Eamcet GT-1 Solutions_02-04-25

1. $\cos 1^\circ . \cos 2^\circ ..... \cos 179^\circ = 0$
(as there will be a term $\cos 90^\circ$ in this series whose value is zero.)

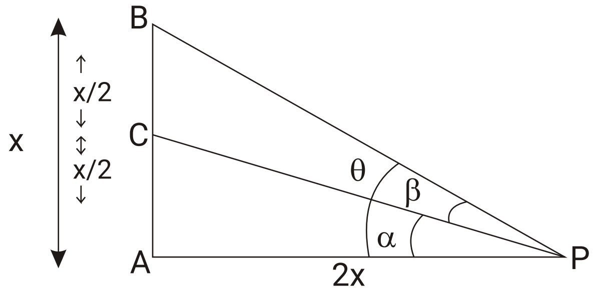
2. Let the angles be $\alpha - d,\ \alpha,\ \alpha + d$
$\therefore\ 3\alpha = 180^\circ \Rightarrow \alpha = 60^\circ$
$\frac{\alpha - d}{\alpha + d} \times \frac{180}{\pi} = \frac{60}{\pi}$
$\Rightarrow d = 30^\circ$
Hence the angles are $30^\circ,\ 60^\circ,\ 90^\circ$

3. $\cos\theta + \sec\theta = -2,\ \theta \in [0,\ 2\pi]$
$\Rightarrow \cos\theta = \sec\theta = -1$
$\theta = \pi$
$\therefore\ \sin^8\theta + \cos^8\theta = 0 + 1 = 1$

4. $\cos\alpha . \sin(\beta - \gamma) + \cos\beta . \sin(\gamma - \alpha) + \cos\gamma . \sin(\alpha - \beta) = \cos\alpha(\sin\beta\cos\gamma - \sin\gamma\cos\beta) + \cos\beta(\sin\gamma\cos\alpha - \sin\alpha\cos\gamma) + \cos\gamma(\sin\alpha\cos\beta - \sin\beta\cos\alpha) = 0$

5. The given expression is equal to
$$x^2 + xy(\tan^2\alpha + \cot^2\alpha) + y^2 - 4xy\frac{\cos^2 2\alpha}{\sin^2 2\alpha}$$
$$= x^2 + y^2 + xy\left[\frac{\sin^4\alpha + \cos^4\alpha}{\sin^2\alpha\cos^2\alpha} - \frac{4(\cos^2\alpha - \sin^2\alpha)^2}{4\sin^2\alpha\cos^2\alpha}\right]$$
$= x^2 + y^2 + xy\left[\frac{2\sin^2\alpha\cos^2\alpha}{\sin^2\alpha\cos^2\alpha}\right] = (x+y)^2$ which is independent of α.

6. $\tan\theta = \frac{4}{5}$
$$\frac{5\sin\theta - 3\cos\theta}{\sin\theta + 2\cos\theta} = \frac{5\tan\theta - 3}{\tan\theta + 2} = \frac{\frac{5\times 4}{5} - 3}{\frac{4}{5} + 2} = \frac{5}{14}$$

7. 

$\tan\theta = \frac{1}{2}, \tan\alpha = \frac{1}{4}, \tan\beta = y$
$\tan\theta = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
$\frac{1}{2} = \frac{\frac{1}{4} + y}{1 - \frac{y}{4}} \Rightarrow \frac{1}{2} = \frac{1 + 4y}{4 - y}$
$4 - y = 2 + 8y$
$\frac{2}{9} = y$

8. $\tan^2\theta\sec^2\theta(\cot^2\theta - \cos^2\theta)$
$= \tan^2\theta\sec^2\theta\left(\frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta\right)$
$= \tan^2\theta\sec^2\theta\cos^2\theta(\mathrm{cosec}^2\theta - 1)$
$= \tan^2\theta\cot^2\theta$
$= 1$

9. $12\tan A - 5 = 0,\ 5\cos B + 3 = 0$
$\Rightarrow \tan A = \frac{5}{12},\ \cos B = -\frac{3}{5}$
$\because\ ABCD$ is cyclic quadrilateral
$\therefore\ A + C = \pi, \quad B + D = \pi$
$\Rightarrow C = -A, \quad D = \pi - B$
$\cos C = -\cos A \quad \tan D = -\tan B$
$= \frac{12}{13} \quad = \frac{4}{3}$
$\therefore$ Equation whose roots are $\cos C$ and $\tan D$ is
$x^2 - \left(-\frac{12}{13} + \frac{4}{3}\right)x + \left(-\frac{12}{13}\right) \times \frac{4}{3} = 0$
$39x^2 - 16x - 48 = 0$

10. Given $\tan\theta = n\tan\phi$
Now $\tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi} = \frac{(n-1)\tan\phi}{1 + n\tan^2\phi}$
$\Rightarrow\ \tan(\theta - \phi) = \frac{n-1}{\cot\phi + n\tan\phi}$
$\Rightarrow\ \tan^2(\theta - \phi) = \frac{(n-1)^2}{\cot^2\phi + n^2\tan^2\phi + 2n}$
$= \frac{(n-1)^2}{(\cot\phi - n\tan\phi)^2 + 4n}$
Denominator is minimum at $\tan^2\phi = \frac{1}{n}$
So, maximum value of
$\tan^2(\theta - \phi) = \frac{(n-1)^2}{0 + 4n} = \frac{(n-1)^2}{4n}$

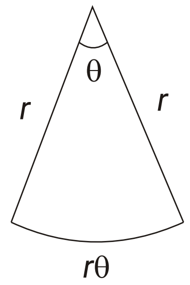
11. Given:$\cos(270 + \theta)\cos(90 - \theta) - \sin(270 - \theta)\cos\theta$
=$\sin\theta . \sin\theta + \cos\theta . \cos\theta$
=$\sin^2\theta + \cos^2\theta = 1$

12. $\sin\theta = \frac{1}{2}$
$\Rightarrow \theta = \frac{\pi}{6} \qquad ..(\mathrm{i})$
Now, $0 < \frac{1}{3} < \frac{1}{2}$
or $\cos\frac{\pi}{2} < \cos\phi < \cos\frac{\pi}{3}$
or $\frac{\pi}{3} < \phi < \frac{\pi}{2} \quad \{\because \cos\theta \text{ is decreasing in } \left[0, \frac{\pi}{2}\right]\} \qquad ..(\mathrm{ii})$
|
From (i) and (ii)
$\theta + \phi = \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

13. $\tan 15^\circ + \tan 15^\circ - \tan 15^\circ + \tan 15^\circ$
$= 2\tan 15^\circ$
$= 2(2 - \sqrt{3}) = 2a \Rightarrow a = 2 - \sqrt{3}$
$\therefore\ \frac{1}{a} + a \Rightarrow (2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$

14. Given :$(\sec A + \tan A - 1)(\sec A - \tan A + 1) - 2\tan A$
= $(\sec A + \tan A)(\sec A - \tan A) + (\sec A + \tan A) - 1(\sec A - \tan A) - 1 - 2\tan A$
= $= (\sec^2 A - \tan^2 A) + \sec A + \tan A - \sec A + \tan A - 2\tan A - 1$
= $(\sec^2 A - \tan^2 A) - 1$
= $1 - 1 = 0$

15. Let the radius of the circle be r. And, let the angle subtended by the sector at the centre be θ (radians).
Thus, the length of the arc = r.



From the figure, we can see that
Perimeter of the sector = $2r + r$.

