

Q1)

- Calculate the open-loop transfer function of this plant. You can use the linearized differential equation to calculate the transfer function.
- Show the plots as subplots next to each other and comment and explain the difference between step and impulse response.

$$\ddot{x} = \frac{3}{5} g \sin(\alpha) = \frac{30}{5} \sin(\alpha)$$

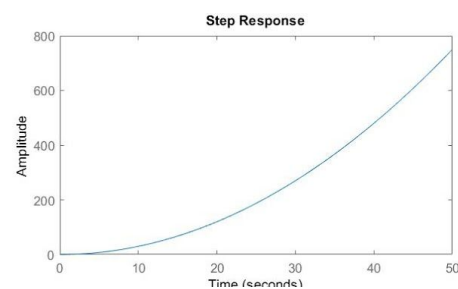
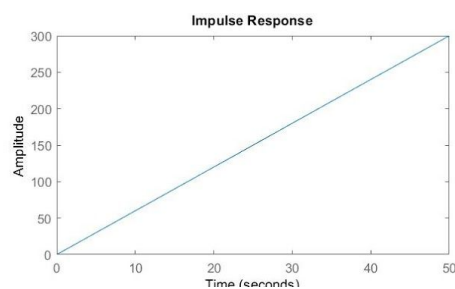
-> Laplace $\Rightarrow s^2 X = \frac{30}{5} \sin(\alpha)$ with initial conditions set to zero

$$[x(0) = \dot{x}(0) = 0]$$

Input: α ; Output: position x

$$\Rightarrow \text{Transfer Function } G = X/\alpha = \frac{30}{5s^2}$$

Because the plate was tilted for an infinitely short time, the ball is put into motion and its velocity stays constant (no acceleration) because the plate is back in a neutral position. That's why the impulse response is linear. With the step response the plate stays at an angle, thereby accelerating the ball and resulting in the step response seen below.



Impulse response and Step response of the ball dynamics system

Q2)

- **Is this response underdamped, critically damped or overdamped? What happens to your response if you increase K_d ? If you consider our real system with visual Feedback, what could be the risk of increasing K_d too high?**

$$(X_{des} - X_{meas})(K_p + 1/s * K_I + s * K_D) = (\text{Alpha})$$

$$X_{meas} = 30 / (5 * s^2) * (\text{Alpha})$$

$$T = K_p + 1/s * K_I + s * K_D$$

$$(X_{des} - 30 / (5 * s^2) * (\text{Alpha})) * T = \text{Alpha}$$

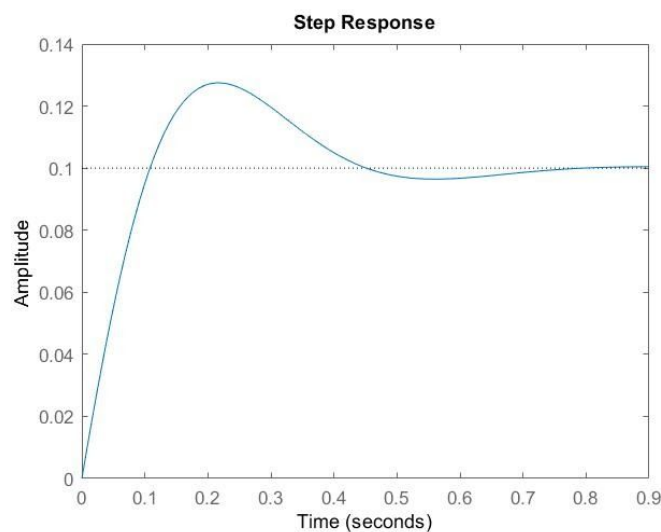
$$X_{des} * T = (30 / (5 * s^2) * T + 1) * (\text{Alpha})$$

$$(\text{Alpha}) / X_{des} = T / ((30 / 5 * s^2) * T + 1) = G$$

- **Hand in the plot for the two different K_d**

The system is underdamped. If we increase K_d , there is less overshoot and the response heads more gradually to a steady-state.

