{prooftree}
\qquad
\begin{prooftree}
\AxiomC{$[A]^i$}
\noLine
\UnaryInfC{$\vdots$}
\noLine
\UnaryInfC{$\perp$}
\RightLabel{$\neg I^i$}
\UnaryInfC{$\neg A$}
\end{prooftree}
\qquad
\begin{prooftree}
\AxiomC{A}
\RightLabel{TI}
\UnaryInfC{$T\langle A \rangle$}
\end{prooftree}
\quad
\begin{prooftree}
\AxiomC{$T\langle A \rangle$}
\RightLabel{TE}
\UnaryInfC{A}
\end{prooftree}
\qquad
\begin{prooftree}
\AxiomC{$a = b$}
\AxiomC{Fa}
\RightLabel{$=E$}
\BinaryInfC{Fb}
\end{prooftree}
$$

GAP VIEWS

GLUT VIEWS

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\lambda = \langle \neg T\lambda \rangle \quad [T\lambda]^1}{T\langle \neg T\lambda \rangle}{=}E}{\neg T\lambda}TE \qquad [T\lambda]^1}{\bot}\cancel{\neg E}}{\neg T\lambda}\neg I^1 \qquad \dfrac{\lambda = \langle \neg T\lambda\rangle \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\lambda = \langle \neg T\lambda \rangle \quad [T\lambda]^2}{T\langle \neg T\lambda \rangle}{=}E}{\neg T\lambda}TE \qquad [T\lambda]^2}{\bot}\cancel{\neg E}}{\neg T\lambda}\neg I^2}{T\langle\neg T\lambda\rangle}TI}{T\lambda}{=}E}{\bot}\cancel{\neg E}$$

$$\dfrac{\neg A \quad A}{\bot}\cancel{\neg E}$$

GLUT VIEWS

$$\dfrac{\begin{matrix}[A]^i \\ \vdots \\ \bot\end{matrix}}{\neg A}\neg I^i \qquad \dfrac{A}{T\langle A\rangle}TI \qquad \dfrac{T\langle A\rangle}{A}TE \qquad \dfrac{a=b \quad Fa}{Fb}{=}E$$

$$
\dfrac{
\dfrac{\dfrac{\dfrac{\dfrac{\lambda=\langle\neg T\lambda\rangle \qquad [T\lambda]^1}{T\langle\neg T\lambda\rangle}{\scriptstyle =E}}{\neg T\lambda}{\scriptstyle \cancel{TE}} \qquad [T\lambda]^1}{\bot}{\scriptstyle \neg E}}{\neg T\lambda}{\scriptstyle \neg I^1}
\qquad
\dfrac{\lambda=\langle\neg T\lambda\rangle \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\lambda=\langle\neg T\lambda\rangle \qquad [T\lambda]^2}{T\langle\neg T\lambda\rangle}{\scriptstyle =E}}{\neg T\lambda}{\scriptstyle \cancel{TE}} \qquad [T\lambda]^2}{\bot}{\scriptstyle \neg E}}{\neg T\lambda}{\scriptstyle \neg I^2}}{T\langle\neg T\lambda\rangle}{\scriptstyle \cancel{TI}}}{T\lambda}{\scriptstyle =E}
}{\bot}{\scriptstyle \neg E}
$$

REVISING TRUTH

$$\dfrac{\neg A \qquad A}{\bot}{\scriptstyle \neg E} \qquad\qquad \dfrac{\begin{matrix}[A]^i\\ \vdots\\ \bot\end{matrix}}{\neg A}{\scriptstyle \neg I^i} \qquad\qquad \cancel{\dfrac{A}{T\langle A\rangle}{\scriptstyle TI} \qquad \dfrac{T\langle A\rangle}{A}{\scriptstyle TE}} \qquad\qquad \dfrac{a=b \qquad Fa}{Fb}{\scriptstyle =E}$$

REVISING TRUTH

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^1}{T\langle \neg T\lambda \rangle}\,{\scriptstyle =E}\,???
}{\neg T\lambda}\,{\scriptstyle TE}
\qquad [T\lambda]^1
}{\bot}\,{\scriptstyle \neg E}
}{\neg T\lambda}\,{\scriptstyle \neg I^1}
\qquad
\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{\lambda = \langle \neg T\lambda \rangle \qquad [T\lambda]^2}{T\langle \neg T\lambda \rangle}\,{\scriptstyle =E}\,???
}{\neg T\lambda}\,{\scriptstyle TE}
\qquad [T\lambda]^2
}{\bot}\,{\scriptstyle \neg E}
}{\neg T\lambda}\,{\scriptstyle \neg I^2}
}{T\langle \neg T\lambda \rangle}\,{\scriptstyle TI}
}{T\lambda}\,{\scriptstyle =E}\,???
}{\bot}\,{\scriptstyle \neg E}
$$

$$
\dfrac{\neg A \qquad A}{\bot}\,{\scriptstyle \neg E}
\qquad\qquad
\dfrac{\begin{matrix}[A]^i \\ \vdots \\ \bot\end{matrix}}{\neg A}\,{\scriptstyle \neg I^i}
\qquad\qquad
\dfrac{A}{T\langle A \rangle}\,{\scriptstyle TI}
\qquad
\dfrac{T\langle A \rangle}{A}\,{\scriptstyle TE}
\qquad\qquad
\overset{???}{\dfrac{a = b \qquad Fa}{Fb}}\,{\scriptstyle =E}
$$