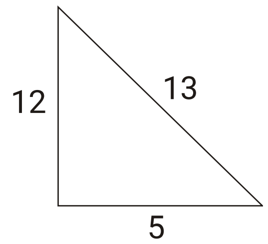
Also, given,
Perimeter of the sector = mr
$\Rightarrow 2r + r = mr$
$\Rightarrow 2 + \theta = m$
$\Rightarrow \theta = m - 2 = (m-2)^c$

16. $\sum_{r=1}^{18} \cos^2(5r)^\circ = \cos^2 5^\circ + \cos^2 10^\circ + \ldots + \cos^2 85^\circ + \cos^2 90^\circ$
$= (\cos^2 5^\circ + \cos^2 85^\circ) + (\cos^2 10^\circ + \cos^2 80^\circ) + \ldots + (\cos^2 40^\circ + \cos^2 50^\circ) + \cos^2 45^\circ$
$= (\cos^2 5^\circ + \sin^2 5^\circ) + (\cos^2 10^\circ + \sin^2 10^\circ) + \ldots + (\cos^2 40^\circ + \sin^2 40^\circ) + \cos^2 45^\circ$
$= \underbrace{1 + 1 + 1 + \ldots\ldots + 1}_{8 \text{ times}} + \frac{1}{2} = 8 + \frac{1}{2} = \frac{17}{2}$

17. For $\theta = \phi = \frac{\pi}{3}$
L.H.S. $= \cos\frac{\pi}{3} . \cos\frac{\pi}{3} . \cos\frac{2\pi}{3} = \frac{1}{2} . \frac{1}{2} . \left(-\frac{1}{2}\right) = \frac{1}{8}$

18. Given that
$\operatorname{cosec} A + \cot A = \frac{11}{2} \quad \ldots\ldots(i)$
we know that
$\operatorname{cosec}^2 A - \cot^2 A = 1$
$\Rightarrow (\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A) = 1$
from (i) put $\operatorname{cosec} A + \cot A = \frac{11}{2}$
$\Rightarrow (\operatorname{cosec} A - \cot A) \times \frac{11}{2} = 1$
$\Rightarrow \operatorname{cosec} A - \cot A = \frac{2}{11} \quad \ldots\ldots(ii)$
by subtracting (i) & (ii)
$\Rightarrow 2\cot A = \frac{11}{2} - \frac{2}{11}$
$\Rightarrow 2\cot A = \frac{117}{22}$
$\Rightarrow \cot A = \frac{117}{44}$
use the formula $\tan A = \frac{1}{\cot A}$
$\Rightarrow \tan A = \frac{44}{117}$

19.



$\sec\alpha = \frac{13}{5},\ 0 < \alpha < \frac{\pi}{2}$
$\therefore \frac{2 - 3\cot\alpha}{4 - 9\sqrt{\sec^2\alpha - 1}} = \frac{2 - 3 \times \frac{5}{12}}{4 - 9 \times \frac{12}{5}} \qquad (\sec^2\alpha - 1 = \tan^2\alpha)$
$= \frac{\frac{3}{4}}{-\frac{88}{5}} = -\frac{15}{352}$

20. $(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$
$= \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta + \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta$
$= (\sin^2\alpha + \cos^2\alpha) + (\sin^2\beta + \cos^2\beta) + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$
$= 2(1 + \cos\alpha\cos\beta + \sin\alpha\sin\beta)$
$= 2(1 + \cos(\alpha - \beta))$
Apply $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$
Hence $2(1 + \cos(\alpha - \beta))$
$= 4\cos^2\frac{\alpha - \beta}{2}$

21. M-1
Given : $\cos 105^\circ + \sin 105^\circ$
$= \cos(90^\circ + 15^\circ) + \sin(90^\circ + 15^\circ)$
Use $\cos(90 + \theta) = -\sin\theta$ and $\sin(90 + \theta) = \cos\theta$
$= \cos 15^\circ - \sin 15^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}} - \frac{\sqrt{3} - 1}{2\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}.$

M-2
Given $\cos 105^\circ + \sin 105^\circ$
$= \cos 105^\circ + \sin(90^\circ + 15^\circ)$
$= \cos 105^\circ + \cos 15^\circ$
Apply $\cos C + \cos D = 2\cos\frac{C + D}{2}\cos\frac{C - D}{2}$
$= 2\cos\frac{105^\circ + 15^\circ}{2}\cos\frac{105^\circ - 15^\circ}{2}$
$= 2\cos 60^\circ \cos 45^\circ$
$= 2\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$
$= \frac{1}{\sqrt{2}}$

22. $2\sin\left(\frac{5\pi}{12}\right)\sin\frac{\pi}{12}$
Use the formula $2\sin A\sin B = \cos(A - B) - \cos(A + B)$
$= \cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right) - \cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right)$
$= \cos\frac{\pi}{3} - \cos\frac{\pi}{2} = \frac{1}{2}$

23. $\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ} = \frac{\cos^2 15^\circ - \sin^2 15^\circ}{\cos^2 15^\circ + \sin^2 15^\circ} = \frac{\cos 30^\circ \cos 0^\circ}{1}$
$= \frac{\sqrt{3}}{2}$

24. Given that
$\sec\theta+\tan\theta=p \quad .....(i)$
We know that
$\sec^2\theta-\tan^2\theta=1$
$\Rightarrow(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)=1$
from (i) put $\sec\theta+\tan\theta=p$
$\Rightarrow(p)(\sec\theta-\tan\theta)=1$
$\Rightarrow\sec\theta-\tan\theta=\dfrac{1}{p} \quad .....(ii)$
by subtracting (ii) from(i), we get,
$\Rightarrow 2\tan\theta=p-\dfrac{1}{p}$
$\Rightarrow 2\tan\theta=\dfrac{p^2-1}{p}$
$\Rightarrow\tan\theta=\dfrac{p^2-1}{2p}$

25. $a_1\cos(\alpha_1+\theta)+a_2\cos(\alpha_2+\theta)....+a_n\cos(\alpha_n+\theta)=0$
$\Rightarrow\ (a_1\cos\alpha_1+a_2\cos\alpha_2+.....+a_n\cos\alpha_n)\cos\theta$
$-(a_1\sin\alpha_1+a_2\sin\alpha_2+...+a_n\sin\alpha_n)\sin\theta=0$
$\Rightarrow\ a_1\sin\alpha_1+a_2\sin\alpha_2+....+a_n\sin\alpha_n=0$ (since $\sin\theta\neq 0$)
and $a_1\cos\alpha_1+a_2\cos\alpha_2+....+a_n\cos\alpha_n=0$
Now,
$a_1\cos(\alpha_1+\lambda)+a_2\cos(\alpha_2+\lambda)+....+a_n\cos(\alpha_n+\lambda)$
$=(a_1\cos\alpha_1+a_2\cos\alpha_2+....+a_n\cos\alpha_n)\cos\lambda$
$-(a_1\sin\alpha_1+a_2\sin\alpha_2+....+a_n\sin\alpha_n)\sin\lambda=0$

26. We have $\dfrac{u_n}{v_n}=\tan n\theta$ and
$\dfrac{v_n-v_{n-1}}{u_{n-1}}=\dfrac{\cos n\theta\sec^n\theta-\cos(n-1)\theta\sec^{n-1}\theta}{\sin(n-1)\theta\sec^{n-1}\theta}$
$=\dfrac{\cos n\theta\sec\theta-\cos(n-1)\theta}{\sin(n-1)\theta}=\dfrac{\cos n\theta-\cos(n-1)\theta\cos\theta}{\cos\theta\sin(n-1)\theta}$
$=\dfrac{\cos(n-1)\theta\cos\theta-\sin(n-1)\theta\sin\theta-\cos(n-1)\theta\cos\theta}{\cos\theta\sin(n-1)\theta}$
$=-\tan\theta$
so that $\dfrac{v_n-v_{n-1}}{u_{n-1}}+\dfrac{1}{n}\dfrac{u_n}{v_n}=-\tan\theta+\dfrac{\tan n\theta}{n}\neq 0$

27. $\tan A+\cot(180^\circ+A)+\cot(90^\circ+A)+\cot(360^\circ-A)$
$=\tan A+\cot A-\tan A-\cot A=0.$

28. $x=\dfrac{-y}{2}=\dfrac{-z}{2}=k$
$xy+yz+zx=-2k^2+4k^2+(-2k^2)=0$

29. $\cos\left(x-\frac{3\pi}{2}\right)+\sin\left(\frac{3\pi}{2}+x\right)+\sin(32\pi+x)-18\cos(19\pi-x)+\cos(56\pi+x)-9\sin(x+17\pi)$
$=-\sin x-\cos x+\sin x+18\cos x+\cos x+9\sin x=18\cos x+9\sin x$
$a+b=27$

