

# *Assertions, Denials Questions, Answers & the Common Ground*

*Greg Restall*



THE UNIVERSITY OF  
MELBOURNE

MCMP SEMINAR / 13 JUNE 2019

To better understand the speech acts  
of *assertion* and *denial*, their  
relationship to *other* speech acts,  
and connections between these speech acts  
and logical notions, including  
the classical sequent calculus.

I want to revisit some themes  
(and revise some of the claims)  
in my 2005 paper “Multiple Conclusions.”

The behaviour of two kinds of speech acts:

*polar (yes/no) questions,*  
and *justification requests.*

Assertion and Denial

Polar Questions

Positions and Rules

Justification Requests

# ASSERTION AND DENIAL

## *Multiple Conclusions*

---

$$X \succ Y$$

Don't *assert* each member of X  
and *deny* each member of Y.

## Defining Rules for Logical Concepts

$$\frac{\frac{X, A, B \succ Y}{X, A \wedge B \succ Y}}{\wedge Df} \quad \frac{\frac{X \succ A, B, Y}{X \succ A \vee B, Y}}{\vee Df} \quad \frac{\frac{X \succ A, Y}{X, \neg A \succ Y}}{\neg Df} \quad \frac{\frac{X, A \succ B, Y}{X \succ A \supset B, Y}}{\supset Df}$$

$$\frac{\frac{X \succ A(n), Y}{X \succ \forall x A(x), Y}}{\forall Df} \quad \frac{\frac{X, A(n) \succ Y}{X, \exists x A(x) \succ Y}}{\exists Df} \quad \frac{\frac{X, Fa \succ Fb, Y \quad X, Fb \succ Fa, Y}{X \succ a = b, Y}}{=Df}$$

*Terms & conditions:* the singular term  $n$  (in  $\forall/\exists Df$ ) and the predicate  $F$  (in  $=Df$ ) do not appear below the line in those rules.



*In taking this approach...*

---

... I was wading into a pre-existing literature about assertion. A *large* literature.

It's been fruitful to think of assertion  
as an act governed by *norms*.

Aim to say what is *true*!

Aim to say what is *true*!

Only say what you *know*!

## For *me*: Production Norms

---

Aim to say what is *true*!

Only say what you *know*!

Be prepared to *back it up* when requested!

The hearer is entitled to re-assert.

The hearer is entitled to re-assert.

You can refer back to the asserter  
to *vouch for* the assertion.

To assert is to bid for the content asserted  
to be added to the COMMON GROUND,  
the body of information that  
we (together) take for granted.



## Stalnaker on Common Ground

*To presuppose something is to take it for granted, or at least to act as if one takes it for granted, as background information as common ground among the participants in the conversation. What is most distinctive about this propositional attitude is that it is a social or public attitude: one presupposes that  $\phi$  only if one presupposes that others presuppose it as well.*

— “Common Ground” *LEP* (2002)

## *What is the relationship between Assertion and Denial?*

---

In “Multiple Conclusions”, I said  
little beyond the claim that  
assertion and denial are incompatible

## *What is the relationship between Assertion and Denial?*

---

In “Multiple Conclusions”, I said  
little beyond the claim that  
assertion and denial are incompatible  
(in some sense).

## *What is the relationship between Assertion and Denial?*

---

In “Multiple Conclusions”, I said  
little beyond the claim that  
assertion and denial are incompatible  
(in some sense).

This does not help distinguish  
*denial* from *retraction*, or  
from other speech acts.

*Let's address this issue...*

---

... by examining polar questions,  
and their answers,  
in the light of our background  
interest in assertion and its norms.

# POLAR QUESTIONS

*Is it the case that p?*

---

This is a distinct speech act  
with its own norms.

*Is it the case that p?*

---

This is a distinct speech act  
with its own norms.

It raises an *issue*.



*There are two ways to settle the issue*

---

*Yes*

*There are two ways to settle the issue*

---

*Yes*

*No*

## *The two ways **clash***

---

If I say *yes* and you say *no*  
to some polar question  $p$ ?,  
then we DISAGREE.

That is, we take *different* positions on  $p$ .

## *The two ways **clash***

---

If I say *yes* and you say *no*  
to some polar question  $p?$ ,  
then we DISAGREE.

That is, we take *different* positions on  $p$ .

There is no *shared* position  
incorporating both of our answers.

