Statistical Inference - course project

02/09/2021

The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is 1/lambda and the standard deviation is also also 1/lambda.

```
set.seed(1)
lambda <- 0.2 # Set lambda = 0.2 for all of the simulations.
n <- 40       # In this simulation, we investigate the distribution of averages
              # of 40 exponentials.
simulations <- 1:1000 # We need to do a thousand or so simulated averages
averages <- sapply(simulations, function(x) { mean(rexp(n, lambda)) })
```

1. Show where the distribution is centered at and compare it to the theoretical center of the distribution.

When we calculate sample and theorithical mean, we see that both lie close together.

```
mean(averages)
```

```
## [1] 4.990025
```

```
1/lambda
```

```
## [1] 5
```

2. Show how variable it is and compare it to the theoretical variance of the distribution.

From the CLT we know that X^bar approximately follows N(mu, sigma^2/n). We know sigma to be 1/lambda. As such it follows that the theoretical standard deviation is:

```
(1/lambda)/sqrt(40) # Theoretical standard deviation
```

```
## [1] 0.7905694
```

```
sd(averages)        # actual standard deviation
```

```
## [1] 0.7817394
```

```r
# And the variances
((1/lambda)/sqrt(40))^2
```

```
## [1] 0.625
```

```r
sd(averages)^2
```

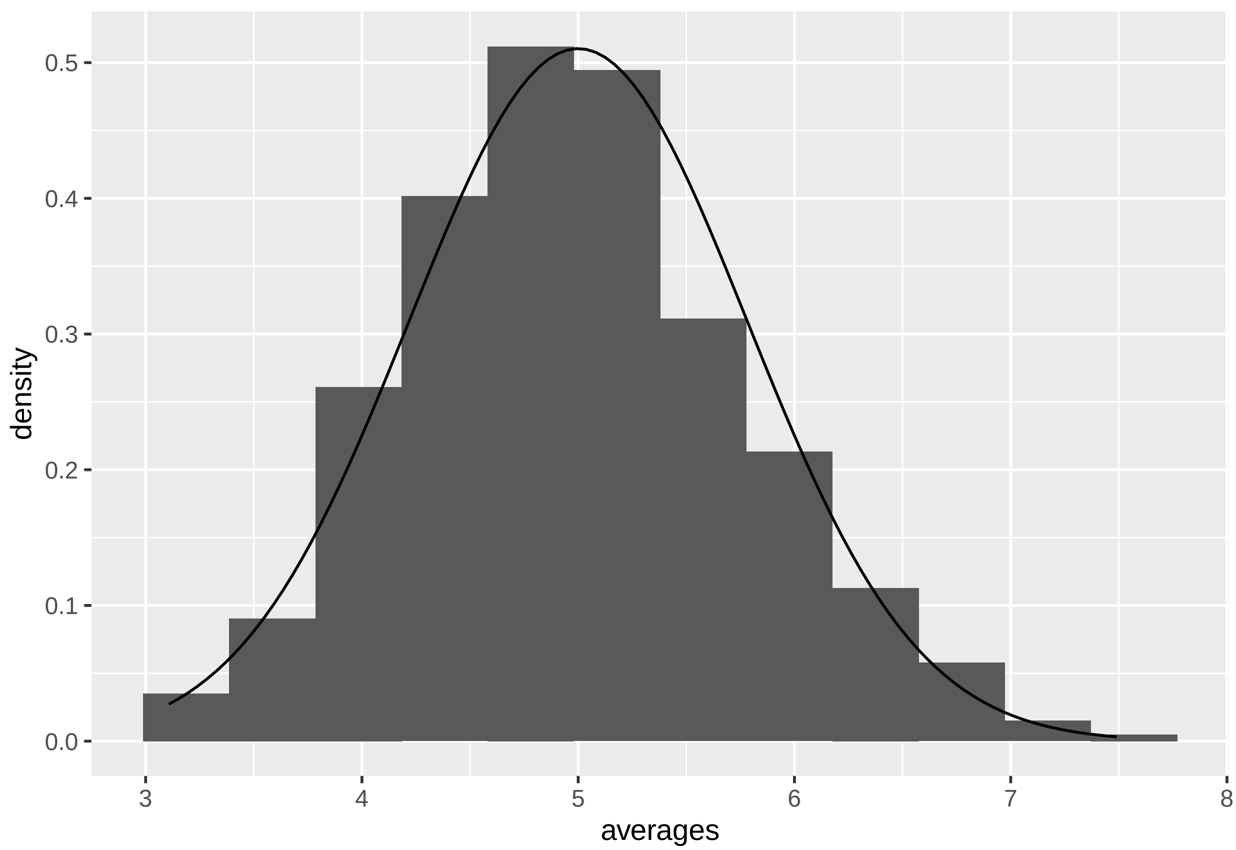
```
## [1] 0.6111165
```

3. Show that the distribution is approximately normal.

To do so, we plot an histogram of thesampled means and overlay the normal distribution with mean 5 and standard deviation 0.7817394 on top of it. We see that the normal distribution indeed closely matches the barplot of the means.

```r
library(ggplot2)
# Sturges' formula
k <- ceiling(log2(length(simulations)) + 1)
bw <- (range(averages)[2] - range(averages)[1]) / k
averages.sd <- sd(averages)

p <- ggplot(data.frame(averages), aes(x=averages))
p <- p + geom_histogram(aes(y=..density..), binwidth=bw)
p <- p + stat_function(fun = dnorm, args=list(mean=5, sd=averages.sd))
p
```



4. Evaluate the coverage.

Evaluate the coverage of the confidence interval for 1/lambda:

$$\bar{X} \pm 1.96 \frac{S}{\sqrt{n}}$$

.

```
mean(averages) + c(-1,1) * 1.96 * sd(averages) / sqrt(length(averages))
```

```
## [1] 4.941572 5.038478
```