

asm binary search

signature:

static	$n : \mathbb{N}$	
controlled	$a : \mathbb{N} \rightarrow \mathbb{Z}$	(uninitialised)
controlled	$x : \mathbb{Z}$	(uninitialised)
controlled	$l : \mathbb{N}$	(initially 0)
controlled	$r : \mathbb{N}$	(initially $n - 1$)
controlled	$pos : \mathbb{N}$	(initially -1)

rule Main:

if $l \leq r \wedge pos = -1$

then let $m = (l+r) \text{ div } 2$ in

 if $a(m) = x$

 then $pos := m$

 else if $a(m) < x$

 then $l := m + 1$

 else $r := m - 1$

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SYMBOLIC EXECUTION

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rule Main:

if $l \leq r \wedge \text{pos} = -1$

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 if $a(m) = x$

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SYMBOLIC EXECUTION

Main^1 (evaluated in S_0)

if $l \leq r \wedge \text{pos} = -1$

then let $m = (l+r) \text{ div } 2$ in

 if $a(m) = x$

 then $\text{pos} := m$

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static	$n : \mathbb{N}$	(defined as 4)
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rule Main:

if $l \leq r$ 1 pos = -1

then let $m = (l+r) \text{ div } 2$ in

if $a(m) = x$

then pos := m

else if $a(m) < x$

then $l := m + 1$

else $r := m - 1$

SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

if $0 \leq l - 1 = -1$

then let $m = (l+r) \text{ div } 2$ in

if $a(m) = x$

then pos := m

else if $a(m) < x$

then $l := m + 1$

else $r := m - 1$

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let $m = (l+r) \text{ div } 2$ in

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if $a(m) = x$

then pos := m

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then $l := m + 1$

else $r := m - 1$

SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

let $m = 1$ in

if $a(m) = x$

then pos := m

else if $a(m) < x$

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rule Main:

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SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

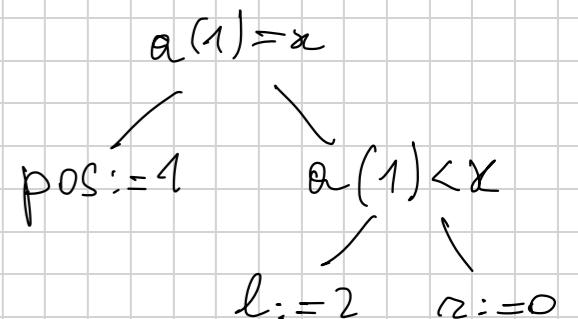
if $a(1) = x$

then pos := 1

else if $a(1) < x$

then $l := 2$

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Main² = Main¹ (S_0) seq Main

if $a(1) = x$

then pos := 1

else if $a(1) < x$

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seq
[Main]

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rule Main:

```

if  $l \leq r$   $\wedge$   $\text{pos} = -1$ 
then let  $m = (l+r) \text{ div } 2$  in
    if  $a(m) = x$ 
    then  $\text{pos} := m$ 
    else if  $a(m) < x$ 
    then  $l := m+1$ 
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```

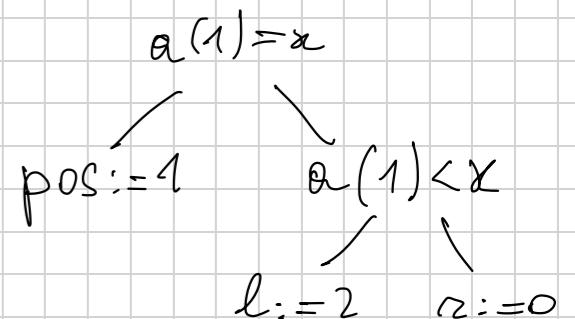
SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

```

if  $a(1) = x$ 
then  $\text{pos} := 1$ 
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```



Main² = Main¹ (S_0) seq Main

```

if  $a(1) = x$ 
then  $\text{pos} := 1$  seq [Main]
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```

if  $l \leq r$   $\wedge$   $\text{pos} = -1$ 
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    if  $a(m) = x$ 
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```

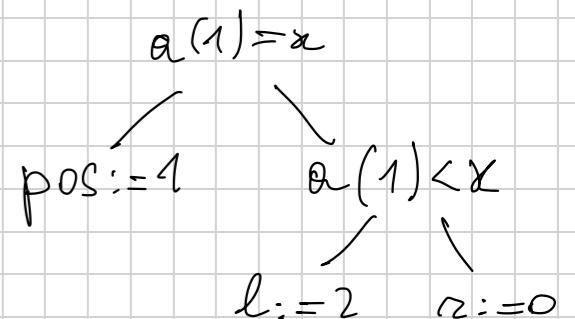
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if  $a(1) = x$ 
then  $\text{pos} := 1$ 
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then  $l := 2$ 
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```



Main² = Main¹ (S_0) seq Main

```

if  $a(1) = x$ 
then  $\text{pos} := 1$  seq [Main]
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```

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if $l \leq r$ $\wedge \text{pos} = -1$

then let $m = (l+r) \text{ div } 2$ in

if $a(m) = x$

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SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