TODAY'S PLAN

FORMALISING one paradoxical argument

LOCATING different positions on this map

FORMALISING a different paradoxical argument

Noticing the PARALLELS

Kripke's MODEL
- why it's needed
- {0, n, 1} & refinement
- stages & the fixed point

THE UPSHOT

t is A

$$\frac{A(t)}{t \in \{x : A(x)\}}\ \in I \qquad\qquad \frac{t \in \{x : A(x)\}}{A(t)}\ \in E$$

t has the property A.

Consider $\{x : x \in x \to p\}$,

for a given statement p.

Does $\{x : x \in x \to p\} \in \{x : x \in x \to p\}$?

Consider $\overbrace{\{x : x \in x \to p\}}^{c}$,

for a given statement p.

Does $\{x : x \in x \to p\} \in \{x : x \in x \to p\}$?

$c \in c$?

$$\frac{c \in c}{c \in c \to p}\,{\in}E \qquad\qquad \frac{c \in c \to p}{c \in c}\,{\in}I$$

$$\dfrac{\dfrac{\dfrac{\dfrac{[c\in c]^1}{c\in c\to p}\,{\in}E \qquad [c\in c]^1}{p}\,{\to}E}{c\in c\to p}\,{\to}I^1 \qquad \dfrac{\dfrac{\dfrac{\dfrac{[c\in c]^2}{c\in c\to p}\,{\in}E \qquad [c\in c]^2}{p}\,{\to}E}{c\in c\to p}\,{\to}I^2}{c\in c}\,{\in}I}{p}\,{\to}E$$

$$c \text{ is } \{x : x\in x\to p\} \qquad \dfrac{A\to B \qquad A}{B}\,{\to}E \qquad \dfrac{\begin{matrix}[A]^i\\ \vdots\\ B\end{matrix}}{A\to B}\,{\to}I^i \qquad \dfrac{A(t)}{t\in\{x:A(x)\}}\,{\in}I \qquad \dfrac{t\in\{x:A(x)\}}{A(t)}\,{\in}E$$

TODAY'S PLAN

FORMALISING one paradoxical argument

LOCATING different positions on this map

FORMALISING a different paradoxical argument

Noticing the PARALLELS

KRIPKE'S MODEL
- why it's needed
- $\{0, n, 1\}$ & refinement
- stages & the fixed point

THE UPSHOT

$$\dfrac{\dfrac{\dfrac{\dfrac{[\cancel{C\in C}]^1}{C\in C\to p}\in E \qquad [\cancel{C\in C}]^1}{p}\to E}{C\in C\to p}\to I^1 \qquad \dfrac{\dfrac{\dfrac{\dfrac{[\cancel{C\in C}]^2}{C\in C\to p}\in E \qquad [\cancel{C\in C}]^2}{p}\to E}{C\in C\to p}\to I^2}{C\in C}\in I}{p}\to E$$

level & no proposition views

C is $\cancel{\{x : x\in x\to p\}}$

$$\dfrac{A\to B \qquad A}{B}\to E \qquad \dfrac{\begin{matrix}[A]^i\\ \vdots \\ B\end{matrix}}{A\to B}\to I^i \qquad \dfrac{A(t)}{t\in\{x : A(x)\}}\in I \qquad \dfrac{t\in\{x : A(x)\}}{A(t)}\in E$$

$$
\begin{prooftree}
\AxiomC{$[c \in c]^1$}
\RightLabel{$\in E$}
\UnaryInfC{$c \in c \to p$}
\AxiomC{$[c \in c]^1$}
\RightLabel{$\to E$}
\BinaryInfC{p}
\RightLabel{$\cancel{\to I^1}$}
\UnaryInfC{$c \in c \to p$}
\AxiomC{$[c \in c]^2$}
\RightLabel{$\in E$}
\UnaryInfC{$c \in c \to p$}
\AxiomC{$[c \in c]^2$}
\RightLabel{$\to E$}
\BinaryInfC{p}
\RightLabel{$\cancel{\to I^2}$}
\UnaryInfC{$c \in c \to p$}
\RightLabel{$\in I$}
\UnaryInfC{$c \in c$}
\RightLabel{$\to E$}
\BinaryInfC{p}
\end{prooftree}
$$

'GAP' views?

