Generality & Existence IV

Modality & Identity

Greg Restall

THE UNIVERSITY OF
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My Aim

To analyse the *quantifiers*

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(including their interactions with *modals*)

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using the tools of *proof theory*

My Aim

To analyse the *quantifiers*
(including their interactions with *modals*)
using the tools of *proof theory*
in order to better understand
quantification, *existence* and *identity*.

My Aim for This Talk

Understanding the interaction between modality and identity.

Today's Plan

Hypersequents & Defining Rules

Identity Rules

Subjunctive Alternatives

Indicative Alternatives

The Status of Worlds Semantics

HYPERSEQUENTS & DEFINING RULES

Subjunctive Alternatives and □

$$\frac{\mathcal{H}[\Gamma \succ \Delta \;\mid\; \succ A]}{\mathcal{H}[\Gamma \succ \Box A, \Delta]}\,[\Box Df]$$

Indicative Alternatives and [e]

$$\frac{\mathcal{H}[\Gamma \succ \Delta \;\parallel\; \succ_{@} A]}{\mathcal{H}[\Gamma \succ [\mathsf{e}]A, \Delta]}\ [[\mathsf{e}]Df]$$

Actual zones and @

$$\dfrac{\mathcal{H}[\Gamma, A \succ_{@} \Delta \mid \Gamma' \succ \Delta']}{\mathcal{H}[\Gamma \succ_{@} \Delta \mid \Gamma', @A \succ \Delta']}\ [@Df]$$

Two Dimensional Hypersequents

$$\begin{array}{ccccccccc}
X^1_1 \succ_{@} Y^1_1 & \mid & X^1_2 \succ Y^1_2 & \mid & \cdots & \mid & X^1_{m_1} \succ Y^1_{m_1} & \| \\
X^2_1 \succ_{@} Y^2_1 & \mid & X^2_2 \succ Y^2_2 & \mid & \cdots & \mid & X^2_{m_2} \succ Y^2_{m_2} & \| \\
\vdots & & \vdots & & & & \vdots & \\
X^n_1 \succ_{@} Y^n_1 & \mid & X^n_2 \succ Y^n_2 & \mid & \cdots & \mid & X^n_{m_n} \succ Y^n_{m_n} &
\end{array}$$

Think of these as *scorecards*, keeping track of assertions and denials.

Free Quantification

$$\begin{array}{c} \mathcal{H}[\Gamma, n \succ A(n), \Delta] \\ \hline\hline \mathcal{H}[\Gamma \succ (\forall x)A(x), \Delta] \end{array}\ [\forall Df] \qquad \begin{array}{c} \mathcal{H}[\Gamma, n, A(n) \succ \Delta] \\ \hline\hline \mathcal{H}[\Gamma, (\exists x)A(x) \succ \Delta] \end{array}\ [\exists Df]$$

Free Quantification

$$\begin{array}{c}\mathcal{H}[\Gamma, n \succ A(n), \Delta] \\ \hline\hline \mathcal{H}[\Gamma \succ (\forall x)A(x), \Delta]\end{array}\ [\forall \mathit{Df}] \qquad \begin{array}{c}\mathcal{H}[\Gamma, n, A(n) \succ \Delta] \\ \hline\hline \mathcal{H}[\Gamma, (\exists x)A(x) \succ \Delta]\end{array}\ [\exists \mathit{Df}]$$

$$\begin{array}{c}\mathcal{H}[t, \Gamma \succ \Delta] \\ \hline\hline \mathcal{H}[t{\downarrow}, \Gamma \succ \Delta]\end{array}\ [{\downarrow}\mathit{Df}] \qquad \begin{array}{c}\mathcal{H}[t_i, \Gamma \succ \Delta] \\ \hline \mathcal{H}[Ft_1 \cdots t_n, \Gamma \succ \Delta]\end{array}\ [\mathrm{F}L]$$