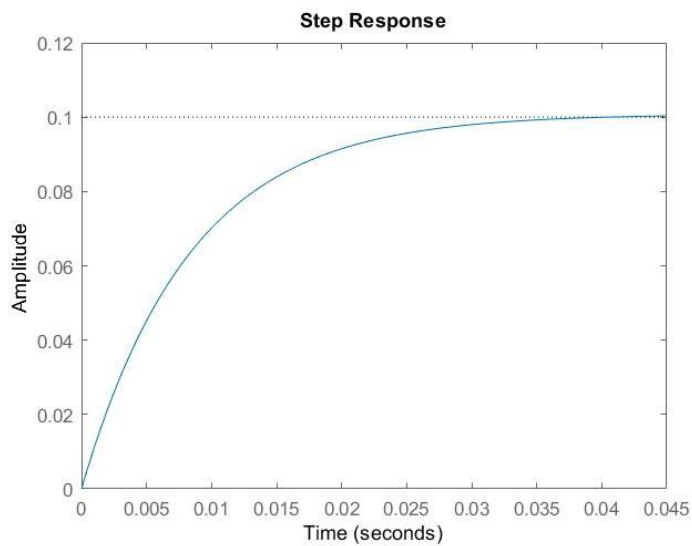
If we choose K_d too big, it takes too long to reach the steady-state and by then the ball would have already fallen off the plate.



Underdamped at $K_d = 2$ ($K_p = 20$, $K_i = 1$)

Prelab 07 - 12.05.2020

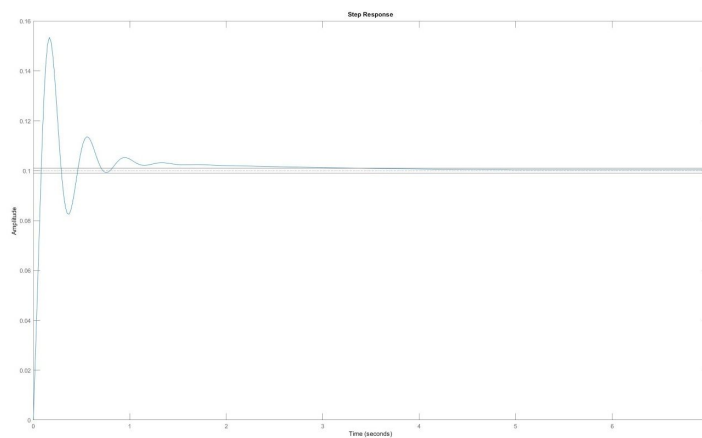
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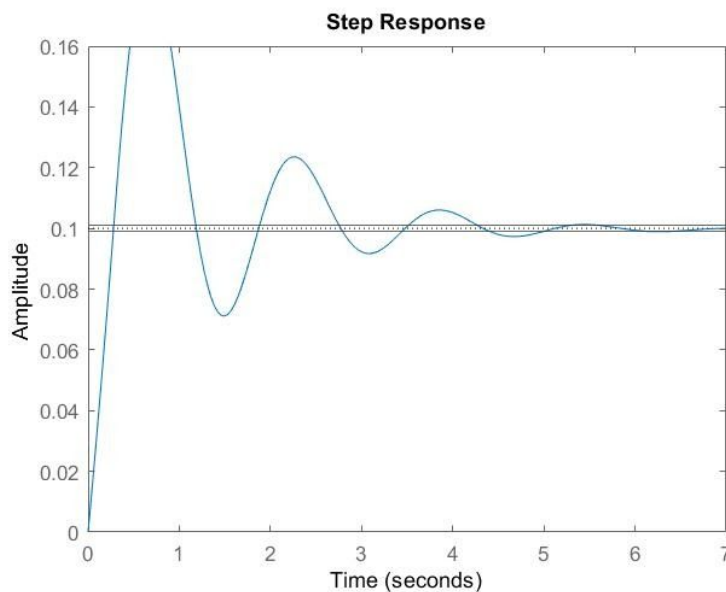
Critically damped at $K_d = 20$ ($K_p = 20$, $K_i = 1$)

Q3)

- **Hand in the plot for your report. What happens if the delay increases to 300 ms? Are you able to stabilize the system within 5 seconds?**



Feedback loop with delay time constant = 0.08 s (80 ms), $K_p = 1.1$, $K_i = 0.2$, $K_d = 2$;



Feedback loop delay time constant = 0.3 s (300 ms)

With the controller parameters from above the amplitude at 5 seconds too high for the controller to compensate. Using $K_p = 0.4$; $K_i = 0.05$; $K_d = 0.54$; we can stabilise the system at 5 s.