---

c is $\{x : x \in x \to p\}$

$$
\begin{prooftree}
\AxiomC{$A \to B$}
\AxiomC{A}
\RightLabel{$\to E$}
\BinaryInfC{B}
\end{prooftree}
\qquad
\cancel{
\begin{prooftree}
\AxiomC{$[A]^i$}
\noLine
\UnaryInfC{$\vdots$}
\noLine
\UnaryInfC{B}
\RightLabel{$\to I^i$}
\UnaryInfC{$A \to B$}
\end{prooftree}
}
\qquad
\begin{prooftree}
\AxiomC{$A(t)$}
\RightLabel{$\in I$}
\UnaryInfC{$t \in \{x : A(x)\}$}
\end{prooftree}
\qquad
\begin{prooftree}
\AxiomC{$t \in \{x : A(x)\}$}
\RightLabel{$\in E$}
\UnaryInfC{$A(t)$}
\end{prooftree}
$$

$$\dfrac{\dfrac{\dfrac{\dfrac{[C\in C]^1}{C\in C\to p}\,{\in}E \qquad [C\in C]^1}{p}\,\cancel{\to E}}{C\in C\to p}\,{\to}I^1 \qquad \dfrac{\dfrac{\dfrac{\dfrac{[C\in C]^2}{C\in C\to p}\,{\in}E \qquad [C\in C]^2}{p}\,\cancel{\to E}}{C\in C\to p}\,{\to}I^2}{C\in C}\,{\in}I}{p}\,{\to}E$$

'GLUT' VIEWS?

C is $\{x : x\in x\to p\}$

$$\cancel{\dfrac{A\to B \qquad A}{B}\,{\to}E} \qquad \dfrac{\begin{matrix}[A]^i\\ \vdots\\ B\end{matrix}}{A\to B}\,{\to}I^i \qquad \dfrac{A(t)}{t\in\{x:A(x)\}}\,{\in}I \qquad \dfrac{t\in\{x:A(x)\}}{A(t)}\,{\in}E$$

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{[c\in c]^1}{c\in c\to p}\,\cancel{\in E}\qquad [c\in c]^1
}{p}\,\to E
}{c\in c\to p}\,\to I^1
\qquad
\dfrac{
\dfrac{
\dfrac{
\dfrac{[c\in c]^2}{c\in c\to p}\,\cancel{\in E}\qquad [c\in c]^2
}{p}\,\to E
}{c\in c\to p}\,\to I^2
}{c\in c}\,\cancel{\in I}
}{p}\,\to E
$$

REVISION about property ascription

c is $\{x : x\in x\to p\}$

$$\dfrac{A\to B\qquad A}{B}\,\to E$$

$$\dfrac{\begin{matrix}[A]^i\\ \vdots\\ B\end{matrix}}{A\to B}\,\to I^i$$

$$\cancel{\dfrac{A(t)}{t\in\{x: A(x)\}}\,\in I}$$

$$\cancel{\dfrac{t\in\{x: A(x)\}}{A(t)}\,\in E}$$

Does your diagnosis of the liar paradox generalise to Curry's paradox?

Should it?

TODAY'S PLAN

FORMALISING one paradoxical argument

LOCATING different positions on this map

FORMALISING a different paradoxical argument

Noticing the PARALLELS

Kripke's MODEL
- why it's needed
- $\{0, n, 1\}$ & refinement
- stages & the fixed point

THE UPSHOT

$m(A \wedge B) = 1$ iff $m(A) = 1$ &
$m(B) = 1$

$m(T\langle A \rangle) =$ .....?

$$m(A \wedge B) = 1 \quad \text{iff} \quad m(A) = 1 \;\&\; m(B) = 1$$

$$m(T\langle A \rangle) = m(A) \;?$$

$$m(A \wedge B) = 1 \text{ iff } m(A) = 1 \;\&\; m(B) = 1$$

$$m(T\langle A \rangle) = m(A) \;?$$

$$\tau = \langle T\tau \rangle : \quad m(T\tau) = m(T\langle T\tau \rangle) = m(T\tau) = m(T\langle T\tau \rangle) = \cdots$$

$$m(A \wedge B) = 1 \text{ iff } m(A) = 1 \;\&\; m(B) = 1$$

$$m(T\langle A \rangle) = m(A) \; ?$$

$$\lambda = \langle \neg T \tau \rangle : \quad m(T\lambda) = m(T\langle \neg T\lambda \rangle)$$
$$= m(\neg T\lambda)$$
$$= m(\neg T\langle \neg T\lambda \rangle)$$
$$= \cdots$$

$$m(A \wedge B) = 1 \text{ iff } m(A) = 1 \;\&\; m(B) = 1$$

$$m(T\langle A \rangle) = m(A) \;?$$

In the presence of self reference, these rules do not assign values of a complex expression in terms of the values of simpler expressions.