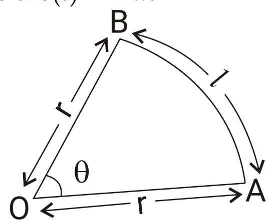
30. $\alpha+\beta$ and $\alpha-\beta$ both are acute angles.
$\cos(\alpha+\beta)=\dfrac{3}{5}$
$\Rightarrow\tan(\alpha+\beta)=\dfrac{4}{3}$
And $\sin(\alpha-\beta)=\dfrac{5}{13}$
$\Rightarrow\tan(\alpha-\beta)=\dfrac{5}{12}$
Now, $\tan 2\alpha=\tan((\alpha+\beta)+(\alpha-\beta))$
$=\dfrac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta).\tan(\alpha-\beta)}$
$=\dfrac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}.\frac{5}{12}}=\dfrac{63}{16}$

31. α, is solution of $4\cos\theta+5\sin\theta=1$,
So, $4\cos\alpha+5\sin\alpha=1$
Divide both sides by $\cos\alpha$
$\left(\text{as }\alpha\neq\frac{\pi}{2},-\frac{\pi}{2},\text{so we can divide by }\cos\alpha\right)$
$\Rightarrow 4+5\tan\alpha=\sec\alpha$
$\Rightarrow 4+5\tan\alpha=\sqrt{1+\tan^2\alpha}$
Square both sides,
$\Rightarrow 16+25\tan^2\alpha+40\tan\alpha=1+\tan^2\alpha$
$\Rightarrow 24\tan^2\alpha+40\tan\alpha+15=0$
$\Rightarrow\tan\alpha=\dfrac{-40\pm\sqrt{1600-4\times 24\times 15}}{48}$
$\Rightarrow\tan\alpha=\dfrac{-40\pm 4\sqrt{10}}{48}$
$\Rightarrow\tan\alpha=\dfrac{-10\pm\sqrt{10}}{12}$

32. We have, $\tan\alpha=\dfrac{m}{m+1}$ and $\tan\beta=\dfrac{1}{2m+1}$
We know $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$
$\Rightarrow\tan(\alpha+\beta)=\dfrac{\frac{m}{m+1}+\frac{1}{2m+1}}{1-\frac{m}{(m+1)}\frac{1}{(2m+1)}}=\dfrac{2m^2+m+m+1}{2m^2+m+2m+1-m}$
$=\dfrac{2m^2+2m+1}{2m^2+2m+1}=1$
$\Rightarrow\tan(\alpha+\beta)=1$
$\Rightarrow\alpha+\beta=\dfrac{\pi}{4}.$
Trick: As $\alpha+\beta$ is independent of m, therefore put $m=1$, then $\tan\alpha=\dfrac{1}{2}$ and $\tan\beta=\dfrac{1}{3}$. Therefore,
$\tan(\alpha+\beta)=\dfrac{\left(\frac{1}{2}\right)+\left(\frac{1}{3}\right)}{1-\left(\frac{1}{6}\right)}=1$. Hence $\alpha+\beta=\dfrac{\pi}{4}$.
(Also check for other values of m).

33. Given: $\sin A=\dfrac{4}{5}$ where$A$ lies in first quadrant
$\Rightarrow\cos A=\sqrt{1-\sin^2 A}=\sqrt{1-\dfrac{16}{25}}$
$\Rightarrow\cos A=\dfrac{3}{5}$
Also $\cos B=-\dfrac{12}{13}$
$\Rightarrow\sin B=-\sqrt{1-\cos^2 B}=-\sqrt{1-\dfrac{144}{169}}$
$\Rightarrow\sin B=-\dfrac{5}{13}$
Now, $\cos(A+B)=\cos A\cos B-\sin A\sin B$
$=\left(\dfrac{3}{5}\right)\left(-\dfrac{12}{13}\right)-\left(\dfrac{4}{5}\right)\left(-\dfrac{5}{13}\right)$
$=-\dfrac{36}{65}+\dfrac{20}{65}=-\dfrac{16}{65}$

34. Given that radius of the circular arc$(r) = 3\ m$ and length of the arc$(l) = 1\ m$



Use the formula
Arc length = [radius] x [angle subtended by the arc on the center of the circle (in radian)]
$\Rightarrow l = r \times \theta$ (where θ is in radian)
$\Rightarrow \theta = \dfrac{l}{r}$
$\Rightarrow \theta = \dfrac{1}{3}$ radian

35. $\sin x = -\dfrac{3}{5}$, where $\pi < x < \dfrac{3\pi}{2}$

$\tan x = \dfrac{3}{4}, \cos x = \dfrac{-4}{5}$

$\therefore\ 80(\tan^2 x - \cos x)$

$= 80\left(\dfrac{9}{16} + \dfrac{4}{5}\right)$

$= 80\left(\dfrac{45+64}{80}\right)$

$= 109$

36. $\sqrt{2\tan\alpha + \dfrac{1}{\cos^2\alpha}}$

$= \sqrt{2\tan\alpha + \sec^2\alpha}$

$= \sqrt{2\tan\alpha + 1 + \tan^2\alpha}$

$= \sqrt{(1+\tan\alpha)^2} = |1 + \tan\alpha|$

(since, $\tan\alpha < -1$ when $\dfrac{\pi}{2} < \alpha < \dfrac{3\pi}{4}$
$\Rightarrow 1 + \tan\alpha < 0$)

$\Rightarrow |1 + \tan\alpha| = -1 - \tan\alpha$

Hence $\sqrt{2\tan\alpha + \dfrac{1}{\cos^2\alpha}} = -1 - \tan\alpha$

37. We have $\sin\theta + \sin^2\theta = 1 \Rightarrow \sin\theta = 1 - \sin^2\theta \Rightarrow \sin\theta = \cos^2\theta$

Now, $\cos^2\theta + \cos^4\theta = \cos^2\theta + (\cos^2\theta)^2$

$\Rightarrow \cos^2\theta + \cos^4\theta = \cos^2\theta + \sin^2\theta$

$\Rightarrow \cos^2\theta + \cos^4\theta = 1$

38. $\tan\alpha + \cot\alpha = a,\ \tan^4\alpha + \cot^4\alpha = ?$

$\tan^4\alpha + \cot^4\alpha = (\tan\alpha + \cot\alpha)^4 - 4\tan\alpha.\cot\alpha.(\tan^2\alpha + \cot^2\alpha) - 6\tan^2\alpha.\cot^2\alpha$

$= a^4 - 4\{(\tan\alpha + \cot\alpha)^2 - 2\tan\alpha.\cot\alpha\} - 6$

$= a^4 - 4a^2 + 2$

39. $y = \sqrt{\dfrac{2\cos^2\alpha}{\sin\alpha\cos\alpha} + \dfrac{1}{\sin^2\alpha}} = \sqrt{2\cot\alpha + \text{cosec}^2\alpha} = |1 + \cot\alpha| = -1 - \cot\alpha$

(As $\alpha \in \left(\dfrac{3\pi}{4}, \pi\right)$)

$\Rightarrow \dfrac{dy}{d\alpha} = \text{cosec}^2\alpha \Rightarrow \left(\dfrac{dy}{d\alpha}\right)$ at $\alpha = \dfrac{5\pi}{6}$ will be $= 4$

40. $5cosA + 3 = 0 \Rightarrow \cos A = -\dfrac{3}{5}$ clearly $A \in (90°, 180°)$

Now roots of equation $9x^2 + 27x + 20 = 0$ are $-\dfrac{5}{3}$ and $-\dfrac{4}{3}$

$\Rightarrow$ Roots $\sec A$ and $\tan A$

41. $1^C = \left(\dfrac{180}{\pi}\right)^0$

$\dfrac{23\pi^c}{4} = \dfrac{180}{\pi} \times \dfrac{23\pi}{4}$

$= 1035°$

42. $\tan\alpha + \tan\beta = \dfrac{1+x}{\sqrt{x}\sqrt{x^2+x+1}}$ and

$\tan\alpha\tan\beta = \dfrac{1}{x^2+x+1}$

$\Rightarrow \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \dfrac{\dfrac{1+x}{\sqrt{x}\sqrt{x^2+x+1}}}{\dfrac{x^2+x}{x^2+x+1}} = \dfrac{\sqrt{x^2+x+1}}{(x)^{3/2}}$

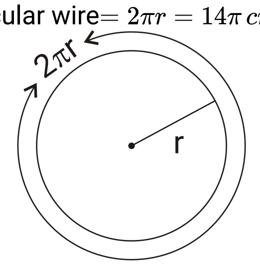
$= \sqrt{\dfrac{1}{x^3} + \dfrac{1}{x^2} + \dfrac{1}{x}} = \tan\gamma$

