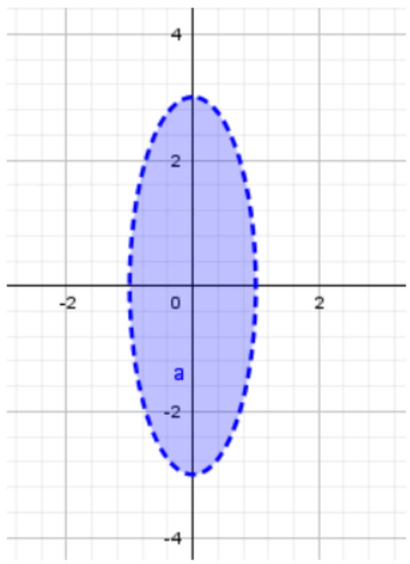
Actividad No. (2)
2024 Cálculo Vectorial.
Nombres: Camilo Rivera, Emerson Tavera, Karen Torres

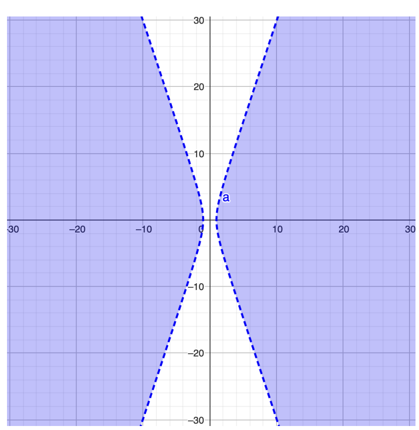
1 Diga si la afirmación es verdadera o falsa. Justifique sus respuestas:

a) La gráfica del dominio de $f(x, y) = \ln(9x^2 - y^2 - 9)$



R// Falso
Pues Dada la función: $f(x, y) = \ln(9x^2 - y^2 - 9)$
$9x^2 - y^2 - 9 > 0$
$9x^2 - y^2 > 9$
$x^2 - \frac{y^2}{9} > 1$

lo que representado gráficamente es:



b) $F_{xx} = \frac{1}{2\cos(x)}$; $Si\ f(x,y) = \int_y^x \ln(\cos(t))\,dt.$

$$f(x,y) = \int_y^x \ln(\cos(t))\,dt$$

f_x teorema fundamental del cálculo

$$f_x = \frac{\partial}{\partial x}\int_y^x \ln(\cos(t))\,dt = \ln(\cos(x))$$

f_{xx} regla de la cadena para diferenciar f_x con respecto a x:

$$f_{xx} = \frac{d}{dx}\ln(\cos(x)) = \frac{1}{\cos(x)}\cdot(-\sin(x)) = -\tan(x)$$

R// Falso $f_{xx} = -\tan(x) \neq \frac{1}{2\cos(x)}$

c) $F_{yy} = \frac{y}{\cos(y)}$; $Si\ f(x,y) = \frac{\sin(x+y)}{x}$

$$\frac{\partial}{\partial y}\left(\frac{\sin(x+y)}{x}\right) = \frac{1}{x}\frac{\partial}{\partial y}\left(\sin\left(x+y\right)\right)$$

Regla cadena

$$\frac{\partial}{\partial u}\left(\sin\left(u\right)\right)\frac{\partial}{\partial y}\left(x+y\right) = \cos\left(u\right)\frac{\partial}{\partial y}\left(x+y\right) =$$
$$\cos\left(x+y\right)\frac{\partial}{\partial y}\left(x+y\right)$$
$$\frac{\partial}{\partial y}\left(x\right) + \frac{\partial}{\partial y}\left(y\right) = 0+1$$
$$\frac{1}{x}\cos\left(x+y\right)\cdot\ 1 = \frac{\cos(x+y)}{x}$$

$$\frac{\partial}{\partial y}\left(\frac{\cos(x+y)}{x}\right) = \frac{1}{x}\frac{\partial}{\partial y}\left(\cos\left(x+y\right)\right)$$