Other responses, like

*Other responses don't settle the issue*

---

Other responses, like  
*maybe*

*Other responses don't settle the issue*

---

Other responses, like

*maybe · I don't know*

## *Other responses don't settle the issue*

---

Other responses, like

*maybe · I don't know · I think so*



## *Other responses don't settle the issue*

---

Other responses, like

*maybe · I don't know · I think so*

are acceptable responses to p?,

## *Other responses don't settle the issue*

---

Other responses, like

*maybe · I don't know · I think so*

are acceptable responses to  $p$ ?,  
but they don't answer the question.  
They don't settle the issue of  $p$ .

## *Settling answers are assertions*

---

A *yes* or a *no* to  $p$ ? counts as an assertion.

A *yes* or a *no* to  $p$ ? counts as an assertion.

(It's governed by all the assertion norms we've already seen.)

*What does a “no” to  $p$ ? **assert**?*

---

Presumably  $\neg p$ .

## *What does a “no” to p? assert?*

---

Presumably  $\neg p$ .

However, I prefer to think of  
a *yes* to p? as ruling p *in*,  
and a *no* to p? as ruling p *out*.

## What does a “no” to $p$ ? *assert*?

Presumably  $\neg p$ .

However, I prefer to think of  
a *yes* to  $p$ ? as ruling  $p$  *in*,  
and a *no* to  $p$ ? as ruling  $p$  *out*.

This way, we can distinguish practices  
where the *issues* are closed under negation  
and those with more limited expressive resources.

## *What does a “no” to p? assert?*

Presumably  $\neg p$ .

However, I prefer to think of  
a *yes* to p? as ruling p *in*,  
and a *no* to p? as ruling p *out*.

This way, we can distinguish practices  
where the *issues* are closed under negation  
and those with more limited expressive resources.

(Nothing important hangs on this distinction.)



$[X : Y]$

$[X : Y]$

- We have ruled *in* everything in X, the POSITIVE COMMON GROUND.

$$[X : Y]$$

- We have ruled *in* everything in X, the POSITIVE COMMON GROUND.
- We have ruled *out* everything in Y, the NEGATIVE COMMON GROUND.

## *Denial and Retraction*

---

ABELARD: Astralabe is in the study.

ELOISE: No, he is in the kitchen.

## *Denial and Retraction*

---

ABELARD: Astralabe is in the study.

ELOISE: No, he is in the kitchen.

ABELARD: Astralabe is in the study.

ELOISE: No, he is *either* in the  
kitchen *or* the study.

## *Denial and Retraction*

---

ABELARD: Astralabe is in the study.

ELOISE: No, he is in the kitchen.

ABELARD: Is Astralabe in the study?

ELOISE: No, he is in the kitchen.

ABELARD: Astralabe is in the study.

ELOISE: No, he is *either* in the  
kitchen *or* the study.

## *Denial and Retraction*

---

ABELARD: Astralabe is in the study.

ELOISE: No, he is in the kitchen.

ABELARD: Is Astralabe in the study?

ELOISE: No, he is in the kitchen.

ABELARD: Astralabe is in the study.

ELOISE: No, he is *either* in the  
kitchen *or* the study.

ABELARD: Is Astralabe in the study?

ELOISE: \*No, he is *either* in the  
kitchen *or* the study.

## *Denial and Retraction*

ABELARD: Astralabe is in the study.

ELOISE: No, he is in the kitchen.

ABELARD: Is Astralabe in the study?

ELOISE: No, he is in the kitchen.

ABELARD: Astralabe is in the study.

ELOISE: No, he is *either* in the kitchen *or* the study.

ABELARD: Is Astralabe in the study?

ELOISE: \*No, he is *either* in the kitchen *or* the study.

INAPPROPRIATE



## Denial and Retraction

ABELARD: Astralabe is in the study.

ELOISE: No, he is in the kitchen.

ABELARD: Is Astralabe in the study?

ELOISE: No, he is in the kitchen.

ABELARD: Astralabe is in the study.

ELOISE: No, he is *either* in the kitchen *or* the study.

ABELARD: Is Astralabe in the study?

ELOISE: \*No, he is *either* in the kitchen *or* the study.

INAPPROPRIATE

ELOISE: Maybe. He's *either* in the kitchen *or* the study.

## *Denial and Retraction*

ABELARD: Astralabe is in the study.

ELOISE: No, he is in the kitchen.

ABELARD: Is Astralabe in the study?

ELOISE: No, he is in the kitchen.

### **STRONG DENIAL**

ABELARD: Astralabe is in the study.

ELOISE: No, he is *either* in the kitchen *or* the study.

ABELARD: Is Astralabe in the study?

ELOISE: \*No, he is *either* in the kitchen *or* the study.

### **INAPPROPRIATE**

ELOISE: Maybe. He's *either* in the kitchen *or* the study.

## Denial and Retraction

ABELARD: Astralabe is in the study.

ELOISE: No, he is in the kitchen.