$\Rightarrow \alpha + \beta = \gamma$

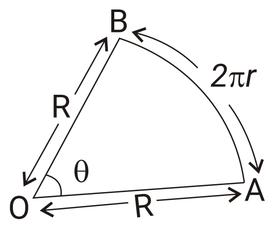
43. Given $\cot(45° + \theta)\cot(45° - \theta)$
= $\tan(90° - 45° - \theta)\cot(45° - \theta)$
$= \tan(45° - \theta)\cot(45° - \theta) = 1.$ {As $\tan A \cdot \cot A = 1$}

Aliter:
Given $\cot(45° + \theta)\cot(45° - \theta)$
Now use
$\cot(A+B) = \dfrac{\cot A\cot B - 1}{\cot B + \cot A}$, $\cot(A+B) = \dfrac{\cot A\cot B + 1}{\cot B - \cot A}$
So $\cot(45° + \theta)\cot(45° - \theta)$
= $\left(\dfrac{\cot 45°\cot B - 1}{\cot B + \cot 45°}\right) \cdot \left(\dfrac{\cot 45°\cot B + 1}{\cot B - \cot 45°}\right)$
= $\left(\dfrac{\cot B - 1}{\cot B + 1}\right) \cdot \left(\dfrac{\cot B + 1}{\cot B - 1}\right)$
= 1

44. Given that the radius of the circular wire= $r = 7\,cm$
Therefore the circumference of the circular wire = the length of the circular wire= $2\pi r = 14\pi\,cm$



Now this wire has been cut & bend again to make a new arc of the circle with a radius= $R = 12\,cm$



Use the formula
Arc length = radius x angle subtended by the arc on the center (in radian)

$\Rightarrow l = R \times \theta$

$\Rightarrow 14\pi = 12 \times \theta$

$\Rightarrow \theta = \frac{14\pi}{12}$

$\Rightarrow \theta = \frac{7\pi}{6}$ radian

$\Rightarrow \theta = \left(\frac{7\pi}{6} \times \frac{180}{\pi}\right)^\circ$

$\Rightarrow \theta = 210^\circ$

45. $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A} = \frac{\tan^2 A}{\tan A - 1} - \frac{1}{\tan A(\tan A - 1)}$

$\Rightarrow \frac{\tan^3 A - 1}{\tan A(\tan A - 1)} = \frac{\tan^2 A + 1 + \tan A}{\tan A}$

$\Rightarrow \frac{\sec^2 A}{\sin A/\cos A} + 1$

$\Rightarrow \sec A \operatorname{cosec} A + 1$

46. $\sin x + \cos x = \frac{1}{2} \Rightarrow (1+\tan x)2 = \sec x \Rightarrow$
$4(1+\tan^2 x + 2\tan x) = \sec^2 x$

$\Rightarrow 4 + 4\tan^2 x + 8\tan x = 1 + \tan^2 x \Rightarrow$
$3\tan^2 x + 8\tan x + 3 = 0$

$\Rightarrow \tan x = \frac{-8 \pm \sqrt{28}}{6} = \frac{-4 \pm \sqrt{7}}{3}$

$\therefore \ \tan x = \frac{-(4+\sqrt{7})}{3}$

47. $\cos 2001\pi + \cot 2001\frac{\pi}{2} + \sec\frac{2001\pi}{3} + \tan\frac{2001\pi}{4} + \operatorname{cosec}\frac{2001\pi}{6}$

$= \cos\pi + \cot\frac{\pi}{2} + \sec 667\pi + \tan\frac{\pi}{4} + \operatorname{cosec}\frac{667\pi}{2} = -1 + 0 - 1 + 1 - 1 = -2$

48. $\therefore \tan\theta = \frac{1-\cos 2\theta}{\sin 2\theta}$

Put $\theta = \frac{15}{2}$

$\tan\frac{15^\circ}{2} = \frac{1-\cos 15^\circ}{\sin 15^\circ} = \frac{1 - \frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

$= \frac{2\sqrt{2}-\sqrt{3}-1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{2\sqrt{6}-3-\sqrt{3}+2\sqrt{2}-\sqrt{3}-1}{3-1}$

$= \frac{2\sqrt{6}-4-2\sqrt{3}+2\sqrt{2}}{2}$

$= \sqrt{6}-\sqrt{4}-\sqrt{3}+\sqrt{2}$

$\therefore \sqrt{3}-\sqrt{2}\times\sqrt{2}-1 = (\sqrt{3}-\sqrt{2})(\sqrt{2}+1)$

$= \sqrt{6}-\sqrt{4}-\sqrt{3}+\sqrt{2} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}+1}$

49. We have, $\sin^6\theta + \cos^6\theta + k\cos^2 2\theta = 1$

$\Rightarrow (\sin^2\theta+\cos^2\theta)(\sin^4\theta - \sin^2\theta\cdot\cos^2\theta + \cos^4\theta) + k\cos^2 2\theta = 1$

$\Rightarrow \sin^4\theta + \cos^4\theta - \sin^2\theta\cdot\cos^2\theta + k\cos^2 2\theta = 1$

$\Rightarrow (\sin^2\theta+\cos^2\theta)^2 - 3\sin^2\theta\cdot\cos^2\theta + k\cos^2 2\theta = 1$

$\Rightarrow 3\sin^2\theta\cdot\cos^2\theta = k\cos^2 2\theta$

$\Rightarrow \frac{3}{4}\sin^2 2\theta = k\cos^2 2\theta$

$\Rightarrow k = \frac{3}{4}\tan^2 2\theta$

50. $\sin 50^\circ + \sin 10^\circ - \sin 70^\circ$
$= 2\sin\left(\frac{60}{2}\right)^\circ \cos\left(\frac{40}{2}\right)^\circ - \sin 70^\circ$
$= (\cos 20^\circ) - \cos 20^\circ$
$= 0$

51. As we have to find the sum of values of 'higher' powers of $\tan$ and $\cot$, we will square both sides of the equation.

So, given,
$\tan\alpha + \cot\alpha = m$
$\Rightarrow \tan^2\alpha + \cot^2\alpha + 2\tan\alpha\cot\alpha = m^2$ (Squaring both sides of the equation)
$\Rightarrow \tan^2\alpha + \cot^2\alpha + 2 = m^2$ (Using $\tan\alpha\cot\alpha = 1$)
$\Rightarrow \tan^2\alpha + \cot^2\alpha = m^2 - 2$
$\Rightarrow (\tan^2\alpha + \cot^2\alpha)^2 = (m^2-2)^2$ (Squaring both sides of the equation)
$\Rightarrow \tan^4\alpha + \cot^4\alpha + 2 = m^4 - 4m^2 + 4$ (Using $\tan\alpha\cot\alpha = 1$)
$\Rightarrow \tan^4\alpha + \cot^4\alpha = m^4 - 4m^2 + 2$

52. $y = \cos(\cos x)$
$y_{\max} = 1 = a$, when $x = \frac{\pi}{2}$
$y_{\min} = \cos 1 = b$ when $x = 0$
$\therefore b = \cos a$

53. $\tan\frac{\pi}{3} + 2\tan\frac{2\pi}{3} + 4\tan\frac{4\pi}{3} + 8\tan\frac{8\pi}{3}$
$= \sqrt{3} - 2\sqrt{3} + 4\sqrt{3} - 8\sqrt{3}$
$= -5\sqrt{3}$

54. $\cot 123^\circ \cot 133^\circ \cot 137^\circ \cot 147^\circ$

$= \cot(90+33)\cot(90+43)\cot(180-43)\cot(180-33) = (-\tan 33)(-\tan 43)(-\cot 43)(-\cot 33) = 1$

55. If $\theta \in \left(\frac{3\pi}{2}, 2\pi\right)$, then $\tan\theta < 0$

Given $a^{\tan\theta} > 1 \Rightarrow 0 < a < 1$

And $b^{\tan\theta} > 1 \Rightarrow 0 < b < 1$

Also $a^{\tan\theta} > b^{\tan\theta}$(given) $\Rightarrow b > a$

Hence $a < b < 1$

56. $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$, which is irrational

$\cos 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$, which is irrational