IDENTITY RULES

Identity Rules

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[{=}L]$$

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta}\,[{=}L^{f}_{r}] \qquad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta}\,[{=}R]$$

$$\frac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta}\,[{=}L^{p}_{r}] \qquad \frac{Fs, \Gamma \succ \Delta}{s = t, Ft, \Gamma \succ \Delta}\,[{=}L^{p}_{l}]$$

$$\begin{array}{c} \Gamma, Fa \succ Fb, \Delta \\ \hline\hline \Gamma \succ a = b, \Delta \end{array}\,[{=}Df] \qquad \frac{\Gamma \succ \Delta}{\Gamma|^{Fx}_{A(x)} \succ \Delta|^{Fx}_{A(x)}}\,[Spec^{Fx}_{A(x)}]$$

Equivalences

$$\frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta}\,[{=}Df] \qquad \frac{\Gamma \succ \Delta}{\Gamma|^{Fx}_{A(x)} \succ \Delta|^{Fx}_{A(x)}}\,[\mathit{Spec}^{Fx}_{A(x)}]$$

$$\mathcal{L}[{=}Df, \mathit{Spec}, \mathit{Cut}]$$

Equivalences

$$\frac{\Gamma \succ A(s), \Delta \qquad \Gamma, A(t) \succ \Delta}{s = t, \Gamma \succ \Delta}\,[{=}L]$$

$$\mathcal{L}[{=}Df, Spec, Cut] \quad = \quad \mathcal{L}[{=}L/R, Cut]$$

Equivalences

$$\frac{\Gamma \succ A(s), \Delta}{s = t, \Gamma \succ A(t), \Delta}\,[{=}L^{f}_{r}]$$

$$\begin{aligned} \mathcal{L}[{=}\mathit{Df}, \mathit{Spec}, \mathit{Cut}] &= \mathcal{L}[{=}L/R, \mathit{Cut}] \\ &= \mathcal{L}[{=}L^{f}_{r}/R, \mathit{Cut}] \end{aligned}$$

Equivalences

$$\frac{\Gamma \succ Fs, \Delta}{s = t, \Gamma \succ Ft, \Delta}[{=}L^p_r] \quad \frac{Fs, \Gamma \succ \Delta}{s = t, Ft, \Gamma \succ \Delta}[{=}L^p_l] \quad \frac{\Gamma, Fa \succ Fb, \Delta}{\Gamma \succ a = b, \Delta}[{=}R]$$

$$\begin{aligned} \mathcal{L}[{=}Df, Spec, Cut] &= \mathcal{L}[{=}L/R, Cut] \\ &= \mathcal{L}[{=}L^f_r/R, Cut] \\ &= \mathcal{L}[{=}L^p_r/L^p_l/R, Cut] \end{aligned}$$

Equivalences

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Each system gives you classical first-order predicate logic with identity.

Decomposing [$=L'$]: conjunctions

$$\dfrac{\dfrac{\dfrac{\Gamma \succ A(s) \land B(s), \Delta}{\Gamma \succ A(s), \Delta}[\land E]}{s = t, \Gamma \succ A(t), \Delta}[{=}L'] \qquad \dfrac{\dfrac{\Gamma \succ A(s) \land B(s), \Delta}{\Gamma \succ B(s), \Delta}[\land E]}{s = t, \Gamma \succ B(t), \Delta}[{=}L']}{s = t, \Gamma \succ A(t) \land B(t), \Delta}[\land R]$$

(Where the [$\land E$] is given by a *Cut* on $A(t) \land B(t) \succ A(t)$, or $A(t) \land B(t) \succ B(t)$.)

Decomposing $[{=}L']$: conjunctions

$$\dfrac{\dfrac{\dfrac{\Gamma \succ A(s) \land B(s), \Delta}{\Gamma \succ A(s), \Delta}\,[\land E]}{s = t, \Gamma \succ A(t), \Delta}\,[{=}L'] \qquad \dfrac{\dfrac{\Gamma \succ A(s) \land B(s), \Delta}{\Gamma \succ B(s), \Delta}\,[\land E]}{s = t, \Gamma \succ B(t), \Delta}\,[{=}L']}{s = t, \Gamma \succ A(t) \land B(t), \Delta}\,[\land R]$$

(Where the $[\land E]$ is given by a *Cut* on $A(t) \land B(t) \succ A(t)$, or $A(t) \land B(t) \succ B(t)$.)

$[{=}L']$ on *conjunctions* is given by $[{=}L']$ on its *conjuncts*.

