Exponential Control k Treatment Model

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Grid Space

We will take the grid-space to be (ξ_1, ξ_2) where $\xi_1 = \log(\lambda_c)$ (λ_c is the control arm's λ), and $\xi_2 = \log(h)$ ($h = \lambda_t/\lambda_c$ is the hazard rate, i.e. the ratio of treatment to control λ parameter). Note that the grid-space is not in the natural parameter space. This results in a slightly different upper-bound formulation, which is outlined below.

Upper Bound

The 0th order Monte Carlo term and its upper bound need no change from this reparametrization.

Gradient Term

Let $\xi = (\xi_1, \xi_2)$, $\lambda = (\lambda_c, \lambda_t)$, and A be the event of false rejection.

$$\begin{aligned}\nabla_\xi P_\lambda(A) = \nabla_\xi \int_A \frac{P_\lambda}{P_{\lambda_0}} dP_{\lambda_0} &= \int_A \nabla_\xi \frac{P_\lambda}{P_{\lambda_0}} dP_{\lambda_0} \\ &= \int_A (D_\xi \lambda)^\top \nabla_\lambda \frac{P_\lambda}{P_{\lambda_0}} dP_{\lambda_0}\end{aligned}$$

If ξ_0 is the point at which we are Taylor expanding, it suffices to compute this gradient at $\xi = \xi_0$ (equivalently, λ_0). This results in

$$\nabla_\xi P_{\lambda_0}(A) = \int_A (D_\xi \lambda(\xi_0))^\top (T - \nabla_\lambda A(\lambda_0)) dP_{\lambda_0}$$

Hence, our gradient Monte Carlo estimate will be

$$\hat{\nabla} f := \frac{1}{N} \sum_{i=1}^N D_\xi \lambda(\xi_0)^\top (T(X_i) - \nabla_\lambda A(\lambda_0)) \mathbb{1}_{X_i \in A}$$

Hessian Term