Q4)

- **How could you use this constraint to see if the PID controller's output exceeds the capabilities of our system?**

If the returned angle α exceeds 25° the system is saturated and it cannot be realised physically.

Q5)

- **When analyzing the system further you notice that the PID controller's output also exceeds the speed limits of your motors. What could be one of the problems you face?**

If the initial offset is too high and the counteracting speed exceeds the motor speed limit we cannot stabilise the system.

Lab 07

Table of Contents

Prelab Q1	1
Prelab 02	2
Prelab 03	3
Prelab 04	5

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Prelab Q1

create the transfer function of the system (you should use the `tf()` function):

```
transBallDynamics = tf([30], [5 0 0])

% set step options to have a step-size of 0.1 instead of 1 (standard),
% use
% stepDataOptions()
opt_StepAmplitude01 = stepDataOptions('StepAmplitude', 0.1);

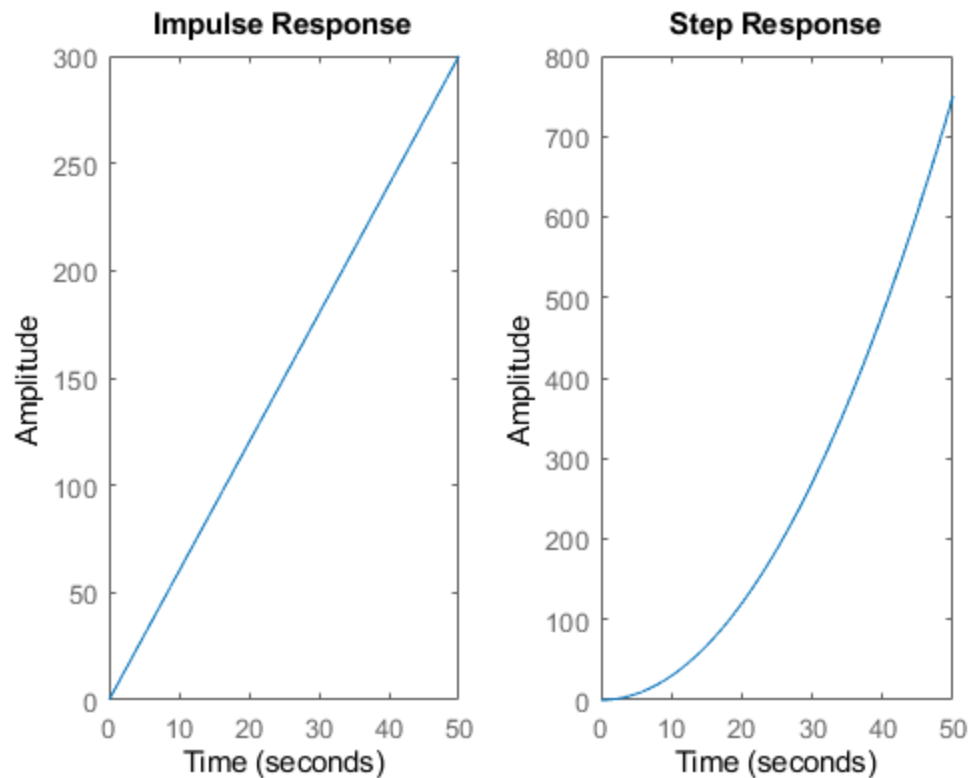
% open loop impulse and step response in one figure (two subplots):
figure(1)
subplot(1,2,1)
impz(transBallDynamics)
subplot(1,2,2)
step(transBallDynamics, opt_StepAmplitude01)
```

transBallDynamics =

30

5 s^2

Continuous-time transfer function.



