

Assertions, Denials Questions, Answers & the Common Ground

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To better understand the speech acts
of *assertion* and *denial*, their
relationship to *other* speech acts,
and connections between these speech acts
and logical notions, including
the classical sequent calculus.

I want to revisit some themes
(and revise some of the claims)
in my 2005 paper “Multiple Conclusions.”

The behaviour of two kinds of speech acts:

polar (yes/no) questions,
and *justification requests.*

Assertion and Denial

Polar Questions

Positions and Rules

Justification Requests

ASSERTION AND DENIAL

Multiple Conclusions

$$X \succ Y$$

Don't *assert* each member of X
and *deny* each member of Y.

In taking this approach...

... I was wading into a pre-existing literature.

It's been fruitful to think of assertion
as an act governed by *norms*.

Aim to say what is *true*!

For *me*: Production Norms

Aim to say what is *true*!

Only say what you *know*!

Be prepared to *back it up* when requested!

The hearer is entitled to re-assert.

For you: Consumption Norms

The hearer is entitled to re-assert.

You can refer back to the asserter
to *vouch for* the assertion.

To assert is to bid for the content asserted
to be added to the COMMON GROUND,
the body of information that
we (together) take for granted.

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assertion and denial are incompatible
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This does not help distinguish
denial from *retraction*, or
from other speech acts.

Let's address this issue...

... by examining polar questions,
and their answers,
in the light of our background
interest in assertion and its norms.

POLAR QUESTIONS

Is it the case that p?

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with its own norms.

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with its own norms.

It raises an *issue*.

There are two ways to settle the issue

Yes

There are two ways to settle the issue

Yes

No

*The two ways **clash***

If I say *yes* and you say *no*
to some polar question $p?$,
then we DISAGREE.

That is, we take *different* positions on p .

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to some polar question $p?$,
then we DISAGREE.

That is, we take *different* positions on p .

There is no *shared* position
incorporating both of our answers.

Other answers don't settle the issue

Other answers, like

Other answers don't settle the issue

Other answers, like
maybe

Other answers don't settle the issue

Other answers, like

maybe · I don't know

Other answers don't settle the issue

Other answers, like

maybe · I don't know · I think so

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count as *acceptable responses* to p?,

Other answers, like

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count as *acceptable responses* to p ?,
but they don't settle the issue of p .

Settling answers are assertions

A *yes* or a *no* to p ? counts as an assertion.

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(It's governed by the assertion norms we saw in the previous section.)

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Presumably $\neg p$.

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This way, we can distinguish practices
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and those with more limited expressive resources.

(Nothing important hangs on this distinction.)

$[X : Y]$

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- We have ruled *in* everything in X, the POSITIVE COMMON GROUND.

$[X : Y]$

- We have ruled *in* everything in X, the POSITIVE COMMON GROUND.
- We have ruled *out* everything in Y, the NEGATIVE COMMON GROUND.

“No” means *No*!

Denial and Retraction

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ELOISE: No, he is in the kitchen.

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ABELARD: Astralabe is in the study.

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ELOISE: Well, he is *either* in the kitchen *or* the study.

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ABELARD: Is Astralabe in the study?

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ABELARD: Astralabe is in the study.

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PARTIAL ANSWER

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WEAK DENIAL

ABELARD: Is Astralabe in the study?

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STRONG DENIAL

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PARTIAL ANSWER

“No” *means No*

These “no”s always mark an objection.

But an objection to *what*?

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- To *strongly deny* p is to bid to add p to the *negative common ground*.
- To *weakly deny* p is to *block* the addition of p to the *positive common ground*, or to bid for its *retraction* if it is already there.

Strong and Weak Denial, and the Common Ground

- Strong *or* weak denials of p are appropriate responses to assertion of p , because the assertion of p is a bid to add p to the positive common ground.

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Strong and Weak Denial, and the Common Ground

- Strong *or* weak denials of p are appropriate responses to assertion of p , because the assertion of p is a bid to add p to the positive common ground.
- While a strong denial of p is one way to settle the question $p?$ — this is generally an appropriate response.
- A weak denial of p is not generally an appropriate response to the polar question $p?$, as the polar question does not place p in the positive common ground, and the question is inappropriate if p is already in the positive common ground, so there is no p to block or retract.

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*Strong and Weak Denials, and Strong and Weak **Assertions***

- STRONG DENIAL: add to the negative common ground.
- STRONG ASSERTION: add to the positive common ground.
- WEAK DENIAL: retract (or block) from the positive common ground.
- WEAK ASSERTION: retract (or block) from the negative common ground. — “Perhaps p.”

That's one way to understand the relationship
between assertion and denial, and how to
distinguish strong denial from other negative speech acts.

Any position $[X, A : A, Y]$
in which A has been
ruled *in* and ruled *out*,
is out of bounds.

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If $X \not\succeq Y$ then $[X : Y]$ is *available*.

