Q1

The efficiency of the Carnot engine is 50% and the temperature of the sink is $500\ \mathrm{K}$. If temperature of the source is kept constant and its efficiency is raised to 60%, then the required temperature of the sink will be

(1) $100\ \mathrm{K}$

(2) $600\ \mathrm{K}$

(3) $400\ \mathrm{K}$

(4) $500\ \mathrm{K}$

Q2

A carrier signal $\mathrm{C}(\mathrm{t}) = 25\sin(2.512 \times 10^{10}\mathrm{t})$ is amplitude modulated by a message signal $\mathrm{m}(\mathrm{t}) = 5\sin(1.57 \times 10^{8}\mathrm{t})$ and transmitted through an antenna.What will be the bandwidth of the modulated signal ?

(1) $8\ \mathrm{GHz}$

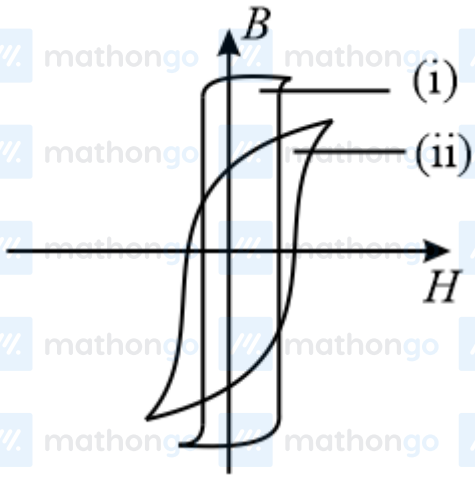
(2) $2.01\ \mathrm{GHz}$

(3) $1987.5\ \mathrm{MHz}$

(4) $50\ \mathrm{MHz}$

Q3

The $B-H$ curve (i) and (ii) shown in the figure is associated with



(1) (i) diamagnetic and (ii) paramagnetic substance

(2) (i) paramagnetic and (ii) ferromagnetic substance

(3) (i) soft iron and (ii) steel respectively

(4) (i) steel and (ii) soft iron respectively

Q4

The relation between Y, η and B is:

Where Y is Young's modulus, B is Bulk modulus and η is modulus of rigidity.

(1) $\frac{1}{\mathrm{Y}}=\frac{1}{3\eta}+\frac{1}{9\mathrm{B}}$

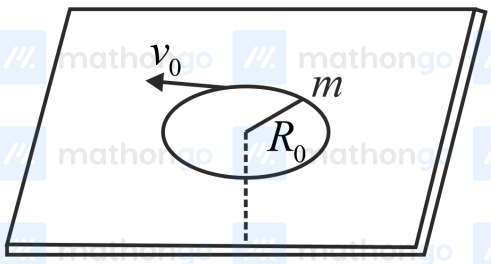
(2) $\frac{9}{\mathrm{Y}}=\frac{1}{\eta}+\frac{3}{\mathrm{B}}$

(3) $\frac{1}{\eta}=\frac{1}{\mathrm{B}}+\frac{1}{\mathrm{Y}}$

(4) $\frac{9}{\mathrm{Y}}=\frac{3}{\eta}+\frac{1}{\mathrm{B}}$

Q5

A mass m moves in a circle on a smooth horizontal plane with velocity v_0 at a radius R_0 . The mass is attached to a string which passes through a smooth hole in plane as shown.



The tension in the string is increased gradually and finally m moves in a circle of radius $\frac{\mathrm{R}_0}{2}$. The final value of the kinetic energy is:

(1) mv_0^2

(2) $\frac{1}{4}\mathrm{mv}_0^2$

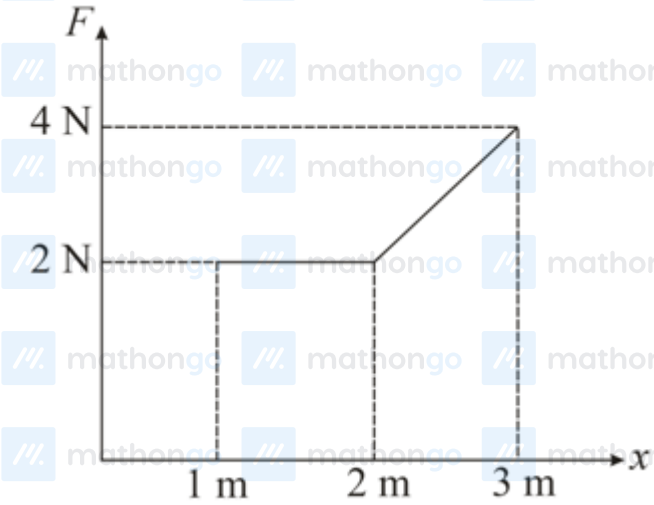
(3) $2\ \mathrm{mv}_0^2$

(4) $\frac{1}{2}mv_0^2$

Q6

Starting from rest, a particle is moving along x-axis. The variation of force with position is shown in figure.

Find the Kinetic energy (KE) of particle when it is at $x = 3$m.



(1) 3.3 J

(2) 4 J

(3) 5 J

(4) 4.2 J

Q7

A sound is produced between two vertical parallel walls. The echo from one wall is heard after 2 s while from the other 2 s after the first echo. The speed of sound in air is 340 m s^{-1}. Choose the correct options.

(1) The distance between two walls is 680 m.

(2) The distance between two walls is 1020 m.

(3) The next echo will be heard after 8 s from the instant original sound was produced.

(4) None of the above.

Q8

The two plates of a parallel plate capacitor are 4 mm apart. A slab of dielectric constant 3 and thickness 3 mm is introduced between the plates with its faces parallel to them. The distance between the plates is so adjusted that the capacitance of the capacitor becomes 2/3rd of its original value. What is the new distance between the plates?

(1)

9 mm

(2)

21 mm

(3)

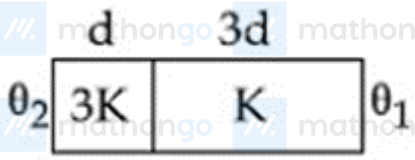
5 mm

(4)

8 mm

Q9

Two materials having coefficients of thermal conductivity 3K and K and thickness d and 3d respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are θ_2 and θ_1 respectively, $(\theta_2 > \theta_1)$. The temperature at the interface is



(1) $\frac{\theta_2+\theta_1}{2}$

(2) $\frac{\theta_1}{6}+\frac{5\theta_2}{6}$

(3) $\frac{\theta_1}{3}+\frac{2\theta_2}{3}$

(4) $\frac{\theta_1}{10}+\frac{9\theta_2}{10}$

Q10

A body is sliding down an inclined plane (angle of inclination 45°). If the coefficient of friction is 0.5 and $g = 9.8\ \text{m s}^{-2}$. then the downward acceleration of the body in m s^{-2} is :-

(1) $\frac{4.9}{\sqrt{2}}$

(2) $4.9\sqrt{2}$

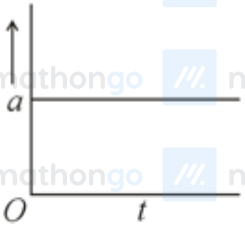
(3) $19.6\sqrt{2}$

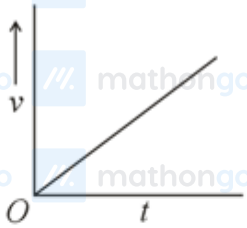
(4) 4.9

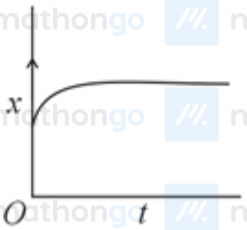
Q11

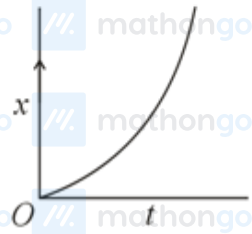
A particle starts from origin O from rest and moves with a uniform acceleration along the positive X -axis. Identify all figures that correctly represent the motion qualitatively.

(a =acceleration, v =velocity, x =displacement, t =time)

(A) 

(B) 

(C) 

(D) 

(1) (A)

(2) (A), (B), (C)

(3) (B), (C)

(4) (A), (B), (D)

Q12

The Binding energy per nucleon of ${}_3^7Li$ and ${}_2^4He$ nucleon are 5.60 MeV and 7.06 M eV, respectively. In the nuclear reaction ${}_3^7Li + {}_1^1H \rightarrow {}_2^4He + {}_2^4He + Q$, the value of energy Q released is

(1) 19.6 MeV

(2) -2.4 MeV

(3) 8.4 MeV

(4) 17.3 MeV

Q13

The dimensional formula of the constant a in Vander Waal's gas equation $\left(P + \frac{a}{V^2}\right)(V - b) = RT$ is:

(1) $[ML^4T^{-1}]$

(2) $[ML^2T^{-2}]$

(3) $[ML^5T^{-3}]$

(4) $[ML^5T^{-2}]$

Q14

A radiowave has a maximum magnetic field induction of 10^{-4} T on arrival at a receiving antenna. The maximum electric field intensity of such a wave is

(1) zero

(2) 3×10^4 V m^{-1}

(3) 5.8×10^{-4} T

(4) 3.0×10^{-5} T

Q15

Find the time required for a 50 Hz alternating current to change its value from zero to the rms value.

(1) 10 ms

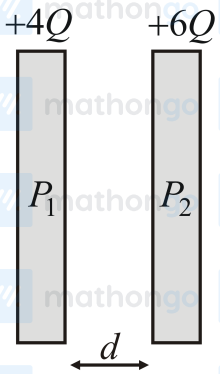
(2) 5 ms

(3) 15 ms

(4) 2.5 ms

Q16

Two identical conducting very large plates P_1 and P_2 having charges $+4Q$ and $+6\ Q$ are placed very closed to each other at separation d. The plate area of either face of the plate is A.



In the above question, if plates P_1 and P_2 are connected by a thin conducting wire, then the amount of heat produced will be

(1) $\frac{Q^2}{A\varepsilon_0}d$

(2) $\frac{5Q^2}{A\varepsilon_0}d$

(3) $\frac{2Q^2}{A\varepsilon_0}d$

(4) None of these

Q17

Magnetic field at the centre of a circular coil of radius R due to i flowing through it is B. The magnetic field at a point along the axis at distance R from the centre is

(1) $\frac{B}{2}$

(2) $\frac{B}{4}$

(3) $\frac{B}{\sqrt{8}}$

(4) $\sqrt{8B}$

Q18

Assertion: Electric appliances with metallic body, e.g., heaters, press etc, have three pin connections whereas an electric bulb has a two pin connection.

Reason: Three pin connections reduce heating of connecting cables.

(1) If both Assertion and Reason are true and Reason is correct explanation of Assertion.

(2) If both Assertion and Reason are true but Reason is not the explanation of Assertion.

(3) If Assertion is true but the Reason is false.

(4) If Assertion is false but Reason is true.

Q19

Choose the correct statement for hydrogen and deuterium atoms (considering the motion of nucleus).

(1) The radius of first Bohr orbit of deuterium is less than that of hydrogen.

(2) The speed of electron in first Balmer line of deuterium is more than that of hydrogen.

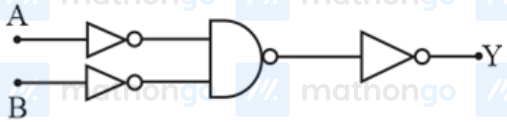
(3) The wavelength of first Balmer line of deuterium is

more than that of hydrogen.

(4) The angular momentum of the electron in the first Bohr orbit of deuterium is more than that of hydrogen.

Q20

The following arrangement performs the logic function of



(1) AND

(2) OR

(3) NAND

(4) NOR

Q21

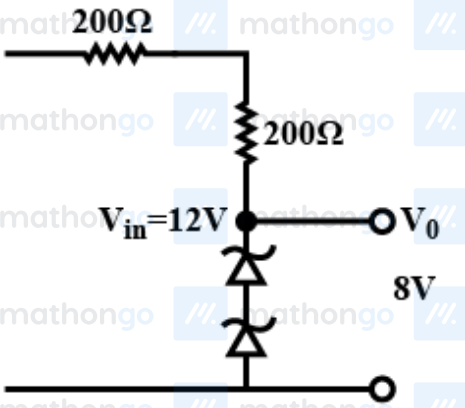
Gas at pressure P_0 is contained in a vessel. If the mass of all the molecules are halved and their speed is doubled then the resulting pressure becomes nP_0. What is the value of ' n'.

Q22

A biconvex lens made of glass $(\mu_g = 1.5)$ has radius of curvature $\frac{2}{3}$m. It is kept in contact with a plane mirror. Find magnitude of power of optical device formed if space between them is filled with water $(\mu_w = 4/3)$ in dioptre.