TODAY'S PLAN

FORMALISING one paradoxical argument

LOCATING different positions on this map

FORMALISING a different paradoxical argument

Noticing the PARALLELS

Kripke's MODEL – $\{0, n, 1\}$ & refinement

why it's needed

stages & the fixed point

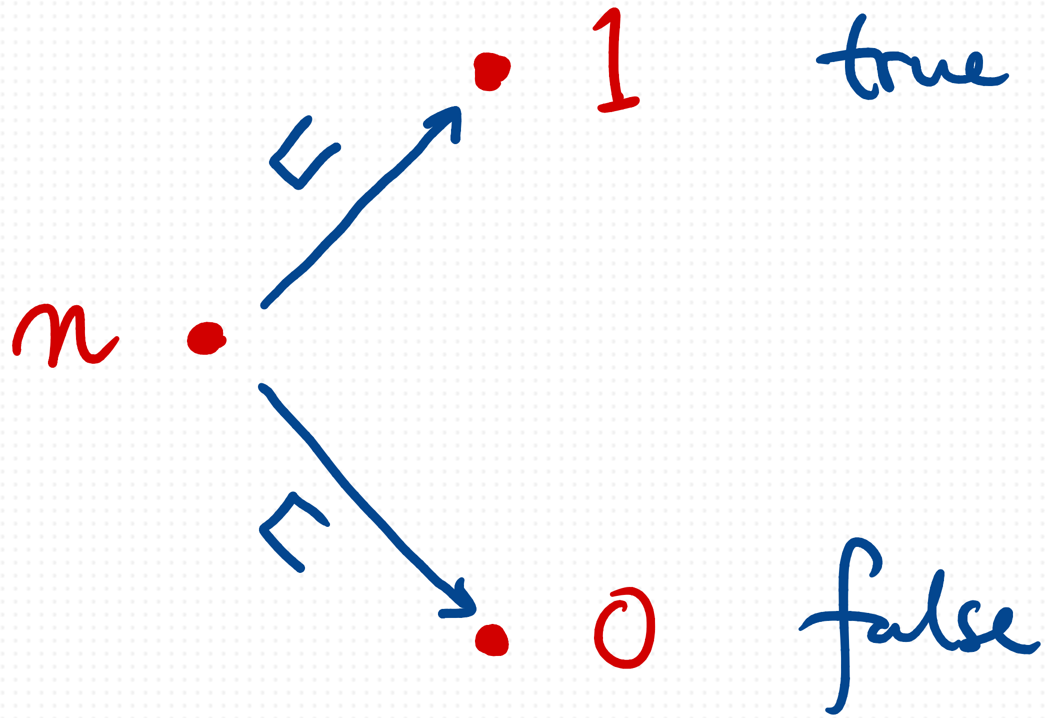
THE UPSHOT

- 1 true
- 0 false

$\bullet\ 1$ true

$n\ \bullet$

$\bullet\ 0$ false



$x \sqsubseteq y$ iff $x \sqsubset y$ or $x = y$

$m(\bot) = 0$

$$m(\neg A) = 1 \text{ iff } m(A) = 0$$
$$= 0 \text{ iff } m(A) = 1$$

$$m(A \wedge B) = 1 \text{ iff } m(A) = 1 \;\&\; m(B) = 1$$
$$= 0 \text{ iff } m(A) = 0 \text{ or } m(B) = 0$$

$$m(A \vee B) = 1 \text{ iff } m(A) = 1 \text{ or } m(B) = 1$$
$$= 0 \text{ iff } m(A) = 0 \;\&\; m(B) = 0$$

$$m(A \to B) = 1 \text{ iff } m(A) = 0 \text{ or } m(B) = 1$$
$$= 0 \text{ iff } m(A) = 1 \;\&\; m(B) = 0$$

A	$\neg A$
0	1
n	n
1	0

$\wedge$	0	n	1
0	0	0	0
n	0	n	n
1	0	n	1

$\vee$	0	n	1
0	0	n	1
n	n	n	1
1	1	1	1

$\rightarrow$	0	n	1
0	1	1	1
n	n	n	1
1	0	n	1

TODAY'S PLAN

FORMALISING one paradoxical argument

LOCATING different positions on this map

FORMALISING a different paradoxical argument

Noticing the PARALLELS

Kripke's MODEL
- why it's needed
- $\{0, n, 1\}$ & refinement
- stages & the fixed point

THE UPSHOT

If $m_1(A) \sqsubseteq m_2(A)$ then $m_1(\neg A) \sqsubseteq m_2(\neg A)$

& if $m_1(B) \sqsubseteq m_2(B)$ then $m_1(A \wedge B) \sqsubseteq m_2(A \wedge B)$

$$m_1(A \vee B) \sqsubseteq m_2(A \vee B)$$

$$m_1(A \to B) \sqsubseteq m_2(A \to B)$$

e.g.

	m_1		m_2
A:	n	$\sqsubseteq$	1
B:	n	$\sqsubseteq$	0
$A \to B$:	n	$\sqsubseteq$	0

	m_1		m_2
A:	n	$\sqsubseteq$	1
B:	n	$\sqsubseteq$	1
$A \to B$:	n	$\sqsubseteq$	1

Adding T to a formal language.