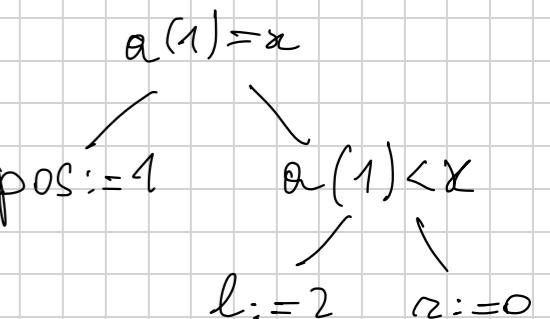
if $a(1) = x$

then $\text{pos} := 1$

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Main² = Main¹ (S_0) seq Main

if $a(1) = x$

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```

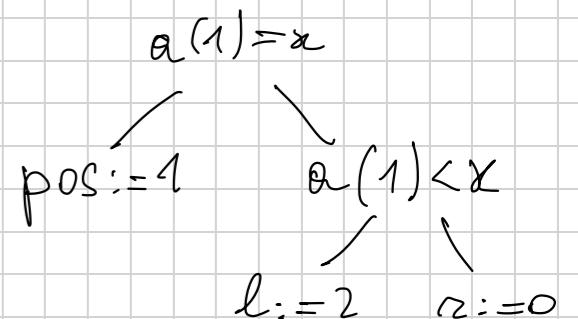
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Main¹ (evaluated in S_0)

```

if  $a(1) = x$ 
then pos := 1
else if  $a(1) < x$ 
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else  $r := 0$ 

```



Main² = Main¹ (S_0) seq Main

```

if  $a(1) = x$ 
then pos := 1 seq
if  $l \leq r$  1 pos = -1
then let  $m = (l+r) \text{ div } 2$  in
    if  $a(m) = x$ 
    then pos := m
    else if  $a(m) < x$ 
    then  $l := m + 1$ 
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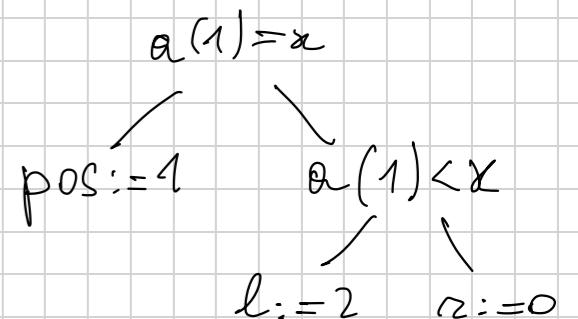
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if  $a(1) = x$ 
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else if  $a(1) < x$ 
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```



Main² = Main¹ (S_0) seq Main

```

if  $a(1) = x$ 
then pos := 1 seq  $\top = \text{false}$ 
if  $l \leq r$  1 pos = -1
then let  $m = (l+r) \text{ div } 2$  in
    if  $a(m) = x$ 
        then pos := m
    else if  $a(m) < x$ 
        then  $l := m+1$ 
    else  $r := m-1$ 
else if  $a(1) < x$ 
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```

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```

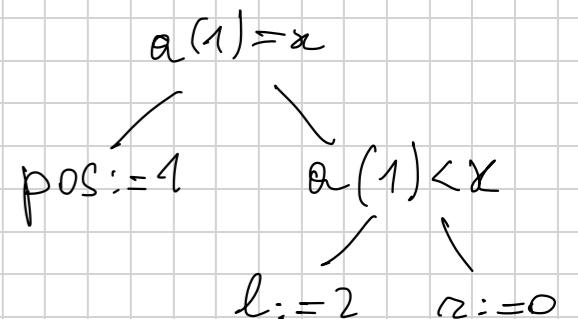
SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