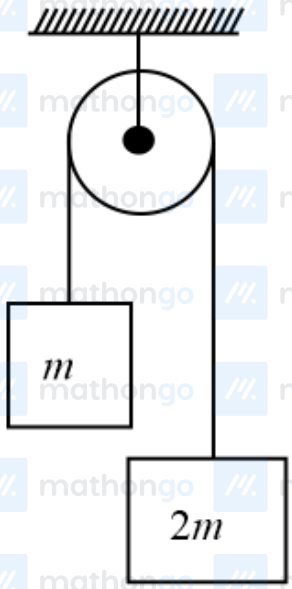
Q23

In the circuit shown below, is working as a 8 V dc regulated voltage source. When 12 V is used as an input, the power dissipated (in mW) in each diode is (Considering both zener diodes are identical)



Q24

A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10$ m s^{-2}, find the work done (in joules) by the string on the block of mass 0.36 kg during the first second after the system is released from rest.

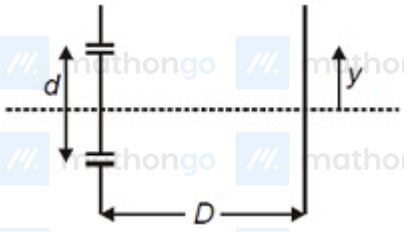


Q25

An inductor of 10 mH is connected to a 20 V battery through a resistor of 10 kΩ and a switch. After a long time, when maximum current is set up in the circuit, the current is switched off. The current in the circuit after 1 μs is $\frac{x}{100}$ mA. Then x is equal to _____ . (Take $e^{-1} = 0.37$)

Q26

In YDSE if incident light consists of two wavelengths λ_1 =4000 Å and λ_2 =5600 Å and is parallel to line SO. The minimum distance y upon screen, measured from point O, will be where the bright fringe due to two wavelengths coincide is $\frac{n\lambda_1 D}{d}$. Find n.



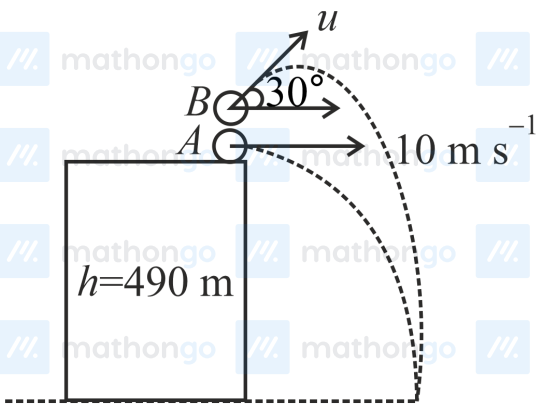
Q27

The surface of a metal is illuminated alternately with photons of energies $E_1 = 4$ eV and $E_2 = 2.5$ eV respectively. The ratio of maximum speeds of the photoelectrons emitted in the two cases is 2. The work

function of the metal in (eV) is.....

Q28

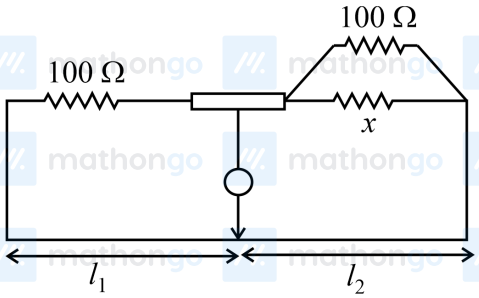
Two balls A and B are projected from the same height as shown in the figure. If the horizontal range of both the balls remains the same, then the speed u of the second ball is ..... $\mathrm{m\ s^{-1}}$. take $\mathrm{g = 9.8\ m\ s^{-2}}$



(roundoff answer to nearest integer)

Q29

In a practical wheat stone bridge circuit as shown, when one more resistance of 100 Ω is connected in parallel with unknown resistance X, then ratio $\frac{l_1}{l_2}$ becomes 2. l_1 is balance length. AB is a uniform wire. Then the value of X(in Ω) must be:



Q30

A force $\vec{F} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ N acts at a point $\left(4\hat{i} + 3\hat{j} - \hat{k}\right)$ m. Then the magnitude of torque about the point $\left(\hat{i} + 2\hat{j} + \hat{k}\right)$ m will be $\sqrt{x}$ N $-$ m.The value of x is.....

Q31

Match List$-$I with List$-$ II

	List$-$I (process)		List$-$ II (catalyst)
(a)	Deacon's process	(i)	ZSM $-$ 5
(b)	Contact process	(ii)	$CuCl_2$
(c)	Cracking of hydrocarbons	(iii)	Ni
(d)	Hydrogenation of vegetable oils	(iv)	V_2O_5

Choose the most appropriate answer from the options given below-

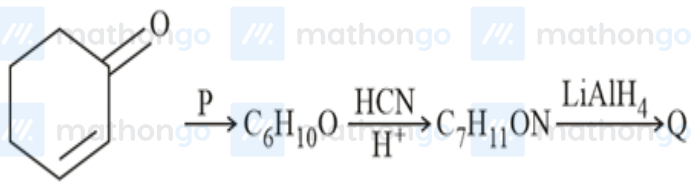
(1) a $-$ ii, b $-$ iv, c $-$ i, d $-$ iii

(2) a $-$ i, b $-$ ii, c $-$ ii, d $-$ iv

(3) a $-$ iii, b $-$ i, c $-$ iv, d $-$ ii

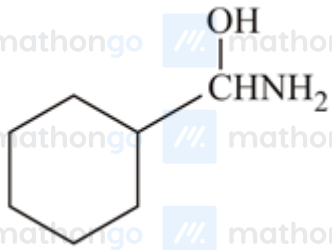
(4) a $-$ iv, b $-$ ii, c $-$ i, d $-$ iii

Q32



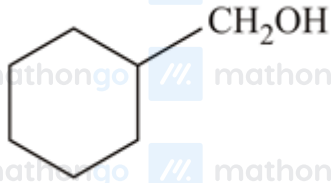
P and Q are -

(1) P is $NaBH_4$



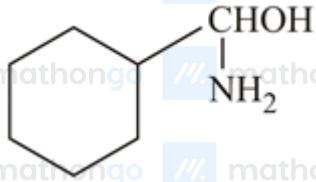
Q is

(2) P is $Ni-H_2$, in ethanol,



Q is

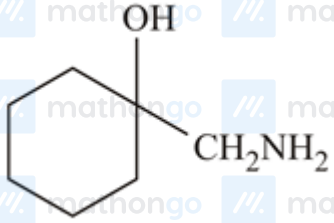
(3) P is $LiAlH_4$,



Q is

(4) P is $Pd-H_2$, in quinoline at room temperature

Q is



Q33

Column I	Column II
A. He	i. Highest electron gain enthalpy
B. Cl	ii. Most electropositive element
C. Ca	iii. Strongest reducing agent

D. Li	iv. Highest ionisation energy

The correct match of the contents in column I with those in column II is

(1) A-(iii), B-(i), C-(ii), D-(iv)

(2) A-(iv), B-(iii), C-(ii), D-(i)

(3) A-(i), B-(ii), C-(iii), D-(iv)

(4) A-(iv), B-(i), C-(ii), D-(iii)

Q34

The froth-flotation process for concentration of sulphide ores is based upon

(1) The difference in magnetic properties of **gangue particles** and **ore particles**.

(2) The difference in specific gravity of **gangue particles and ore particles.**

(3) difference in reactivity of **gangue and ore particles**

(4) preferential wetting properties with the frothing agent and water

Q35

Which is an electron deficient hydride among the following?

(1) LiH

(2) CaH_2

(3) AlH_3

(4) SiH_4

Q36

Addition of sodium hydroxide solution to a weak acid (HA) results in a buffer of pH 6. If ionization constant of HA is 10^{-5}, the ratio of salt to acid concentration in the buffer solution will be:

(1) 10 : 1

(2) 4 : 5

(3) 5 : 4

(4) 1 : 10

Q37

Which oxo−acid of sulphur contains the S− S bond in its structure?

(1) Disulphurous acid

(2) Disulphuric acid

(3) Perdisulphuric acid

(4) Hydrosulphurous acid

Q38

Among the following complexes the one which shows Zero crystal field stabilizations energy (CFSE)

(1) $\left[Ni(H_2O)_6\right]^{3+}$

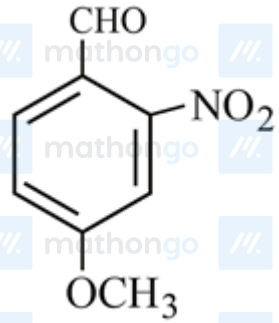
(2) $\left[Fe(H_2O)_6\right]^{3+}$

(3) $\left[Co(H_2O)_6\right]^{2+}$

(4) $\left[Co(H_2O)_6\right]^{3+}$

Q39

What is the correct IUPAC name of



(1) 4-methoxy-2-nitrobenzaldehyde

(2) 4-formyl-3-nitro anisole

(3) 4-methoxy-6-nitrobenzaldehyde

(4) 2-formyl-5-methoxy nitrobenzene

Q40

Which of the following statements is not true?

(1) Density of solid gets increased due to interstitial defects

(2) Frenkel defects do not alter the density of the solid

(3) Non-stoichiometric defects modify the formula of the compound

(4) Non-stoichiometric defects do not alter the density of the solid

Q41

The pollution by _____ causes minamata disease.

(1) Organic waste into drinking water

(2) Oil spill in water

(3) Industrial waste mercury into fishing water

(4) Arsenic into the atmosphere

Q42

Balmer gives an equation for wavelength of visible radiation of H-spectrum as $\lambda = \frac{kn^2}{n^2-4}$. The value of k in terms of Rydberg constant, R, is

(1) R

(2) 4R

(3) $\frac{R}{4}$

(4) $\frac{4}{R}$

Q43

Given below are two statements:

Statement I : None of the alkaline earth metal hydroxides dissolve in alkali.

Statement II : Solubility of alkaline earth metal hydroxides in water increases down the group.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is correct but Statement II is incorrect.

(2) Statement I is incorrect but Statement II is correct.

(3) Statement I and Statement II both are incorrect.

(4) Statement I and Statement II both are correct.

Q44

Which of the following is TRUE regarding hybridisation in PCl_5?

(1) All the bond angles in PCl_5 are equivalent.

(2) Axial and equatorial bonds have the same bond length.

(3) On orbital overlap between phosphorus and chlorine, five sp^3 d-p sigma covalent bonds are formed.

(4) Axial and equatorial bonds have equivalent bond energies.

Q45

Drugs used as pain killers are called ____.

(1) analgesics

(2) tranquilizers

(3) antacids

(4) antiseptics

Q46

If one strand of DNA has the sequence ATGCTTGA, the sequence in the complementary strand would be

(1) TACGAACT.

(2) TCCGAACT.

(3) TACGTACT.

(4) TACGTAGT.

Q47

Given below are two statements :

Statement I : Aniline is less basic than acetamide.

Statement II : In aniline, the lone pair of electrons on nitrogen atom is delocalised over benzene ring due to resonance and hence less available to a proton.

Choose the most appropriate option ;

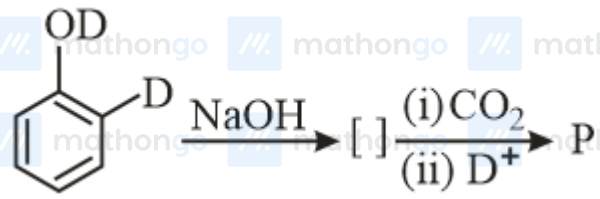
(1) Statement I is true but statement II is false.

(2) Statement I is false but statement II is true.

(3) Both statement I and statement II are true.

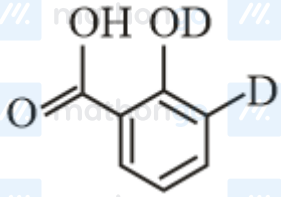
(4) Both statement I and statement II are false.

Q48

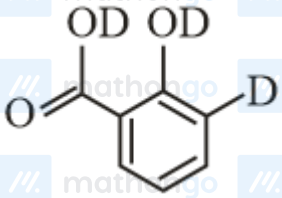


Here, P is;

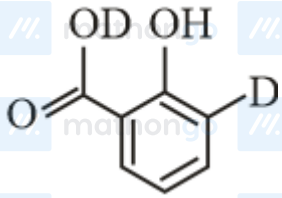
(1)



(2)



(3)



(4) Reaction not possible.

Q49

Consider the Assertion and Reason given below.

Assertion (A) : Ethene polymerized in the presence of Ziegler Natta Catalyst at high temperature and pressure is used to make buckets and dustbins.

Reason (R) : High density polymers are closely packed and are chemically inert. Choose the correct answer from the following:

(1) (A) is correct but (R) is wrong.

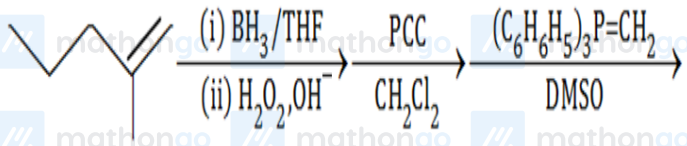
(2) Both (A) and (R) are correct but (R) is not the correct explanation of (A).

(3) Both (A) and (R) are correct and (R) is the correct explanation of (A).

(4) (A) and (R) both are wrong.