$\mathcal{M}_0$

- interpret the non T sentences however you like in $\{0, n, 1\}$.
- assign Tx the value n

We treat T-sentences as undetermined at stage 0, and we progressively refine them over stages.

$$m_0(\text{non T atom}) = m_1(\text{non-T atom})$$

so, $m_0(p) \sqsubseteq m_1(p)$

$$m_0(A) = m_1(T\langle A\rangle)$$

so, $\underset{= n}{m_0(T\langle A\rangle)} \sqsubseteq m_1(T\langle A\rangle)$

so, $m_0 \sqsubseteq m_1$

$$m_i(\text{non T atom}) = m_{i+1}(\text{non-T atom})$$

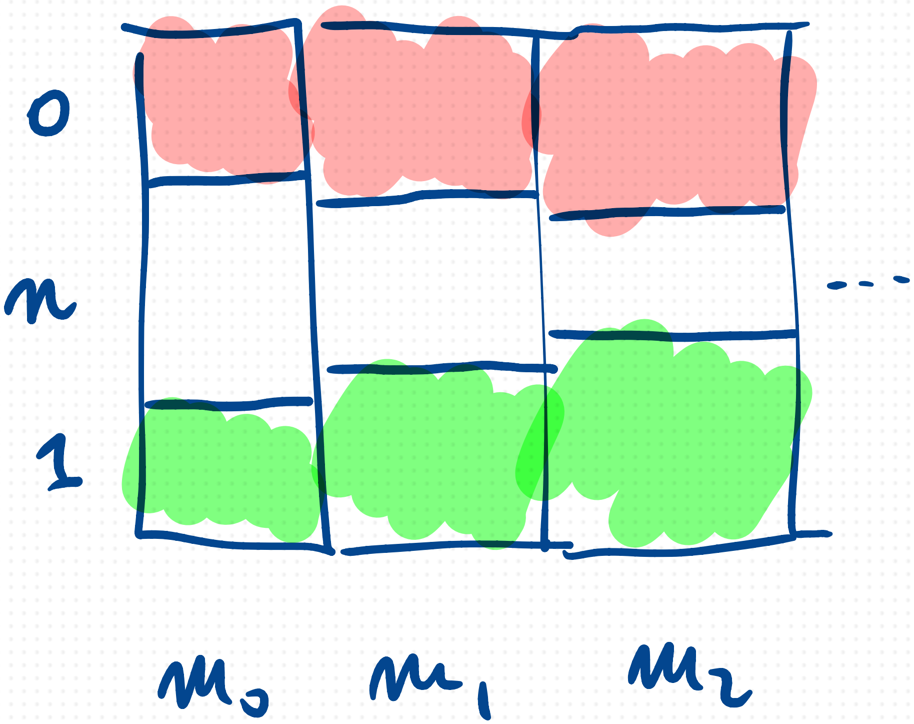
so, $m_i(p) \sqsubseteq m_{i+1}(p)$

$$m_i(A) = m_{i+1}(T\langle A\rangle)$$

so, $m_i(T\langle A\rangle) \qquad m_{i+1}(T\langle A\rangle)$

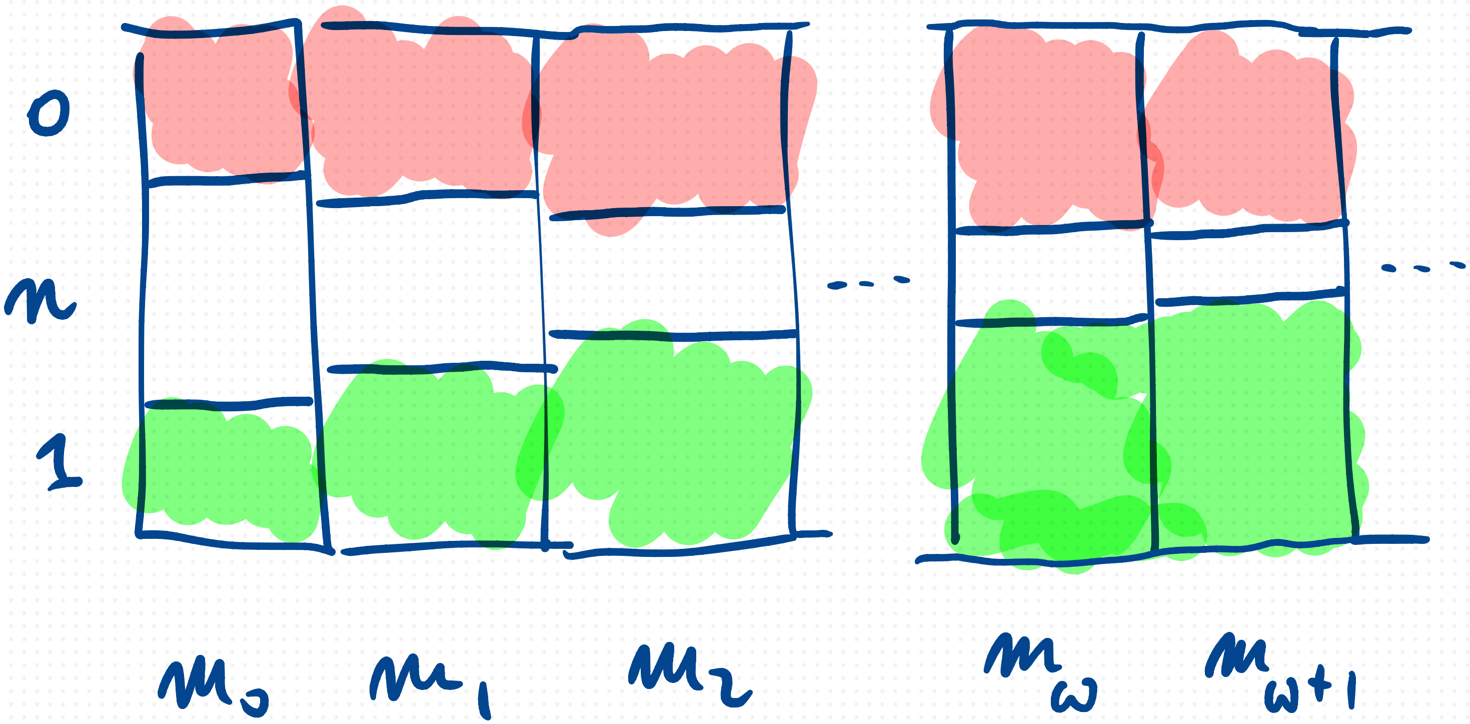
