Actividad No. (2)
2024 Cálculo Vectorial.
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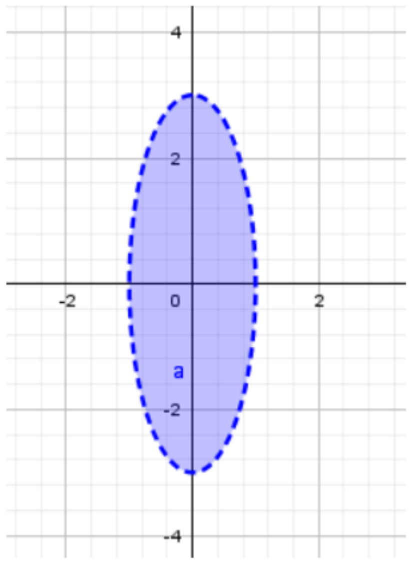
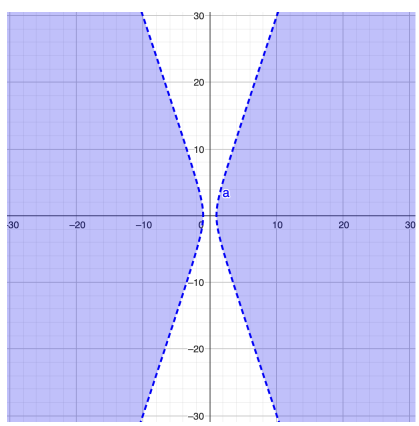
1 Diga si la afirmación es verdadera o falsa. Justifique sus respuestas:

a) La gráfica del dominio de $f(x,y) = \ln(9x^2 - y^2 - 9)$



R// Falso
Pues Dada la función: $f(x,y) = \ln(9x^2 - y^2 - 9)$
$9x^2 - y^2 - 9 > 0$
$9x^2 - y^2 > 9$
$x^2 - \frac{y^2}{9} > 1$

lo que representado gráficamente es:



b) $F_{xx} = \frac{1}{2\cos(x)}$; $Si\ f(x,y) = \int_y^x \ln(\cos(t))\,dt$.

$f(x,y) = \int_y^x \ln(\cos(t))\,dt$

f_x teorema fundamental del cálculo
$f_x = \frac{\partial}{\partial x}\int_y^x \ln(\cos(t))\,dt = \ln(\cos(x))$

f_{xx} regla de la cadena para diferenciar f_x con respecto a x:
$f_{xx} = \frac{d}{dx}\ln(\cos(x)) = \frac{1}{\cos(x)} \cdot (-\sin(x)) = -\tan(x)$

R// Falso $f_{xx} = -\tan(x) \neq \frac{1}{2\cos(x)}$

c) $F_{yy} = \frac{y}{\cos(y)};\ Si\ f(x,y) = \frac{\sin(x+y)}{x}$

$\frac{\partial}{\partial\, y}\left(\frac{\sin(x+y)}{x}\right) = \frac{1}{x}\frac{\partial}{\partial\, y}\left(\sin\left(x+y\right)\right)$

Regla cadena
$\frac{\partial}{\partial\, u}\left(\sin\left(u\right)\right)\frac{\partial}{\partial\, y}\left(x+y\right) = \cos\left(u\right)\frac{\partial}{\partial\, y}\left(x+y\right) =$
$\cos\left(x+y\right)\frac{\partial}{\partial\, y}\left(x+y\right)$
$\frac{\partial}{\partial\, y}\left(x\right) + \frac{\partial}{\partial\, y}\left(y\right) = 0+1$
$\frac{1}{x}\cos\left(x+y\right)\cdot\ 1 = \frac{\cos(x+y)}{x}$

$\frac{\partial}{\partial\, y}\left(\frac{\cos(x+y)}{x}\right) = \frac{1}{x}\frac{\partial}{\partial\, y}\left(\cos\left(x+y\right)\right)$
Regla cadena
$\frac{\partial}{\partial\, y}\left(\cos\left(x+y\right)\right)$
$\frac{\partial}{\partial\, u}\left(\cos\left(u\right)\right)\frac{\partial}{\partial\, y}\left(x+y\right) =$
$-\sin\left(u\right)\frac{\partial}{\partial\, y}\left(x+y\right) =$
$-\sin\left(x+y\right)\frac{\partial}{\partial\, y}\left(x+y\right)$
$= \frac{1}{x}\left(-\sin\left(x+y\right)\cdot\ 1\right) = -\frac{\sin(x+y)}{x}$