*A Word on **Cut***

$$\frac{X \succ A, Y \quad X, A \succ Y}{X \succ Y} \textit{Cut}$$

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In any available position $[X : Y]$, if one way to settle $A?$ is *not* available, then the other way to settle it *is* available.

POSITIONS AND RULES

Defining Rules

$$\frac{\frac{X, A, B \succ Y}{\quad}}{X, A \wedge B \succ Y} \wedge Df$$

$$\frac{\frac{X \succ A, B, Y}{\quad}}{X \succ A \vee B, Y} \vee Df$$

$$\frac{\frac{X \succ A, Y}{\quad}}{X, \neg A \succ Y} \sim Df$$

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These are kinds of *definitions*, showing how to treat assertions or denials of the *defined* concept in terms of the assertions or denials of their components.

Derivations

$$\frac{\neg p \succ \neg p}{\succ p, \neg p} \neg Df$$
$$\frac{\succ p, \neg p}{\succ p \vee \neg p} \vee Df$$

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$$\frac{p, \neg p \succ}{p \wedge \neg p \succ} \wedge Df$$

Derivations

$$\begin{array}{c}
 \neg p \succ \neg p \\
 \hline
 \succ p, \neg p \quad \neg Df \\
 \hline
 \succ p \vee \neg p \quad \vee Df
 \end{array}
 \qquad
 \begin{array}{c}
 p \succ p \\
 \hline
 \succ p, \neg p \succ \quad \neg Df \\
 \hline
 p \wedge \neg p \succ \quad \wedge Df
 \end{array}$$

$$\begin{array}{c}
 \frac{p, q \vee r \succ p \wedge q, q \vee r}{p, q \vee r \succ p \wedge q, r, q} \vee Df \qquad \frac{p \wedge q, q \vee r \succ p \wedge q, r}{q, p, q \vee r \succ p \wedge q, r} \wedge Df \\
 \hline
 \frac{\qquad \qquad \qquad}{p, q \vee r \succ p \wedge q, r} Cut \\
 \hline
 \frac{p, q \vee r \succ p \wedge q, r}{p, q \vee r \succ (p \wedge q) \vee r} \vee Df \\
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- They don't have the same *shape* as proofs.
- (Where is the *conclusion* in $p \vee q \succ p, q$?)
- A endsequent $X \succ A$ doesn't tell you to *infer* A from X — it merely tells you to not assert all members of X and deny A .

Let's make this problem *sharp*

“Well, now, let's take a little bit of the argument in that First Proposition—just *two* steps, and the conclusion drawn from them. Kindly enter them in your note-book. And in order to refer to them conveniently, let's call them *A*, *B*, and *Z*:—

(*A*) Things that are equal to the same are equal to each other.

(*B*) The two sides of this Triangle are things that are equal to the same.

(*Z*) The two sides of this Triangle are equal to each other.

Readers of Euclid will grant, I suppose, that *Z* follows logically from *A* and *B*, so that any one who accepts *A* and *B* as true, *must* accept *Z* as true?”

“Undoubtedly! The youngest child in a High School—as soon as High Schools are invented, which will not be till some two thousand years later—will grant *that*.”

“And if some reader had *not* yet accepted *A* and *B* as true, he might still accept the *sequence* as a *valid* one, I suppose?”

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“And if some reader had *not* yet accepted *A* and *B* as true, he might still accept the *sequence* as a *valid* one, I suppose?”

The Tortoise never asserts *A* and $A \supset Z$ while denying *Z*,
but he doesn't accept *A* and $A \supset Z$ as a *reason* for *Z*.

JUSTIFICATION REQUESTS

What is a justification request?

ABELARD: Astralabe is in the study.

ELOISE: *Really?*

ABELARD: I saw him there five minutes ago.

ELOISE: OK.

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ELOISE: *Are you sure?* He's been in the kitchen with me for the last half hour.

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ELOISE: *Are you sure?* He's been in the kitchen with me for the last half hour.

ABELARD: Astralabe is in the study.

ELOISE: *Really?*

ABELARD: I saw him there five minutes ago.

ELOISE: Yes, but I saw him in the kitchen two minutes ago.

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Justification Requests and Norms for Assertion

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That's a JUSTIFICATION REQUEST.

What is a justification request?

A justification request for a strong assertion [or strong denial] is an attempt to block the addition to the common ground, until a *reason* is given.

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A justification request for a strong assertion [or strong denial] is an attempt to block the addition to the common ground, until a *reason* is given.

This reason is another assertion [or denial] which must be *granted*, (added to the common ground) in order for the request to be met.

What is a justification request?

A justification request for a strong assertion [or strong denial] is an attempt to block the addition to the common ground, until a *reason* is given.

This reason is another assertion [or denial] which must be *granted*, (added to the common ground) in order for the request to be met.

Granting the given reason is *necessary* but not *sufficient* for satisfying the justification request.

Definitions and Justification Requests

ACHILLES So ... this is an *equilateral* triangle.

