$P: (n+100)^3 \leq 8n^3 \quad \forall n \geq 101$

$1 > 0$

$n+1 > n$ for all n

$\frac{1}{n} > \frac{1}{n+1}$ for all $n > 0$

for all $n > 0$,

$n > -1/2$

$2n > -1$

$2n+1 > 0$

$n^2 + 2n + 1 > n^2$ → n^2 added to both sides

$(n+1)^2 > n^2$

→ $\frac{1}{n^2} > \frac{1}{(n+1)^2} \quad \forall n > 0$

For all $n>0$

$3n>0$

$3n^2>0$

$3n^2+3n>0$

$3n^2+3n+1>1>0$

$n^3+3n^2+3n+1>n^3$ → n^3 added to both sides

$(n+1)^3>n^3$

$$\rightarrow \frac{1}{n^3}>\frac{1}{(n+1)^3} \quad \forall n>0$$

Let $f(n) = 300\frac{1}{n}+30000\frac{1}{n^2}+1000000\frac{1}{n^3}$

For all $n>0$,

$$\frac{1}{n+1}<\frac{1}{n}, \quad \frac{1}{(n+1)^2}<\frac{1}{n^2}, \quad \frac{1}{(n+1)^3}<\frac{1}{n^3}$$

$$300\frac{1}{n+1}<300\frac{1}{n}, \quad 30000\frac{1}{(n+1)^2}<30000\frac{1}{n^2},$$

$$1000000\frac{1}{(n+1)^3}<1000000\frac{1}{n^3}$$

$$300\frac{1}{n+1}+30000\frac{1}{(n+1)^2}+1000000\frac{1}{(n+1)^3}<300\frac{1}{n}+30000\frac{1}{n^2}+1000000\frac{1}{n^3}$$

$$\rightarrow f(n+1)<f(n) \quad \forall n>0$$

$f(100) = 300\frac{1}{100} + 30000\frac{1}{100^2} + 1000000\frac{1}{100^3} = 7$

$f(100) = 7$

For all $n > 100$

$f(n) < f(100)$ (shown on Pg 2)

for all $n > 100$

$f(n) < f(100) = 7 \Rightarrow f(n) < 7$

$300\frac{1}{n} + 30000\frac{1}{n^2} + 1000000\frac{1}{n^3} < 7 \quad \forall n > 100$

$300n^2 + 30000n + 1000000 < 7n^3 \quad \forall n > 100$

(multiplied both sides by n^3)

$n^3 + 300n^2 + 30000n + 1000000 < 8n^3 \quad \forall n > 100$

(added n^3 to both sides)

$(n+100)^3 < 8n^3 \quad \forall n > 100$

$(n+100)^3 \leq 8n^3 \quad \forall n \geq 101$

$P: n^3 \leq (n+100)^3 \quad \forall n \geq 1$

$n \geq 1$

$n > 0$

$\rightarrow 30\,000n > 0$

$n^2 > 0 \quad \forall n > 0$

$\rightarrow 300n^2 > 0$

$\rightarrow 1000\,000 > 0$

$300n^2 + 30\,000n + 1000\,000 > 0 \quad \forall n > 0$

Add n^3 to both sides:

$n^3 + 300n^2 + 30\,000n + 1000\,000 > n^3 \quad \forall n > 0$

$(n+100)^3 > n^3 \quad \forall n > 0$

1 is greater than 0, and $>$ is a subset of $\geq$, so this becomes

$(n+100)^3 \geq n^3 \quad \forall n \geq 1$