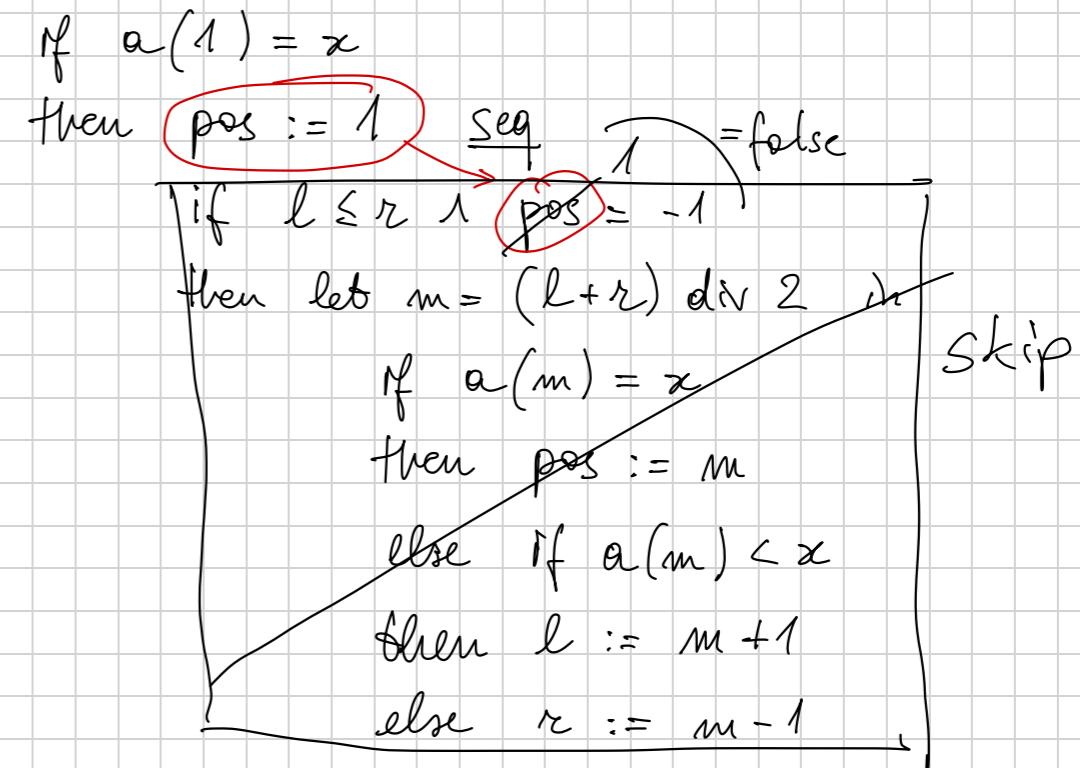
```

if  $a(1) = x$ 
then pos := 1
else if  $a(1) < x$ 
then  $l := 2$ 
else  $r := 0$ 

```



Main² = Main¹ (S_0) seq Main



```

else if  $a(1) < x$ 
then  $l := 2$  seq [Main]
else  $r := 0$  seq [Main]

```

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controlled	$\text{pos} : \mathbb{N}$	(initially -1)

rule Main:

```

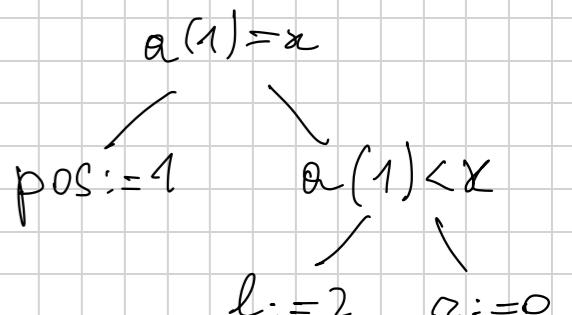
if  $l \leq r$  1 pos = -1
then let  $m = (l+r) \text{ div } 2$  in
    if  $a(m) = x$ 
    then pos := m
    else if  $a(m) < x$ 
    then l := m + 1
    else r := m - 1
  
```

SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

```

if  $a(1) = x$ 
then pos := 1
else if  $a(1) < x$ 
then l := 2
else r := 0
  
```



Main² = Main¹ (S_0) seq Main

```

if  $a(1) = x$ 
then pos := 1
  
```

~~seq skip~~

```

else if  $a(1) < x$ 
then l := 2
  
```

seq [Main]

```

else r := 0
  
```

seq [Main]

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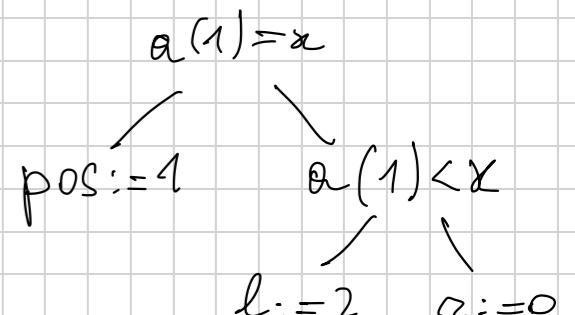
if  $l \leq r$  1 pos = -1
then let  $m = (l+r) \text{ div } 2$  in
    if  $a(m) = x$ 
    then pos := m
    else if  $a(m) < x$ 
    then l := m + 1
    else r := m - 1
  
```

SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

```

if  $a(1) = x$ 
then pos := 1
else if  $a(1) < x$ 
then l := 2
else r := 0
  
```



Main² = Main¹ (S_0) seq Main

```

if  $a(1) = x$ 
then pos := 1 seq skip
else if  $a(1) < x$ 
then l := 2 seq
    if  $l \leq r$  1 pos = -1
    then let  $m = (l+r) \text{ div } 2$  in
        if  $a(m) = x$ 
        then pos := m
        else if  $a(m) < x$ 
        then l := m + 1
        else r := m - 1
      else r := 0 [Main]
    
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then let  $m = (l+r) \text{ div } 2$  in
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    then pos := m
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    else  $r := m - 1$ 

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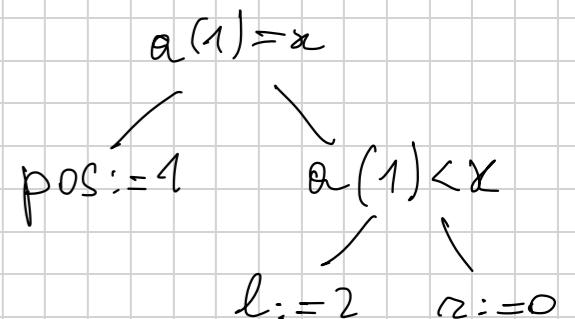
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Main¹ (evaluated in S_0)