Q50

What is the product of the reaction sequence below?



(1) 2-methyl-1-hexene

(2) 2,3-dimethyl-2-pentene

(3) 2-methyl-2-hexene

(4) 3-methyl-1-hexene

Q51

Rate of diffusion of H_2 Gas is $3\sqrt{3}$ Times that of a hydrocarbon gas (C_nH_{2n-2}) Under identical conditions. Calculate value of n.

Q52

For the reaction

$A + B \rightleftharpoons 2C$

the value of equilibrium constant is 100 at 298 K. If the initial concentration of all the three species is 1 M each, then the equilibrium concentration of C is $x \times 10^{-1}$ M. The value of x is _____. (Nearest integer)

Q53

A KCl solution of conductivity 0.14 S m^{-1} shows a resistance of 4.19Ω in a conductivity cell. If the same cell is filled with an HCl solution, the resistance drops to 1.03Ω. The conductivity of the HCl solution is ____ $\times 10^{-2}$ S m^{-1}. (Round off to the Nearest Integer).

Q54

The reaction $2\,A + B_2 \rightarrow 2\,AB$ is an elementary reaction.

For a certain quantity of reactants, if the volume of the reaction vessel is reduced by a factor of 3, the rate of the reaction increases by a factor of _____ . (Round off to the Nearest Integer).

Q55

In neutral or faintly alkaline solution, 8 moles of permanganate anion quantitatively oxidize thiosulphate anions to produce X moles of a sulphur containing product. The magnitude of X is

Q56

How many ethers will be formed when a mixture of $C_2H_5\,OH$ and methyl alcohol are treated with concentrated $H_2\,SO_4$?

Q57

How many of the following are optically inactive?

i. trans $-\left[Co\,(en)_2(NH_3)Cl\right]^{2+}$

ii. cis $-\left[Co\,(en)_2\,Br_2\right]^{+}$

iii. $\left[Co\,(NH_3)_3\,Cl_3\right]$

iv. trans $-\left[Co\,(NH_3)_4\,Cl_2\right]^{+}$

v. trans $-\left[CoCl_2\,(C_2O_4)_2\right]^{3-}$

Q58

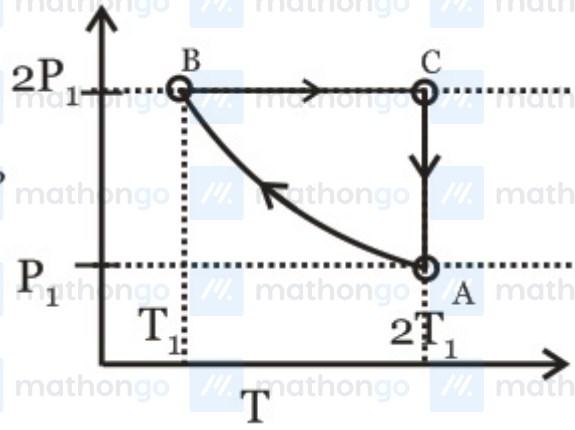
The percentage of anhydrous salt is 56.25 by weight. In the crystalline solid $MSO_4.nH_2\,O$ of molar mass $288\ g\ mol^{-1}$. The value of 'n' is:

Q59

15 mL of aqueous solution of Fe^{2+} in acidic medium completely reacted with 20 mL of 0.03 M aqueous $Cr_2O_7^{2-}$. The molarity of the Fe^{2+} solution is _____ $\times 10^{-2}$M (Round off to the Nearest Integer).

Q60

Two moles of an ideal gas (monatomic) are taken through cycle ABCA. During process AB, PT = constant T1 = 300 K. The work done in the process may be (150 R)x, the value of x is:



Q61

If the system of equations $\binom{n}{3}x+\binom{n}{4}y+35z=0$, $\binom{n}{4}x+35y+\binom{n}{3}z=0$ and $35x+\binom{n}{3}y+\binom{n}{4}z=0$ has a non-trivial solution, then the value of n is equal to $(\forall n \in N,\ n \geq 4)$

(1) 6

(2) 7

(3) 8

(4) 9

Q62

Coefficient of variation of two distributions are 60% and 75%, and their standard deviation are 18 and 15 respectively, then their arithmetic means respectively are

(1) 30, 30

(2) 30, 20

(3) $20, 30$

(4) $20, 20$

Q63

If $\int \frac{3\tan\left(x-\frac{\pi}{4}\right)}{\cos^2 x\sqrt{\tan^3 x+\tan^2 x+\tan x}}dx = k\tan^{-1}(\sqrt{\tan x+1+\cot x})+C$, then the value of k is: [where C is constant of integration.]

(1)

2

(2)

3

(3)

6

(4)

8

Q64

Let $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ be three non-zero vectors satisfying $\vec{\mathbf{a}} = \vec{\mathbf{b}} \times \vec{\mathbf{c}} + 2\vec{\mathbf{b}}$ where $|\vec{\mathbf{b}}| = |\vec{\mathbf{c}}| = 2$ and $|\vec{\mathbf{a}}| \leq 4$. The sum of possible value(s) of $|2\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}|$ is:

(1)

8

(2)

12

(3)

20

(4)

32

Q65

Let $f(x)=\sin^{-1}(\sin x)$ and $g(x)=\cos^{-1}[x+1]+\sin^{-1}[x]$. Then the number of solution of $f(x)+g(x)=4$, lies in the interval

(where [*] denotes the greatest integer function)

(1) $(-3,-1)$

(2) $(1,5)$

(3) $(-2,1)$

(4) $(0,2)$

Q66

Let matrix $\mathrm{A}=\begin{bmatrix}\mathrm{x} & \mathrm{y} & -\mathrm{z}\\ 1 & 2 & 3\\ 1 & 1 & 2\end{bmatrix}$, where $\mathrm{x},\ \mathrm{y},\ \mathrm{z}\in \mathrm{N}$. If $|\mathrm{adj}|\mathrm{adj}|\mathrm{adj}|\mathrm{adjA}||||=4^{8}\cdot 5^{16}$, then the number of such matrices A is equal to (where, $|M|$ represents determinant of a matrix M)

(1) 28

(2) 42

(3) 20

(4) 36

Q67

If the value of $\lim_{n\to\infty}\sum_{k=0}^{n}\frac{{}^{n}C_{k}}{n^{k}(k+3)}$ equals L. Then $[L]$ is equal to:

[Note: Where [k] denotes greatest integer function less than or equal to k.]

(1)
0

(2)
1

(3)

2

(4)

3

Q68

If the expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, (where $\odot, \oplus \in \{\vee, \wedge\}$,) then the ordered pair $(\oplus, \odot)$ is equivalent to :

(1) $(\vee, \wedge)$

(2) $(\wedge, \wedge)$

(3) $(\vee, \vee)$

(4) $(\wedge, \vee)$

Q69

The number of values of k for which the equation $\left(x^2 + (2k-6)x + 7 - 3k\right)\left(x^2 + (2k-2)x + 3k - 5\right) = 0$ has two different pairs of equal roots, is equal to:

(1)

0

(2)

1

(3)

2

(4)

more than 2

Q70

Let a, b and c be non-negative real numbers satisfying $a+b+c=9$. If the maximum value of the expression $a^2b^3c^4$ can be expressed as 2^x3^y, where x and y are natural numbers, then the value of $\log_{10}(x^y)$, is:

(1)
2

(2)
3

(3)
4

(4)
6

Q71

The number of integral points on the circle, touching the parabola $y^2=8x$ at $(2,4)$ and passing through $(0,4)$, are equal to

(1) 1

(2) 2

(3) 4

(4) 3

Q72

If $S_n=\left(1^2-1+1\right)(1!)+\left(2^2-2+1\right)(2!)+\ldots+\left(n^2-n+1\right)(n!)$, then S_{50} is:

(1) $52!$

(2) $1+49\times 51!$

(3) $52!-1$

(4) $50\times 51!-1$

Q73

Let z and w be complex numbers such that $\bar{z} + i\overline{w} = 0$ and $\arg zw = \pi$. Then $\arg z$ equals

(1) $\frac{5\pi}{4}$

(2) $\frac{\pi}{2}$

(3) $\frac{3\pi}{4}$

(4) $\frac{\pi}{4}$

Q74

The function $f(x)$ satisfies the functional equation $3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30$ for all real $x \neq 1$. The value of $f(7)$ is:

(1) 8

(2) 4

(3) -8

(4) 11

Q75

The image of the line $\frac{x}{2} = \frac{y-1}{5} = \frac{z+1}{3}$ in the plane $x + y + 2z = 3$ meets the $xz-$plane at the point (a, b, c), then the value of c is equal to

(1) $\frac{11}{6}$

(2) $\frac{129}{6}$

(3) $\frac{115}{6}$

(4) $\frac{232}{3}$

Q76

$S = \{1, 2, 3\}, f : S \to S$ satisfies the property : $\forall x \in S, f(f(x)) = f(x)$. How many different functions are there for $f(x)$?

(1)
8

(2)
10

(3)
1

(4)
4

Q77

What can be said about the quadratic equation $ax^2 + bx + c = 0$ if $2a + 3b + 6c = 0$, $(a, b, c \in R)$?

(1) At least one root in $(0, 1)$

(2) At least one root in $(2, 3)$

(3) At least one root in $(4, 5)$

(4) None of these

Q78

Let $z_1,\ z_2$ and z_3 are the points on the argand plane which lie on the circle with equation $|z - z_0| = \frac{4}{\sqrt{3}}$ (where z_0 is the centre of the circle). If $z_1 = 0,\ z_2 = -4$ and $z_3 = 4 + 3z_0$, then $|\arg(z_0)|$ is equal to (where $\arg Z \in (-\pi, \pi]$)

(1) $\frac{\pi}{6}$

(2) $\frac{5\pi}{12}$

(3) $\frac{5\pi}{6}$

(4) $\frac{2\pi}{3}$

Q79

Let PQ be the common chord of the circles $S_1 : x^2 + y^2 + 2x + 3y + 1 = 0$ and $S_2 : x^2 + y^2 + 4x + 3y + 2 = 0$, then the perimeter (in units) of the triangle C_1PQ is equal to $\left(\text{where, } C_1 = \left(-1, \frac{-3}{2}\right)\right)$

(1) $\frac{9}{2}$

(2) $2\sqrt{2} + 3$

(3) $3\sqrt{2} + 2$

(4) $\frac{3}{2} + 2\sqrt{2}$

Q80

The value of $\lim\limits_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} \frac{(2j-1)+8n}{(2j-1)+4n}$ is equal to:

(1) $5 + \log_e\left(\frac{3}{2}\right)$

(2) $2 - \log_e\left(\frac{2}{3}\right)$

(3) $3 + 2\log_e\left(\frac{2}{3}\right)$

(4) $1 + 2\log_e\left(\frac{3}{2}\right)$

Q81

Given $f(x) = \begin{cases} x|x| & \text{for } x \le -1 \\ [x+1]+[1-x] & \text{for } -1 < x < 1 \\ -x|x| & \text{for } x \ge 1 \end{cases}$, where [.] denotes the greatest integer function. If $I = \int\limits_{-2}^{2} f(x)\, dx$, then $|3I| =$

Q82

If the solution of the differential equation $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$ is $x + kye^{\frac{x}{y}} = C$ (where, C is an arbitrary constant), then the value of k is equal to

Q83

Let a fair coin be tossed 6 times. A, B and C three events are defined as

A : exactly 4 heads are obtained.

B : 4^{th} head obtained on 5^{th} toss.

C : tail is obtained, on 2^{nd} or 4^{th} toss

If conditional probability, $\mathrm{P}\left(\frac{\mathrm{B}}{\mathrm{A} \cap \mathrm{C}}\right)=\frac{m}{n}$ where $m, n \in N$ then find the least value of $(n-m)$

Q84

Suppose a function $f:[0,10] \rightarrow R$ is continuous and differentiable everywhere in its domain. If $f(10)=19$ and $|f'(x)-5| \leq 4 \forall x$ in domain. Find maximum value of $f(0)$.

Q85

Let $f(x)=\left[x-\frac{1}{4}\right]+x[x]+|x(x-4) \sin x|+(2x-1)^{1/3}$. Find the number of points in $(0, 2\pi)$ where $f(x)$ is non-derivable.

[Note: $[\cdot]$ denotes the greatest integer function.]

Q86

The plane denoted by $P_1 : 4x + 7y + 4z + 81 = 0$ is rotated through a right angle about its line of intersection with the plane $P_2 : 5x + 3y + 10z = 25$. If the plane in its new position be denoted by P, and the distance of this plane from the origin is k, then the value of $\left[\frac{k}{2}\right]$ (where $[.]$ represents the greatest integer less than or equal to k) is

Q87

If $A_1, A_2, A_3 \ldots\ldots A_{20}$ are 20 skew-symmetric matrices of same order and $B=\sum_{r=1}^{20} 2r(A_r)^{(2r+1)}$, then the sum of the principal diagonal elements of matrix B is equal to

Q88

The number obtained after dividing the number formed by the last three digits of 17^{256} by 3 is

Q89

The area in sq. units inside the parabola $5x^2 - y = 0$ but outside the parabola $2x^2 - y + 9 = 0$ is A. Find $\frac{A^2}{12}$.

