Assertions, Denials Questions, Answers & the Common Ground

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My ***Aim***

To better understand the speech acts of *assertion* and *denial*, their relationship to *other* speech acts, and connections between these speech acts and logical notions, including the classical sequent calculus.

I want to revisit some themes
(and revise some of the claims)
in my 2005 paper "Multiple Conclusions."

The behaviour of two kinds of speech acts:

polar (yes/no) *questions*,

and *justification requests*.

My ***Plan***

Assertion and Denial

Polar Questions

Positions and Rules

Justification Requests

ASSERTION AND DENIAL

$$X \succ Y$$

Don't *assert* each member of X
and *deny* each member of Y.

Defining Rules for Logical Concepts

$$\frac{X, A, B \succ Y}{X, A \wedge B \succ Y}\ {\wedge}Df \qquad \frac{X \succ A, B, Y}{X \succ A \vee B, Y}\ {\vee}Df \qquad \frac{X \succ A, Y}{X, \neg A \succ Y}\ {\neg}Df \qquad \frac{X, A \succ B, Y}{X \succ A \supset B, Y}\ {\supset}Df$$

$$\frac{X \succ A(n), Y}{X \succ \forall x A(x), Y}\ {\forall}Df \qquad \frac{X, A(n) \succ Y}{X, \exists x A(x) \succ Y}\ {\exists}Df \qquad \frac{X, Fa \succ Fb, Y \quad X, Fb \succ Fa, Y}{X \succ a = b, Y}\ {=}Df$$

Terms & conditions: the singular term n (in $\forall/\exists$*Df*) and the predicate F (in $=$*Df*) do not appear below the line in those rules.

… I was wading into a pre-existing literature about assertion. A *large* literature.

It's been fruitful to think of assertion as an act governed by *norms*.

Aim to say what is *true*!

*For **me**: Production Norms*

Aim to say what is *true*!

Only say what you *know*!

*For **me**: Production Norms*

Aim to say what is *true*!

Only say what you *know*!

Be prepared to *back it up* when requested!

*For **you**: Consumption Norms*

The hearer is entitled to re-assert.

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You can refer back to the asserter
to *vouch for* the assertion.

*For **us**: Our Common Ground*

To assert is to bid for the content asserted
to be added to the COMMON GROUND,
the body of information that
we (together) take for granted.

Stalnaker on Common Ground

To presuppose something is to take it for granted, or at least to act as if one takes it for granted, as background information as common ground among the participants in the conversation. What is most distinctive about this propositional attitude is that it is a social or public attitude: one presupposes that ϕ *only if one presupposes that others presuppose it as well.*

— "Common Ground" *L&P* (2002)

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This does not help distinguish *denial* from *retraction*, or from other speech acts.

… by examining polar questions,
and their answers,
in the light of our background
interest in assertion and its norms.

POLAR QUESTIONS

Is it the case that p?

This is a distinct speech act
with its own norms.

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with its own norms.

It raises an *issue*.

There are two ways to settle the issue

Yes

There are two ways to settle the issue

Yes *No*

If I say *yes* and you say *no*
to some polar question p?,
then we DISAGREE.

That is, we take *different* positions on p.

The two ways ***clash***

If I say *yes* and you say *no*
to some polar question p?,
then we DISAGREE.

That is, we take *different* positions on p.

There is no *shared* position
incorporating both of our answers.

Other responses, like

Other responses, like

maybe

Other responses, like

maybe · *I don't know*

Other responses don't settle the issue

Other responses, like

maybe · *I don't know* · *I think so*

Other responses, like

maybe · *I don't know* · *I think so*

are acceptable responses to p?,

Other responses, like

maybe · *I don't know* · *I think so*

are acceptable responses to p?,

but they don't answer the question.
They don't settle the issue of p.

Settling answers are assertions

A *yes* or a *no* to p? counts as an assertion.

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(It's governed by all the assertion norms we've already seen.)

*What does a "no" to p? **assert**?*

Presumably $\neg p$.

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However, I prefer to think of
a *yes* to p? as ruling p *in*,
and a *no* to p? as ruling p *out*.

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This way, we can distinguish practices
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and those with more limited expressive resources.

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(Nothing important hangs on this distinction.)

$$[X : Y]$$

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- We have ruled *in* everything in X, the POSITIVE COMMON GROUND.

$$[X : Y]$$

- We have ruled *in* everything in X, the POSITIVE COMMON GROUND.
- We have ruled *out* everything in Y, the NEGATIVE COMMON GROUND.