STRONG DENIAL

ABELARD: Astralabe is in the study.

ELOISE: No, he is *either* in the kitchen *or* the study.

ABELARD: Is Astralabe in the study?

ELOISE: No, he is in the kitchen.

STRONG DENIAL

ABELARD: Is Astralabe in the study?

ELOISE: \*No, he is *either* in the kitchen *or* the study.

INAPPROPRIATE

ELOISE: Maybe. He's *either* in the kitchen *or* the study.

## Denial and Retraction

ABELARD: Astralabe is in the study.

ELOISE: No, he is in the kitchen.

STRONG DENIAL

ABELARD: Astralabe is in the study.

ELOISE: No, he is *either* in the kitchen *or* the study.

ABELARD: Is Astralabe in the study?

ELOISE: No, he is in the kitchen.

STRONG DENIAL

ABELARD: Is Astralabe in the study?

ELOISE: \*No, he is *either* in the kitchen *or* the study.

INAPPROPRIATE

ELOISE: Maybe. He's *either* in the kitchen *or* the study.

PARTIAL ANSWER

## Denial and Retraction

ABELARD: Astralabe is in the study.

ELOISE: No, he is in the kitchen.

STRONG DENIAL

ABELARD: Astralabe is in the study.

ELOISE: No, he is *either* in the kitchen *or* the study.

WEAK DENIAL

ABELARD: Is Astralabe in the study?

ELOISE: No, he is in the kitchen.

STRONG DENIAL

ABELARD: Is Astralabe in the study?

ELOISE: \*No, he is *either* in the kitchen *or* the study.

INAPPROPRIATE

ELOISE: Maybe. He's *either* in the kitchen *or* the study.

PARTIAL ANSWER

## *Strong and Weak Denial*

---

- To *strongly deny*  $p$  is to bid to add  $p$  to the *negative common ground*.

## *Strong and Weak Denial*

- To *strongly deny*  $p$  is to bid to add  $p$  to the *negative common ground*.
- To *weakly deny*  $p$  is to *block* the addition of  $p$  to the *positive common ground*, or to bid for its *retraction* if it is already there.

## *Strong and Weak Denial, and the Common Ground*

---

- Strong *or* weak denials of  $p$  are appropriate responses to an assertion of  $p$ , because the assertion of  $p$  is a bid to add  $p$  to the positive common ground.



## *Strong and Weak Denial, and the Common Ground*

- Strong *or* weak denials of  $p$  are appropriate responses to an assertion of  $p$ , because the assertion of  $p$  is a bid to add  $p$  to the positive common ground.
- A strong denial of  $p$  is one way to settle the question  $p$ ? — this is generally an appropriate response.

## *Strong and Weak Denial, and the Common Ground*

- Strong *or* weak denials of  $p$  are appropriate responses to an assertion of  $p$ , because the assertion of  $p$  is a bid to add  $p$  to the positive common ground.
- A strong denial of  $p$  is one way to settle the question  $p?$  — this is generally an appropriate response.
- A weak denial of  $p$  is not generally an appropriate response to the polar question  $p?$ , as the polar question does not place  $p$  in the positive common ground, and the question is inappropriate if  $p$  is already in the positive common ground, so there is no  $p$  to block or retract.

## *Strong and Weak Denials, and Strong and Weak **Assertions***

---

- **STRONG DENIAL:** add to the negative common ground.

## *Strong and Weak Denials, and Strong and Weak **Assertions***

---

- **STRONG DENIAL:** add to the negative common ground.
- **STRONG ASSERTION:** add to the positive common ground.

## *Strong and Weak Denials, and Strong and Weak **Assertions***

---

- **STRONG DENIAL:** add to the negative common ground.
- **STRONG ASSERTION:** add to the positive common ground.
- **WEAK DENIAL:** retract (or block) from the positive common ground.

## *Strong and Weak Denials, and Strong and Weak **Assertions***

- **STRONG DENIAL**: add to the negative common ground.
- **STRONG ASSERTION**: add to the positive common ground.
- **WEAK DENIAL**: retract (or block) from the positive common ground.
- **WEAK ASSERTION**: retract (or block) from the negative common ground.

## *Strong and Weak Denials, and Strong and Weak **Assertions***

- STRONG DENIAL: add to the negative common ground.
- STRONG ASSERTION: add to the positive common ground.
- WEAK DENIAL: retract (or block) from the positive common ground.
- WEAK ASSERTION: retract (or block) from the negative common ground. — “Perhaps p.”

*That's* one way to understand the relationship  
between assertion and denial, and how to  
distinguish strong denial  
from other negative speech acts.