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if  $a(1) = x$ 
then pos := 1
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```



Main² = Main¹ (S_0) seq Main

```

if  $a(1) = x$ 
then pos := 1
else if  $a(1) < x$ 
then  $l := 2$  seq
    if  $l \leq r$  1 pos = -1
    then let  $m = (l+r) \text{ div } 2$  in
        if  $a(m) = x$ 
        then pos := m
        else if  $a(m) < x$ 
        then  $l := m + 1$ 
        else  $r := m - 1$ 
    else r := 0 seq [Main]

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if $l \leq r$ 1 pos = -1

then let $m = (l+r) \text{ div } 2$ in

if $a(m) = x$

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Main¹ (evaluated in S_0)

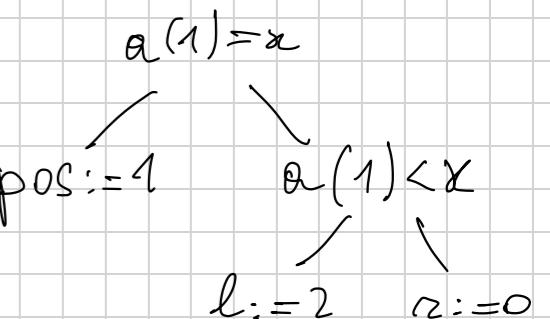
if $a(1) = x$

then pos := 1

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then $l := 2$

else $r := 0$



Main² = Main¹ (S_0) seq Main

if $a(1) = x$

then pos := 1

else if $a(1) < x$

then $l := 2$ seq

if $2 \leq 3$ 1 -1 = -1

then let $m = (2+3) \text{ div } 2$ in

if $a(m) = x$

then pos := m

else if $a(m) < x$

then $l := m + 1$

else $r := m - 1$

else $r := 0$ seq [Main]

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rule Main:

```

if  $l \leq r$   $\wedge$   $\text{pos} = -1$ 
then let  $m = (l+r) \text{ div } 2$  in
    if  $a(m) = x$ 
    then  $\text{pos} := m$ 
    else if  $a(m) < x$ 
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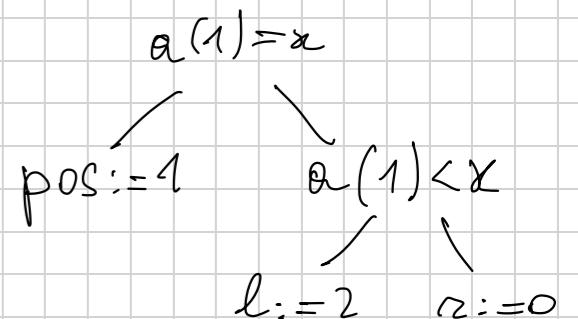
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if  $a(1) = x$ 
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then  $\text{pos} := 1$ 
else if  $a(1) < x$ 
then  $l := 2$  seq

```

$= 2$

let $m = \overbrace{(2+3) \text{ div } 2}^{= 2}$ in

```

if  $a(m) = x$ 
then  $\text{pos} := m$ 
else if  $a(m) < x$ 
then  $l := m+1$ 
else  $r := m-1$ 
else  $r := 0$  seq [Main]

```

asm binary search

signature:

static	$n : \mathbb{N}$	(defined as 4)
controlled	$a : \mathbb{N} \rightarrow \mathbb{Z}$	(uninitialised)
controlled	$x : \mathbb{Z}$	(uninitialised)
controlled	$l : \mathbb{N}$	(initially 0)
controlled	$r : \mathbb{N}$	(initially $n - 1$)
controlled	$\text{pos} : \mathbb{N}$	(initially -1)

rule Main:

```

if  $l \leq r$   $\wedge$   $\text{pos} = -1$ 
then let  $m = (l+r) \text{ div } 2$  in
    if  $a(m) = x$ 
    then  $\text{pos} := m$ 
    else if  $a(m) < x$ 
    then  $l := m+1$ 
    else  $r := m-1$ 

```

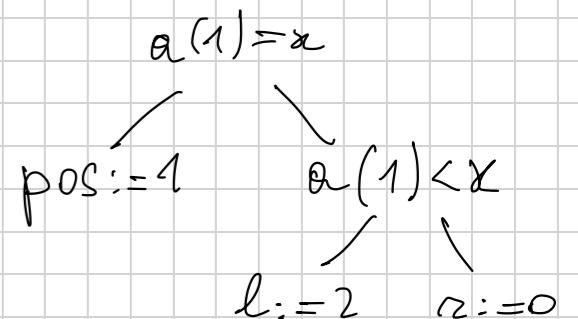
SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

```

if  $a(1) = x$ 
then  $\text{pos} := 1$ 
else if  $a(1) < x$ 
then  $l := 2$ 
else  $r := 0$ 

```



Main² = Main¹ (S_0) seq Main