Denial and Retraction

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ELOISE: No, he is in the kitchen.

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PARTIAL ANSWER

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WEAK DENIAL

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- To *strongly deny* p is to bid to add p to the *negative common ground*.

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- To *weakly deny* p is to *block* the addition of p to the *positive common ground*, or to bid for its *retraction* if it is already there.

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- Strong *or* weak denials of p are appropriate responses to an assertion of p, because the assertion of p is a bid to add p to the positive common ground.

- A strong denial of p is one way to settle the question p? — this is generally an appropriate response.

- A weak denial of p is not generally an appropriate response to the polar question p?, as the polar question does not place p in the positive common ground, and the question is inappropriate if p is already in the positive common ground, so there is no p to block or retract.

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- STRONG ASSERTION: add to the positive common ground.

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Strong and Weak Denials, and Strong and Weak ***Assertions***

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- STRONG ASSERTION: add to the positive common ground.
- WEAK DENIAL: retract (or block) from the positive common ground.
- WEAK ASSERTION: retract (or block) from the negative common ground. — "Perhaps p."

That's one way to understand the relationship between assertion and denial, and how to distinguish strong denial from other negative speech acts.

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(what we, together, *take for granted*)
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ELOISE: No. The interior angles of *this* triangle add up to 180 degrees. Can you prove the general case?

Eloise here seems to block from the common ground (weakly deny)
a logical consequence of claims in the common ground (the axioms of geometry),
for the same general reason as for other weak denials.

Any position $[X, A : A, Y]$
in which A has been
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If $X \not\succ Y$ then $[X : Y]$ is *available*.

A Word on ***Cut***

$$\frac{X \succ A, Y \quad X, A \succ Y}{X \succ Y}\,\textit{Cut}$$

$$\frac{X \succ A, Y \qquad X, A \succ Y}{X \succ Y}\,\mathit{Cut}$$

In any available position $[X : Y]$, if one way to settle A? is *not* available, then the other way to settle it *is* available.

POSITIONS AND RULES

Defining Rules

$$\frac{X, A, B \succ Y}{X, A \land B \succ Y}\ {\land}Df \qquad \frac{X \succ A, B, Y}{X \succ A \lor B, Y}\ {\lor}Df$$

$$\frac{X \succ A, Y}{X, \neg A \succ Y}\ {\neg}Df \qquad \frac{X, A \succ B, Y}{X \succ A \supset B, Y}\ {\supset}Df$$

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$$\frac{X \succ A, Y}{X, \neg A \succ Y} \neg Df \qquad \frac{X, A \succ B, Y}{X \succ A \supset B, Y} \supset Df$$

These are kinds of *definitions*, showing how to treat assertions or denials of the *defined* concept in terms of the assertions or denials of their components.

Derivations

$$\dfrac{\dfrac{\neg p \succ \neg p}{\succ p, \neg p}\,\neg Df}{\succ p \vee \neg p}\,\vee Df$$

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\dfrac{\dfrac{\neg p \succ \neg p}{\succ p, \neg p}\,{\scriptstyle \neg Df}}{\succ p \lor \neg p}\,{\scriptstyle \lor Df}
\qquad\qquad
\dfrac{\dfrac{p \succ p}{p, \neg p \succ}\,{\scriptstyle \neg Df}}{p \land \neg p \succ}\,{\scriptstyle \land Df}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{p, q \lor r \succ p \land q, q \lor r}{p, q \lor r \succ p \land q, r, q}\,{\scriptstyle \lor Df} \quad \dfrac{p \land q, q \lor r \succ p \land q, r}{q, p, q \lor r \succ p \land q, r}\,{\scriptstyle \land Df}}{p, q \lor r \succ p \land q, r}\,{\scriptstyle Cut}}{p, q \lor r \succ (p \land q) \lor r}\,{\scriptstyle \lor Df}}{p \land (q \lor r) \succ (p \land q) \lor r}\,{\scriptstyle \land Df}
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- They don't have the same *shape* as proofs.
- (Where is the *conclusion* in $p \lor q \succ p, q$?)
- A endsequent $X \succ A$ doesn't tell you to *infer* A *from* X — it merely tells you to not assert all members of X and deny A.