Q90

If the curve $f(x) = 3x^3 + ax^2 + bx$ where a, b are non negative integers, cuts the x-axis at 3 distinct points.

Find the minimum value of $(a + b)$.

Answer Key

Q1 (3)	**Q2** (4)	**Q3** (3)	**Q4** (4)
Q5 (3)	**Q6** (3)	**Q7** (2)	**Q8** (4)
Q9 (4)	**Q10** (1)	**Q11** (4)	**Q12** (4)
Q13 (4)	**Q14** (2)	**Q15** (4)	**Q16** (4)
Q17 (3)	**Q18** (3)	**Q19** (1)	**Q20** (4)
Q21 (2)	**Q22** (2)	**Q23** (40)	**Q24** (8)
Q25 (74)	**Q26** (7)	**Q27** (2)	**Q28** (11)
Q29 (100)	**Q30** (195)	**Q31** (1)	**Q32** (4)
Q33 (4)	**Q34** (4)	**Q35** (3)	**Q36** (1)
Q37 (1)	**Q38** (2)	**Q39** (1)	**Q40** (4)
Q41 (3)	**Q42** (4)	**Q43** (2)	**Q44** (3)
Q45 (1)	**Q46** (1)	**Q47** (2)	**Q48** (3)
Q49 (3)	**Q50** (4)	**Q51** (4)	**Q52** (25)
Q53 (57)	**Q54** (27)	**Q55** (6)	**Q56** (3)
Q57 (4)	**Q58** (7)	**Q59** (24)	**Q60** (8)
Q61 (2)	**Q62** (2)	**Q63** (3)	**Q64** (2)
Q65 (3)	**Q66** (4)	**Q67** (1)	**Q68** (4)

Q69 (2)

Q70 (2)

Q71 (3)

Q72 (2)

Q73 (3)

Q74 (2)

Q75 (2)

Q76 (2)

Q77 (1)

Q78 (3)

Q79 (2)

Q80 (4)

Q81 (8)

Q82 (1)

Q83 (7)

Q84 (9)

Q85 (15)

Q86 (7)

Q87 (0)

Q88 (227)

Q89 (36)

Q90 (4)

Q1

Efficiency (η) of a Carnot engine is given by $\eta = 1 - \frac{T_2}{T_1}$, where T_1 is the temperature of the source and T_2 is the temperature of the sink.

Here, $T_2 = 500$ K.

$\therefore \ 0.5 = 1 - \frac{500}{T_1} \Rightarrow T_1 = 1000 \ \text{K}$

Now, $\eta' = 0.6 = 1 - \frac{T_2'}{1000}$ (T_2' is the new sink temperature)

$T_2' = 400$ K

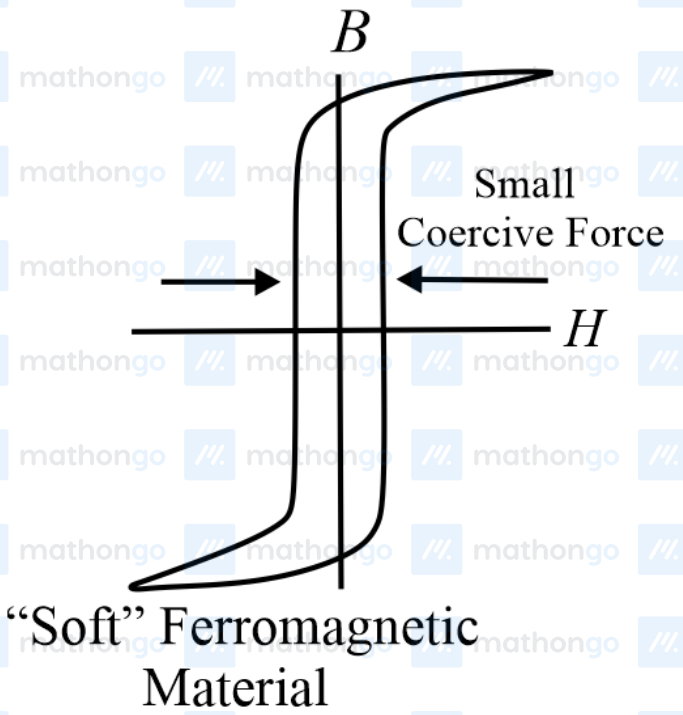
Q2

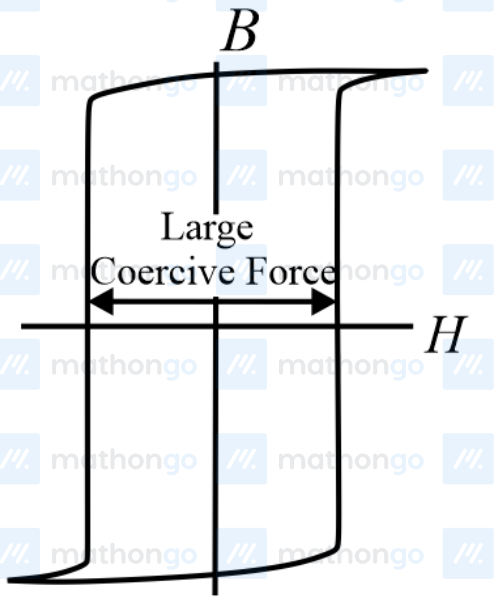
Band width $= 2\text{f}_\text{m}$

$\omega_\text{m} = 1.57 \times 10^8 = 2\pi \text{f}_\text{m}$

$\text{BW} = 2\text{f}_\text{m} = \frac{10^8}{2}$ Hz $= 50$ MHz

Q3





From the hysteresis curve of soft iron we know Soft iron has high retentivity and low coercive force therefore the loop (i) is for soft iron and the loop (ii) is for steel.

Q4

The relation between Y, η and B is $\frac{9}{Y}=\frac{1}{B}+\frac{3}{\eta}$

Where Y is Young's modulus, B is Bulk modulus and η is modulus of rigidity.

Q5

From conservation of angular momentum.

$\mathrm{mv_0R_0}=\mathrm{mv}'\left(\frac{\mathrm{R_0}}{2}\right)$

$\Rightarrow \mathrm{v}'=2\mathrm{v_0}$

Hence, final $\mathrm{KE}=\frac{1}{2}\mathrm{mv}'^2=\frac{1}{2}\mathrm{m}(2\mathrm{v_0})^2$

$=2\mathrm{mv_0^2}$

Q6

From Work Energy Theorem,

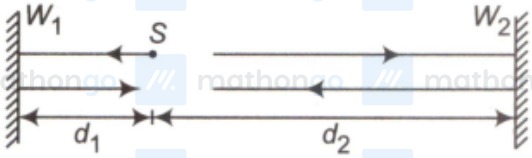
Change in Kinetic energy $(KE)=$ Work done

$KE_f-KE_i=$ area under the graph of F vs x

$KE_f-0=5$

$KE_f=5\ \mathrm{J}$

Q7



$2d_1=340\times t_1$

$\therefore\ d_1=340\ \mathrm{m}\quad(t=2\ \mathrm{s})$

$2d_2=340\times t_2$

$\therefore d_2 = 680\ \text{m} (t_2 = t_1 + 2 = 4\ \text{s})$

$\therefore$ Distance between walls $= d_1 + d_2 = 1020\ \text{m}.$

Next echo will be heard at 6 s not at 8 s. Because sound wave reflected from W_2 will be reflected by W_1 in next 2 s.

Q8

Here, distance between parallel plates d = 4 mm = 0.004 m, K = 3, thickness t = 3 mm = 0.003 m and d_1 =?

$\therefore C = \frac{\varepsilon_0 A}{d}$ and $C_1 = \frac{\varepsilon_0 A}{d_1 - t\left(1 - \frac{1}{K}\right)}$

since $C_1 = \frac{2}{3}C$ (given)

$\therefore \quad \frac{\varepsilon_0 A}{d_1 - t\left(1 - \frac{1}{K}\right)} = \frac{2}{3}\frac{\varepsilon_0 A}{d}$

$\frac{1}{d_1 - t\left(1 - \frac{1}{K}\right)} = \frac{2}{3d}$

$\frac{1}{d_1 - 0.003\left(1 - \frac{1}{3}\right)} = \frac{2}{3 \times 0.004}$

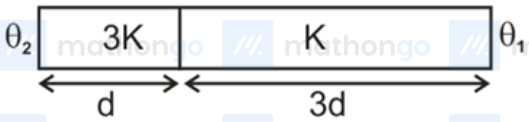
$\frac{1}{d_1 - 0.003 \times \frac{2}{3}} = \frac{1}{0.006}$

$\frac{1}{d_1 - 0.002} = \frac{1}{0.006}$

$d_1 - 0.002 = 0.006$

$d_1 = 0.006 + 0.002 = 0.008\ \text{m} = 8\ \text{mm}.$

Q9



Let the temperature of the junction T $^\circ$C .

Rate of heat flow in Rod 1 = rate of heat flow in Rod 2

$\frac{3\,\text{kA}}{\text{d}}(\theta_2 - \text{T}) = \frac{\text{kA}}{3\text{d}}(\text{T} - \theta_1)$

$\Rightarrow 9(\theta_2 - \text{T}) = (\text{T} - \theta_1)$

$\Rightarrow 10\mathrm{T}=9\theta_2+\theta_1$

$\Rightarrow \mathrm{T}=\frac{9\theta_2+\theta_1}{10}=\frac{\theta_1}{10}+\frac{9\theta_2}{10}$

Q10

Acceleration of body$=g\sin\theta-\mu g\cos\theta$

$=9.8[\sin 45^{\circ}-0.5\cos 45^{\circ}]=\frac{4.9}{\sqrt{2}}\ \mathrm{m\ sec}^{-2}$

Q11

Since, the particle starts from rest, this means, initial velocity, $u=0$

Also, it moves with uniform acceleration along positive X -axis. This means, its acceleration (a) is constant.

$\therefore$ Given, $a-t$ graph in (A) is correct. As we know, for velocity-time graph, slope $=$ acceleration.

Since, the given $v-t$ graph in (B) represents that its slope is constant and non-zero.

Also, the displacement of such a particle w.r.t. time is given by

$x=ut+\frac{1}{2}at^2=0+\frac{1}{2}at^2$

$\Rightarrow\ x\propto t^2$

So, x versus t graph would be a parabola with starting from origin.

This is correctly represented in displacement time graph given in (D).

Q12

${}_4Li^7+{}_1H^1\rightarrow{}_2\left({}_2He^4\right)$

BE of products$=((5.6\ \mathrm{MeV})\times 7)+0$

$=39.2\ \mathrm{MeV}$

$E_i=-39.2\ \mathrm{MeV}$

BE of reactant$=(7.06)\times 4\times 2$

$=56.48$ MeV

$E_f=-56.48$ MeV

As nuclear energy decreases, some energy will be released.

$Q_{\text{release}}=E_i-E_f=(-39.2)-(-56.48)=17.28$ MeV

Q13

Here, the dimension of $\frac{a}{V^2}$ will be equal to pressure so $\frac{a}{(L^3)^2}=ML^{-1}T^{-2}$ [Principle of homogeneity]

$\therefore \quad [a]=\left[ML^5T^{-2}\right]$

Aliter:

According to gas equation, for one mol of a real gas.

$\left[P+\frac{a}{V^2}\right](V-b)=RT$

$PV+\frac{a}{V}-Pb+\frac{ab}{V^2}=RT$

As this equation is dimensionally correct, each term on either side will have same dimensions, i.e.,

$\left[\frac{a}{V}\right]=[PV]$

or $\quad [a]=\left[ML^{-1}T^{-2}\right]\left[L^3\right]\left[L^3\right]=\left[ML^5T^{-2}\right]$

and $[P\times b]=(PV)$

or $\quad [b]=[V]=[L]^3$

Note: Actually vander Waals equations for μ mol is

$\left[P+\frac{\mu^2a}{V^2}\right][V-\mu b]=\mu RT$

So that $[\mu b]=[V] \quad i.e., \quad [b]=\left[L^3\mu^{-1}\right]$ with units m^3/mol

and $\left[\mu^2a\right]=\left[PV^2\right] \quad i.e., \quad [a]=\left[ML^5T^{-2}\mu^{-2}\right]$ with units $J\ m^3/\text{mol}^2$

Q14

Given that,

$B_0=10^{-4}$ T,

the speed of the electromagnetic wave is

$c=\frac{E_0}{B_0}$

$(c=3\times 10^8\ \text{m s}^{-1})$

$E_0 = cB_0$

$E_0 = 3 \times 10^8 \times 10^{-4}$

$E_0 = 3 \times 10^4 \text{ V m}^{-1}$

Q15

Instantaneous current in AC circuit, at instant t

(Assuming $I = 0$ at $t = 0$).