```

if  $a(1) = x$ 
then  $\text{pos} := 1$ 
else if  $a(1) < x$ 
then  $l := 2$  seq

```

```

if  $a(2) = x$ 
then  $\text{pos} := 2$ 
else if  $a(2) < x$ 
then  $l := 2 + 1$ 
else  $r := 2 - 1$ 
else  $r := 0$  seq [Main]

```

asm binary search

signature:

static	$n : \mathbb{N}$	(defined as 4)
controlled	$a : \mathbb{N} \rightarrow \mathbb{Z}$	(uninitialised)
controlled	$x : \mathbb{Z}$	(uninitialised)
controlled	$l : \mathbb{N}$	(initially 0)
controlled	$r : \mathbb{N}$	(initially $n - 1$)
controlled	$\text{pos} : \mathbb{N}$	(initially -1)

rule Main:

```

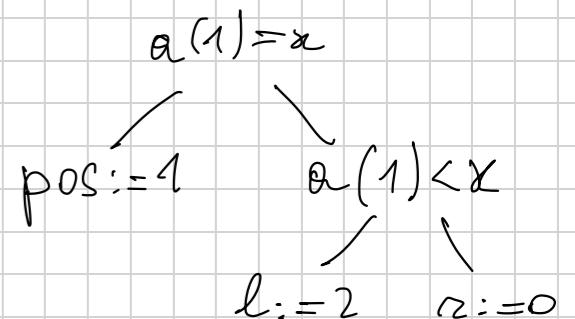
if  $l \leq r$  1 pos = -1
then let  $m = (l+r) \text{ div } 2$  in
    if  $a(m) = x$ 
    then pos := m
    else if  $a(m) < x$ 
    then l := m + 1
    else r := m - 1
  
```

SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

```

if  $a(1) = x$ 
then pos := 1
else if  $a(1) < x$ 
then l := 2
else r := 0
  
```



Main² = Main¹ (S_0) seq Main

```

if  $a(1) = x$ 
then pos := 1
else if  $a(1) < x$ 
then l := 2 seq
  
```

```

if  $a(2) = x$ 
then pos := 2
else if  $a(2) < x$ 
then l := 3
else r := 1
else r := 0 seq [Main]
  
```

asm binary search

signature:

static	$n : \mathbb{N}$	(defined as 4)
controlled	$a : \mathbb{N} \rightarrow \mathbb{Z}$	(uninitialised)
controlled	$x : \mathbb{Z}$	(uninitialised)
controlled	$l : \mathbb{N}$	(initially 0)
controlled	$r : \mathbb{N}$	(initially $n - 1$)
controlled	$\text{pos} : \mathbb{N}$	(initially -1)

rule Main:

```

if  $l \leq r$  1 pos = -1
then let  $m = (l+r) \text{ div } 2$  in
    if  $a(m) = x$ 
    then pos := m
    else if  $a(m) < x$ 
    then  $l := m + 1$ 
    else  $r := m - 1$ 

```

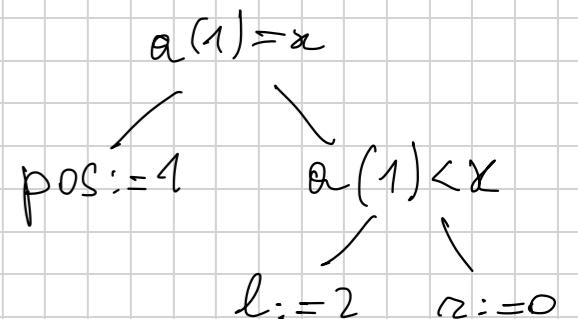
SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

```

if  $a(1) = x$ 
then pos := 1
else if  $a(1) < x$ 
then  $l := 2$ 
else  $r := 0$ 

```



Main² = Main¹ (S_0) seq Main

```

if  $a(1) = x$ 
then pos := 1
else if  $a(1) < x$ 
then  $l := 2$  seq

```

$l := 2$ seq


```

if  $a(2) = x$ 
then  $l := 2$  seq pos := 2
else if  $a(2) < x$ 
then  $l := 2$  seq  $l := 3$ 
else  $l := 2$  seq  $r := 1$ 

```

$l := 2$ seq

$l := 3$ seq

$r := 1$ seq