$\sin 15^\circ \cos 15^\circ = \left(\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right)\left(\dfrac{\sqrt{3}+1}{2\sqrt{2}}\right) = \dfrac{(\sqrt{3})^2-1^2}{(2\sqrt{2})^2}$

= $\dfrac{1}{4}$, which is rational

$\sin 15^\circ \cos 75^\circ = \left(\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right)\left(\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right) = \dfrac{(\sqrt{3}-1)^2}{(2\sqrt{2})^2} = \dfrac{4-2\sqrt{3}}{8}$

= $\dfrac{2-\sqrt{3}}{4}$, which is irrational

57. $1^c = \left(\dfrac{180}{\pi}\right)^\circ$

$= \dfrac{180 \times 7}{22} = 57.27^\circ$

$\because \sin 1^\circ < \sin 57.27^\circ$

58. On expansion we get

$\cos(\theta_1-\theta_2)\cos(\theta_3-\theta_4)+\cos(\theta_1+\theta_2)\cos(\theta_3+\theta_4)=0$

$\Rightarrow\ (\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2)(\cos\theta_3\cos\theta_4+\sin\theta_3\sin\theta_4)$

$+(\cos\theta_1\cos\theta_2-\sin\theta_1\sin\theta_2)(\cos\theta_3\cos\theta_4-\sin\theta_3\sin\theta_4)=0$

$\Rightarrow\ 2\cos\theta_1\cos\theta_2\cos\theta_3\cos\theta_4+2\sin\theta_1\sin\theta_2\sin\theta_3\sin\theta_4=0$

$\therefore \tan\theta_1\tan\theta_2\tan\theta_3\tan\theta_4=-1$

59. Let

$\therefore\ \sin\theta+\sin 2\theta+\sin 4\theta+\sin 5\theta$

$=\sin\theta+\sin 5\theta+\sin 2\theta+\sin 4\theta$

$=2\sin 3\theta\cos 2\theta+2\sin 3\theta\cos\theta$

$=2\sin 3\theta(\cos 2\theta+\cos\theta)$

$=2\sin\dfrac{3\pi}{18}\left(\cos\dfrac{2\pi}{18}+\cos\dfrac{\pi}{18}\right)$

$=\cos\dfrac{\pi}{9}+\cos\dfrac{\pi}{18}$

$=\sin\left(\dfrac{\pi}{2}-\dfrac{\pi}{9}\right)+\sin\left(\dfrac{\pi}{2}-\dfrac{\pi}{18}\right)$

$=\sin\dfrac{7\pi}{18}+\sin\dfrac{8\pi}{18}$

$=\sin\dfrac{7\pi}{18}+\sin\dfrac{4\pi}{9}$

60. Given $x\sin 45^\circ\cos^2 60^\circ = \dfrac{\tan^2 60^\circ \operatorname{cosec} 30^\circ}{\sec 45^\circ \cot^2 30^\circ}$

$\Rightarrow x = \dfrac{\tan^2 60^\circ \operatorname{cosec} 30^\circ}{\sec 45^\circ \cot^2 30^\circ \sin 45^\circ \cos^2 60^\circ}$

It is known $\tan^2 60^\circ = \cot^2 30^\circ$,hence

$\Rightarrow x = \dfrac{\operatorname{cosec} 30^\circ}{\sec 45^\circ \sin 45^\circ \cos^2 60^\circ}$

$\Rightarrow x = \dfrac{2}{(\sqrt{2})\left(\dfrac{1}{\sqrt{2}}\right)\left(\dfrac{1}{2}\right)^2}$

$\Rightarrow x = 8$

61. We have

$K_1=\tan 27\theta-\tan\theta=(\tan 27\theta-\tan 9\theta)+(\tan 9\theta-\tan 3\theta)+(\tan 3\theta-\tan\theta)$

Now, $\tan 3\theta-\tan\theta=\dfrac{\sin 2\theta}{\cos 3\theta\cos\theta}=\dfrac{2\sin\theta}{\cos 3\theta}$

Similarly, $\tan 9\theta-\tan 3\theta=\dfrac{2\sin 3\theta}{\cos 9\theta}$ and

$\tan 27\theta-\tan 9\theta=\dfrac{2\sin 9\theta}{\cos 27\theta}$

$\therefore K_1=2\left[\dfrac{\sin 9\theta}{\cos 27\theta}+\dfrac{\sin 3\theta}{\cos 9\theta}+\dfrac{\sin\theta}{\cos 3\theta}\right]=2K_2$

62. If$\sin\theta_1+\sin\theta_2+\sin\theta_3=3$ is true only if

$\sin\theta_1=1\Rightarrow\theta_1=\dfrac{\pi}{2}$

$\sin\theta_2=1\Rightarrow\theta_2=\dfrac{\pi}{2}$

$\sin\theta_3=1\Rightarrow\theta_3=\dfrac{\pi}{2}$

$\cos\theta_1+\cos\theta_2+\cos\theta_3=\cos\dfrac{\pi}{2}+\cos\dfrac{\pi}{2}+\cos\dfrac{\pi}{2}=0$

63. Observe : $\dfrac{2\pi}{5}-\dfrac{\pi}{15}=\dfrac{\pi}{3}$

$\Rightarrow \dfrac{\tan\left(\dfrac{2\pi}{5}\right)-\tan\left(\dfrac{\pi}{15}\right)}{1+\tan\left(\dfrac{2\pi}{15}\right)\tan\left(\dfrac{\pi}{15}\right)}=\tan\dfrac{\pi}{3}=\sqrt{3}$

$\Rightarrow \tan\left(\dfrac{2\pi}{5}\right)-\tan\left(\dfrac{\pi}{15}\right)-\sqrt{3}\tan\left(\dfrac{2\pi}{5}\right)\tan\left(\dfrac{\pi}{15}\right)=\sqrt{3}$

64. $3A=2A+A$

$\Rightarrow \tan 3A=\tan(2A+A)$

$\Rightarrow \tan 3A=\frac{\tan 2A+\tan A}{1-\tan 2A.\tan A}$

$\Rightarrow \tan 3A-\tan 3A\tan 2A\tan A=\tan 2A+\tan A$

$\Rightarrow \tan 3A-\tan 2A-\tan A=\tan 3A\tan 2A\tan A$

65. $2r+I=\dfrac{1}{2}2\pi r$ where r is radius of the circle and I is the length of the arc i.e. $I=r\theta$

$2r+r\theta=\pi r$

$\Rightarrow \theta=(\pi-2)\,\text{radian}$

66. Diameter of the wire $=10\,cm$

Length of wire $=10\,\pi\,cm$

Another circle is of diameter $=1\,m=100\,cm$

$\therefore$ Radius $=50\,cm$

$\text{Required angle}=\dfrac{\text{Length of arc}}{\text{Radius}}=\dfrac{10\,\pi}{50}=\dfrac{\pi}{5}$

67. $\sin\alpha=-\dfrac{1}{\sqrt{2}}\Rightarrow\alpha=\pi+\dfrac{\pi}{4},\ 2\pi-\dfrac{\pi}{4}$

$\cos\beta=-\dfrac{1}{\sqrt{2}}\Rightarrow\beta=\pi-\dfrac{\pi}{3},\ \pi-\dfrac{\pi}{3}$

$\tan\gamma=-\sqrt{3}\Rightarrow\gamma=\pi-\dfrac{\pi}{3},\ 2\pi-\dfrac{\pi}{3}$

$\therefore$ Sum of all values $=8\pi-\dfrac{2\pi}{3}=\dfrac{22\pi}{3}$

68. We have,

$\tan(A-B)=\tan\dfrac{\pi}{4}$

or $\dfrac{\tan A-\tan B}{1+\tan A\tan B}=1$

or $\tan A-\tan B=1+\tan A\tan B$

or $\tan A-\tan B-1-\tan A\tan B=0$

Adding 2 both sides, we get

$\tan A-\tan B+1-\tan A\tan B=2$

or $(1+\tan A)-\tan B(1+\tan A)=2$

or $(1+\tan A)(1-\tan B)=2$

69. $\sin 2\sin 3\sin 5\quad\{\because 1^c=57^\circ 17^1\}$

= (+ve)(+ve)(−ve)

= Negative

70. $\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \ldots\ldots\ldots\ldots + \sin^2 80^\circ + \sin^2 85^\circ + \sin^2 90^\circ$

