Assignment 4

1st Iteration:

- $V_0(s_1) = 10$, $V_0(s_2) = 1,0$, $V_0(s_3) = 0,0$

$V_1(s) = B^*(V_0(s))$

$$B^*(V)(s) = \max_{a \in \mathcal{A}} \{ \sum_{s' \in \mathcal{S}} \sum_{r \in \mathcal{D}} \mathcal{P}_R(s, a, r, s') \cdot (r + \gamma \cdot W(s')) \} \text{ for all } s \in \mathcal{N}$$

$Q(a,s)$

$Q_1(s_1, a_1) = P(s_1, a_1 s_1)\{R(s_1, a_1) + \gamma V_0(s_1)\}$
$+ P(s_1, a_1, s_2)\{R(s_1, a_1) + \gamma V_0(s_2)\}$
$+ P(s_1, a_1, s_3)\cdot\{R(s_1, a_1) + \gamma V_0(s_3)\}$
$= 0,2 \cdot [8 + 10] + 0,6 \cdot [8 + 1,0] +$
$+ 0,2\,[8 + 0] = \frac{18}{5} + \frac{27}{5} + \frac{8}{5} =$
$= \frac{53}{5} = 10,6$

$Q_1(s_1, a_2) = 0,1 \cdot [10 + 10) + 0,2 \cdot [10 + 1] +$
$0,7 \cdot [10 + 0] = 2 + \frac{11}{5} + 7 = 11,2$

$V_1(s_1) = \underline{11,2}$

$Q_1(s_2, a_1) = 0,3 \cdot [1 + \frac{10}{10}] + 0,3 \cdot [1 + 1] + 0,4[1 + 0]$
$= \frac{3 \cdot 13}{10} + 0,4 = \underline{4,3}$

$$Q_1(s_2, a_2) = 0{,}5 \cdot [-1 + 10] + 0{,}3[-1 + 1) + 0{,}2 \cdot [-1 + 0] = 4{,}5 - 0{,}2 = \underline{4{,}3}$$

↳ $V_1(s_2) = \underline{4{,}3}$ $\quad V_2(s_3) = \underline{0}$

↳ $\pi_1(s_1) = a_2$ & $\pi_2(s_2) = a_1 \,||\, a_2$

2nd Iteration:

$$Q_2(s_1, a_1) = 0{,}2(8 + 11{,}2) + 0{,}6(8 + 4{,}3) + 0{,}2 \cdot 8 = 8 + \frac{11{,}2}{5} + 4{,}3 \cdot 0{,}6 = 12{,}82$$

$$Q_2(s_1, a_2) = 0{,}1 \cdot [10 + 11{,}2) + 0{,}2 \cdot (10 + 4{,}3) + 0{,}7 \cdot 10 = 10 + 1{,}91 = 11{,}91$$

$V_2(s_2) = \underline{12{,}82}$

$$Q_1(s_2, a_1) = 0{,}3[1 + 11{,}2) + 0{,}3[1 + 4{,}3) + 0{,}4 \cdot 1 = 5{,}65$$

$$Q_2(s_2, a_2) = 0{,}5(-1 + 11{,}2) + 0{,}3(-1 + 4{,}3) + 0{,}2 \cdot -1 = 5{,}89$$

↳ $V_2(s_2) = 5,00$

$\pi_2(s_1) = a_1$ & $\pi_2(s_2) = a_2$

$$q_4(s,a) \overset{①}{R(s,a)} + \gamma \overset{②}{P(s'|a,s)} V(s')$$

a: $q_k(s_1, a_1) > q_k(s_1, a_2) \quad \forall\, k \geq 2$

b: $q_k(s_2, a_1) < q_k(s_2, a_2) \quad \forall\, k \geq 2$