## *One Consequence*

---

The common ground  
(what we, together, *take for granted*)  
seems to be *very* finely grained.

## One Consequence

The common ground  
(what we, together, *take for granted*)  
seems to be *very* finely grained.

*Abelard is being tutored by  
Eloise in geometry. He is  
reasoning about a triangle  
with interior angles of 40, 60  
and 80 degrees. He adds up  
the angles, and notices that  
they sum to 180 degrees ...*

## One Consequence

The common ground  
(what we, together, *take for granted*)  
seems to be *very* finely grained.

*Abelard is being tutored by  
Eloise in geometry. He is  
reasoning about a triangle  
with interior angles of 40, 60  
and 80 degrees. He adds up  
the angles, and notices that  
they sum to 180 degrees ...*

ABELARD: The interior angles of  
triangles add up to 180 degrees.

## One Consequence

The common ground  
(what we, together, *take for granted*)  
seems to be *very* finely grained.

*Abelard is being tutored by  
Eloise in geometry. He is  
reasoning about a triangle  
with interior angles of 40, 60  
and 80 degrees. He adds up  
the angles, and notices that  
they sum to 180 degrees...*

ABELARD: The interior angles of  
triangles add up to 180 degrees.

ELOISE: No. The interior angles of *this*  
triangle add up to 180 degrees. Can you  
prove the general case?

## One Consequence

The common ground  
(what we, together, *take for granted*)  
seems to be *very* finely grained.

*Abelard is being tutored by  
Eloise in geometry. He is  
reasoning about a triangle  
with interior angles of 40, 60  
and 80 degrees. He adds up  
the angles, and notices that  
they sum to 180 degrees...*

ABELARD: The interior angles of  
triangles add up to 180 degrees.

ELOISE: No. The interior angles of *this*  
triangle add up to 180 degrees. Can you  
prove the general case?

Eloise here seems to block from the common ground (weakly deny)  
a logical consequence of claims in the common ground (the axioms of geometry),  
for the same general reason as for other weak denials.

Any position  $[X, A : A, Y]$   
in which  $A$  has been  
asserted and denied,  
is out of bounds.

Any position  $[X, A : A, Y]$   
in which  $A$  has been  
asserted and denied,  
is out of bounds.

$$X, A \succ A, Y$$

Any position  $[X, A : A, Y]$   
in which  $A$  has been  
asserted and denied,  
is out of bounds.

$$X, A \succ A, Y$$

If  $X \not\succeq Y$  then  $[X : Y]$  is *available*.



## *A Word on **Cut***

---

$$\frac{X \succ A, Y \quad X, A \succ Y}{X \succ Y} \textit{Cut}$$

## *A Word on **Cut***

$$\frac{X \succ A, Y \quad X, A \succ Y}{X \succ Y} \text{Cut}$$

In any available position  $[X : Y]$ , if one way to settle  $A?$  is *not* available, then the other way to settle it *is* available.

# POSITIONS AND RULES

## Defining Rules

$$\frac{X, A, B \succ Y}{\frac{}{X, A \wedge B \succ Y}} \wedge^{Df}$$

$$\frac{X \succ A, B, Y}{\frac{}{X \succ A \vee B, Y}} \vee^{Df}$$

$$\frac{X \succ A, Y}{\frac{}{X, \neg A \succ Y}} \neg^{Df}$$

$$\frac{X, A \succ B, Y}{\frac{}{X \succ A \supset B, Y}} \supset^{Df}$$

## Defining Rules

$$\frac{\frac{X, A, B \succ Y}{X, A \wedge B \succ Y}}{\wedge Df}$$

$$\frac{\frac{X \succ A, B, Y}{X \succ A \vee B, Y}}{\vee Df}$$

$$\frac{\frac{X \succ A, Y}{X, \neg A \succ Y}}{\neg Df}$$

$$\frac{\frac{X, A \succ B, Y}{X \succ A \supset B, Y}}{\supset Df}$$

These are kinds of *definitions*, showing how to treat assertions or denials of the *defined* concept in terms of the assertions or denials of their components.