Prelab 02

Controller Parameters:

```
K_P = 20;
K_I = 1;
K_D = 2;
K_D_ = 20;
```

```
% create the transfer function of the Controller:
```

```
transPID_Controller = tf([K_D_ K_P K_I], [1 0])
```

```
transForward = transPID_Controller * transBallDynamics;
```

```
% create the transfer function of the feedback loop
```

```
% trans_Q2_feedback = tf([6*K_D 6*K_P 6*K_I], [1 6*K_D 6*K_P 6*K_I]);
```

```
trans_Q2_feedback = feedback(transForward, 1)
```

```
% Plot the closed loop step response
```

```
figure(2)
```

```
step(trans_Q2_feedback, opt_StepAmplitude01)
```

```
% under damped using K_d = 2, and critically damped using K_d = 20;
```

```
transPID_Controller =
```

```
20 s^2 + 20 s + 1
```

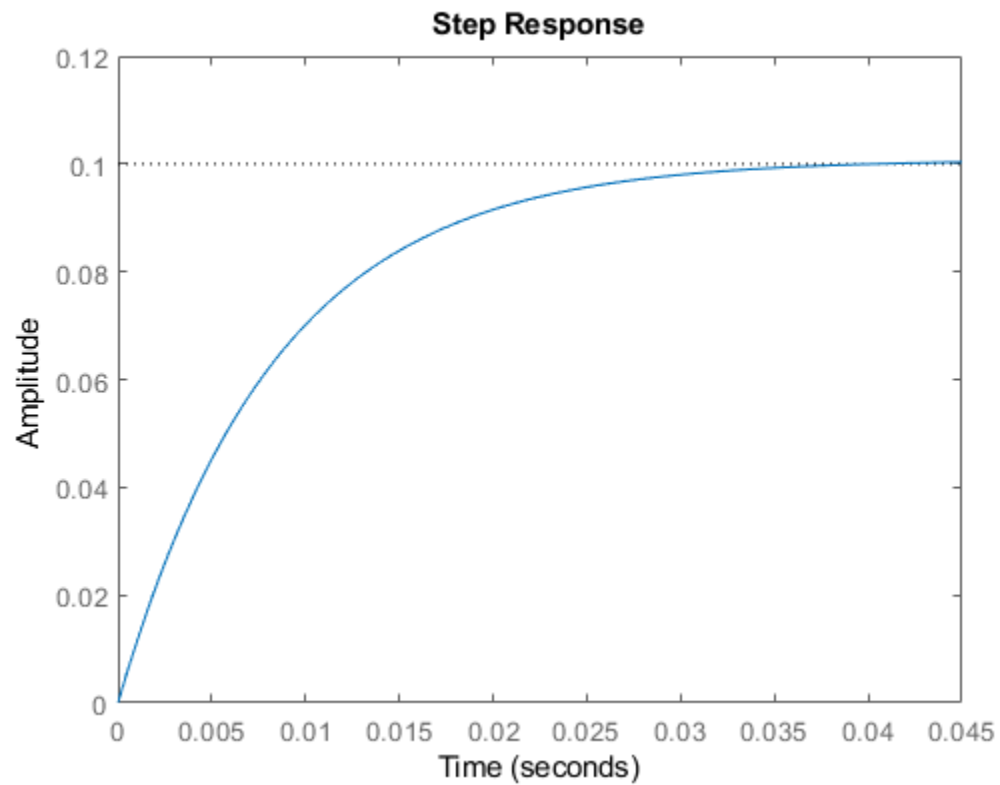
$$\frac{\quad}{s}$$

Continuous-time transfer function.

`trans_Q2_feedback =`

$$\frac{600 s^2 + 600 s + 30}{5 s^3 + 600 s^2 + 600 s + 30}$$

Continuous-time transfer function.



Prelab 03

controller constants:

```
Kp = 0.4;  
Ki = 0.05;  
Kd = 0.54;
```

```
% create the transfer function of the Controller  
transPID_ControllerDelay = tf([Kd Kp Ki], [1 0])
```

```
% Variables for delay:
DELAY=0.08; % 80ms
DELAY_=0.3; % 300ms

% create the transfer function of the delay in the system
% trans_Q3_delay = exp(-DELAY * s);
transDelay = tf(1, 1, 'InputDelay', DELAY_)

% create the transfer function of the feedback loop with delay
transForwardDelay = transPID_ControllerDelay * transBallDynamics;
transFeedbackDelay = feedback(transForwardDelay, transDelay)

% Plot the closed loop step response of the system with delay
y_top = 0.16;
y_bot = 0;
time_bot = 0;
time_top = 7;

figure(3)
step(transFeedbackDelay, opt_StepAmplitude01)
axis([time_bot time_top y_bot y_top])
yline(0.1+0.001)
yline(0.1-0.001)
% check design criteria for "steady state" error at 5 seconds
```

transPID_ControllerDelay =

$$\frac{0.54 s^2 + 0.4 s + 0.05}{s}$$

Continuous-time transfer function.

