

2.13

If $(c_{prod}, p_{prod}) = (c_1, p_1) \times (c_2, p_2)$ and $c_1, p_1, c_2, p_2 > 0$,

$$(\text{lower-bound}_{prod}, \text{upper-bound}_{prod}) = ((c_1 - (c_1 p_1))(c_2 - (c_2 p_2)), (c_1 + (c_1 p_1))(c_2 + (c_2 p_2))) \quad (1)$$

$$(\text{lower-bound}_{prod}, \text{upper-bound}_{prod}) = ((c_1 c_2 (1 - p_1)(1 - p_2), c_1 c_2 (1 + p_1)(1 + p_2))) \quad (2)$$

$$(3)$$

$$c_{prod} = \frac{1}{2}(c_1 c_2 (1 - p_1)(1 - p_2) + c_1 c_2 (1 + p_1)(1 + p_2)) \quad (4)$$

$$c_{prod} = \frac{c_1 c_2}{2}((1 - p_1)(1 - p_2) + (1 + p_1)(1 + p_2)) \quad (5)$$

$$c_{prod} = \frac{c_1 c_2}{2}(1 - p_1 - p_2 + p_1 p_2 + 1 + p_1 + p_2 + p_1 p_2) \quad (6)$$

$$c_{prod} = \frac{c_1 c_2}{2}(2 + 2p_1 p_2) \quad (7)$$

$$c_{prod} = c_1 c_2 (1 + p_1 p_2) \quad (8)$$

$$(9)$$

$$p_{prod} = \frac{\text{upper-bound}_{prod} - c_{prod}}{c_{prod}} \quad (10)$$

$$p_{prod} = \frac{c_1 c_2 (1 + p_1)(1 + p_2) - c_1 c_2 (1 + p_1 p_2)}{c_1 c_2 (1 + p_1 p_2)} \quad (11)$$

$$p_{prod} = \frac{(1 + p_1)(1 + p_2) - (1 + p_1 p_2)}{1 + p_1 p_2} \quad (12)$$

$$p_{prod} = \frac{1 + p_1 + p_2 + p_1 p_2 - 1 - p_1 p_2}{1 + p_1 p_2} \quad (13)$$

$$p_{prod} = \frac{p_1 + p_2}{1 + p_1 p_2} \quad (14)$$

For small values of p_1 and p_2 , $p_1 p_2 \rightarrow 0$
 $\therefore p_{prod} \approx p_1 + p_2$.