$I = I_0 \sin(wt) = I_0 \sin(2\pi ft)$

I_0, w *and* $f$are peak current, angular frequency and frequency, respectively.

We know, rms current $I_{rms} = \frac{I_0}{\sqrt{2}}$. if rms current is equal to the instantaneous current at time t, then

$I = \frac{I_0}{\sqrt{2}} = I_0 \sin(wt), \ \sin(wt) = \frac{1}{\sqrt{2}}$

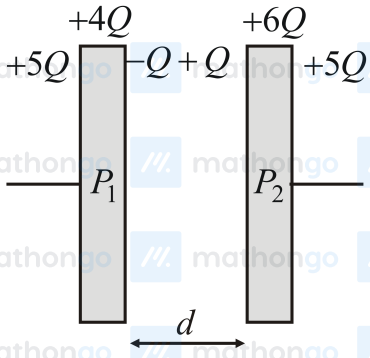
$\Rightarrow wt = 2\pi ft = \frac{\pi}{4}$

$\Rightarrow t = \frac{\pi}{4(2\pi f)} = \frac{\pi}{4(2\pi \times 50)} = \frac{1}{400}$

$\Rightarrow t = 2.5 \text{ ms}$

Q16

From the above questions charges on different plates will be,



When we connected both the plates with the wires, the charges will start flowing from one plate to the other and the final charges are as shown in the figure. Energy stored in the capacitor will be zero finally, i.e., $U_f = 0$, but initially, the energy will be stored in the electric field between the charges which can be given by,

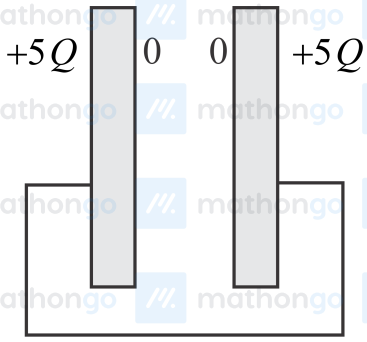
$$E = \frac{Q}{2A\varepsilon_0} + \frac{Q}{2A\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

So, the initial energy in the plates will be,

$$U_{\mathrm{i}} = \frac{1}{2}\varepsilon_0 E^2 \times Ad = \frac{1}{2} \times \varepsilon_0 \times \left(\frac{Q}{A\varepsilon_0}\right)^2 \times Ad$$

The energy lost in heat will be,

$$\Delta H = U_{\mathrm{i}} - U_{\mathrm{f}} = \frac{Q^2}{2A\varepsilon_0} d$$



Q17

The magnetic field at a point along the axis at distance R from the centre of a circular coil of radius R carrying i is, $B = \frac{\mu_0 i R^2}{2\left(R^2+R^2\right)^{3/2}}$, by using formula magnetic field at axial point at a distance equal to the radius of coil.

$= \frac{\mu_0 i}{2\sqrt{8}R} = \frac{B}{\sqrt{8}} \left[B_{\text{centre}} = B = \frac{\mu_0 i}{2R}\right]$

Q18

The electrical appliances with metallic body like heater, press, etc, have three pin connections. Two pins are for supply line and the third pin is for earth connection for safety purposes.

Q19

We know that, radius of Bohr orbit is $r_{\mathrm{n}} = \left(\frac{n^2h^2}{4\pi^2 mkZe^2}\right)$

where, m is the reduced mass of the electron.

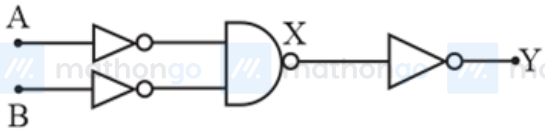
$m = \frac{m_e M}{m_e + M}$

where, m_e is the mass of the electron and M is the mass of the nucleus.

The mass of deuterium is more than that of hydrogen. So, the reduced mass of electron is more for deuterium than that for hydrogen.

Hence, the radius of first Bohr orbit of deuterium is less than that of hydrogen.

Q20



$Y = \overline{\overline{A} \cdot \overline{B}} = \overline{A} \cdot \overline{B} = \overline{A + B}$

Q21

Pressure of a gas is given by $P = \frac{1}{3}\frac{mN}{V}(v_{\text{rms}})^2$.

Where, m =mass of the gas,

N = Number of gas molecules,

V =Volume of the vessel,

v_{rms} = RMS speed of gas molecules.

So, $P_0 = \frac{1}{3}\frac{mN}{V}(v_{\text{rms}})^2$.

If the mass of all the molecules are halved and their speed is doubled,

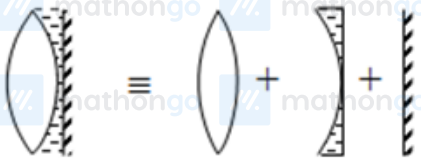
$P = \frac{1}{3}\frac{(m/2)N}{V}(2v_{\text{rms}})^2$

$\Rightarrow P = 2\left[\frac{1}{3}\frac{mN}{V}(v_{\text{rms}})^2\right]$

$\Rightarrow P = 2P_0$

Therefore, $n = 2$.

Q22



focal length of convex mirror $(f_1) = Rm$

focal length of concave lens $(f_2) = -3Rm$

effective lens $(f_{eq}) = \frac{f_1 f_2}{f_1 + f_2} = \frac{-3R}{-2} = \frac{3R}{2} = 1\text{m}$

$\frac{1}{f} = \frac{1}{f_M} - \frac{2}{f_1} = \frac{1}{\infty} - \frac{2}{1}$

$f = -\frac{1}{2}\text{m}$

$P = -2D$

Q23

$i = \frac{(12-8)}{(200+200)}\text{A} = \frac{4}{400} = 10^{-2}\text{ A}$

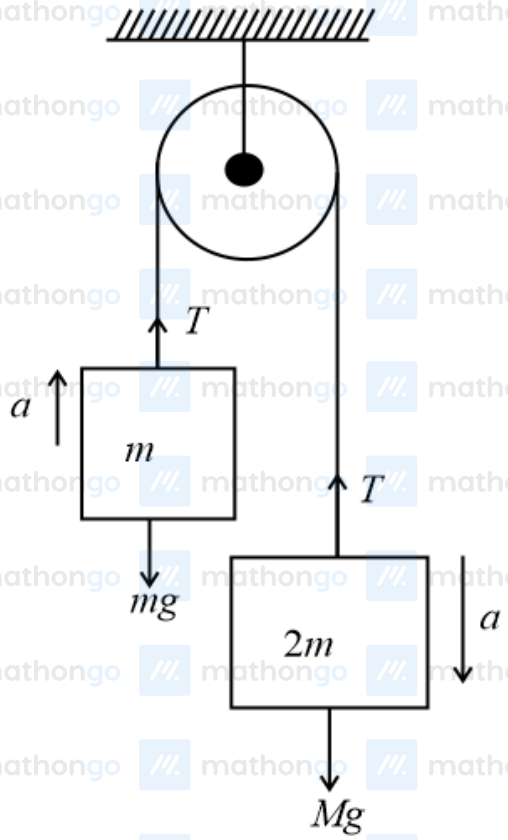
Power loss in each diode $= (4)\left(10^{-2}\right)\text{ W} = 40\text{ mW}$

Q24

Given, $m = 0.36$ kg and $M = 2m = 0.72$ kg

Let a be the acceleration when the system is released.

Forces on m and M are shown in figure.



From the figure, we have

$T - mg = ma \quad \ldots(1)$

and $Mg - T = Ma \quad \ldots(2)$

Adding above two equation,

$g(M - m) = (M + m)a$

$\Rightarrow a = \frac{g(M-m)}{(M+m)}$

$a = \frac{g(0.72-0.36)}{(0.72+0.36)} = \frac{g \times 0.36}{1.08} = \frac{g}{3}$

Putting these values in (1),

$T = \left(m \times \frac{g}{3}\right) + (m \times g)$

$T = \frac{4mg}{3}$

Now, displacement of block is $s = ut + \frac{1}{2}at^2$

Here, initial velocity $u = 0$, then $s = \frac{1}{2}at^2$.

Work done by the string on the block is $W = T \times s = T \times \frac{1}{2}at^2$

$= \frac{4mg}{3} \times \frac{1}{2} \times \frac{g}{3} \times t^2$

$= \frac{4 \times 0.36 \times 10}{3} \times \frac{1}{2} \times \frac{10}{3} \times 1$

$W = 8\ \text{J}$

Q25

$I_{\max} = \frac{V}{R} = \frac{20\ \text{V}}{10\ \text{K}\Omega} = 2\ \text{mA}$

For LR-decay circuit,

$I = I_{\max} e^{-Rt/L}$

$I = 2\ \text{mA}\ \text{e}^{\frac{-10 \times 10^3 \times 1 \times 10^{-6}}{10 \times 10^{-3}}}$

$I = 2\ \text{mA}\ \text{e}^{-1}$

$I = 2 \times 0.37\ \text{mA}$

$I = \frac{74}{100}\ \text{mA}$

$x = 74$

Q26

$n_1 \overline{X}_1 = n_2 \overline{X}_2$

$n_1 \frac{D\lambda_1}{d} = n_2 \frac{D\lambda_2}{d}$

$n_1 \lambda_1 = n_1 \lambda_2$

$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} = \frac{4000}{5600} = \frac{40}{58} = \frac{5}{7}$

$\therefore \quad \frac{n_2}{n_1} = \frac{5}{7} \Rightarrow \frac{X_1}{X_2} = \frac{7}{5}$

$\therefore \quad y = 7\overline{X}_1 = \frac{7D\lambda_1}{d}$

Q27

Given,

Energy of two photons are $E_1 = 4$ eV and $E_2 = 2.5$ eV

The ratio of maximum speeds of the photoelectrons emitted in the two cases is $\frac{v_1}{v_2} = 2$

Using Einstein equation of photoelectric effect,

$KE_{max} = \frac{1}{2}mv^2 = E - \phi \ \ldots \left(1\right)$

Where, ϕ is the work function of metal and E is the energy of photon

Now using equation for both the cases we get,

$\frac{1}{2}mv_1^2 = 4 - \phi \ \ldots \left(2\right)$

$\frac{1}{2}mv_2^2 = 2.5 - \phi \ \ldots \left(3\right)$

Dividing equation (2) and (3) and substitute given values, we get,

$\frac{{v_1}^2}{{v_2}^2} = \frac{4-\phi}{2.5-\phi} = (2)^2$

$\Rightarrow 3\phi = 6$

$\Rightarrow \phi = 2$ eV

Q28

The ball, B, follows horizontal and angular projectile and the ball A follows only horizontal projectile,

the height of the tower is, $h = 490$ m, and both the particle follows the same range,

now for particle A,

$R = u\sqrt{\frac{2h}{g}} = 10 \times \sqrt{\frac{2\times 490}{9.8}} = 100 \text{ m}$

and for oblique projectile,

$R = 100 \text{ m} = u\cos\theta \times t + u\cos\theta \times \left(490 + \frac{u\sin^2\theta}{2g}\right)$

it means,

$$R = u\cos\theta \times \frac{u\sin\theta}{g} + u\cos\theta \times \sqrt{\frac{\left(490+\frac{u^2\sin^2\theta}{2g}\right)}{g}}$$

$$\Rightarrow R = 100 = u\cos 30^\circ \times \frac{u\sin 30^\circ}{9.8} + u\cos 30^\circ \times \sqrt{\frac{\left(490+\frac{u^2\sin^2 30^\circ}{2\times 9.8}\right)}{9.8}}$$

$$\Rightarrow u = 10.9 \text{ m s}^{-1}$$

Q29

The given Wheat stone's bridge is in a balanced condition

$$\frac{P}{Q} = \frac{R}{S}$$

$$\Rightarrow \frac{100}{l_1} = \frac{\left(\frac{100\,x}{100+x}\right)}{l_2}$$

$\because \quad \frac{l_1}{l_2} = 2$ So, $\frac{100}{\left(\frac{100x}{100+x}\right)} = 2$

$\Rightarrow$ The unknown resistance is $\mathrm{x} = 100\ \Omega$

Q30

$$\vec{r} = (4-1)\hat{i} + (3-2)\hat{j} + (-1-1)\hat{k}$$

$$= 3\hat{i} + \hat{j} - 2\hat{k}$$

$$\tau = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & j & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix}$$

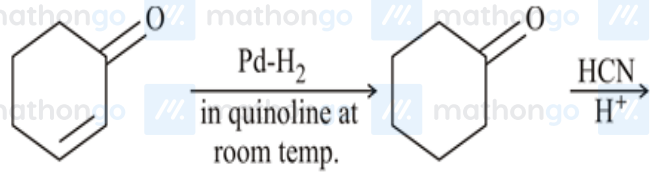
$$= \hat{i}(7) - \hat{j}(11) + \hat{k}(5) = 7\hat{i} - 11\hat{j} + 5\hat{k}$$

$$= \sqrt{49 + 121 + 25} = \sqrt{195}$$

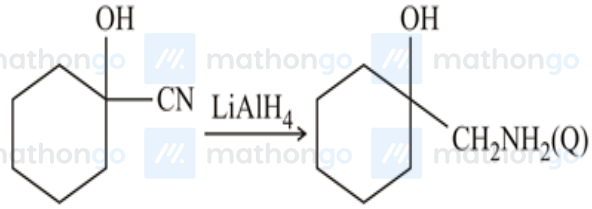
Q31

In manufacture of $H_2\,SO_4$ (contact process), V_2O_5 is used as a catalyst. Ni catalysts enables the hydrogenation of fats. $CuCl_2$ is used as catalyst in Deacon's process. $ZSM-5$ used as catalyst in cracking of hydrocarbons.