## *Derivations*

$$\frac{\neg p \succ \neg p}{\succ p, \neg p} \neg Df$$
$$\frac{\succ p, \neg p}{\succ p \vee \neg p} \vee Df$$

## *Derivations*

$$\frac{\neg p \succ \neg p}{\succ p, \neg p} \neg Df$$
$$\frac{\succ p, \neg p}{\succ p \vee \neg p} \vee Df$$

$$\frac{p \succ p}{p, \neg p \succ} \neg Df$$
$$\frac{p, \neg p \succ}{p \wedge \neg p \succ} \wedge Df$$

## Derivations

$$\begin{array}{c} \neg p \succ \neg p \\ \hline \succ p, \neg p \quad \neg Df \\ \hline \succ p \vee \neg p \quad \vee Df \end{array} \qquad \begin{array}{c} p \succ p \\ \hline \succ p, \neg p \succ \quad \neg Df \\ \hline p \wedge \neg p \succ \quad \wedge Df \end{array}$$

$$\begin{array}{c} \frac{p, q \vee r \succ p \wedge q, q \vee r}{p, q \vee r \succ p \wedge q, r, q} \vee Df \qquad \frac{p \wedge q, q \vee r \succ p \wedge q, r}{q, p, q \vee r \succ p \wedge q, r} \wedge Df \\ \hline \qquad \qquad \qquad \frac{p, q \vee r \succ p \wedge q, r}{p, q \vee r \succ (p \wedge q) \vee r} \vee Df \\ \hline \qquad \qquad \qquad \frac{p, q \vee r \succ (p \wedge q) \vee r}{p \wedge (q \vee r) \succ (p \wedge q) \vee r} \wedge Df \end{array} \quad \text{Cut}$$



## *Sequent Derivations aren't exactly **Proofs***

---

- They don't have the same *shape* as proofs.

## *Sequent Derivations aren't exactly **Proofs***

- They don't have the same *shape* as proofs.
- (Where is the *conclusion* in  $p \vee q \succ p, q$ ?)

## *Sequent Derivations aren't exactly **Proofs***

- They don't have the same *shape* as proofs.
- (Where is the *conclusion* in  $p \vee q \succ p, q$ ?)
- A endsequent  $X \succ A$  doesn't tell you to *infer A from X* — it merely tells you to not assert all members of  $X$  and deny  $A$ .

## Let's make this problem *sharp*

“Well, now, let's take a little bit of the argument in that First Proposition—just *two* steps, and the conclusion drawn from them. Kindly enter them in your note-book. And in order to refer to them conveniently, let's call them *A*, *B*, and *Z*:—

(*A*) Things that are equal to the same are equal to each other.

(*B*) The two sides of this Triangle are things that are equal to the same.

(*Z*) The two sides of this Triangle are equal to each other.

Readers of Euclid will grant, I suppose, that *Z* follows logically from *A* and *B*, so that any one who accepts *A* and *B* as true, *must* accept *Z* as true?”

“Undoubtedly! The youngest child in a High School—as soon as High Schools are invented, which will not be till some two thousand years later—will grant *that*.”

“And if some reader had *not* yet accepted *A* and *B* as true, he might still accept the *sequence* as a *valid* one, I suppose?”

## Let's make this problem *sharp*

“Well, now, let's take a little bit of the argument in that First Proposition—just *two* steps, and the conclusion drawn from them. Kindly enter them in your note-book. And in order to refer to them conveniently, let's call them *A*, *B*, and *Z*:—

(*A*) Things that are equal to the same are equal to each other.

(*B*) The two sides of this Triangle are things that are equal to the same.

(*Z*) The two sides of this Triangle are equal to each other.

Readers of Euclid will grant, I suppose, that *Z* follows logically from *A* and *B*, so that any one who accepts *A* and *B* as true, *must* accept *Z* as true?”

“Undoubtedly! The youngest child in a High School—as soon as High Schools are invented, which will not be till some two thousand years later—will grant *that*.”

“And if some reader had *not* yet accepted *A* and *B* as true, he might still accept the *sequence* as a *valid* one, I suppose?”

The Tortoise never asserts *A* and  $A \supset Z$  while denying *Z*,  
but he doesn't accept *A* and  $A \supset Z$  as a *reason* for *Z*.

# JUSTIFICATION REQUESTS

## *What is a justification request?*

---

ABELARD: Astralabe is in the kitchen.

ELOISE: *Really?*

ABELARD: I saw him there five minutes ago.

ELOISE: OK.

## *What is a justification request?*

ABELARD: Astralabe is in the kitchen.

ELOISE: *Really?*

ABELARD: I saw him there five minutes ago.

ELOISE: OK.

ABELARD: Astralabe is in the kitchen.

ELOISE: *Really?*

ABELARD: I saw him there five minutes ago.

ELOISE: *Are you sure?* He's been in the study with me for the last half hour.



## *What is a justification request?*

ABELARD: Astralabe is in the kitchen.

ELOISE: *Really?*

ABELARD: I saw him there five minutes ago.

ELOISE: OK.

ABELARD: Astralabe is in the kitchen.

ELOISE: *Really?*

ABELARD: I saw him there five minutes ago.

ELOISE: *Are you sure?* He's been in the study with me for the last half hour.