$$\begin{array}{ccc} m_i(T\langle A\rangle) & & m_{i+1}(T\langle A\rangle) \\ \parallel & & \parallel \\ m_{i-1}(A) & \sqsubseteq & m_i(A) \end{array}$$

so, $m_i \sqsubseteq m_{i+1}$



0
n
1
m0
m1
m2
...

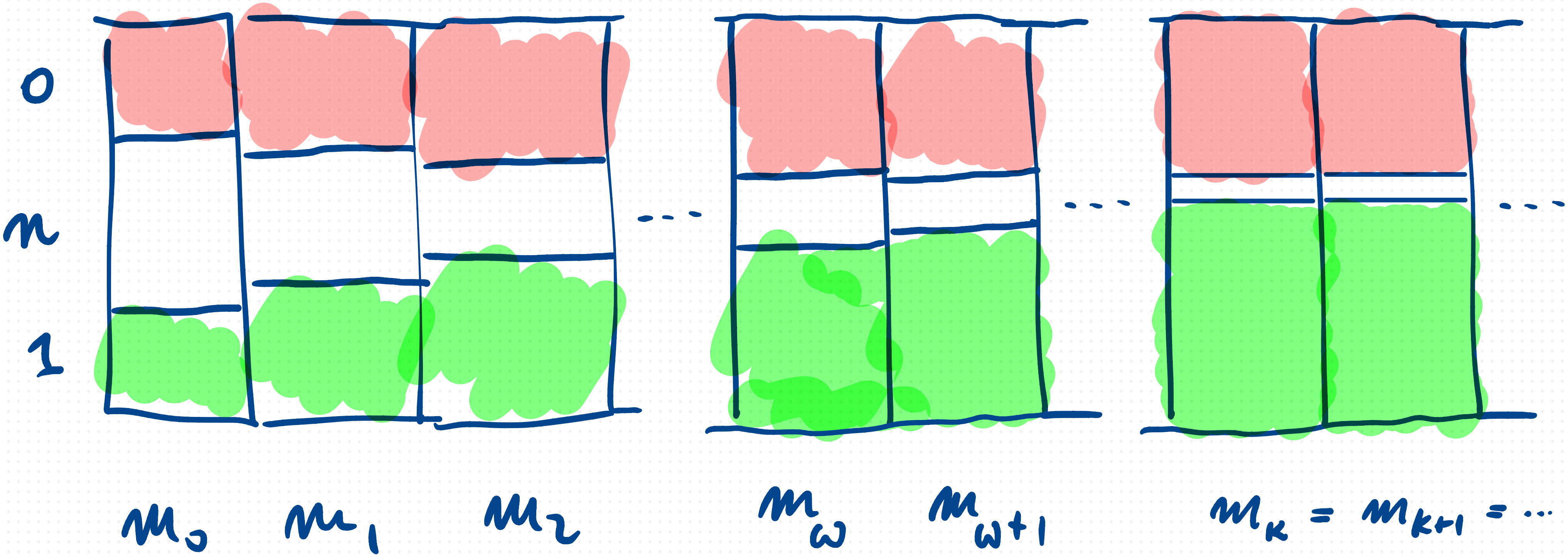
0	p, $\neg q$	p, $T\langle p\rangle$, $\neg q$	p, $T\langle p\rangle$, $T\langle T\langle p\rangle\rangle$, $\neg q$	
n	$T\langle p\rangle$, $T\langle q\rangle$	$T\langle T\langle p\rangle\rangle$, $T\langle T\langle q\rangle\rangle$	$T\langle T\langle T\langle p\rangle\rangle\rangle$	$\dots$
1	q, $\neg p$	$T\langle q\rangle$, $\neg p$, q, $T\langle \neg p\rangle$	$T\langle q\rangle$, $\neg p$, $T\langle T\langle q\rangle\rangle$, q, $T\langle \neg p\rangle$	
	m_0	m_1	m_2	