But for *negation*...

$$\dfrac{\dfrac{\dfrac{\Gamma \succ \neg A(s), \Delta}{\Gamma, A(s) \succ \Delta}\,[\neg Df]}{s = t, A(t), \Gamma \succ \Delta}\,[{=}L' \text{ on the wrong side!}]}{s = t, \Gamma \succ \neg A(t), \Delta}\,[\neg Df]$$

SUBJUNCTIVE ALTERNATIVES

The Defining Rule for Identity in Hypersequents

$$\frac{\mathcal{H}[\Gamma, Fa \succ Fb, \Delta]}{\mathcal{H}[\Gamma \succ a = b, \Delta]}\ [{=}Df]$$

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How general is the F in $[{=}Df]$?

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How general is the F in $[{=}Df]$?

$$\frac{\Gamma \succ \Delta}{\Gamma|^{Fx}_{A(x)} \succ \Delta|^{Fx}_{A(x)}}\ [Spec^{Fx}_{A(x)}]$$

Let's allow $A(x)$ to contain $\Box$ and $\Diamond$.

The Defining Rule for Identity in Hypersequents

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How general is the F in $[{=}\mathit{Df}]$?

$$\frac{\Gamma \succ \Delta}{\Gamma|^{Fx}_{A(x)} \succ \Delta|^{Fx}_{A(x)}} \; [\mathit{Spec}^{Fx}_{A(x)}]$$

Let's allow $A(x)$ to contain $\Box$ and $\Diamond$.

Call this [*Modal Spec*].

$[{=}L^{\Box}]$

$$\frac{\mathcal{H}[\Gamma \succ A(s), \Delta]}{\mathcal{H}[s = t, \Gamma \succ A(t), \Delta]}\,[{=}L^{\Box}]$$

(Where $A(x)$ can contain $\Box$.)

Decomposing $[{=}L^{\Box}]$ with [*Modal Spec*]: necessities

$$\cfrac{\cfrac{\cfrac{\mathcal{H}[s = t, \Gamma \succ \Box A(s), \Delta]}{\mathcal{H}[s = t, \Gamma \succ \Delta \mid \quad \succ A(s)]}\,[\Box Df]}{\mathcal{H}[s = t, \Gamma \succ \Delta \mid \quad \succ A(t)]}\,[{=}L']}{\mathcal{H}[s = t, \Gamma \succ \Box A(t), \Delta]}\,[\Box Df]$$

Identity across Subjunctive Alternatives

$$\frac{\mathcal{H}[\Gamma \succ \Delta \mid \Gamma' \succ Fs, \Delta']}{\mathcal{H}[s = t, \Gamma \succ \Delta \mid \Gamma' \succ Ft, \Delta']} \; [{=}L^{p}_{|r}]$$

$$\frac{\mathcal{H}[\Gamma \succ \Delta \mid \Gamma', Fs \succ \Delta']}{\mathcal{H}[s = t, \Gamma \succ \Delta \mid \Gamma', Ft \succ \Delta']} \; [{=}L^{p}_{|l}]$$

Identity across Subjunctive Alternatives

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$$\frac{\mathcal{H}[\Gamma \succ \Delta \mid \Gamma', Fs \succ \Delta']}{\mathcal{H}[s = t, \Gamma \succ \Delta \mid \Gamma', Ft \succ \Delta']}\,[{=}L^{p}_{|l}]$$

This makes sense in planning contexts.

Equivalences

$$\mathcal{L}[{=}\mathit{Df}, \mathit{Modal\ Spec}, \mathit{Cut}] = \mathcal{L}[{=}L^{\Box}/R, \mathit{Cut}] = \mathcal{L}[{=}L^{\mathsf{p}}_{|\mathsf{l},\mathsf{r}}/R, \mathit{Cut}]$$