ABELARD: Astralabe is in the kitchen.

ELOISE: *Really?*

ABELARD: I saw him there five minutes ago.

ELOISE: Yes, but he was in the study two minutes ago.

We should *expect* the need for justification requests  
given the commitments and entitlements  
involved in assertion.

We should *expect* the need for justification requests  
given the commitments and entitlements  
involved in assertion.

If I give you permission to  
ask *me* to vouch for my assertion  
you should to be able to call me on it.

We should *expect* the need for justification requests  
given the commitments and entitlements  
involved in assertion.

If I give you permission to  
ask *me* to vouch for my assertion  
you should to be able to call me on it.

That's a JUSTIFICATION REQUEST.

## *What is a justification request?*

---

A justification request for a strong assertion [or strong denial] is an attempt to block the addition to the common ground, until a *reason* is given.

## *What is a justification request?*

A justification request for a strong assertion [or strong denial] is an attempt to block the addition to the common ground, until a *reason* is given.

This reason is another assertion [or denial] which must be *granted*, (added to the common ground) in order for the request to be met.

## *What is a justification request?*

A justification request for a strong assertion [or strong denial] is an attempt to block the addition to the common ground, until a *reason* is given.

This reason is another assertion [or denial] which must be *granted*, (added to the common ground) in order for the request to be met.

Granting the given reason is *necessary* but not *sufficient* for satisfying the justification request.

## Definitions and Justification Requests

ACHILLES So ... this is an *equilateral* triangle.

TORTOISE I'm sorry, I don't follow, my heroic friend. I've not heard that word before: what does '*equilateral*' mean?

ACHILLES Oh, that's easy to explain. '*Equilateral*' means having sides of the same length. An *equilateral* triangle is a triangle with all three sides the same length.

TORTOISE OK. That sounds good. You may continue with your reasoning.

ACHILLES Well, as I was saying, the sides of this triangle are all one cubit in length, so it is an equilateral triangle.

TORTOISE Perhaps you will forgive me, Achilles, but I still don't follow. I grant to you that the sides of this triangle all have the same length. I fail to see, however, that it *follows* that it is an equilateral triangle. Could you explain why it is?



If I accept the definition  $A =_{df} B$ ,  
then I should accept granting  $A$  as meeting  
a justification request for the assertion of  $B$   
and ruling out  $A$  as meeting a justification  
request for  $B$ 's denial and *vice versa*.

A failure to accept this is a sign  
that I have not mastered the definition.

What goes for a definition of the form  $A =_{df} B$   
can also go for defining rules:

$$\frac{X, A, B \succ Y}{X, A \wedge B, \succ Y} \wedge Df$$

What goes for a definition of the form  $A =_{df} B$   
can also go for defining rules:

$$\frac{X, A, B \succ Y}{X, A \wedge B, \succ Y} \wedge Df$$

It is a mistake to grant  $A$  and grant  $B$   
and to look for something more to discharge  
a justification request for an assertion of  $A \wedge B$ ,  
if you take  $\wedge Df$  as a *definition*.

## *Justification Requests and Defining Rules*

$$\frac{X, A \succ B, Y}{X \succ A \supset B, Y} \supset Df$$

## *Justification Requests and Defining Rules*

$$\frac{X, A \succ B, Y}{X \succ A \supset B, Y} \supset Df$$

It is a mistake to rule A in and rule B out  
and to look for something more to discharge  
a justification request for a denial of  $A \supset B$   
if you accept  $\supset Df$  as a definition.

## *Justification Requests, Defining Rules and Derivations*

A *little* more work is required to show why granting  $A$  and  $A \supset Z$  is enough to meet a justification request for  $Z$ 's assertion.

## Justification Requests, Defining Rules and Derivations

A *little* more work is required to show why granting  $A$  and  $A \supset Z$  is enough to meet a justification request for  $Z$ 's assertion.

Consider this *focussed* derivation:

$$\frac{A \supset Z \succ A \supset Z}{A \supset Z, A \succ Z} \supset Df$$

- Read the *premise* as telling us that in a position in which  $A \supset Z$  is already ruled in, we have an answer to the justification request for  $A \supset Z$ 's assertion.

## Justification Requests, Defining Rules and Derivations

A *little* more work is required to show why granting  $A$  and  $A \supset Z$  is enough to meet a justification request for  $Z$ 's assertion.