0
n
1
m_0
m_1
m_2
m_ω
$m_{\omega+1}$

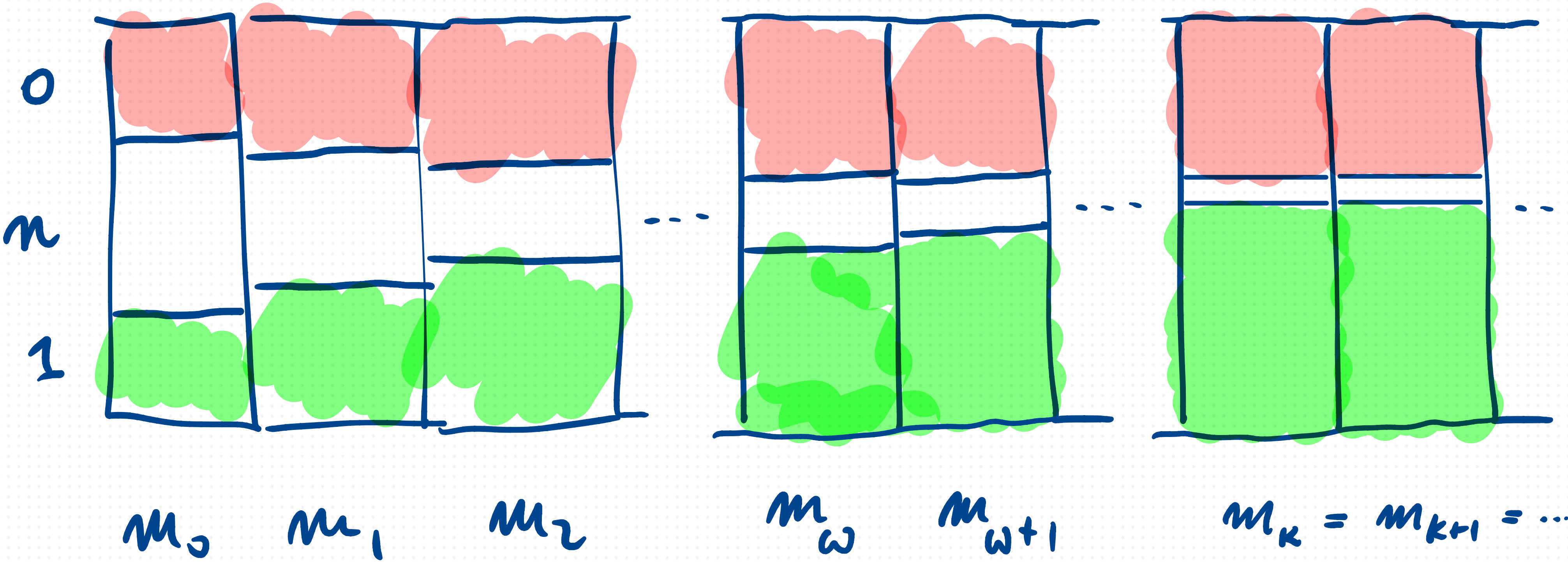
(!)

We eventually reach a fixed-point.



(!)

We eventually reach a fixed-point.

