Actividad No. (3)
2024 Cálculo Vectorial.
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1 : De la página 963 del libro que seguimos de Cálculo de Varias Variables de Stewart, entre el ejercicio 3 y el ejercicio 12, escoja y resuelva 5 ejercicios.

a : $f(x,y) = 3x + y; x^2 + y^2 = 10$

$F(x,y) = 3x + y$
$G(x,y) = x^2 + y^2 - 10$

$\nabla F < Fx, Fy > = < 3, 1 >; \nabla G < Gx, Gy > = < 2x, 2y >$
$\nabla F = \lambda \nabla G$

$2\lambda x = 3$
$2\lambda y = 1$
$x^2 + y^2 = 10$

$$(i) x = -\frac{3}{2\lambda} \qquad (ii) y = -\frac{1}{2\lambda} \qquad (iii) \left(-\frac{3}{2\lambda}\right)^2 + \left(-\frac{1}{2\lambda}\right)^2 = 10$$

$$\left(-\frac{3}{2\lambda}\right)^2 + \left(-\frac{1}{2\lambda}\right)^2 = 10$$

$\frac{9}{4\lambda^2} + \frac{1}{4\lambda^2} = 10$
$\frac{9}{4\lambda^2} \cdot 4\lambda^2 + \frac{1}{4\lambda^2} \cdot 4\lambda^2 = 10 \cdot 4\lambda^2 \qquad simplificando : 10 = 40\lambda^2$

$\lambda^2 = \frac{1}{4}$
$\lambda = \sqrt{\frac{1}{4}},\ \lambda = -\sqrt{\frac{1}{4}}$
$\lambda = \frac{1}{2},\ \lambda = -\frac{1}{2}$

$Para : \lambda = \frac{1}{2}$
$x = -\frac{3}{2\lambda} = -\frac{3}{2*\left(\frac{1}{2}\right)} = -3 \qquad y = -\frac{1}{2\lambda} = -\frac{1}{2*\left(\frac{1}{2}\right)} = -1$

$Para :\ \lambda = -\frac{1}{2}$
$x = -\frac{3}{2\lambda} = -\frac{3}{2*\left(-\frac{1}{2}\right)} = 3 \qquad y = -\frac{1}{2\lambda} = -\frac{1}{2*\left(-\frac{1}{2}\right)} = 1$

R// Puntos críticos: $(-3,-1)\, y\, (3,1) \qquad \lambda = -\frac{1}{2}; \frac{1}{2}$

b : $f(x,y) = x^2 + y^2; xy = 1$

$F(x,y) = x^2 + y^2$
$G(x,y) = xy - 1$

$\frac{\partial}{\partial x}\left(x^2 + y^2\right) == \frac{\partial}{\partial x}\left(x^2\right) + \frac{\partial}{\partial x}\left(y^2\right) = 2x \qquad \frac{\partial}{\partial y}\left(x^2 + y^2\right) = \frac{\partial}{\partial y}\left(x^2\right) + \frac{\partial}{\partial y}\left(y^2\right) = 2y$
$\frac{\partial}{\partial x}(xy - 1) = \frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial x}(1) = y \qquad \frac{\partial}{\partial y}(xy - 1) = \frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial y}(1) = x$

$\nabla F < Fx, Fy > = < 2x, 2y >; \nabla G < Gx, Gy > = < y, x >$
$\nabla F = \lambda \nabla G$

$2x + \lambda y = 0$
$2y + \lambda x = 0$
$xy - 1 = 0$

$(i) \quad \lambda = -\frac{2x}{y}$
$(ii) \quad 2y = -\frac{\left(-\frac{2x}{y}\right)^2 y}{2} - \frac{2x^2}{y}$
$2y^2 = -2x^2$
$y = \pm x$

$(iii) \quad xy = 1$
$x\dfrac{1}{x} = 1$ se cumple sí $x \neq 0$
$\lambda = -\frac{2(1)}{(1)} = -2$
$\lambda = -\frac{2(-1)}{(-1)} = -2$

R// Puntos críticos: $(1,1)\, y\, (-1,-1) \qquad \lambda = -2$

c : $f\,(x,\ y) \ = \ e^{xy};\ x^3 \ + \ y^3 = 16$

$F(x,y) = e^{xy}$
$G(x,y) = x^3 \ + \ y^3 - 16$

$\frac{\partial}{\partial\, x}\left(e^{xy}\right) = e^{xy}\frac{\partial}{\partial\, x}\left(xy\right) = e^{xy}y; \qquad \frac{\partial}{\partial\, y}\left(e^{xy}\right) = e^{xy}\frac{\partial}{\partial\, y}\left(xy\right) = e^{xy}x;$

$\frac{\partial}{\partial\, x}\left(x^3 + y^3 - 16\right) = \frac{\partial}{\partial\, x}\left(x^3\right) + \frac{\partial}{\partial\, x}\left(y^3\right) - \frac{\partial}{\partial\, x}\left(16\right) = 3x^2; \qquad \frac{\partial}{\partial\, y}\left(x^3 + y^3 - 16\right) = \frac{\partial}{\partial\, y}\left(x^3\right) + \frac{\partial}{\partial\, y}\left(y^3\right) - \frac{\partial}{\partial\, y}\left(16\right) = 3y^2;$

$\nabla F < Fx, Fy > = < e^{xy}y, e^{xy}x >; \nabla G < Gx, Gy > = < 3x^2, 3y^2 >$
$\nabla F = \lambda \nabla G$

$e^{xy}y = \lambda 3x^2$
$e^{xy}x = \lambda 3y^2$
$x^3 \ + \ y^3 - 16 = 0$

$(i) \quad \lambda = \dfrac{e^{xy}y}{3x^2}; \quad x \neq \ 0 \qquad (ii) \quad \lambda = \dfrac{e^{xy}x}{3y^2}; \quad y \neq \ 0 \rightarrow \qquad \dfrac{e^{xy}y}{3x^2} = \dfrac{e^{xy}x}{3y^2} = \lambda \rightarrow \qquad ; \dfrac{y}{x^2} = \dfrac{x}{y^2}; \rightarrow \qquad y^3 = x^3;$

$(iii) \quad x^3 \ + \ x^3 - 16 = 0 \rightarrow \qquad 2x^3 = 16 \rightarrow \qquad x^3 = 8 \rightarrow \qquad x = 2;$

$(ii) \quad y^3 = (2)^3 \rightarrow \qquad y^3 = 8 \rightarrow \qquad y = \sqrt[3]{8} = 2$

$(i) \quad \lambda = \dfrac{e^{xy}y}{3x^2} \rightarrow \qquad \lambda = \frac{e^{2(2)} \cdot 2}{3(2)^2} \rightarrow \qquad \lambda = \frac{e^4}{6} = 9{,}0996$

R// Punto crítico: $(2,2) \qquad \lambda = \frac{e^4}{6} = 9{,}0996$