Let's make this problem ***sharp***

"Well, now, let's take a little bit of the argument in that First Proposition—just *two* steps, and the conclusion drawn from them. Kindly enter them in your note-book. And in order to refer to them conveniently, let's call them A, B, and Z:—

(A) Things that are equal to the same are equal to each other.

(B) The two sides of this Triangle are things that are equal to the same.

(Z) The two sides of this Triangle are equal to each other.

Readers of Euclid will grant, I suppose, that Z follows logically from A and B, so that any one who accepts A and B as true, *must* accept Z as true?"

"Undoubtedly! The youngest child in a High School—as soon as High Schools are invented, which will not be till some two thousand years later—will grant *that*."

"And if some reader had *not* yet accepted A and B as true, he might still accept the *sequence* as a *valid* one, I suppose?"

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(*Z*) The two sides of this Triangle are equal to each other.

Readers of Euclid will grant, I suppose, that *Z* follows logically from *A* and *B*, so that any one who accepts *A* and *B* as true, *must* accept *Z* as true?"

"Undoubtedly! The youngest child in a High School—as soon as High Schools are invented, which will not be till some two thousand years later—will grant *that*."

"And if some reader had *not* yet accepted *A* and *B* as true, he might still accept the *sequence* as a *valid* one, I suppose?"

The Tortoise never asserts A and $A \supset Z$ while denying Z, but he doesn't accept A and $A \supset Z$ *as a reason* for Z.

JUSTIFICATION REQUESTS

What is a justification request?

ABELARD: Astralabe is in the kitchen.

ELOISE: *Really*?

ABELARD: I saw him there five minutes ago.

ELOISE: OK.

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ELOISE: *Are you sure*? He's been in the study with me for the last half hour.

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ABELARD: Astralabe is in the kitchen.
ELOISE: *Really*?
ABELARD: I saw him there five minutes ago.
ELOISE: Yes, but he was in the study two minutes ago.

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That's a JUSTIFICATION REQUEST.

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What is a justification request?

A justification request for a strong assertion [or strong denial] is an attempt to block the addition to the common ground, until a *reason* is given.

This reason is another assertion [or denial] which must be *granted*, (added to the common ground) in order for the request to be met.

A justification request for a strong assertion [or strong denial] is an attempt to block the addition to the common ground, until a *reason* is given.

This reason is another assertion [or denial] which must be *granted*, (added to the common ground) in order for the request to be met.

Granting the given reason is *necessary* but not *sufficient* for satisfying the justification request.

Definitions and Justification Requests

ACHILLES So … this is an *equilateral* triangle.

TORTOISE I'm sorry, I don't follow, my heroic friend. I've not heard that word before: what does '*equilateral*' mean?

ACHILLES Oh, that's easy to explain. '*Equilateral*' means having sides of the same length. An *equilateral* triangle is a triangle with all three sides the same length.

TORTOISE OK. That sounds good. You may continue with your reasoning.

ACHILLES Well, as I was saying, the sides of this triangle are all one cubit in length, so it is an equilateral triangle.

TORTOISE Perhaps you will forgive me, Achilles, but I still don't follow. I grant to you that the sides of this triangle all have the same length. I fail to see, however, that it *follows* that it is an equilateral triangle. Could you explain why it is?

If I accept the definition $A =_{\mathrm{df}} \mathrm{B}$,
then I should accept granting A as meeting
a justification request for the assertion of B
and ruling out A as meeting a justification
request for B's denial and *vice versa*.

A failure to accept this is a sign
that I have not mastered the definition.

What goes for a definition of the form $A =_{df} B$ can also go for defining rules:

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It is a mistake to grant A and grant B
and to look for something more to discharge
a justification request for an assertion of $A \land B$,
if you take $\land Df$ as a *definition*.

Justification Requests and Defining Rules

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It is a mistake to rule A in and rule B out
and to look for something more to discharge
a justification request for a denial of $A \supset B$
if you accept $\supset Df$ as a definition.

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$$\frac{A \supset Z \succ \boxed{A \supset Z}}{A \supset Z, A \succ \boxed{Z}} \supset Df$$

- Read the *premise* as telling us that in a position in which $A \supset Z$ is already ruled in, we have an answer to the justification request for $A \supset Z$'s assertion.

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- Read the *premise* as telling us that in a position in which $A \supset Z$ is already ruled in, we have an answer to the justification request for $A \supset Z$'s assertion.
- Then applying $\supset Df$ we see why we have an answer to the request concerning Z's assertion, in a context in which $A \supset Z$ and A have both been ruled in. (In granting $A \supset Z$ and A we have settled Z positively. Its denial is ruled out, since to assert A and deny Z amounts to denying $A \supset Z$.)