TORTOISE I'm sorry, I don't follow, my heroic friend. I've not heard that word before: what does '*equilateral*' mean?

ACHILLES Oh, that's easy to explain. '*Equilateral*' means having sides of the same length. An *equilateral* triangle is a triangle with all three sides the same length.

TORTOISE OK. That sounds good. You may continue with your reasoning.

ACHILLES Well, as I was saying, the sides of this triangle are all one cubit in length, so it is an equilateral triangle.

TORTOISE Perhaps you will forgive me, Achilles, but I still don't follow. I grant to you that the sides of this triangle all have the same length. I fail to see, however, that it *follows* that it is an equilateral triangle. Could you explain why it is?

Definitions and Justification Requests

If I accept the definition $A =_{df} B$,
then I should accept granting A as meeting
a justification request for the assertion of B
and ruling out A as meeting a justification
request for B 's denial and *vice versa*.

A failure to accept this is a sign
that I have not mastered the definition.

What goes for a definition of the form $A =_{df} B$
can also go for defining rules:

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It is a mistake to grant A and grant B
and to look for something more to discharge
a justification request for an assertion of $A \wedge B$,
if you take $\wedge Df$ as a *definition*.

Justification Requests and Defining Rules

$$\frac{X, A \succ B, Y}{X \succ A \supset B, Y} \supset Df$$

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It is a mistake to rule A in and rule B out
and to look for something more to discharge
a justification request for a denial of $A \supset B$
if you accept $\supset Df$ as a definition.

Justification Requests, Defining Rules and Derivations

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Consider this *focussed* derivation:

$$\frac{A \supset Z \succ A \supset Z}{A \supset Z, A \succ Z} \supset Df$$

- Read the *premise* as telling us that in a position in which $A \supset Z$ is already ruled in, we have an answer to the justification request for $A \supset Z$'s assertion.

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- Read the *premise* as telling us that in a position in which $A \supset Z$ is already ruled in, we have an answer to the justification request for $A \supset Z$'s assertion.
- Then applying $\supset Df$ we see why we have an answer to the request concerning Z 's assertion, in a context in which $A \supset Z$ and A have both been ruled in. (In granting $A \supset Z$ and A we have settled Z positively. Its denial is ruled out, since to assert A and deny Z amounts to denying $A \supset Z$.)

SLOGAN: A derivation of $X \succ A$, Y shows us how to meet a justification request for the assertion of A in any available position extending $[X : Y]$.

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A derivation of X , $A \succ Y$ shows us how to meet a justification request for the denial of A in any available position extending $[X : Y]$.

Focussed Structural Rules

$$X, A \succ \boxed{A}, Y \quad X, \boxed{A} \succ A, Y$$

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$$\frac{X, A, \boxed{A} \succ Y}{X, \boxed{A} \succ Y} \textit{W}$$

$$\frac{X \succ A, \boxed{A}, Y}{X \succ \boxed{A}, Y} \textit{W}$$

Swap

$$\frac{\frac{X \succ A, B, Y \quad X, A \succ A, B, Y}{X \succ A, B, B, Y} \text{Cut}}{X \succ A, B, Y} W$$

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Focussed Defining Rules

$$\frac{X, A, B \succ Y}{X, A \wedge B \succ Y} \wedge Df$$

$$\frac{X, A, B \succ Y}{X, A \wedge B \succ Y} \wedge Df$$

$$\frac{X \succ A, B, Y}{X \succ A \vee B, Y} \vee Df$$

$$\frac{X \succ A, B, Y}{X \succ A \vee B, Y} \vee Df$$

$$\frac{X \succ A, Y}{X, \neg A \succ Y} \neg Df$$

$$\frac{X, A \succ B, Y}{X \succ A \supset B, Y} \supset Df$$

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Focussed Derivations

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$$\frac{p, \neg p \succ}{p \wedge \neg p \succ} \wedge Df \quad \frac{\succ p, \neg p}{\succ p \vee \neg p} \vee Df$$

Proof and Supposition

$$\frac{X, A \succ B, Y}{X \succ A \supset B, Y} \supset Df \qquad \frac{X, A \succ B, Y}{X \succ A \supset B, Y} \supset Df$$

To prove $A \supset B$, *rule A in* (suppose it) and prove B.

Or, *rule B out* (suppose it), and refute A.

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- The making of an *inference* is a (possibly preemptive) answer to a justification request.
- A derivation of a sequent $X \succ A, Y [X, A \succ Y]$ can be transformed into a *procedure* for meeting a justification request for an assertion of A [denial of A] in any available position, appealing only what is granted in $[X : Y]$, and to the defining rules used in that derivation.

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- The *negative* view of the bounds is seen in the clash between assertion and denial, and the *positive* view is found in the answers we can give to justification requests.
- Adopting *defining rules* is one way to be *very* precise about the norms governing the concepts so defined, combining *safety*, *univocity* and *expressive power*.

THANK YOU!