```

else  $r := 0$  seq [Main]

```

asm binary search

signature:

static	$n : \mathbb{N}$	(defined as 4)
controlled	$a : \mathbb{N} \rightarrow \mathbb{Z}$	(uninitialised)
controlled	$x : \mathbb{Z}$	(uninitialised)
controlled	$l : \mathbb{N}$	(initially 0)
controlled	$r : \mathbb{N}$	(initially $n - 1$)
controlled	$\text{pos} : \mathbb{N}$	(initially -1)

rule Main:

if $l \leq r$ $\wedge \text{pos} = -1$

then let $m = (l+r) \text{ div } 2$ in

if $a(m) = x$

then $\text{pos} := m$

else if $a(m) < x$

then $l := m + 1$

else $r := m - 1$

SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

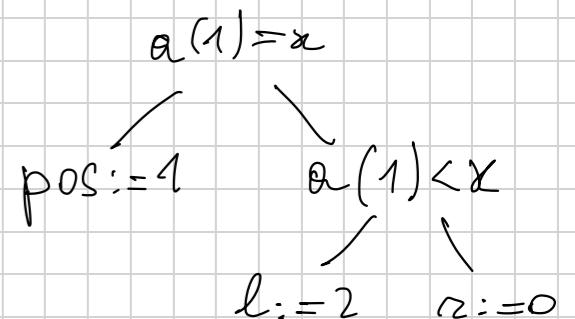
if $a(1) = x$

then $\text{pos} := 1$

else if $a(1) < x$

then $l := 2$

else $r := 0$



Main² = Main¹ (S_0) seq Main

if $a(1) = x$

then $\text{pos} := 1$

else if $a(1) < x$

then if $a(2) = x$

then $l := 2$ seq $\text{pos} := 2$

else if $a(2) < x$

then $\cancel{l := 2}$ seq $l := 3$

else $\cancel{l := 2}$ seq $r := 1$

else $r := 0$ seq [Main]

asm binary search

signature:

static	$n : \mathbb{N}$	(defined as 4)
controlled	$a : \mathbb{N} \rightarrow \mathbb{Z}$	(uninitialised)
controlled	$x : \mathbb{Z}$	(uninitialised)
controlled	$l : \mathbb{N}$	(initially 0)
controlled	$r : \mathbb{N}$	(initially $n - 1$)
controlled	$\text{pos} : \mathbb{N}$	(initially -1)

rule Main:

if $l \leq r$ 1 pos = -1

then let $m = (l+r) \text{ div } 2$ in

if $a(m) = x$

then pos := m

else if $a(m) < x$

then $l := m + 1$

else $r := m - 1$

SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

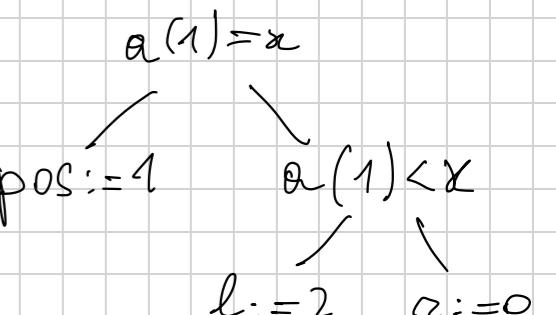
if $a(1) = x$

then pos := 1

else if $a(1) < x$

then $l := 2$

else $r := 0$



Main² = Main¹ (S_0) seq Main

if $a(1) = x$

then pos := 1

else if $a(1) < x$

then if $a(2) = x$

then { $l := 2$, pos := 2 }

else if $a(2) < x$

then $l := 3$

else { $l := 2$, $r := 1$ }

else $r := 0$ seq [Main]

asm binary search

signature:

static	$n : \mathbb{N}$	(defined as 4)
controlled	$a : \mathbb{N} \rightarrow \mathbb{Z}$	(uninitialised)
controlled	$x : \mathbb{Z}$	(uninitialised)
controlled	$l : \mathbb{N}$	(initially 0)
controlled	$r : \mathbb{N}$	(initially $n - 1$)
controlled	$\text{pos} : \mathbb{N}$	(initially -1)

rule Main:

```

if  $l \leq r \wedge \text{pos} = -1$ 
then let  $m = (l+r) \text{ div } 2$  in
    if  $a(m) = x$ 
    then  $\text{pos} := m$ 
    else if  $a(m) < x$ 
    then  $l := m+1$ 
    else  $r := m-1$ 

```

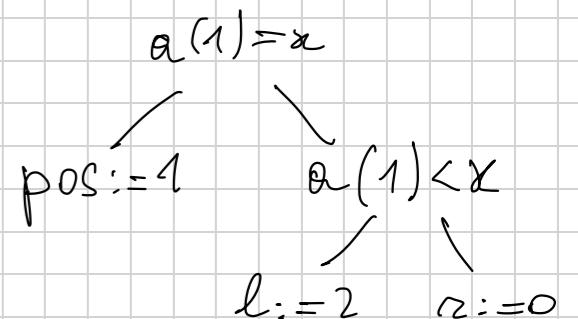
SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

```

if  $a(1) = x$ 
then  $\text{pos} := 1$ 
else if  $a(1) < x$ 
then  $l := 2$ 
else  $r := 0$ 

```



Main² = Main¹ (S_0) seq Main

```

if  $a(1) = x$ 
then  $\text{pos} := 1$ 
else if  $a(1) < x$ 
then if  $a(2) = x$ 
then  $\{l := 2, \text{pos} := 2\}$ 
else if  $a(2) < x$ 
then  $l := 3$ 
else  $\{l := 2, r := 1\}$ 
else  $r := 0$  seq

```

```

if  $l \leq r \wedge \text{pos} = -1$ 
then let  $m = (l+r) \text{ div } 2$  in

```