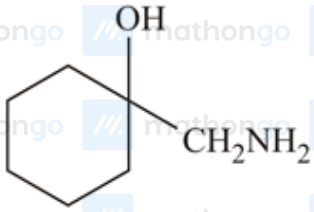
Q32



(P)



$\therefore$ P is $Pd-H_2$, in quinoline at room temp.,



Q is

Q33

He $\Rightarrow$ Inert gas

Cl $\Rightarrow$ Electron gain enthalpy is highest

Ca $\Rightarrow$ Most electropositive metal

Li $\Rightarrow$ Strong reducing agent

He has the electronic configuration of $1s^2$. It is completely filled electronic configuration, due to this, it has the highest ionisation enthalpy.

$Cl \longrightarrow Cl^-$ gives the highest $\Delta_{eg}H$ as Cl^- after getting one electron, it will achieve the stable electronic configuration $18[Ar]$. Due to this, large amount of energy is released during this process.

$$_{20}Ca \xrightarrow[-2e^-]{} Ca^{2+}$$

Alkaline earth metal and present in the fourth period, the outermost electron is far away from the nucleus. As a result, less energy is required to remove the electron and easily donate the electron, hence, shows maximum metallic nature.

E^{o}_{red} of Li is -3.07 V, a strong reducing agent.

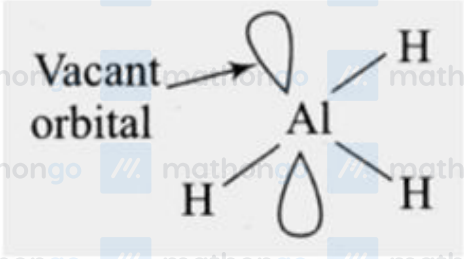
It has very high hydration energy due to smaller in size. The hydration energy is directly proportional to the charge density of the ion.

Q34

On being preferrentially wetted by oil, the ore particles rise to the surface in the form of froth and from there we can separate them.

Q35

AlH_3 is an electron deficient hydride.



It is generally formed by the group 13 element, and have lesser number of electrons than that required for writing its Lewis structure. Being electron deficient, this hydride generally behaves as a Lewis acid, which act as electron acceptor. This is a polynuclear hydride. According to the octet rule, each element tends to completely fill it's outermost shell with $8e^-$ in it. The electronic configuration of aluminium is $2, 8, 3$, and it still needs 5 more electrons to complete its octet. Al has 3 valence electrons to complete its octet, while each hydrogen has one valence electron.

Q36

$pH = pk_a + \log\left[\frac{\text{Salt}}{\text{Acid}}\right]$ $(\because [\text{Salt}]=[\text{Anion}])$

$\Rightarrow 6 = 5 + \log \frac{\text{Salt}}{\text{Acid}}$

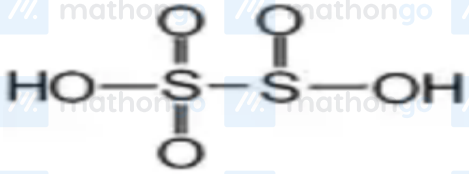
$\Rightarrow 1 = \log \frac{\text{Salt}}{\text{Acid}}$

$\Rightarrow \log 10 = \log \frac{\text{Salt}}{\text{Acid}}$

$\frac{\text{Salt}}{\text{acid}} = \frac{10}{1}$

Q37

Disulphurous acid ($H_2S_2O_5$) contains S – S in its structure.



Q38

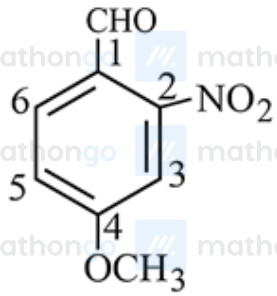
$\left[Fe(H_2O)_6\right]^{3+}$

$Fe^{+2} = 3d^5\left(t_{2g}^{1,1,1}e_g^{1,1}\right)$

So C.F.S.E is $= [-0.4 \times 3 + 0.6 \times 2]\Delta_0 = 0$

Q39

The -CHO functional group is the highest priority functional group. The carbons of the benzene ring are numbered accordingly.



4, methoxy-2 nitrobenzaldehyde

Q40

Non – stoichiometric Schottky defect is a type of point defect. This defect is forms when oppositely charged ion leave their lattice site creating vacancies, thus lowers the density of crystal.

Q41

Mercury poisoning often produces a crippling and fatal disease called Minamata disease.

Q42

$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ (Rydberg equation)

For H-atom, Z = 1

For visible radiation, $n_1 = 2$

$\therefore \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{R(n^2-4)}{4n^2}$

Or $\lambda = \frac{4n^2}{R(n^2-4)} = \frac{kn^2}{n^2-4}$

$\therefore k = \frac{4}{R}$

Q43

Statement-I is incorrect

$Be(OH)_2$ dissolve in alkali due to it's amphoteric nature.

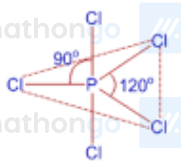
Statement-II is correct

Solubility of alkaline earth metal hydroxide in water increases down the group due to rapid decreases in lattice energy as compared to hydration energy.

Q44

On orbital overlap between phosphorus and chlorine, five sp^3 d-p sigma covalent bonds are formed.

This statement is true, as PCl_5 carries five sigma bonds and all these sigma bonds are used for hybridisation according to the valence shell electron pair repulsion theory. Structure of PCl_5 is shown below:



And we can see here, that axial bonds and equatorial bonds, both are not of the same length.

Q45

A substance which is used for the purpose of diagnosis, prevention, cure or relief of a disease is called drug. Drugs are the chemicals of low molecular masses, these interact with macromolecular targets and produce a response. When the biological response is therapeutic and useful, these chemicals are called medicines and are used in diagnosis, prevention and treatment of diseases.

Analgesics are the drugs used to reduce or abolish pain without causing impairment of consciousness, mental confusion or paralysis or some other nervous system disturbances.

Q46

We can say that if the sequence of bases in one strand of DNA is I, then, the sequence in the second strand should be II. The base pairs on one of the strands of DNA bind with the base pairs of the other strand very specifically. Adenine always pairs with thymine with two hydrogen bonds and guanine always pairs with cytosine with three hydrogen bonds.

$$\mathrm{A:T:G:C:T:T:G:A} \rightarrow \mathrm{I}$$

$$\mathrm{T:A:C:G:A:A:C:T} \rightarrow \mathrm{II}$$

Q47

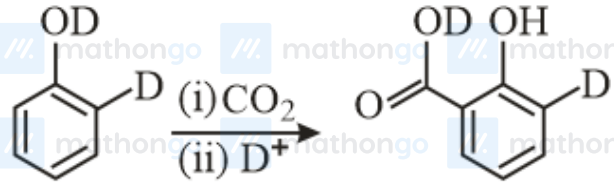
Explanation :- aniline is more basic than acetamide because in acetamide, lone pair of nitrogen is delocalised to more electronegative element oxygen.

In Aniline lone pair of nitrogen delocalised over benzene ring.

Q48

In the first step of the reaction, NaOH is given which is a strong base, abstracts the hydrogen and phenoxide ion is formed, which is more reactive than phenol. It further undergoes electrophilic substitution reaction with CO_2 to form

salicylic acid. But further reaction with D^+ abstracts the hydrogen of acid to form deuterated acid. It follows the mechanism of Kolbe's reaction.

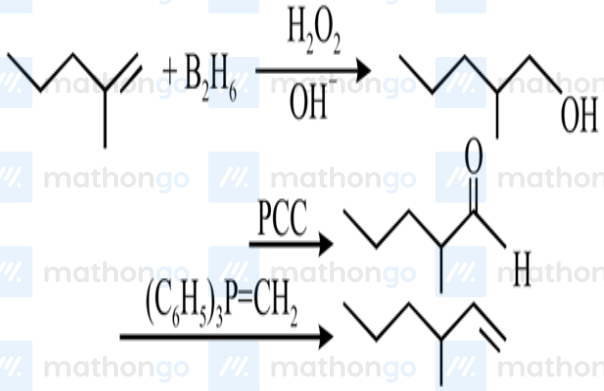


Q49

High density polythene: It is formed when addition polymerisation of ethene takes place in a hydrocarbon solvent in the presence of a catalyst such as triethylaluminium and titanium tetrachloride (Ziegler-Natta catalyst) at a temperature of 333 K to 343 K and under a pressure of 6-7 atmospheres. High density polythene (HDP) thus produced, consists of linear molecules and has a high density due to close packing. It is also chemically inert and more tougher and harder. It is used for manufacturing buckets, dustbins, bottles, pipes, etc.

Q50

Here the final product is 3-methyl-1-hexene and it is formed as follows



Q51

$$\frac{R_{H_2}}{R_{H.C}} = \sqrt{\frac{M_{H.C}}{2}} = 3\sqrt{3}$$

$$M_{H.C} = 54\ g/mol$$

$$M_{H.C} = 12n + (2n - 2) = 54$$

$n = 4$

Q52

$$\begin{array}{ccccc} A & + & B & \rightleftharpoons & 2C \\ 1 & & 1 & & 1 \\ -x & & -x & & 2x \\ \hline 1-x & & 1-x & & 1+2x \end{array}$$

$K = \frac{[C]_{eq}^2}{[A]_{eq}[B]_{eq}} = \frac{(1+2x)^2}{(1-x)(1-x)}$

$100 = \left(\frac{1+2x}{1-x}\right)^2$

$\left(\frac{1+2x}{1-x}\right) = 10$

$x = \frac{3}{4}$

$[C]_{eq.} = 1 + 2x$

$= 1 + 2\left(\frac{3}{4}\right)$

$= 2.5\ M$

$= 25 \times 10^{-1}\ M$

Q53

$\kappa = \frac{1}{R} \cdot G^*$

For same conductivity cell, G^* is constant and hence $\kappa \cdot R. =$ constant.

$\therefore 0.14 \times 4.19 = \kappa \times 1.03$

or, $= \frac{0.14 \times 4.19}{1.03}$

$= 0.5695\ Sm^{-1}$

$= 56.95 \times 10^{-2}\,\mathrm{Sm}^{-1} \approx 57 \times 10^{-2}\,\mathrm{Sm}^{-1}$

Q54

Reaction : $2\,\mathrm{A} + \mathrm{B}_2 \longrightarrow 2\,\mathrm{AB}$

As the reaction is elementary, the rate of reaction is

$\mathrm{r} = \mathrm{K}.\left[\mathrm{A}\right]^2[\,\mathrm{B}_2]$

on reducing the volume by a factor of 3, the concentrations of A and B_2 will become 3 times and hence, the rate becomes $3^2 \times 3 = 27$ times of initial rate.

Q55

$8\mathrm{MnO}_4^- + 3\mathrm{S}_2\mathrm{O}_3^{2-} + \mathrm{H}_2\mathrm{O} \rightarrow 8\mathrm{MnO}_2 + 6\mathrm{SO}_4^{2-} + 2\mathrm{OH}^{\ominus}$

Q56

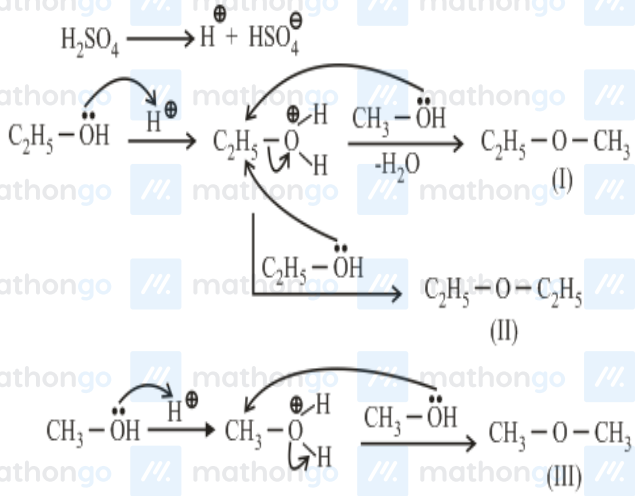
When alcohol is treated with concentrated $\mathrm{H}_2\,\mathrm{SO}_4$, ethers are formed by dehydration of alcohols.

Same alcohol form symmetric ether.

Different alcohol form unsymmetric ether.

The reaction follows SN^2 mechanism.

Mechanism:



Hence, three different alcohol are formed.

Q57

Complexes (i), (iii),(iv) and (v) are optically inactive due to the presence of plane of symmetry.