transDelay =

$$\exp(-0.3s) * (1)$$

Continuous-time transfer function.

transFeedbackDelay =

$$A = \begin{array}{c|ccc} & x1 & x2 & x3 \\ \hline x1 & -3.24 & -2.4 & -0.3 \\ x2 & 1 & 0 & 0 \\ x3 & 0 & 1 & 0 \end{array}$$

$$B = \begin{array}{c|c} & u1 \\ \hline x1 & 2 \end{array}$$


```
x2    0
x3    0
```

```
C =
```

```
      x1      x2      x3
y1  1.62    1.2    0.15
```

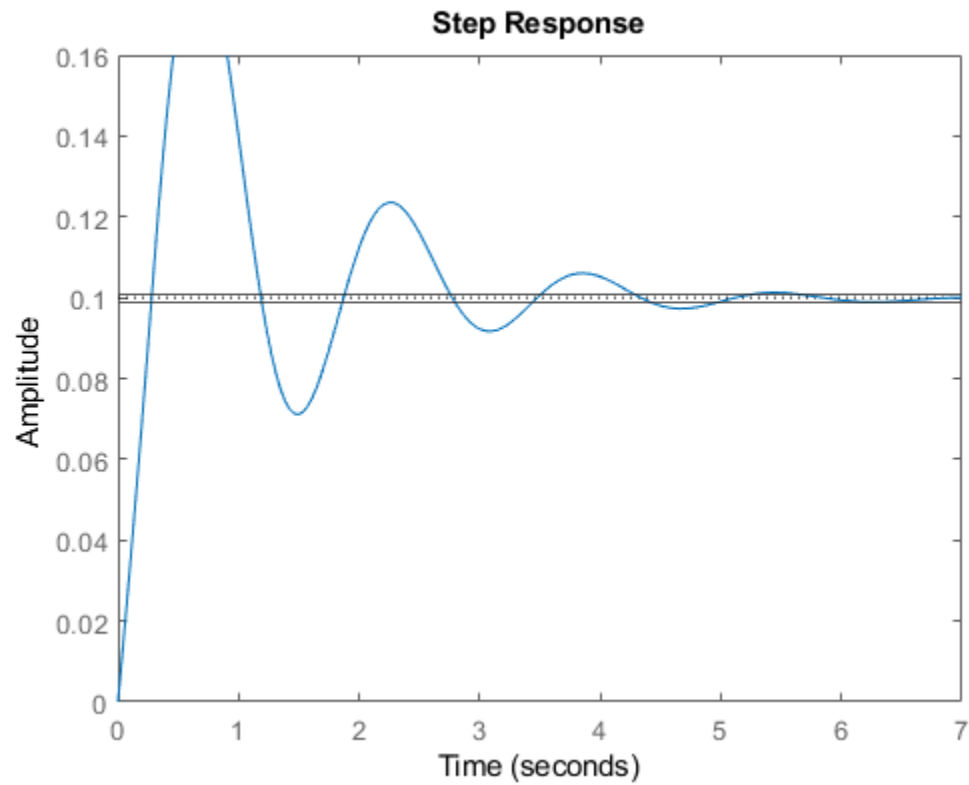
```
D =
```

```
      u1
y1    0
```

(values computed with all internal delays set to zero)

Internal delays (seconds): 0.3

Continuous-time state-space model.



Prelab 04

```
% Hint:
% - Make use of the linearized differential equations
GRAVITY = 9.81;

syms x(alpha)
equation = diff(x, alpha) == 3/5 * GRAVITY;
```

```
solution = dsolve(eqation);  
% - Filter out the first data points from the time delay to get rid of  
% the  
% incontinuity at the beginning of the sytem-response (otherwise the  
% derivative explodes)  
[x_output, t_output] = step(transFeedbackDelay, opt_StepAmplitude01);  
  
% - Make sure to use the right units (rad, degree, ...) --> check if  
% your  
% values make sense if you are not sure. E.g. a free-falling ball  
% has 9.81 m/s^2 acceleration
```

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