$= (\sin^2 5^\circ + \sin^2 85^\circ) + (\sin^2 10^\circ + \sin^2 80^\circ) + \ldots\ldots + (\sin^2 40^\circ + \sin^2 50^\circ) + \sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$

$= 7 + \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1$

$= 9\frac{1}{2}$

71. $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$

$= \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} \cdot \frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}$

$= \frac{(1+\sin\alpha)^2 - \cos^2\alpha}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$

$= \frac{1+2\sin\alpha+\sin^2\alpha-(1-\sin^2\alpha)}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$

$= \frac{2\sin a(1+\sin a)}{(1+\cos a+\sin a)(1+\sin a)}$

$= \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = x$

72. $A + C = \pi \Rightarrow C = (\pi - A)$

$B + D = \pi \Rightarrow D = (\pi - B)$

$\cos A - \cos B + \cos(\pi - A) - \cos(\pi - B)$

$\cos A - \cos B - \cos A + \cos B = 0$

73. $LHS = (1+\tan 1^\circ)(1+\tan 44^\circ)(1+\tan 2^\circ)(1+\tan 43^\circ)\ldots\ldots(1+\tan 45^\circ)$

As if $A + B = \frac{\pi}{4}$, then

$(1+\tan A)(1+\tan B) = 2\left\{\tan\frac{\pi}{4} = \frac{\tan A+\tan B}{1-\tan A\cdot\tan B}\right\}$

$\Rightarrow (1+\tan 1^\circ+\tan 44^\circ+\tan 1^\circ\tan 44^\circ)+(1+\tan 2^\circ+\tan 43^\circ+\tan 2^\circ+\tan 43^\circ)\ldots\ldots(1+\tan 45^\circ)$

$\Rightarrow (1+1)(1+1)\ldots\ldots(1+1)$23 times $= 2^{23}$

$\Rightarrow n = 23$

74. $\sin 1 \quad \cos 2 \quad \tan 3 \quad \cot 4 \quad \sec 5 \quad \text{cosec}\, 6 \; + \quad - \quad - \quad + \quad + \quad -$

= negative

75. We have to eliminate θ, to arrive at an equation independent of θ.

Given,

$x = a\sin\theta \Rightarrow \sin\theta = \frac{a}{x}$ ...(1)

$y = b\tan\theta \Rightarrow \tan\theta = \frac{y}{b}$ ...(2)

Recall the three basic trigonometric identities. The one which fits over here would give a relation between $\sin\theta$ and $\tan\theta$, or between $\frac{1}{\sin\theta}(=\text{cosec}\,\theta)$ and $\frac{1}{\tan\theta}(=\cot\theta)$.

We know that

$\text{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow \frac{1}{\sin^2\theta} - \frac{1}{\tan^2\theta} = 1$

$\Rightarrow \left(\frac{a}{x}\right)^2 - \left(\frac{b}{y}\right)^2 = 1$ (From (1) and (2))

$\Rightarrow \frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$

76. $a\cos\theta = b,\ c\sin\theta = d$

$\Rightarrow \cos\theta = \frac{b}{a},\ \sin\theta = \frac{d}{c}$

Squaring and adding,

$\frac{b^2}{a^2} + \frac{d^2}{c^2} = 1$

$\Rightarrow b^2c^2 + a^2d^2 = a^2c^2$

77. $a\cos\theta + b\sin\theta = 3$

$a\sin\theta - b\cos\theta = 4$

Squaring and adding, $a^2 + b^2 = 25$

78. Given: $\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ}$

Using the result

$\tan(90^\circ - \theta) = \cot\theta$, $\cot(90^\circ - \theta) = \tan\theta$, we get

$= \frac{\cot 54^\circ}{\tan(90^\circ - 54^\circ)} + \frac{\tan 20^\circ}{(\cot 90^\circ - 20^\circ)}$

$\frac{\cot 54^\circ}{\cot 54^\circ} + \frac{\tan 20^\circ}{\tan 20^\circ} = 1 + 1 = 2$

79. $P_n = \cos^n\theta + \sin^n\theta \Rightarrow$

$P_n - P_{n-2} = \cos^n\theta + \sin^n\theta - \cos^{n-2}\theta - \sin^{n-2}\theta$

$= \cos^{n-2}\theta(\cos^2\theta - 1) + \sin^{n-2}\theta(\sin^2\theta - 1) = \cos^{n-2}\theta(-\sin^2\theta) + \sin^{n-2}\theta(-\cos^2\theta)$

$= -\sin^2\theta\cos^2\theta(\cos^{n-4}\theta + \sin^{n-4}\theta) = -\sin^2\theta\cos^2\theta\, P_{n-4}$

80. $\tan^4 x - 2\tan^3 x - \tan^2 x + 2\tan x + 1$

$= \tan^4 x + \tan^2 x - 2\tan^3 x + 2\tan x - 2\tan^2 x + 1$

$= (\tan^2 - \tan x)^2 + 2(\tan x - \tan^2 x) + 1 = 4.$

81. $\because \mathrm{I} = \frac{\mathrm{Gm}}{\mathrm{r}^2} \rightarrow \text{here m} \rightarrow \text{same}$

$\Rightarrow \mathrm{I} \propto \frac{1}{\mathrm{r}^2}$

$\Rightarrow \frac{\mathrm{I}_1}{\mathrm{I}_2} = \frac{\mathrm{r}_2^2}{\mathrm{r}_1^2}$

$\Rightarrow \frac{40}{\mathrm{I}_2} = \frac{(10)^2}{(5)^2}$

$\Rightarrow \mathrm{I}_2 \propto \frac{40}{4}$

$\Rightarrow \mathrm{I}_2 = \frac{10\mathrm{N}}{\mathrm{kg}}$

82. Distance $\mathrm{s} = (2.5)\mathrm{t}^2$

Speed $\mathrm{v} = \frac{\mathrm{ds}}{\mathrm{dt}} = (2.5)(2\mathrm{t})$

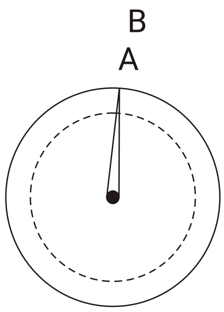
$\mathrm{v} = 5\mathrm{t}$

At $\mathrm{t} = 5,\ \mathrm{v} = 5 \times 5 = 25$ m/s

83. Factual. The magnitude of distance is always greater than the magnitude of displacement.

84. The second hand of the clock in minute covers an angle of 360° and the initial and final positions are same.

So, Displacement = 0



85. $\frac{6\pi}{36} + \theta = \frac{\pi}{2}$
$\Rightarrow \theta = \frac{\pi}{3}$

86. The term geostationary comes from the fact that such a satellite appears nearly stationary in the sky as seen by a ground-based observer. It can be installed over any city on the equator.

87. A is correct because $F = \frac{Gm_1m_2}{r^2}$.
D is correct because $T^2 \propto r^3$
$\therefore$ A and D are correct

88. Bodies having masses m_1, m_2 experience gravitational force due to Earth m_1g, m_2g respectively. So both bodies travel with the same gravitational acceleration.
$\therefore$ Time taken any body to reach ground level, from level of height h is, $t = \sqrt{\frac{2h}{g}}$.
Given that bodies are dropped from heights h_1, h_2 respectively.
$\Rightarrow \frac{t_1}{t_2} = \sqrt{\frac{h_1}{h_2}}$

89. The gravitational force passes through the center of the Sun. So, the torque is zero.

90. In uniform circular motion, the velocity and acceleration vectors keep on changing because their directions change even if magnitudes don't.

91. Gravitational potential energy of two particle system $= \frac{-Gm_1m_2}{d}$.
If m_1 is doubled and d is halved, potential energy becomes four times.

92. no force acts in horizontal direction only gravitational force acts along vertical direction so no horizontal acceleration .along the zero acceleration direction body have uniform motion or in rest

93. Kepler's first law,
Law of Orbits : All planets move in elliptical orbits, with the sun at one of the foci of the ellipse.

94. The gravitational field intensity at the center of a uniform solid sphere of radius R and mass M is zero.

95. Gravitational constant does not depend on the nature of medium in which bodies are kept.

96. Statement-I is correct. This is mainly due to the rotation of the Earth on its own axis.
Statement-II is incorrect because the acceleration due to gravity decreases as we go below the surface of the Earth.

