$x \sim \mathcal{N}(\mu, \sigma^2)$

$$\mathbb{E}[U(x)] = \mathbb{E}\left[x - \frac{\alpha x^2}{2}\right] = \mathbb{E}[x] - \frac{\alpha}{2}\mathbb{E}[x^2] =$$

$$= \mu - \frac{\alpha}{2}\mathbb{E}[x^2]$$

$$\sigma^2 = \mathbb{E}[x^2] - \mathbb{E}[x]^2$$

$$\sigma^2 = \mathbb{E}[x^2] - \mu^2 \Rightarrow \mathbb{E}[U(x)] = \mu - \frac{\alpha}{2}(\sigma^2 + \mu^2)$$

$$x_{CE} \Rightarrow U(x_{CE}) = \mathbb{E}(U(x))$$

$$x_{CE} - \frac{\alpha}{2}x_{CE}^2 = \mu - \frac{\alpha}{2}(\sigma^2 + \mu^2)$$

~~Given that~~ we know that $x \geq 0$ and $\alpha > 0$ (concave)

$$\hookrightarrow \quad x_{CE} = \frac{1 \pm \sqrt{\alpha^2(\mu^2 + \sigma^2) - 2\alpha\mu + 1}}{\alpha}$$

It has to be ≥ 0

$$\hookrightarrow \quad x_{CE} = \frac{1 + \sqrt{\alpha^2(\mu^2 + \sigma^2) - 2\alpha\mu + 1}}{\alpha}$$

$$\pi_A = \mathbb{E}[x] - x_{CE} = \mu - x_{CE}$$

---

z risky asset : z : $\sim \mathcal{N}(\mu, \sigma^2)$

$1 - z \rightarrow r$

$$\hookrightarrow \quad \omega \sim \mathcal{N}(1 + r + z(\mu - r), z^2\sigma^2)$$

$$u(w) = w - \frac{\alpha w^2}{2}$$

maximize x_{CE}

3)

$p : x(1+\alpha)$

$1-p : x(1-\beta)$

↳ ~~return~~

Expected ~~return~~ Outcome : $p \cdot (1+\alpha)x + (1-p)(1-\beta)x$

a) $o_1 = f \cdot W_0(1+\alpha) + (1-f)W_0 = W_0(1+f\alpha)$

$o_2 = fW_0(1-\beta) + (1-f)W_0 = W_0(1-f\beta)$

b) $\log(o_1)$ & $\log(o_2)$

c) $\mathbb{E}(\log W) = p\log(W_0(1+f\alpha)) + (1-p)\log(W_0(1-\beta f))$

d) $\frac{d}{df}\mathbb{E}(\log(W)) = \frac{p}{W_0(1+f\alpha)}\alpha W_0 - \frac{(1-p)}{W_0(1-\beta f)}\beta W_0 \overset{!}{=} 0$

$$p\alpha \cancel{W_0} W_0 (1-\beta f) = (1-p)\beta \cancel{W_0} W_0 (1+f\alpha)$$

$$p\alpha - p\alpha\beta f = (1-p)\beta + (1-p)\alpha\beta f$$

$$p\alpha = (1-p)\beta + \alpha\beta f$$

$$f^* = \frac{p\alpha - (1-p)\beta}{\alpha\beta} = \frac{p}{\beta} - \frac{(1-p)}{\alpha}$$

$$\frac{d^2 E(\log(\omega))}{df^2} = \frac{d}{df}\left[p\alpha \frac{1}{1+f\alpha} - (1-p)\beta \frac{1}{1-\beta f}\right]$$

$$= p\alpha\left[-\frac{1}{(1+f\alpha)^2}\alpha - (1-p)\beta\frac{1}{(1-\beta f)^2}\cdot\beta\right] < 0$$

e) intuitive: if $\alpha \uparrow$ $\rightarrow$ invest ~~little~~ a lot ✓

if $\beta \uparrow$ $\rightarrow$ invest little