Q58

Considering 100gof solid, mass of anhydrous slat $= 56.25\ g$ and mass of water $= 100 - 56.25$

$43.75\ g$

Moles of H_2O in $288\ g$ of solid $= \frac{43.75}{100} \times 288$

$= 126\ g$

$\therefore$ Moles of $H_2O = \frac{126}{18} = 7\ mol$

$\therefore n = 7$

Q59

$n_{eq}\, Fe^{2+} = n_{eq}\, Cr_2\, O_7^{2-}$

or $\left(\frac{15 \times M_{Fe^{2+}}}{1000}\right) \times 1 = \left(\frac{20 \times 0.03}{1000}\right) \times 6$

$\therefore M_{Fe^{2+}} = 0.24M = 24 \times 10^{-2}M$

Q60

$PdV + VdP = nRdT$ ...(i)

along $AB \rightarrow PT = \text{constant}$

$PdT + TdP = 0$

$dP = -\frac{P}{T}dT$

Substitute in (i)

$PdV + V\left(-\frac{P}{T}dT\right) = nRdT$

$PdV = nRdT + \frac{PV}{T}dT = 2nRdT$

$W_{AB} = -\int_{V_1}^{V_2} PdV = -R\int_{2T_1}^{T_1} 2ndT = -2nR[T_1 - 2T_1]$

$= -2 \times 2 \times R[300 - 600]$

$= 1200\ R = (150\ R)x$

$\therefore\ x = 8$

Q61

For non trival solution, $\Delta = 0$

$$\begin{vmatrix} {}^nC_3 & {}^nC_4 & 35 \\ {}^nC_4 & 35 & {}^nC_3 \\ 35 & {}^nC_3 & {}^nC_4 \end{vmatrix} = 0$$

$\Rightarrow {}^nC_3 + {}^nC_4 + 35 = 0$ (not possible)

or ${}^nC_3 = {}^nC_4 = 35 \Rightarrow n = 7$

Q62

Given, $CV_1 = 60, CV_2 = 75, \sigma_1 = 18$ and $\sigma_2 = 15$

Let $\bar{x}_1$ and $\bar{x}_2$ be the means of 1^{st} and 2^{nd} distribution respectively.

Then, $CV_1 = \frac{\sigma_1}{\bar{x}_1} \times 100 \Rightarrow \bar{x}_1 = \frac{18 \times 100}{60} = 30$

$CV_2 = \frac{\sigma_2}{\bar{x}_2} \times 100 \Rightarrow \bar{x}_2 = \frac{15 \times 100}{75} = 20$

Hence, $\bar{x}_1 = 30$ and $\bar{x}_2 = 20$

Q63

$$I=\int\frac{3(\tan x-1)\sec^2 x}{(\tan x+1)\sqrt{\tan^3 x+\tan^2 x+\tan x}}dx=3\int\frac{(t-1)}{(t+1)\sqrt{t^3+t^2+t}}dt$$

$$=3\int\frac{\left(1-\frac{1}{t^2}\right)}{\left(t+\frac{1}{t}+2\right)\sqrt{t+\frac{1}{t}+1}}dt \quad \text{Let } t+\frac{1}{t}+1=z^2\Rightarrow\left(1-\frac{1}{t^2}\right)dt=2zdz$$

$$=6\int\frac{dz}{(z^2+1)}=6\tan^{-1}\sqrt{1+\frac{1}{t}+t}+C$$

Q64

$\vec{a}=\vec{b}\times\vec{c}+2\vec{b}$

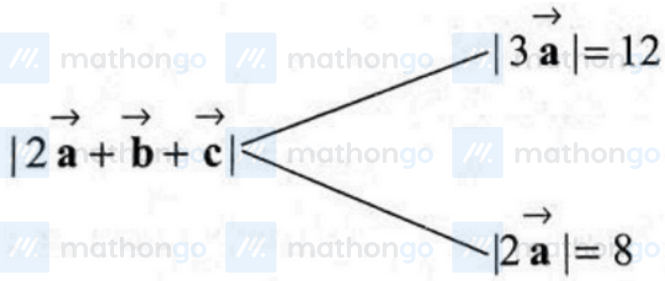
Taking dot product with $\vec{b}$

$\vec{a}\cdot\vec{b}=2|\vec{b}|^2\Rightarrow|\vec{a}||\vec{b}|\cos\theta=2|\vec{b}|^2$

$\Rightarrow\quad\cos\theta=\frac{4}{|\vec{a}|}\Rightarrow|\vec{a}|=4\Rightarrow\theta=0^\circ$

$\Rightarrow\quad\vec{a}=2\vec{b}$

Now, $\vec{b}\times\vec{c}=0\Rightarrow\vec{b}=\vec{c}$ or $\vec{b}=-\vec{c}$



$\therefore$ Required sum $=12+8=20$

Q65

Given $g(x)=\cos^{-1}[x+1]+\sin^{-1}[x]$

For domain

$-1\le[x+1]\le 1$ and $-1\le[x]\le 1$

$\Rightarrow\quad -1\le x+1<2$ and $-1\le x<2$

$\Rightarrow -2 \leq x < 1$ and $-1 \leq x < 2$

$\Rightarrow x \in [-1, 1)$

Now $g(x) = \begin{cases} \frac{\pi}{2} - \frac{\pi}{2} & ; \quad -1 \leq x < 0 \\ 0 + 0 & ; \quad 0 \leq x < 1 \end{cases}$

$g(x) = 0; \ x \in [-1, 1)$

Now $f(x) + g(x) = 4$

$\Rightarrow \sin^{-1}(\sin x) + 0 = 4,$

$\Rightarrow \sin^{-1}(\sin x) = 4$, which is not possible. Hence no solution.

Q66

$\det.(\text{adj}(\text{adj}(\text{adj}(\text{adjA})))) = |\text{A}|^{(3-1)^4}$

$= |\text{A}|^{16} = 4^8 . 5^{16}$

$\Rightarrow |\text{A}| = \pm 10$

$|\text{A}| = \begin{vmatrix} \text{x} & \text{y} & -\text{z} \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{vmatrix} = \text{x} + \text{y} + \text{z} = \pm 10$

$\because \text{x}, \text{y}, \text{z} \in \text{N} \Rightarrow \text{x} + \text{y} + \text{z} = -10$ (not possible)

Hence, $\text{x} + \text{y} + \text{z} = 10$

The number of such matrices $= {}^9\text{C}_2$

$= 36$

Q67

$$\lim_{n\to\infty} \sum_{k=0}^{n} \frac{{}^nC_k}{n^k} \int_0^1 x^{k+2} dx = \int_0^1 \left[\lim_{n\to\infty} \sum_{k=0}^{n} {}^nC_k \left(\frac{x}{n}\right)^k x^2 \right] dx = \int_0^1 \left[\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n x^2 \right] dx$$

$$= \int_0^1 e^x x^2 dx = \int_0^1 e^x \left(x^2 + 2x\right) dx - 2\int_0^1 e^x (x+1) dx + 2\int_0^1 e^x dx$$

$$= e^x \left(x^2 - 2x + 2\right)\Big]_0^1 = e - 2$$

Q68

1) $(p \vee q) \wedge (\sim p \wedge q)$

$\equiv [(p \vee q) \wedge \sim p] \wedge q$ (Using associative property)

$\equiv [q \wedge \sim p] \wedge q$

$\equiv \sim p \wedge q$

2) $= (p \wedge q) \wedge (\sim p \wedge q)$

$\equiv (p \wedge \sim p) \wedge q$

$\equiv F \wedge q \equiv F$

3) $(p \vee q) \wedge (\sim p \vee q)$

$\equiv (p \wedge \sim p) \vee q$

$\Rightarrow F \vee q \equiv q$

4) $(p \wedge q) \wedge (\sim p \vee q)$

$\equiv (q \wedge p) \wedge (\sim p \vee q)$

$\equiv q \wedge [p \wedge (\sim p \vee q)]$ (using associative property)

$\equiv q \wedge (p \wedge q)$

$\equiv p \wedge q$

Q69

Case-I : Both equations have both the roots in common.

i.e., $\frac{1}{1}=\frac{2k-6}{2k-2}=\frac{7-3k}{3k-5} \Rightarrow$ no value of k

Case-II : Equation $x^2+(2k-6)x+7-3k=0$ has equal roots and equation $x^2+(2k-2)x+(3k-5)=0$ has equal roots

$\therefore \quad (2k-6)^2-4(7-3k)=0 \Rightarrow 4k^2-12k+8=0 \Rightarrow k^2-3k+2=0 \Rightarrow k=1,2$

$(2k-2)^2-4(3k-5)=0 \Rightarrow 4k^2-20k+24=0 \Rightarrow (k-2)(k-3)=0 \Rightarrow k=2$

$\therefore k=2$

Q70

Consider the numbers $a/2, a/2, b/3, b/3, b/3, c/4, c/4, c/4, c/4$ using A.M. $\geq$ G.M. we get

$\frac{a+b+c}{9} \geq \left(\frac{a^2b^3c^4}{2^{10}3^3}\right)^{1/9}$

$\Rightarrow$ maximum value of $a^2b^3c^4$ is $2^{10} \times 3^3$

Hence $x=10$ and $y=3$

$\therefore \quad \log_{10}(x^y)=\log_{10}\left(10^3\right)=3$

Q71

Equation of tangent at $(2,4)$ on the parabola $y^2=8x$ is

$y(4)=8\left(\frac{x+2}{2}\right) \Rightarrow y=x+2$

Let the equation of the circle touching line $y=x+2$ at $(2,4)$ is

$(x-2)^2+(y-4)^2+\lambda(x-y+2)=0$ which passes through $(0,4)$

$\Rightarrow 4+0+\lambda(0-4+2) \Rightarrow \lambda=2$

$\Rightarrow$ Required circle is $x^2+y^2-2x-10y+24=0$

$\Rightarrow (x-1)^2+(y-5)^2=2$

If x and y are integers, then

$(x-1)^2=1=(y-5)^2$

$\Rightarrow x=0,2$ and $y=4,6$

$\Rightarrow 4$ integral points lie on the circle

Q72

Given,

$S_n = \left(1^2 - 1 + 1\right)\left(1!\right) + \left(2^2 - 2 + 1\right)\left(2!\right) + \cdots + \left(n^2 - n + 1\right)\left(n!\right)$

Its general term t_r is given as,

$\left(r^2 - r + 1\right)r!$

$t_r = \left[\left(r^2 - 1\right) - \left(r - 2\right)\right]\left(r!\right)$

$t_r = (r-1)\,(r+1)! - (r-2)(r)!$

$S_n = \sum_{r=1}^{n} t_r$

$S_n = (0 - (-1)) + (3! - 0) + (2(4)! - 3!) + \ldots + (n-1)(n+1)! - (n-2)n!$

$S_{50} = 1 + 49(51)!\,.$

Q73

We have,

$\arg zw = \pi$

$\Rightarrow \arg z + \arg w = \pi \quad \ldots(1)$

Now,

$\bar{z} + i\overline{w} = 0 \Rightarrow \bar{z} = -i\overline{w}$

$\therefore z = iw$

$\Rightarrow \arg z = \arg i + \arg w$

$\Rightarrow \arg z = \frac{\pi}{2} + \arg w$

$\Rightarrow \arg z = \frac{\pi}{2} + \pi - \arg z \text{ [from (1)]}$

$\therefore \arg z = \frac{3\pi}{4}$

Q74

$\because \quad 3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30 \ldots(1)$

Replace x by $\frac{x+59}{x-1}$, we get

$3f\left(\frac{x+59}{x-1}\right) + 2f(x) = 10\left(\frac{x+59}{x-1}\right) + 30 \ldots(2)$

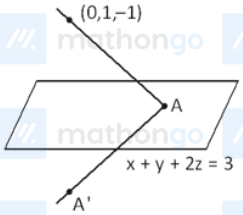
From eqn. $(1) \times 3$ - eqn. $(2) \times 2$, we get

$$5f(x) = 30x - 20\left(\frac{x+59}{x-1}\right) + 30$$

$5f(7) = 210 - 20 \times 11 + 30 = 20 \quad \Rightarrow \quad f(7) = 4$

Q75

Any general point on the line is $(2\lambda, 5\lambda + 1, 3\lambda - 1)$



On satisfying this point on the plane, we get,

$2\lambda + 5\lambda + 1 + 6\lambda - 2 = 3$

$13\lambda = 4 \Rightarrow \lambda = \frac{4}{13}$

So, coordinates of the point are $\left(\frac{8}{13}, \frac{33}{13}, \frac{-1}{13}\right)$

This point also lies on the image of the line

Image of point $(0,1,-1)$ also lies on the image of the line

$\frac{x-0}{1} = \frac{y-1}{1} = \frac{z+1}{2} = -2\frac{(-4)}{6}$

$x = \frac{4}{3}, y = \frac{7}{3}, z = \frac{5}{3}$

Point is $\left(\frac{4}{3}, \frac{7}{3}, \frac{5}{3}\right)$

Equation of image of the line is $\frac{x-\frac{4}{3}}{28} = \frac{y-\frac{7}{3}}{-8} = \frac{z-\frac{5}{3}}{68}$