R// Falso $-\frac{\sin(x+y)}{x} \neq \frac{\sin(x+y)}{x}$

D) $F_{yx} = F_{xy};\ Si\ f(x,y) = \frac{\sin(x+y)}{x}$

$\frac{\partial}{\partial\, y}\left(\frac{\sin(x+y)}{x}\right) = \frac{\cos(x+y)}{x}$

$\frac{\partial}{\partial\, x}\left(\frac{\cos(x+y)}{x}\right) = \frac{\frac{\partial}{\partial\, x}(\cos(x+y))x - \frac{\partial}{\partial\, x}(x)\cos(x+y)}{x^2}$

$\frac{\partial}{\partial\, x}\left(\cos\left(x+y\right)\right) =$
$-\sin\left(x+y\right)\frac{\partial}{\partial\, x}\left(x+y\right) = -\sin\left(x+y\right)\frac{\partial}{\partial\, x}\left(x+y\right) =$
$-\sin\left(x+y\right)\cdot\ 1 = -\sin\left(x+y\right)$
$\frac{\partial}{\partial\, x}\left(x\right) = 1$

$\frac{(-\sin(x+y))x - 1\cdot\cos(x+y)}{x^2} = \frac{-x\sin(x+y) - \cos(x+y)}{x^2}$

$\frac{\partial}{\partial\, x}\left(\frac{\sin(x+y)}{x}\right) = \frac{\frac{\partial}{\partial\, x}(\sin(x+y))x - \frac{\partial}{\partial\, x}(x)\sin(x+y)}{x^2}$

$\frac{\partial}{\partial\, x}\left(\sin\left(x+y\right)\right) = \cos\left(x+y\right)\frac{\partial}{\partial\, x}\left(x+y\right)$
$\cos\left(x+y\right)\cdot\ 1 = \cos\left(x+y\right)$

$\frac{\partial}{\partial\, x}\left(x\right) = 1$

$= \frac{\cos(x+y)x - 1\cdot\sin(x+y)}{x^2} = \frac{x\cos(x+y) - \sin(x+y)}{x^2}$

$\frac{\partial}{\partial y}\left(\frac{x\cos(x+y)-\sin(x+y)}{x^2}\right) = \frac{1}{x^2}\frac{\partial}{\partial y}\left(x\cos\left(x+y\right)-\sin\left(x+y\right)\right) =$
$\frac{1}{x^2}\left(\frac{\partial}{\partial y}\left(x\cos\left(x+y\right)\right)-\frac{\partial}{\partial y}\left(\sin\left(x+y\right)\right)\right) =$

$\frac{\partial}{\partial y}\left(x\cos\left(x+y\right)\right) = x\frac{\partial}{\partial y}\left(\cos\left(x+y\right)\right) =$
$-\sin\left(x+y\right)\frac{\partial}{\partial y}\left(x+y\right) = x\left(-\sin\left(x+y\right)\cdot\ 1\right) = -x\sin\left(x+y\right)$

$\frac{\partial}{\partial y}\left(\sin\left(x+y\right)\right) = \cos\left(x+y\right)\frac{\partial}{\partial y}\left(x+y\right) =$
$\cos\left(x+y\right)\cdot\ 1 == \cos\left(x+y\right)$

$= \frac{1}{x^2}\left(-x\sin\left(x+y\right)-\cos\left(x+y\right)\right) = \frac{-x\sin(x+y)-\cos(x+y)}{x^2}$

R// Verdadero $\frac{-x\sin(x+y)-\cos(x+y)}{x^2} = \frac{-x\sin(x+y)-\cos(x+y)}{x^2}$

2 Halle los extremos relativos de:

A) $F_{y,x} = -x^3+4xy-2y^2+1$

Puntos criticos
$\frac{\partial}{\partial x}\left(-x^3+4xy-2y^2+1\right) = -\frac{\partial}{\partial x}\left(x^3\right)+\frac{\partial}{\partial x}\left(4xy\right)-\frac{\partial}{\partial x}\left(2y^2\right)+\frac{\partial}{\partial x}\left(1\right) = 4y-3x^2$
$\frac{\partial}{\partial y}\left(-x^3+4xy-2y^2+1\right) = -\frac{\partial}{\partial y}\left(x^3\right)+\frac{\partial}{\partial y}\left(4xy\right)-\frac{\partial}{\partial y}\left(2y^2\right)+\frac{\partial}{\partial y}\left(1\right) = 4x-4y$
$\nabla f\left(x,\ y\right)\left(4y-3x^2,\ 4x-4y\right)$