```

if  $a(m) = x$ 
then  $\text{pos} := m$ 
else if  $a(m) < x$ 
then  $l := m+1$ 
else  $r := m-1$ 

```

asm binary search

signature:

static	$n : \mathbb{N}$	(defined as 4)
controlled	$a : \mathbb{N} \rightarrow \mathbb{Z}$	(uninitialised)
controlled	$x : \mathbb{Z}$	(uninitialised)
controlled	$l : \mathbb{N}$	(initially 0)
controlled	$r : \mathbb{N}$	(initially $n - 1$)
controlled	$\text{pos} : \mathbb{N}$	(initially -1)

rule Main:

```

if  $l \leq r$  1 pos = -1
then let  $m = (l+r) \text{ div } 2$  in
    if  $a(m) = x$ 
    then pos := m
    else if  $a(m) < x$ 
    then  $l := m+1$ 
    else  $r := m-1$ 

```

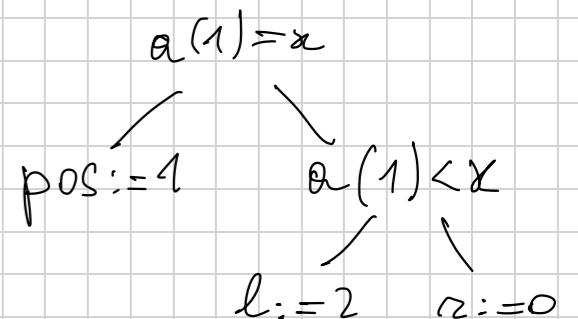
SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

```

if  $a(1) = x$ 
then pos := 1
else if  $a(1) < x$ 
then  $l := 2$ 
else  $r := 0$ 

```



Main² = Main¹ (S_0) seq Main

```

if  $a(1) = x$ 
then pos := 1
else if  $a(1) < x$ 
then if  $a(2) = x$ 
then {  $l := 2$ , pos := 2 }
else if  $a(2) < x$ 
then  $l := 3$ 
else {  $l := 2$ ,  $r := 1$  }
else if  $a(0) = x$ 
then { pos := 0,  $r := 0$  }
else if  $a(0) < x$ 
then {  $l := 1$ ,  $r := 0$  }
else {  $r := -1$  }

```

asm binary-search

signature:

static	$n : \mathbb{N}$	(defined as 4)
controlled	$a : \mathbb{N} \rightarrow \mathbb{Z}$	(uninitialised)
controlled	$x : \mathbb{Z}$	(uninitialised)
controlled	$l : \mathbb{N}$	(initially 0)
controlled	$r : \mathbb{N}$	(initially $n - 1$)
controlled	$\text{pos} : \mathbb{N}$	(initially -1)

rule Main:

```

if  $l \leq r$  1 pos = -1
then let  $m = (l+r) \text{ div } 2$  in
    if  $a(m) = x$ 
    then pos := m
    else if  $a(m) < x$ 
    then  $l := m+1$ 
    else  $r := m-1$ 

```

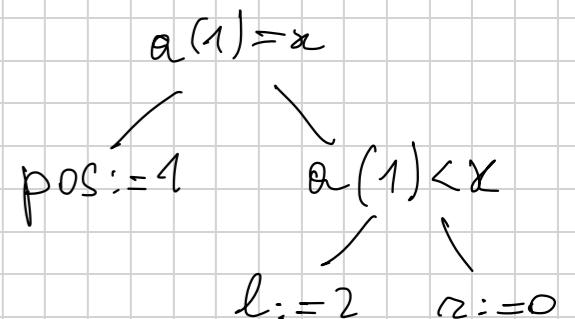
SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)

```

if  $a(1) = x$ 
then pos := 1
else if  $a(1) < x$ 
then  $l := 2$ 
else  $r := 0$ 

```

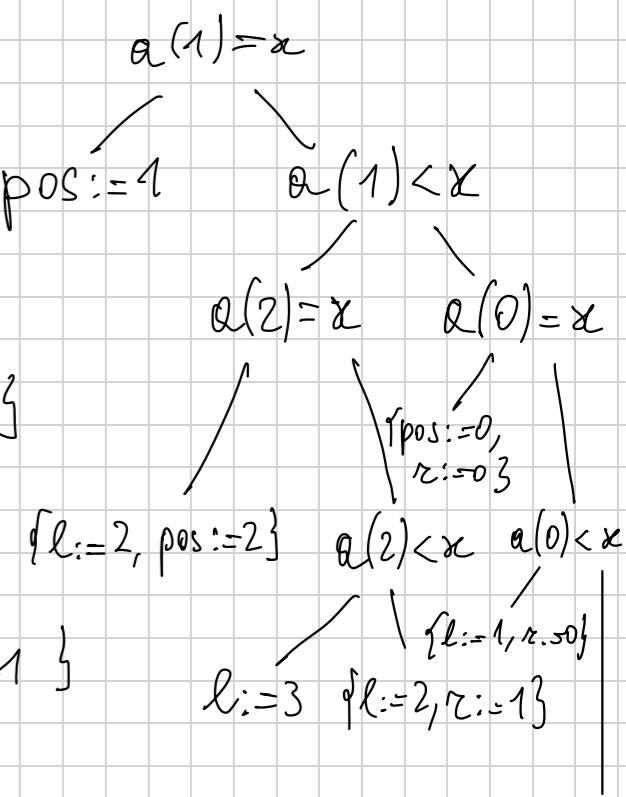


Main² = Main¹ (S_0) seq Main