Consider this *focussed* derivation:

$$\frac{A \supset Z \succ A \supset Z}{A \supset Z, A \succ Z} \supset Df$$

- Read the *premise* as telling us that in a position in which  $A \supset Z$  is already ruled in, we have an answer to the justification request for  $A \supset Z$ 's assertion.
- Then applying  $\supset Df$  we see why we have an answer to the request concerning  $Z$ 's assertion, in a context in which  $A \supset Z$  and  $A$  have both been ruled in. (In granting  $A \supset Z$  and  $A$  we have settled  $Z$  positively. Its denial is ruled out, since to assert  $A$  and deny  $Z$  amounts to denying  $A \supset Z$ .)



**SLOGAN:** A derivation of  $X \succ A$ ,  $Y$  shows us how to meet a justification request for the assertion of  $A$  in any available position extending  $[X : Y]$ .

**SLOGAN:** A derivation of  $X \succ A$ ,  $Y$  shows us how to meet a justification request for the assertion of  $A$  in any available position extending  $[X : Y]$ .

A derivation of  $X$ ,  $A \succ Y$  shows us how to meet a justification request for the denial of  $A$  in any available position extending  $[X : Y]$ .

## *Focussed Structural Rules*

---

$$X, A \succ \boxed{A}, Y \quad X, \boxed{A} \succ A, Y$$

## *Focussed Structural Rules*

$X, A \succ \boxed{A}, Y$        $X, \boxed{A} \succ A, Y$

$X, A \succ A, \boxed{B}, Y$        $X, A, \boxed{B} \succ A, Y$

## *Focussed Structural Rules*

$$X, A \succ \boxed{A}, Y \qquad X, \boxed{A} \succ A, Y$$

$$X, A \succ A, \boxed{B}, Y \qquad X, A, \boxed{B} \succ A, Y$$

$$\frac{X \succ \boxed{A}, Y \quad X, A \succ \boxed{B}, Y}{X \succ \boxed{B}, Y} \textit{Cut}$$

$$\frac{X \succ \boxed{A}, Y \quad X, A, \boxed{B} \succ Y}{X, \boxed{B} \succ Y} \textit{Cut}$$

## Focussed Structural Rules

$$X, A \succ \boxed{A}, Y \quad X, \boxed{A} \succ A, Y$$

$$X, A \succ A, \boxed{B}, Y \quad X, A, \boxed{B} \succ A, Y$$

$$\frac{X \succ \boxed{A}, Y \quad X, A \succ \boxed{B}, Y}{X \succ \boxed{B}, Y} \text{Cut}$$

$$\frac{X \succ \boxed{A}, Y \quad X, A, \boxed{B} \succ Y}{X, \boxed{B} \succ Y} \text{Cut}$$

$$\frac{X, A, \boxed{A} \succ Y}{X, \boxed{A} \succ Y} W$$

$$\frac{X \succ A, \boxed{A}, Y}{X \succ \boxed{A}, Y} W$$

# Swap

$$\frac{\frac{X \succ \boxed{A}, B, Y \quad X, A \succ A, \boxed{B}, Y}{X \succ A, B, \boxed{B}, Y} \text{Cut}}{X \succ A, \boxed{B}, Y} W$$

# Swap

$$\frac{X \succ \boxed{A}, B, Y \quad X, A \succ A, \boxed{B}, Y}{X \succ A, B, \boxed{B}, Y} \text{Cut}$$
$$\frac{X \succ A, B, \boxed{B}, Y}{X \succ A, \boxed{B}, Y} W$$

$$\frac{X \succ \boxed{A}, B, Y}{X \succ A, \boxed{B}, Y} \text{Swap}$$



## Focussed Defining Rules

$$\frac{X, A, B \succ Y}{X, A \wedge B \succ Y} \wedge Df$$

$$\frac{X, A, B \succ Y}{X, A \wedge B \succ Y} \wedge Df$$

$$\frac{X \succ A, B, Y}{X \succ A \vee B, Y} \vee Df$$

$$\frac{X \succ A, B, Y}{X \succ A \vee B, Y} \vee Df$$

$$\frac{X \succ A, Y}{X, \neg A \succ Y} \neg Df$$

$$\frac{X, A \succ B, Y}{X \succ A \supset B, Y} \supset Df$$

$$\frac{X, A \succ B, Y}{X \succ A \supset B, Y} \supset Df$$

## *Focussed Derivations*

$$\frac{p \succ p}{p, \neg p \succ} \neg Df$$
$$\frac{p, \neg p \succ}{p \wedge \neg p \succ} \wedge Df$$

## *Focussed Derivations*

$$\frac{\frac{p \succ p}{p, \neg p \succ} \neg Df}{p \wedge \neg p \succ} \wedge Df \qquad \frac{\frac{\neg p \succ \neg p}{\succ p, \neg p} \neg Df}{\succ p \vee \neg p} \vee Df$$

## *Proof and Supposition*

$$\frac{X, A \succ B, Y}{X \succ A \supset B, Y} \supset Df \qquad \frac{X, A \succ B, Y}{X \succ A \supset B, Y} \supset Df$$

To prove  $A \supset B$ , *rule A in* (suppose it) and prove B.