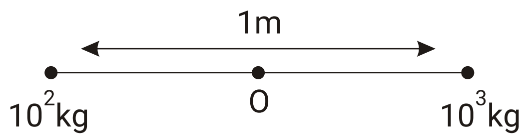
97. $x = 4t^2$
$v = \frac{dx}{dt} = 8t$
v at $t = 5\ s = 40$ m/s

98. Gravitation field intensity due to a particle of mass m at position $\vec{r}$ with respect to itself is
$\vec{I} = \frac{Gm}{r^2}(-\hat{r})$

99. $R = \frac{u^2}{g}\sin 2\theta$
R is maximum for $2\theta = 90°$.

100. The relative velocity of the stone w.r.t. the ground will be zero at the highest point.

101.



$\because$ We know that $V = -\frac{GM}{r}$

$\Rightarrow V_{net} = -\frac{GM_1}{\frac{r}{2}} - \frac{GM_2}{\frac{r}{2}}$

$\Rightarrow V_{net} = -\frac{2G}{r}(M_1 + M_2)$

$\Rightarrow V_{net} = -\frac{2G}{r}(10^2 + 10^3)$

$\Rightarrow V_{net} = -2200 \times 6.67 \times 10^{-11} J/kg$

$\Rightarrow V_{net} = -146.74 \times 10^{-9} J/kg$

102. $\because PE = -\frac{GM_1M_2}{r}$

$\Rightarrow PE = -\frac{G \times 2 \times 4}{10}$

$\Rightarrow PE = -\frac{4G}{5}$

103. Given that two satellites S_1 and S_2 are revolving around a planet in circular orbits. They have periods of revolution 1 hr. and 8 hr. respectively.
The ratio of time period is given by, $\frac{T_1}{T_2} = \frac{1}{8}$.
Angular velocity is inversely proportional to the time period and is given by $T = \frac{2\pi}{\omega_1}$, hence
$\Rightarrow \frac{\omega_2}{\omega_1} = \frac{1}{8}$
$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{8}{1}$

104. $\because v_e = \sqrt{\frac{2GM_e}{R_e}}$

$\because g = \frac{GM_e}{Re^2}$

$\Rightarrow GM_e = gR_{e^2}$

$\Rightarrow v_e = \sqrt{\frac{2gRe^2}{Re}}$

$\Rightarrow v_e = \sqrt{2gR_e}$

So escape velocity is independent of mass escape velocity is varies with mass "m^0".

105. $v = \sqrt{\frac{GM}{r}}$

$v \propto \frac{1}{\sqrt{r}} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{3r}{r}} = \sqrt{3} : 1$

106. $x = 5t^2 - 4t + 5$
$v = 10t - 4$
At $t = 2$ s, $v = 16$ m/s

107. $v_{1,2} = v_1 - v_2$.
Taking eastwards as positive, we get,
$v_{1,2} = 10 - 15 = -5$ m/s.
So, the answer is 5 m/s westwards.

108. The intensity of the gravitational field due to a non-uniform ring at its geometrical center depends on the mass distribution. It can be zero or non-zero.

109. $v = u + at$
$60 = 10 + 2t$
$t = 25$ sec

110. When the gravitational potential at infinity is zero, the potential at any other point is negative.

111. a_A = 8 m/s^2 & a_B = 4 m/s^2

$a_{BA} = a_B - a_A = 4 - 8 = -$ 4m/s^2

112. $\Rightarrow g' = \frac{gR^2}{r^2} = \frac{gR^2}{\left(R + \frac{R}{4}\right)^2} = \frac{16g}{25}$

$\therefore$ Weight $= \frac{16}{25} \times 100 = 64$ N

113. $I_g = \frac{F}{m}$

$= \frac{3}{60 \times 10^{-3}} = 50$ N/kg

114. $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 5^2 = 125\,m$

115. $V_e = \sqrt{\frac{2GM}{R}} \Rightarrow V_e \propto \sqrt{\frac{M}{R}}$

As $\frac{M}{R}$ increases, V_e increases.

Statement I is correct.

Also, $V_e \propto \frac{1}{\sqrt{R}}$.

Statement II is incorrect.

116. The SI unit of gravitational potential is Joule/kg.

117. $S_{n^{th}} = u + \frac{a}{2}(2n - 1)$

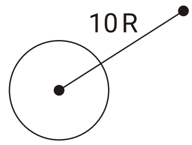
$19 = 0 + \frac{a}{2}(2 \times 10 - 1)$

$a = 2$ m/s^2

118. Force of gravitation is independent of the medium. Force is F when masses are in vacuum. When masses are dipped in water, the force will be same.

119. Gravitation is the phenomenon of interaction between any arbitrary shaped bodies.

120.



$g' = \frac{GM}{(10R)^2} = \frac{g}{100}$

$W' = \left(\frac{W}{100}\right)$

121. The structure and bond in the allene (C_3H_4) can be seen below:

$CH_2 = C = CH_2$

The terminal carbon is forming 3 $\sigma-$bond and 1 $\pi-$bond.

⇒ Steric number of terminal carbon = 3

⇒ Hybridization of terminal carbon = sp^2

The middle carbon is forming 2 $\sigma-$bond and 2 $\pi-$bond.

⇒ Steric number of terminal carbon = 2

⇒ Hybridization of terminal carbon = sp

⇒ The 2 carbon atoms are in sp^2 and 1 carbon atom is in sp.

⇒ Option (B) is ***CORRECT.***

122. Na^+, Mg^{2+} and Al^{3+} are isoelectronic species.

All the species have 10 e^-

The number of protons in Na^+ = 11

The number of protons in Mg^{2+} = 12

The number of protons in Al^{3+} = 13

⇒ In Na^+ 10 e^- are experiencing the charge of 11 protons.

⇒ In Mg^{2+} 10 e^- are experiencing the charge of 12 protons.

⇒ In Al^{3+} 10 e^- are experiencing the charge of 13 protons.

⇒ The attraction is strongest in Al^{3+} and weakest in Na^+

We know as the attraction increases, the ionic radius decreases.

⇒ The ionic radii order is: $Na^+ > Mg^{2+} > Al^{3+}$

⇒ 1.02 Å > $Mg^{2+} > Al^{3+}$

⇒ Option (B) is ***CORRECT.***

123. Based on his experiments on X-ray spectrum using different elements, Moseley's gave the Modern Periodic Law.

The Modern Periodic Table is based on the Modern Periodic Law which states that:

"The physical and chemical properties of elements are periodic functions of their atomic number".

124. According to modern periodic law physical and chemical properties are periodic function of their atomic number.

125. Increases from left to right since the size decreases.

126. The electron gain enthalpy generally decreases down the group as the size of the atom increases.
But 3p elements have higher electron gain enthalpy than the corresponding 2p elements.
This is because of the smaller size of the 2p orbital and when an electron is added, it suffers significant repulsion with other electrons present in that level.

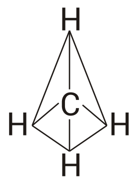
⇒ Cl has a higher electron gain enthalpy than F.

⇒ Order of electron gain enthalpy (Magnitude or Absolute value)
Cl > F > Br > I

⇒ Option (C) is ***CORRECT.***

127. Isoelectronic species:
O^{2-}, F^-, Na^+, Mg^{2+} (All contain 10 electrons)

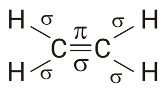
128. CH_4 is tetrahedral



sp^3 hybridization
(tetrahedral)

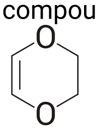
129. Discovery of noble gases meant that now every ninth element resembled the first element and not every eight elements.
This led to the loss of similarity with the musical notes which resulted in the abolishing of Newland's law of octaves.

130. Ethylene is



, it has 5σ bonds and 1π bond.

131. Heterocyclic compounds are organic compounds with a ring structure that contains carbon atom(s) and at least one different atom from carbon, such as N, O, S etc.
The atoms that are forming the ring are carbon and oxygen in the following compound:



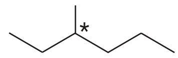
For all other compounds the ring is made up of on Carbon atoms.