Regla cadena

$$\frac{\partial}{\partial y}\left(\cos\left(x+y\right)\right)$$
$$\frac{\partial}{\partial u}\left(\cos\left(u\right)\right)\frac{\partial}{\partial y}\left(x+y\right) =$$
$$-\sin\left(u\right)\frac{\partial}{\partial y}\left(x+y\right) =$$
$$-\sin\left(x+y\right)\frac{\partial}{\partial y}\left(x+y\right)$$
$$= \frac{1}{x}\left(-\sin\left(x+y\right)\cdot\ 1\right) = -\frac{\sin(x+y)}{x}$$

R// Falso $-\frac{\sin(x+y)}{x} \neq \frac{\sin(x+y)}{x}$

D) $F_{yx} = F_{xy};\ Si\ f(x,y) = \frac{\sin(x+y)}{x}$

$$\frac{\partial}{\partial\, y}\left(\frac{\sin(x+y)}{x}\right) = \frac{\cos(x+y)}{x}$$

$$\frac{\partial}{\partial\, x}\left(\frac{\cos(x+y)}{x}\right) = \frac{\frac{\partial}{\partial\, x}(\cos(x+y))x - \frac{\partial}{\partial\, x}(x)\cos(x+y)}{x^2}$$

$$\frac{\partial}{\partial\, x}\left(\cos\left(x+y\right)\right) =$$
$$-\sin\left(x+y\right)\frac{\partial}{\partial\, x}\left(x+y\right) = -\sin\left(x+y\right)\frac{\partial}{\partial\, x}\left(x+y\right) =$$
$$-\sin\left(x+y\right)\cdot\ 1 = -\sin\left(x+y\right)$$
$$\frac{\partial}{\partial\, x}\left(x\right) = 1$$

$$\frac{(-\sin(x+y))x - 1\cdot\cos(x+y)}{x^2} = \frac{-x\sin(x+y) - \cos(x+y)}{x^2}$$

$$\frac{\partial}{\partial\, x}\left(\frac{\sin(x+y)}{x}\right) = \frac{\frac{\partial}{\partial\, x}(\sin(x+y))x - \frac{\partial}{\partial\, x}(x)\sin(x+y)}{x^2}$$

$$\frac{\partial}{\partial\, x}\left(\sin\left(x+y\right)\right) = \cos\left(x+y\right)\frac{\partial}{\partial\, x}\left(x+y\right)$$
$$\cos\left(x+y\right)\cdot\ 1 = \cos\left(x+y\right)$$

$$\frac{\partial}{\partial\, x}\left(x\right) = 1$$

$$= \frac{\cos(x+y)x - 1\cdot\sin(x+y)}{x^2} = \frac{x\cos(x+y) - \sin(x+y)}{x^2}$$

$$\frac{\partial}{\partial\, y}\left(\frac{x\cos(x+y) - \sin(x+y)}{x^2}\right) = \frac{1}{x^2}\frac{\partial}{\partial\, y}\left(x\cos\left(x+y\right) - \sin\left(x+y\right)\right) =$$
$$\frac{1}{x^2}\left(\frac{\partial}{\partial\, y}\left(x\cos\left(x+y\right)\right) - \frac{\partial}{\partial\, y}\left(\sin\left(x+y\right)\right)\right) =$$

$$\frac{\partial}{\partial\, y}\left(x\cos\left(x+y\right)\right) = x\frac{\partial}{\partial\, y}\left(\cos\left(x+y\right)\right) =$$
$$-\sin\left(x+y\right)\frac{\partial}{\partial\, y}\left(x+y\right) = x\left(-\sin\left(x+y\right)\cdot\ 1\right) = -x\sin\left(x+y\right)$$

$$\frac{\partial}{\partial\, y}\left(\sin\left(x+y\right)\right) = \cos\left(x+y\right)\frac{\partial}{\partial\, y}\left(x+y\right) =$$
$$\cos\left(x+y\right)\cdot\ 1 == \cos\left(x+y\right)$$

$$= \frac{1}{x^2}\left(-x\sin\left(x+y\right) - \cos\left(x+y\right)\right) = \frac{-x\sin(x+y) - \cos(x+y)}{x^2}$$

R// Verdadero $\frac{-x\sin(x+y)-\cos(x+y)}{x^2} = \frac{-x\sin(x+y)-\cos(x+y)}{x^2}$