Equivalences

$$\mathcal{L}[{=}Df, \mathit{Modal\ Spec}, \mathit{Cut}] = \mathcal{L}[{=}L^{\Box}/R, \mathit{Cut}] = \mathcal{L}[{=}L^{p}_{|l,r}/R, \mathit{Cut}]$$

$$\dfrac{\dfrac{\dfrac{\succ \;\; | \;\; \mathsf{Fs} \succ \mathsf{Fs}}{\succ \;\; | \;\; \succ \mathsf{Fs} \supset \mathsf{Fs}}\,[\supset Df]}{\succ \Box(\mathsf{Fs} \supset \mathsf{Fs})}\,[\Box Df]}{\mathsf{s} = \mathsf{t} \succ \Box(\mathsf{Fs} \supset \mathsf{Ft})}\,[{=}L^{\Box}]$$

Equivalences (*cont.*)

$$\mathcal{L}[{=}\mathit{Df}, \mathit{Modal\ Spec}, \mathit{Cut}] = \mathcal{L}[{=}L^{\Box}/R, \mathit{Cut}] = \mathcal{L}[{=}L^{p}_{|l,r}/R, \mathit{Cut}]$$

$$\dfrac{\overline{\mathrm{s} = \mathrm{t} \succ \Box(\mathrm{Fs} \supset \mathrm{Ft})} \qquad \dfrac{\dfrac{\overline{\mathrm{Ft} \succ \mathrm{Ft}} \qquad \mathcal{H}[\Gamma \succ \Delta \mid \Gamma' \succ \mathrm{Fs}, \Delta']}{\mathcal{H}[\Gamma \succ \Delta \mid \Gamma', \mathrm{Fs} \supset \mathrm{Ft} \succ \mathrm{Ft}, \Delta']}\,[{\supset}L]}{\mathcal{H}[\Box(\mathrm{Fs} \supset \mathrm{Ft}), \Gamma \succ \Delta \mid \Gamma' \succ \mathrm{Ft}, \Delta']}\,[\Box L]}{\mathcal{H}[\mathrm{s} = \mathrm{t}, \Gamma \succ \Delta \mid \Gamma' \succ \mathrm{Ft}, \Delta']}\,[\mathit{Cut}]$$

Fully Refined Positions with Identity

$$\frac{\mathcal{H}[a = b, \Gamma \succ \Delta \mid \Gamma', Fa, Fb \succ \Delta']}{\mathcal{H}[a = b, \Gamma \succ \Delta \mid \Gamma', Fb \succ \Delta']}\ [{=}L^{p}_{|l}]$$

$$\frac{\mathcal{H}[a = b, \Gamma \succ \Delta \mid \Gamma' \succ Fa, Fb, \Delta']}{\mathcal{H}[a = b, \Gamma \succ \Delta \mid \Gamma' \succ Fb, \Delta']}\ [{=}L^{p}_{|r}]$$

$$\frac{\mathcal{H}[\Gamma, Fa \succ a = b, Fb, \Delta]}{\mathcal{H}[\Gamma \succ a = b, \Delta]}\ [{=}R^{*}, F \text{ new}]$$

Free Quantification and Contingent Existence

NON-COMMITAL: $$\frac{\mathcal{H}[\Gamma, Fs \succ Ft, \Delta]}{\mathcal{H}[\Gamma \succ s =_n t, \Delta]} \; [=_n Df]$$

Free Quantification and Contingent Existence

NON-COMMITAL: $$\frac{\mathcal{H}[\Gamma, Fs \succ Ft, \Delta]}{\mathcal{H}[\Gamma \succ s =_n t, \Delta]}\ [=_n Df]$$

COMMITTAL: $$\frac{\mathcal{H}[\Gamma \succ s, \Delta] \quad \mathcal{H}[\Gamma \succ t, \Delta] \quad \mathcal{H}[\Gamma, Fs \succ Ft, \Delta]}{\mathcal{H}[\Gamma \succ s =_c t, \Delta]}\ [=_c Df]$$

We'll work with non-committal identity for the rest of this talk.
(Committal identity $s =_c t$ is definable as $(s =_n t) \land s{\downarrow} \land t{\downarrow}$.)