For $xz-$plane, putting $y = 0$, we get,

$\frac{x-\frac{4}{3}}{28} = \frac{7}{24} = \frac{z-\frac{5}{3}}{68}$

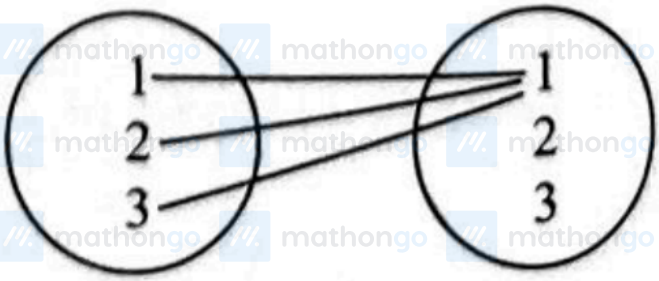
$\Rightarrow z = \frac{129}{6}$

Q76

$f[f(x)] = f(x) \forall x \in S = \{1, 2, 3\}$

I. When range contains 1 element

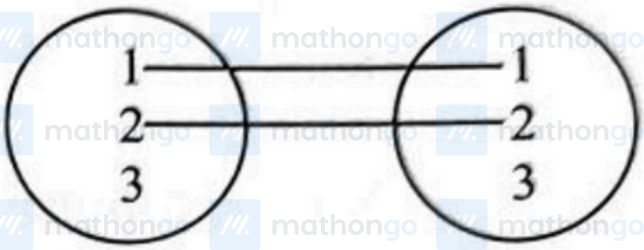
${}^3C_1 \times 1 = 3$



II. When range contains 2 elements

e.g., let $f(1) = 1$ and $f(2) = 2$

Let $f(3) = 1$ OR Let $f(3) = 2$



(i) If $x = 1$, LHS $= f[f(x)] = f(1)$

RHS $= 1$

In this case also LHS = RHS $\forall x \in S$

(ii) If $x = 2$, LHS = RHS

(iii) If $x = 3$, LHS = RHS

$\therefore$ ${}^3C_2 \times 2 = 6$

Remaining element can be mapped 2 ways.

III. When range contains 3 elements

$f(1) = 1 \quad f(2) = 2 \quad f(3) = 3$

Q77

$f(x)= \frac{ax^3}{3} + \frac{bx^2}{2} + cx$

$f(0)= 0$

$f(1)= \frac{a}{3} + \frac{b}{2} + c$

$= \frac{2a+3b+6c}{6} = 0$

From Rolle's theorem,

$\exists$ at least one point $x = \alpha$ in $(0, 1)$ such that $f'(\alpha)= 0$

Where $f'(x)= ax^2 + bx + c$

$\Rightarrow ax^2 + bx + c$ has at least one root in $(0, 1)$.

Q78

Here $\frac{z_1+z_2+z_3}{3} = z_0 \Rightarrow z_3 = 4 + 3z_0$

Therefore, center coincides with the circumcentre

$\Rightarrow$ Triangle is equilateral $\Rightarrow |z_1 - z_2|= 4$

Clearly, z_3 either lie in the second or third quadrant

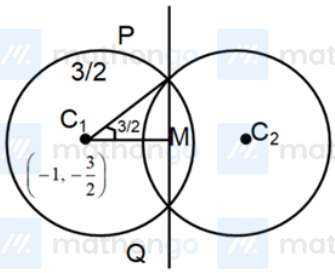
So the centre z_0 also lies in the second or third quadrant.

$\therefore z_0$ can be $= -2 + \frac{2}{\sqrt{3}}i, -2 - \frac{2}{\sqrt{3}}i$

$\Rightarrow \arg(z_0)= \frac{5\pi}{6}$

Q79

Equation of the common chord is $2x + 1 = 0$



$C_1M = \left|\frac{-2+1}{2}\right| = \frac{1}{2}$

$PM = \sqrt{\frac{9}{4} - \frac{1}{4}} = \sqrt{2}$

Length of the common chord $= 2\sqrt{2}$

Hence, the perimeter of $\Delta C_1PQ = \frac{3}{2} + \frac{3}{2} + 2\sqrt{2}$

$= 3 + 2\sqrt{2}$ units

Q80

$\lim\limits_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} \frac{\left(\frac{2j}{n} - \frac{1}{n} + 8\right)}{\left(\frac{2j}{n} - \frac{1}{n} + 4\right)}$

$\int_0^1 \frac{2x+8}{2x+4} dx = \int_0^1 dx + \int_0^1 \frac{4}{2x+4} dx$

$= 1 + 4\frac{1}{2}\left(\ln|2x+4|\right)\Big|_0^1$

$= 1 + 2\ell n\left(\frac{3}{2}\right)$

Q81

$f(x) = -x^2$ for $x \leq -1$,
$f(x) = 1$ for $-1 < x < 0$,

$f(x) = 2$ for $x = 0$,
$f(x) = 1$ for $0 < x < 1$ and
$f(x) = -x^2$ for $x \geq 1$

$\Rightarrow f(x)$ is even

$\therefore I = 2\int\limits_0^2 f(x)\, dx = 2\int\limits_0^1 f(x)\, dx + 2\int\limits_1^2 f(x)\, dx = 2\int\limits_0^1 (1)\, dx + 2\int\limits_1^2 -x^2 dx = \frac{-8}{3}$

$\therefore |3I| = |-8| = 8$

Q82

The given equation is

$e^{\frac{x}{y}}\left(dx - \frac{x}{y}dy\right) + e^{\frac{x}{y}}dy + dx = 0$

or $e^{\frac{x}{y}} y d\left(\frac{x}{y}\right) + e^{\frac{x}{y}}dy + dx = 0$

$\Rightarrow d\left(e^{\frac{x}{y}}y\right) + dx = 0$

On integrating, we get,

$e^{\frac{x}{y}}y + x = C$

$\Rightarrow k = 1$

Q83

$\mathrm{P}\left(\frac{\mathrm{B}}{\mathrm{A}\cap\mathrm{C}}\right) = \frac{\mathrm{P}(\mathrm{A}\cap\mathrm{B}\cap\mathrm{C})}{\mathrm{P}(\mathrm{A}\cap\mathrm{C})}$

$\mathrm{A}\cap\mathrm{C}: \left(-\underline{\mathrm{T}}---\right)\ (---\underline{\mathrm{T}}--) - (-\underline{\mathrm{T}}-\underline{\mathrm{T}}--)$

$\mathrm{P}\left(\mathrm{A}\cap\mathrm{C}\right) = {}^5\mathrm{C}_4 \cdot \left(\frac{1}{2}\right)^5 \cdot \frac{1}{2} + {}^5\mathrm{C}_4 \cdot \left(\frac{1}{2}\right)^5 \cdot \frac{1}{2} - \left(\frac{1}{2}\right)^6$

$= \frac{9}{2^6}$

$\mathrm{A}\cap\mathrm{B}\cap\mathrm{C}: (\underline{\mathrm{H}}\ \ \underline{\mathrm{T}}\ \ \underline{\mathrm{HHHT}})$

or $(\underline{\mathrm{H}}, \underline{\mathrm{H}}\ \ \underline{\mathrm{HTHT}})$

$\mathrm{P}\left(\mathrm{A}\cap\mathrm{B}\cap\mathrm{C}\right) = \frac{2}{2^6}$

Required probability $= \frac{\frac{2}{2^6}}{\frac{9}{2^6}} = \frac{2}{9} = \frac{m}{n}$

$\therefore n - m = 7$

Q84

$y = f(x)$ in $[0, 10]$

Use L.M.V.T.

$$f'(x) = \frac{f(10) - f(0)}{10} = \frac{19 - f(0)}{10}$$

$$-4 \le \frac{19 - f(0)}{10} - 5 \le 4$$

$$1 \le \frac{19 - f(0)}{10} \le 9$$

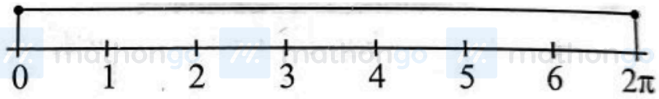
$$10 \le 19 - f(0) \le 90$$

$$-71 \le f(0) \le 9$$

$\therefore \quad f(0)|_{\text{max.}} = 9$

Q85

$x = I + \frac{1}{4}\left\{\frac{1}{4}, \frac{5}{4}, \frac{9}{4}, \frac{13}{4}, \frac{17}{4}, \frac{21}{4}, \frac{25}{4}\right\}$



$[x] \to \{1, 2, 3, 4, 5, 6\}$

$(2x-1)^{1/3} \to \left\{\frac{1}{2}\right\}$

$\sin x \to \pi$

$\Rightarrow$ Total 15 points

Q86

$4x + 7y + 4z + 81 = 0 \quad \ldots.(\text{i})$

$5x + 3y + 10z = 25 \quad \ldots.(\text{ii})$

Equation of plane passing through line of intersection of planes (i) and (ii) is

$(4x + 7y + 4z + 81) + \lambda(5x + 3y + 10z - 25) = 0 \quad \ldots.(\text{iii})$

or

$(4 + 5\lambda)x + (7 + 3\lambda)y + (4 + 10\lambda)z + 81 - 25\lambda = 0 \quad \ldots(\text{iv})$

Normal vector of plane $= 4\hat{i} + 7\hat{j} + 4\hat{k}$

(iv) is perpendicular to plane (i). So,

$$\left(4\hat{\mathrm{i}} + 7\hat{\mathrm{j}} + 4\hat{\mathrm{k}}\right) \cdot \left((4 + 5\lambda)\hat{\mathrm{i}} + (7 + 3\lambda)\hat{\mathrm{j}} + (4 + 10\lambda)\hat{\mathrm{k}}\right) = 0$$

$4(4 + 5\lambda) + 7(7 + 3\lambda) + 4(4 + 10\lambda) = 0$

$\therefore \lambda = -1$

From (iii), equation of plane is

$-x + 4y - 6z + 106 = 0 \quad \ldots(\text{v})$

Distance of (v) from $(0,0,0)$, i.e.,

$k=\frac{0+0+0+106}{\sqrt{1^2+4^2+6^2}}=\frac{106}{\sqrt{1+16+36}}=\frac{106}{\sqrt{53}}$

Therefore,

$\left[\frac{k}{2}\right]=\left[\frac{2\sqrt{53}}{2}\right]=\left[\sqrt{53}\right]=7$

$\Rightarrow\left[\frac{k}{2}\right]=7$

Q87

$A_1^T=-A_1,\ A_2^T=-A_2,\ \ldots\ldots,\ A_{20}^T=-A_{20}$

$B=\sum_{r=1}^{20}2r(A_r)^{2r+1}$

$B^T=\left(\sum_{r=1}^{20}2r(A_r)^{2r+1}\right)^T$

$=\sum_{r=1}^{20}2r\left(A_r^T\right)^{2r+1}$

$=\sum_{r=1}^{20}2r(-A_r)^{2r+1}$

$=-\sum_{r=1}^{20}2r(A_r)^{2r+1}$

$=-B$

$\Rightarrow B$ is skew-symmetric

Hence, the sum of principal diagonal elements of $B=0$

Q88

We can have $17^{256}=(290-1)^{128}$

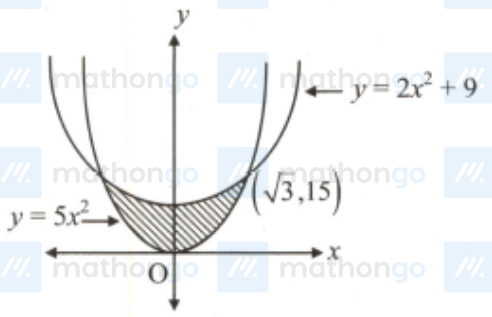
$=1000I+{}^{128}C_2(290)^2-{}^{128}C_1(290)+1$, where I is an integer

$=1000I+128(290)(18415-1)+1$

$=1000m+681$

Hence, $681/3=227$

Q89



Solving $y = 5x^2$ and $y = 2x^2 + 9$ we get $x = \pm\sqrt{3}$

Area, $A = \int_0^{\sqrt{3}} \left((2x^2 + 9) - 5x^2\right) dx$

$= 2\int_0^{\sqrt{3}} (9 - 3x^2) dx$

$= 2\left[9x - x^3\right]_0^{\sqrt{3}}$

$= 2\left(9\sqrt{3} - 3\sqrt{3}\right) = 12\sqrt{3}$

$\Rightarrow \frac{A^2}{12} = \frac{\left(12\sqrt{3}\right)^2}{12} = 36$

Q90

$$f(x) = x\left(2x^2 + ax + b\right)$$

$$D > 0$$

$$a^2 - 8b > 0$$

$$a^2 > 8b$$

$$(a, b)|_{\text{min.}} = (3, 1)$$

Note that b can not be zero. (think!)

$\therefore \quad a + b|_{\text{min.}} = 4$