⇒ Option (C) is ***CORRECT.***

132. Mercury (Hg) and Bromine (Br) are both liquids at room temperature while Gallium (Ga) is a liquid above 30^0C.
Among Hg and Br, Hg is metal and Br is a non-metal.

Thus, among the given options, the element which is a non-metal and exists as a liquid at room temperature is Bromine.

133.



sp^3-hybridization

134. Atomic numbers of N, O, F and Na are 7, 8, 9 and 11 respectively.

Therefore, total number of electrons in each of N^{3-}, O^{2-}, F^- and Na^+ is 10 and hence they are isoelectronic.

135. Mendeleev's periodic table had 8 groups.

Groups 1 to 7 were divided into two subgroups A and B while group VIII had 3 sets of elements.

It must be noted that the noble gases were not a part of Mendeleev's periodic table and were later added in the modified periodic table.

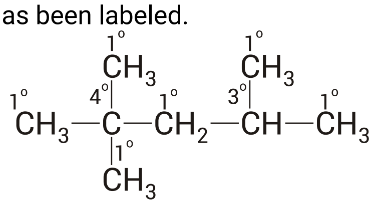
136. Lanthanides are called Rare Earth Metals.

Note: Lanthanides and Actinides both are f-Block elements of which only Lanthanides are called Rare earth metals.

137. The nature of given compounds is given below:
$B(OH)_3$ – Acidic
H_3PO_3 – Acidic
$Be(OH)_2$ – Amphoteric
$Al(OH)_3$ – Amphoteric
NaOH – Basic
$Ca(OH)_2$ – Basic

Option (B) has $B(OH)_3$ and $Al(OH)_3$. $B(OH)_3$ is acidic and $Al(OH)_3$ is amphoteric in nature. They have different nature.
⇒ Option (B) is ***CORRECT.***

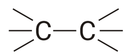
138. A tertiary carbon atom is a carbon atom bound to three other carbon atoms.
The primary, tertiary, and quaternary carbon in the molecule has been labeled.



⇒ It contains only one 3^0 or tertiary carbon atom.
⇒ Option (D) is ***CORRECT.***

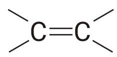
139.

(A) ethane



1σ

(B) ethene



$1\sigma, 1\pi$

(C) C_2 $\quad \mathrm{C=C}\ 2\pi$, (as per MOT)

(D) ethyne $\quad -\mathrm{C \equiv C}-1\sigma, 2\pi$

140. Higher oxidation states are stabilized by elements of high electronegativity like fluorine and oxygen as they have a high tendency of attracting electrons toward themselves thus making the high oxidation state stable.

141. **The rules for the nomenclature of Elements of Atomic Numbers greater than 100:**

(1) The name is derived directly from the atomic number of the element using the following numerical roots:
0 = nil
1 = un
2 = bi
3 = tri
4 = quad
5 = pent
6 = hex
7 = sept
8 = oct
9 = enn
(2) The roots are put together in the order of the digits which make up the atomic number and terminated by 'ium' to spell out the name. The final 'n' of 'enn' is elided when it occurs before 'nil', and the final 'i' of 'bi' and of 'tri' when it occurs before 'ium'.

$\underset{un}{1}\ \ \underset{nil}{0}\ \ \underset{enn+ium}{9}$

Hence the atomic number of the element unnilennium is 109.

142. All members of homologous series have same general formula but different molecular and structural formula physical properties generally have regular variation in homologous series.
For example general formula for alkane series is C_nH_{2n+2}.

143. The degree of unsaturation in C_3H_6O is,

D.O.U. = $\dfrac{(2 \times 3 + 2) - 6}{2} = 1$

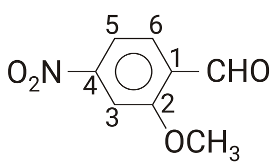
The compound has one degree of unsaturation with one oxygen atom.

⇒ The compound is acyclic Aldehyde.

⇒ Option (D) is ***CORRECT.***

Ester and carboxylic acid requires 2 oxygen atoms and in cyclic ketone, D.O.U. is 2 (one ring and one double bond of ketone)

144. The carbon chain in the given compound should be numbered as follows:



⇒ The IUPAC name of this compound is:
2-Methoxy-4-nitrobenzaldehyde.

145. Given ions are: P^{3-}, S^{2-}, Ca^{2+}, K^+, Cl^-
We can see that the given species are isoelectronic species and all have 18 electrons.
In P^{3-} 18 e^- experiences the charge of 15 protons. (atomic number of P)
In S^{2-} 18 e^- experiences the charge of 16 protons.
In Ca^{2+} 18 e^- experiences the charge of 20 protons.
In K^+ 18 e^- experiences the charge of 19 protons.
In Cl^- 18 e^- experiences the charge of 17 protons.

As the number of protons increases in the nucleus, effective nuclear charge increases, and ionic radius decreases.

⇒ The correct order for ionic radius is:
$P^{3-} > S^{2-} > Cl^- > K^+ > Ca^{2+}$

⇒ Option (A) is ***CORRECT.***

NOTE: In isoelectronic species, size increases with the increase of negative charge, and size decreases with an increase in positive charge.

146. $CH_2 = CH_2$ both the carbon atoms are sp^2 hybridised.

147. • Mendeleev was the author of the textbook **"Principles of Chemistry"** which help him to propose the Periodic law and to construct his periodic table of elements.
• He invented the accurate barometer.
• At the time of D.I. Mendeleev, the structure of the atom was **not** known.
• Element with atomic number 101 is known as Mendelevium, which is named after him

⇒ The statement given in option (C) is not true.

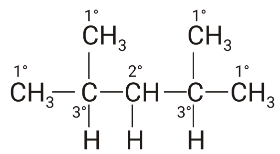
⇒ Option (C) is ***CORRECT.***

148. sp^2 and sp^3

$$\underset{sp^3}{CH_3}-\underset{sp^2}{CH}=\underset{sp^2}{CH}-\underset{sp^3}{CH_2}-\underset{sp^3}{CH_3}$$

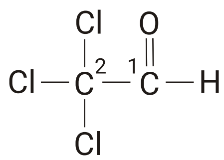
149. Eka – Aluminium = Gallium
Eka – Silicon = Germanium
[Eka means have configuration similar] Name given by Mendleev

150.



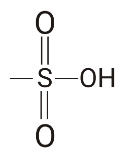
Only one 2° carbon is present in this compound.

151. The Carbon atom of the aldehyde group should be given higher priority than the C atom having the Cl groups.



⇒ The IUPAC name of $Cl_3C.CHO$ is 2,2,2-Trichloroethanal.

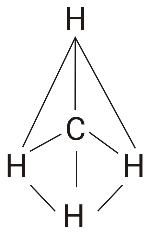
152.



Group present in sulphonic acids

153. Due to his full-filled configuration.

154.



(CH_4)

Tetrahedral

155. $I_{p_1} + I_{p_2} = 178 + 348 = +526$

156. In general, Electronegativity increases on moving from left to right across a period except for noble gases.

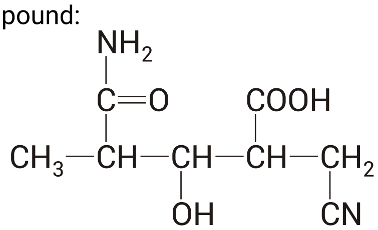
⇒ The most electronegative element in any period belongs to the 17th group i.e. the halogen family.

The general electronic configuration of the halogen family is: ns^2, np^5

⇒ The most electronegative element in a period has the electronic configuration of ns^2, np^5

157. As the e^- is to be removed from stable configuration.

158. Given compound:



Functional groups present are $-CONH_2$ (Amide), $-OH$ (Alcohol), $-COOH$ (carboxylic acid), and $-CN$ (cyanide)

⇒ Total number of functional group = 4

⇒ Option (B) is ***CORRECT.***

159. Isoelectronic means same number of electrons.

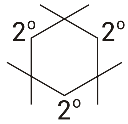
$Ca^{2+} = 18$

$Ar = 18$

160. The hydrogen attached to the 2^0 carbon is called 2^0 hydrogen.

The carbon which is connected to 2 carbon atoms is known as 2^0 carbon.

The 2^0 carbon has been marked in the molecule.



Number of 2^0 carbon = 3

⇒ Number of 2^0 hydrogen = 3 × 2 = 6

⇒ Answer = 6