Essential Properties

$$F^{\Box}t =_{df} \Box(t{\downarrow} \supset Ft)$$

Fully Refined Positions with Contingent Existence

$$[(\exists y)\Box(\forall x)(F^{\Box}x \equiv x = y) : \Box(\exists x)F^{\Box}x]$$

INDICATIVE ALTERNATIVES

Identity across Indicative Alternatives?

$$\frac{\mathcal{H}[\Gamma \succ \Delta \;\|\; \Gamma' \succ_{@} \mathrm{F}s, \Delta']}{\mathcal{H}[s = t, \Gamma \succ \Delta \;\|\; \Gamma' \succ_{@} \mathrm{F}t, \Delta']}\,[{=}L^{p}_{\|r}]$$

This makes *less* sense for identity.

Disagreeing over Identities

$$\frac{\mathcal{H}[\Gamma \succ \Delta \;\;\|\;\; \Gamma' \succ_{@} Fs, \Delta']}{\mathcal{H}[s = t, \Gamma \succ \Delta \;\;\|\;\; \Gamma' \succ_{@} Ft, \Delta']}\,[{=}L^{p}_{\|r}]$$

Disagreeing over Identities

$$\dfrac{\mathcal{H}[\Gamma \succ \Delta \ \parallel\ \Gamma' \succ_{@} Fs, \Delta']}{\mathcal{H}[s = t, \Gamma \succ \Delta \ \parallel\ \Gamma' \succ_{@} Ft, \Delta']}\,[{=}L^{p}_{\parallel r}]$$

$$\dfrac{\dfrac{\succ_{@} \ \parallel\ Fa \succ_{@} Fa}{a = b \succ_{@} \ \parallel\ Fa \succ_{@} Fb}\,[{=}L^{p}_{\parallel r}]}{a = b \succ_{@} \ \parallel\ \succ_{@} a = b}\,[{=}L]$$

Disagreeing over Identities

$$\frac{\mathcal{H}[\Gamma \succ \Delta \;\parallel\; \Gamma' \succ_{@} Fs, \Delta']}{\mathcal{H}[s = t, \Gamma \succ \Delta \;\parallel\; \Gamma' \succ_{@} Ft, \Delta']}\,[{=}L^{p}_{\parallel r}] \qquad \dfrac{\dfrac{\succ_{@} \;\parallel\; Fa \succ_{@} Fa}{a = b \succ_{@} \;\parallel\; Fa \succ_{@} Fb}\,[{=}L^{p}_{\parallel r}]}{a = b \succ_{@} \;\parallel\; \succ_{@} a = b}\,[{=}L]$$

If we admit $[{=}L^{p}_{\parallel r}]$, we rule out coherent disagreement over identities.

Fully Refined Positions with 2D Sequents

$$F^{[e]}t =_{df} [e](t{\downarrow} \supset Ft)$$

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How Intensional is Quantification?

$$[(\exists x)(a = x \wedge \neg[e](a = x)) :\ ]_{@}$$

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How Intensional is Quantification?

$$[(\exists x)(a = x \land \neg[e](a = x)) :\ ]_{@}$$

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$$[a, a = b : [e](a = b)]_{@}$$

$$[a, a = b :\ ]_{@} \ \| \ [\ : a = b]_{@}$$

$$[a, a = b :\ ]_{@} \ \| \ [Fa : a = b, Fb]_{@}$$

Intensional Identity

$$\frac{\mathcal{H}[\Gamma \succ \Delta \ \| \ \Gamma' \succ_{@} Fs, \Delta']}{\mathcal{H}[s \equiv t, \Gamma \succ \Delta \ \| \ \Gamma' \succ_{@} Ft, \Delta']}\,[{\equiv}L^{p}_{\|r}]$$

THE STATUS OF WORLDS SEMANTICS

What are worlds?

WORLDS IN A MODEL: Components of fully refined positions

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WORLDS: Components of fully refined positions starting from the *truth*.

What are worlds? and *possibilia*?

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POSSIBILIA IN A MODEL: Terms occuring positively
in some component in a fully refined position, identified by $=$.

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POSSIBILIA: Terms occuring positively in some
subjunctive alternative component of the starting location
in a fully refined position starting from the *truth*, identified by $=$.

What are worlds? and *possibilia*?

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POSSIBILIA: Terms occuring positively in some subjunctive alternative component of the starting location in a fully refined position starting from the *truth*, identified by $=$.

INDIVIDUAL CONCEPTS: Terms occuring in a fully refined position starting from the *truth*, identified by $\equiv$.

THANK YOU!

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