SLOGAN: A derivation of $X \succ \boxed{A}, Y$ shows us how to meet a justification request for the assertion of A in any available position extending $[X : Y]$.

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A derivation of $X, \boxed{A} \succ Y$ shows us how to meet a justification request for the denial of A in any available position extending $[X : Y]$.

Focussed Structural Rules

$$X, A \succ \boxed{A}, Y \qquad X, \boxed{A} \succ A, Y$$

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$$\frac{X \succ \boxed{A}, Y \quad X, A \succ \boxed{B}, Y}{X \succ \boxed{B}, Y}\ \mathit{Cut} \qquad \frac{X \succ \boxed{A}, Y \quad X, A, \boxed{B} \succ Y}{X, \boxed{B} \succ Y}\ \mathit{Cut}$$

$$\frac{X, A, \boxed{A} \succ Y}{X, \boxed{A} \succ Y}\ W \qquad \frac{X \succ A, \boxed{A}, Y}{X \succ \boxed{A}, Y}\ W$$

Swap

$$\dfrac{\dfrac{X \succ \boxed{A}, B, Y \qquad X, A \succ A, \boxed{B}, Y}{X \succ A, B, \boxed{B}, Y}\,\mathit{Cut}}{X \succ A, \boxed{B}, Y}\,W$$

Swap

$$\dfrac{\dfrac{X \succ \boxed{A}, B, Y \quad X, A \succ A, \boxed{B}, Y}{X \succ A, B, \boxed{B}, Y}\,\textit{Cut}}{X \succ A, \boxed{B}, Y}\,\textit{W}$$

$$\dfrac{X \succ \boxed{A}, B, Y}{X \succ A, \boxed{B}, Y}\,\textit{Swap}$$

Focussed Defining Rules

$$\dfrac{X, \boxed{A}, B \succ Y}{\dfrac{}{X, \boxed{A \land B} \succ Y}}\ \land Df \qquad \dfrac{X, A, \boxed{B} \succ Y}{\dfrac{}{X, \boxed{A \land B} \succ Y}}\ \land Df$$

$$\dfrac{X \succ \boxed{A}, B, Y}{\dfrac{}{X \succ \boxed{A \lor B}, Y}}\ \lor Df \qquad \dfrac{X \succ A, \boxed{B}, Y}{\dfrac{}{X \succ \boxed{A \lor B}, Y}}\ \lor Df$$

$$\dfrac{X \succ \boxed{A}, Y}{\dfrac{}{X, \boxed{\neg A} \succ Y}}\ \neg Df \qquad \dfrac{X, A \succ \boxed{B}, Y}{\dfrac{}{X \succ \boxed{A \supset B}, Y}}\ \supset Df \qquad \dfrac{X, \boxed{A} \succ B, Y}{\dfrac{}{X \succ \boxed{A \supset B}, Y}}\ \supset Df$$

Focussed Derivations

$$\dfrac{\dfrac{p \succ \boxed{p}}{p, \boxed{\neg p} \succ}\,\neg Df}{\boxed{p \land \neg p} \succ}\,\land Df$$

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Proof and ***Supposition***

$$\frac{X, A \succ \boxed{B}, Y}{X \succ \boxed{A \supset B}, Y}\supset\!Df \qquad \frac{X, \boxed{A} \succ B, Y}{X \succ \boxed{A \supset B}, Y}\supset\!Df$$

To prove $A \supset B$, *rule* A *in* (suppose it) and prove B.

Or, *rule* B *out* (suppose it), and refute A.

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- The making of an *inference* is a (possibly preemptive) answer to a justification request.
- A derivation of a sequent $X \succ A, Y$ [$X, A \succ Y$] can be transformed into a *procedure* for meeting a justification request for an assertion of A [denial of A] in any available position, appealing only what is granted in [$X : Y$], and to the defining rules used in that derivation.

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- Derivations are one way we can *grasp* complex bounds and *enforce* them.
- The *negative* view of the bounds is seen in the clash between assertion and denial, and the *positive* view is found in the answers we can give to justification requests.
- Adopting *defining rules* is one way to be *very* precise about the norms governing the concepts so defined, combining *safety*, *univocity* and *expressive power*.

THANK YOU!