Or, *rule B out* (suppose it), and refute A.

## *Answers!*

Now that we see how focussed derivations can be seen as procedures for meeting justification requests, we have answers to our original questions and concerns about the sequent calculus.

## Answers!

Now that we see how focussed derivations can be seen as procedures for meeting justification requests, we have answers to our original questions and concerns about the sequent calculus.

- If we understand a *conclusion* of a *proof* the meeting of a justification request, we can see why this kind of conclusion is *single*.

## Answers!

Now that we see how focussed derivations can be seen as procedures for meeting justification requests, we have answers to our original questions and concerns about the sequent calculus.

- If we understand a *conclusion* of a *proof* the meeting of a justification request, we can see why this kind of conclusion is *single*.

## Answers!

Now that we see how focussed derivations can be seen as procedures for meeting justification requests, we have answers to our original questions and concerns about the sequent calculus.

- If we understand a *conclusion* of a *proof* the meeting of a justification request, we can see why this kind of conclusion is *single*.
- Since both assertions and denials can be the target of a justification request, this single conclusion can be in the *right* or the *left* of a sequent.



## Answers!

Now that we see how focussed derivations can be seen as procedures for meeting justification requests, we have answers to our original questions and concerns about the sequent calculus.

- If we understand a *conclusion* of a *proof* the meeting of a justification request, we can see why this kind of conclusion is *single*.
- Since both assertions and denials can be the target of a justification request, this single conclusion can be in the *right* or the *left* of a sequent.

## Answers!

Now that we see how focussed derivations can be seen as procedures for meeting justification requests, we have answers to our original questions and concerns about the sequent calculus.

- If we understand a *conclusion* of a *proof* the meeting of a justification request, we can see why this kind of conclusion is *single*.
- Since both assertions and denials can be the target of a justification request, this single conclusion can be in the *right* or the *left* of a sequent.
- The making of an *inference* is a (possibly preemptive) answer to a justification request.

## Answers!

Now that we see how focussed derivations can be seen as procedures for meeting justification requests, we have answers to our original questions and concerns about the sequent calculus.

- If we understand a *conclusion* of a *proof* the meeting of a justification request, we can see why this kind of conclusion is *single*.
- Since both assertions and denials can be the target of a justification request, this single conclusion can be in the *right* or the *left* of a sequent.
- The making of an *inference* is a (possibly preemptive) answer to a justification request.

## Answers!

Now that we see how focussed derivations can be seen as procedures for meeting justification requests, we have answers to our original questions and concerns about the sequent calculus.

- If we understand a *conclusion* of a *proof* the meeting of a justification request, we can see why this kind of conclusion is *single*.
- Since both assertions and denials can be the target of a justification request, this single conclusion can be in the *right* or the *left* of a sequent.
- The making of an *inference* is a (possibly preemptive) answer to a justification request.
- A derivation of a sequent  $X \succ A, Y$  [ $X, A \succ Y$ ] can be transformed into a *procedure* for meeting a justification request for an assertion of  $A$  [denial of  $A$ ] in any available position, appealing only what is granted in  $[X : Y]$ , and to the defining rules used in that derivation.

## *The value of derivations*

---

- The bounds, by themselves, can transcend our grasp.

## *The value of derivations*

---

- The bounds, by themselves, can transcend our grasp.
- Is  $[PA : GC]$  out of bounds? Who knows?

## *The value of derivations*

- The bounds, by themselves, can transcend our grasp.
- Is  $[PA : GC]$  out of bounds? Who knows?
- Derivations are one way we can *grasp* complex bounds and *enforce* them.

## *The value of derivations*

- The bounds, by themselves, can transcend our grasp.
- Is [PA : GC] out of bounds? Who knows?
- Derivations are one way we can *grasp* complex bounds and *enforce* them.
- The *negative* view of the bounds is seen in the clash between assertion and denial, and the *positive* view is found in the answers we can give to justification requests.



## *The value of derivations*

- The bounds, by themselves, can transcend our grasp.
- Is [PA : GC] out of bounds? Who knows?
- Derivations are one way we can *grasp* complex bounds and *enforce* them.
- The *negative* view of the bounds is seen in the clash between assertion and denial, and the *positive* view is found in the answers we can give to justification requests.
- Adopting *defining rules* is one way to be *very* precise about the norms governing the concepts so defined, combining *safety*, *univocity* and *expressive power*.

THANK YOU!