$\begin{bmatrix} 4y-3x^2=0 \\ 4x-4y=0 \end{bmatrix}$

$4y-3x^2+\left(4x-4y\right) = 0+0$
$-3x^2+4x = 0$
$x_{1,\,2} = \frac{-4\pm\sqrt{4^2-4(-3)\cdot\,0}}{2(-3)}\ \left(x=0,\ x=\frac{4}{3}\right)$

$4y-3\cdot\ 0^2 = 0$
$y = 0$

$4y-3\left(\frac{4}{3}\right)^2 = 0$
$y = \frac{4}{3}$

$\begin{pmatrix} x=0, & y=0 \\ x=\frac{4}{3}, & y=\frac{4}{3} \end{pmatrix}\ \left(0,\ 0\right),\ \left(\frac{4}{3},\ \frac{4}{3}\right)$

$\frac{\partial^2}{\partial x^2}\left(-x^3+4xy-2y^2+1\right) = -\frac{\partial}{\partial x}\left(x^3\right)+\frac{\partial}{\partial x}\left(4xy\right)-\frac{\partial}{\partial x}\left(2y^2\right)+\frac{\partial}{\partial x}\left(1\right) =$
$-3x^2+4y-0+0$
$\frac{\partial}{\partial x}\left(4y-3x^2\right) = -6x$

$\frac{\partial^2}{\partial y^2}\left(-x^3+4xy-2y^2+1\right) = -\frac{\partial}{\partial y}\left(x^3\right)+\frac{\partial}{\partial y}\left(4xy\right)-\frac{\partial}{\partial y}\left(2y^2\right)+\frac{\partial}{\partial y}\left(1\right) =$
$-0+4x-4y+0$
$\frac{\partial}{\partial y}\left(4x-4y\right) = -4$

$\frac{\partial^2}{\partial x\partial y}\left(-x^3+4xy-2y^2+1\right)$
$\frac{\partial}{\partial x}\left(-x^3+4xy-2y^2+1\right) = -3x^2+4y-0+0 = 4y-3x^2$
$\frac{\partial}{\partial y}\left(4y-3x^2\right) = 4$

$D\left(x,\ y\right) = \left(-6x\right)\left(-4\right)-\left(4\right)^2 = 24x-16$
$D\left(x,\ y\right) = 24x-16$ en $\left(0,\ 0\right)$: Negativo
Silla $\left(0,\ 0\right)$
$D\left(x,\ y\right) = 24x-16$ en $\left(\frac{4}{3},\ \frac{4}{3}\right)$: Positivo

Máximo $\left(\frac{4}{3},\ \frac{4}{3}\right)$

B) $F_{y,x} = x^2 - y^2 - x - y$

Puntos criticos

$f = x^2 - y^2 - x - y$

$\frac{\partial}{\partial\, x}\left(x^2 - y^2 - x - y\right) = \frac{\partial}{\partial\, x}\left(x^2\right) - \frac{\partial}{\partial\, x}\left(y^2\right) - \frac{\partial}{\partial\, x}\left(x\right) - \frac{\partial}{\partial\, x}\left(y\right) = 2x - 1$

$\frac{\partial}{\partial\, y}\left(x^2 - y^2 - x - y\right) = \frac{\partial}{\partial\, y}\left(x^2\right) - \frac{\partial}{\partial\, y}\left(y^2\right) - \frac{\partial}{\partial\, y}\left(x\right) - \frac{\partial}{\partial\, y}\left(y\right) = -2y - 1$

$\nabla f\left(x,\ y\right) = \left(2x - 1,\ -2y - 1\right)$

$\begin{bmatrix} 4y - 3x^2 = 0 \\ 4x - 4y = 0 \end{bmatrix};\ 4y - 3x^2 + (4x - 4y) = 0 + 0 = -3x^2 + 4x = 0$

$x_{1,\,2} = \frac{-4 \pm \sqrt{4^2 - 4(-3) \cdot 0}}{2(-3)}\ \left(x = 0,\ x = \frac{4}{3}\right)$