```

if  $a(1) = x$ 
then pos := 1
else if  $a(1) < x$ 
then if  $a(2) = x$ 
then if  $l := 2, pos := 2 \}$ 
else if  $a(2) < x$ 
then  $l := 3$ 
else if  $l := 2, r := 1 \}$ 
else if  $a(0) = x$ 
then { pos := 0, r := 0 }
else if  $a(0) < x$ 
then { l := 1, r := 0 }
else { r := -1 }

```



$z := -1$

asm binary search

signature:

static	$n : \mathbb{N}$	(defined as 4)
controlled	$a : \mathbb{N} \rightarrow \mathbb{Z}$	(uninitialised)
controlled	$x : \mathbb{Z}$	(uninitialised)
controlled	$l : \mathbb{N}$	(initially 0)
controlled	$r : \mathbb{N}$	(initially $n - 1$)
controlled	$\text{pos} : \mathbb{N}$	(initially -1)

rule Main:

if $l \leq r$ 1 pos = -1

then let $m = (l+r) \text{ div } 2$ in

if $a(m) = x$

then pos := m

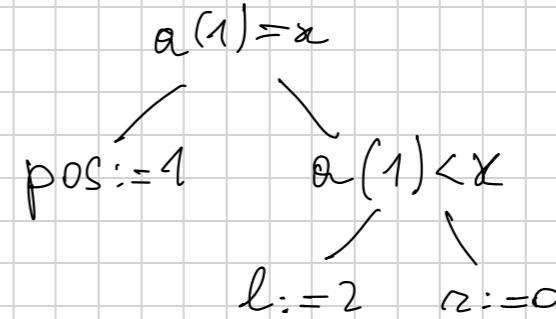
else if $a(m) < x$

then $l := m + 1$

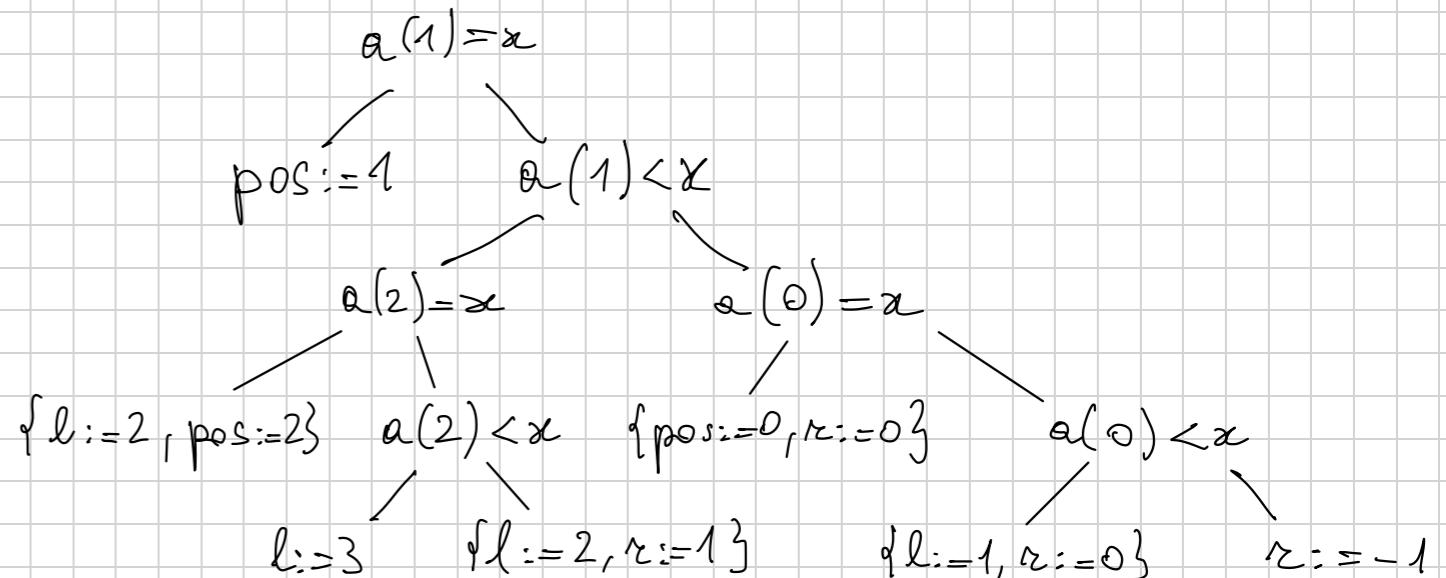
else $r := m - 1$

SYMBOLIC EXECUTION

Main¹ (evaluated in S_0)



Main² = Main¹ (S_0) seq Main



Main³ = Main² (S_0) seq Main