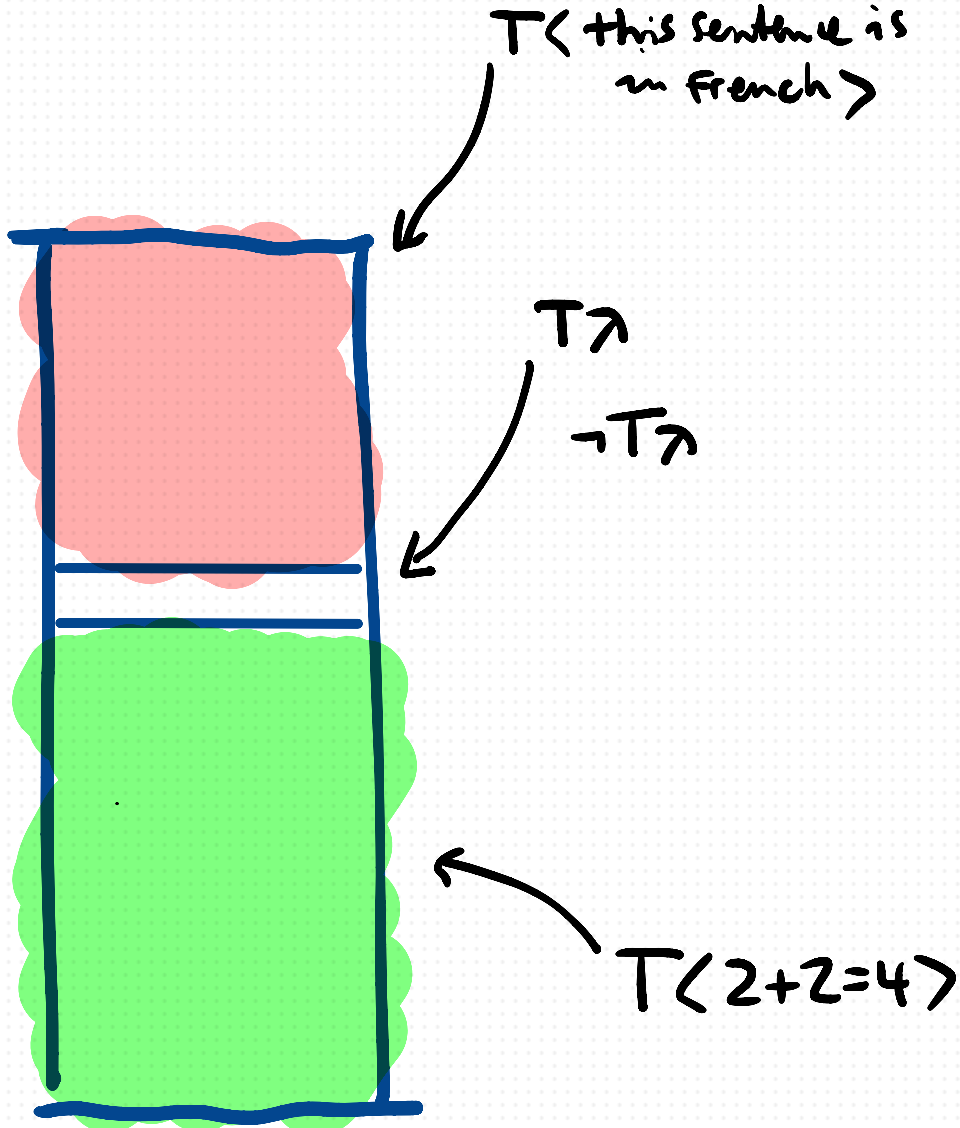


$$m_k(A) = m_{k+1}(T\langle A\rangle) = m_k(T\langle A\rangle)$$

by definition

mk is a fixed point

$$\lambda = \langle \neg T\lambda \rangle$$



TODAY'S PLAN

FORMALISING one paradoxical argument

LOCATING different positions on this map

FORMALISING a different paradoxical argument

Noticing the PARALLELS

Kripke's MODEL
- why it's needed
- $\{0, n, 1\}$ & refinement
- stages & the fixed point

THE UPSHOT

NOT a 'SOLUTION' to the LIAR PARADOX

The naïve reading $\left\{\begin{array}{ll} 0 & \text{TRUE} \\ n & \text{NEITHER} \\ 1 & \text{FALSE} \end{array}\right\}$ seems susceptible to a revenge paradox.

λ is neither true nor false

$\neg T\lambda$ $\qquad\qquad$ $\neg\neg T\lambda$

BOTH n.

MORE THAN JUST THE LIAR PARADOX

- It's a **MODEL**: it gives a safety guarantee for the language it interprets.

- Nothing special about **negation**. Refinement is doing all the work ~ and it applies equally to → and the Curry paradox, as to the Liar

- Nothing special about **truth**. This technique applies equally well to property ascription and any other circular definitions.

NEXT WEEK

What can these models mean?

Can we be neutralist about these paradoxes?

Thank You!

https://consequently.org/