$2x - 1 = 0 \quad x = \frac{1}{2}\ ; \qquad 2y - 1 = 0 \quad y = -\frac{1}{2}$

$\frac{\partial^2}{\partial\, y^2}\left(x^2 - y^2 - x - y\right) = \frac{\partial}{\partial\, y}\left(x^2\right) - \frac{\partial}{\partial\, y}\left(y^2\right) - \frac{\partial}{\partial\, y}\left(x\right) - \frac{\partial}{\partial\, y}\left(y\right) = -2y - 1$

$\frac{\partial}{\partial\, y}\left(-2y - 1\right) = -\frac{\partial}{\partial\, y}\left(2y\right) - \frac{\partial}{\partial\, y}\left(1\right) = -2$

$\frac{\partial^2}{\partial\, x^2}\left(x^2 - y^2 - x - y\right) = \frac{\partial}{\partial\, x}\left(2x - 1\right) = 2$

$\frac{\partial^2}{\partial\, y^2}\left(x^2 - y^2 - x - y\right) = \frac{\partial}{\partial\, y}\left(-2y - 1\right) = -2$

$\frac{\partial^2}{\partial\, x \partial\, y}\left(x^2 - y^2 - x - y\right)$

$\frac{\partial}{\partial\, x}\left(x^2 - y^2 - x - y\right) = 2x - 1$

$\frac{\partial}{\partial\, y}\left(2x - 1\right) = 0$

$D\left(x,\ y\right) = 2\left(-2\right) - \left(0\right)^2 = -4$

$D\left(x,\ y\right) = -4$ en $\left(\frac{1}{2},\ -\frac{1}{2}\right):$ Negativo

$D < 0\ en\ \left(\frac{1}{2},\ -\frac{1}{2}\right),$ Silla $\left(\frac{1}{2},\ -\frac{1}{2}\right)$

C) $F_{y,x} = x^2 - y^2 - 3xy$

Puntos criticos

$f = x^2 - y^2 - 3xy$

$\frac{\partial}{\partial\, x}\left(x^2 - y^2 - 3xy\right) = \frac{\partial}{\partial\, x}\left(x^2\right) - \frac{\partial}{\partial\, x}\left(y^2\right) - \frac{\partial}{\partial\, x}\left(3xy\right) = 2x - 3ys$

$\frac{\partial}{\partial\, y}\left(x^2 - y^2 - 3xy\right) = \frac{\partial}{\partial\, y}\left(x^2\right) - \frac{\partial}{\partial\, y}\left(y^2\right) - \frac{\partial}{\partial\, y}\left(3xy\right) = -3x - 2y$

$\nabla f\left(x,\ y\right) = \left(2x - 3y,\ -3x - 2y\right)$

$\begin{bmatrix} 2x - 3y = 0 \\ -3x - 2y = 0 \end{bmatrix}$

$2x - 3y = 0x = \frac{3y}{2}$

$-3 \cdot \frac{3y}{2} - 2y = 0 \qquad -\frac{13y}{2} = 0 \qquad x = 0,\ y = 0$

$\frac{\partial^2}{\partial\, y^2}\left(x^2 - y^2 - 3xy\right)$

$\frac{\partial}{\partial\, y}\left(x^2 - y^2 - 3xy\right) = \frac{\partial}{\partial\, y}\left(x^2\right) - \frac{\partial}{\partial\, y}\left(y^2\right) - \frac{\partial}{\partial\, y}\left(3xy\right) = -3x - 2y$

$\frac{\partial}{\partial y}\left(-3x-2y\right)=-\frac{\partial}{\partial y}\left(3x\right)-\frac{\partial}{\partial y}\left(2y\right)=-0-2=-2$

$f\left(x,\ y\right)=x^2-y^2-3xy$
$\frac{\partial^2}{\partial x^2}\left(x^2-y^2-3xy\right)=\frac{\partial}{\partial x}\left(2x-3y\right)=2$
$\frac{\partial^2}{\partial y^2}\left(x^2-y^2-3xy\right)=\frac{\partial}{\partial y}\left(-3x-2y\right)=-2$
$\frac{\partial^2}{\partial x\partial y}\left(x^2-y^2-3xy\right)=\frac{\partial}{\partial y}\left(2x-3y\right)=-3$

$D\left(x,\ y\right)=2\left(-2\right)-\left(\left(-3\right)\right)^2=-13$

$D\left(x,\ y\right)=-13\,\mathrm{en}\ \left(0,\ 0\right):$ Negativo
$D\left(x,\ y\right)<0,\ \mathrm{Silla}\left(0,\ 0\right)$