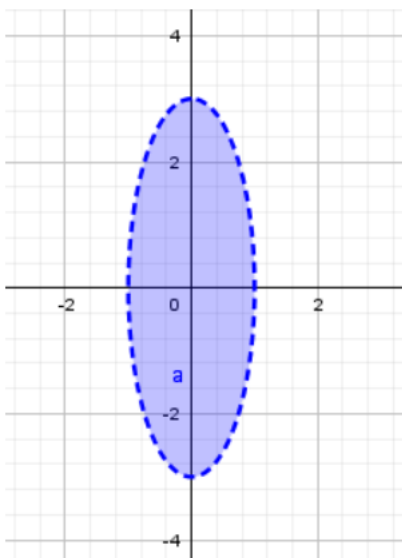


Actividad No. (2)
2024 Cálculo Vectorial.
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1 Diga si la afirmación es verdadera o falsa. Justifique sus respuestas:

a) La gráfica del dominio de $f(x, y) = \ln(9x^2 - y^2 - 9)$



R// Falso

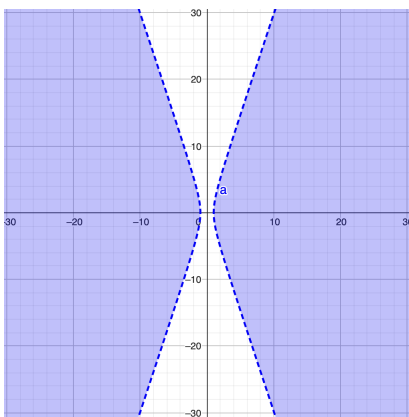
Pues Dada la función: $f(x, y) = \ln(9x^2 - y^2 - 9)$

$$9x^2 - y^2 - 9 > 0$$

$$9x^2 - y^2 > 9$$

$$x^2 - \frac{y^2}{9} > 1$$

lo que representado gráficamente es:



b) $F_{xx} = \frac{1}{2\cos(x)}$; Si $f(x, y) = \int_y^x \ln(\cos(t)) dt$.

$$f(x, y) = \int_y^x \ln(\cos(t)) dt$$

f_x teorema fundamental del cálculo

$$f_x = \frac{\partial}{\partial x} \int_y^x \ln(\cos(t)) dt = \ln(\cos(x))$$

f_{xx} regla de la cadena para diferenciar f_x con respecto a x :

$$f_{xx} = \frac{d}{dx} \ln(\cos(x)) = \frac{1}{\cos(x)} \cdot (-\sin(x)) = -\tan(x)$$

$$\mathbf{R// Falso} \quad f_{xx} = -\tan(x) \neq \frac{1}{2\cos(x)}$$

c) $F_{yy} = \frac{y}{\cos(y)}$; Si $f(x, y) = \frac{\sin(x+y)}{x}$

$$\frac{\partial}{\partial y} \left(\frac{\sin(x+y)}{x} \right) = \frac{1}{x} \frac{\partial}{\partial y} (\sin(x+y))$$

Regla cadena

$$\frac{\partial}{\partial u} (\sin(u)) \frac{\partial}{\partial y} (x+y) = \cos(u) \frac{\partial}{\partial y} (x+y) =$$

$$\cos(x+y) \frac{\partial}{\partial y} (x+y)$$

$$\frac{\partial}{\partial y} (x) + \frac{\partial}{\partial y} (y) = 0 + 1$$

$$\frac{1}{x} \cos(x+y) \cdot 1 = \frac{\cos(x+y)}{x}$$

$$\frac{\partial}{\partial y} \left(\frac{\cos(x+y)}{x} \right) = \frac{1}{x} \frac{\partial}{\partial y} (\cos(x+y))$$

Regla cadena

$$\frac{\partial}{\partial y} (\cos(x+y))$$

$$\frac{\partial}{\partial u} (\cos(u)) \frac{\partial}{\partial y} (x+y) =$$

$$-\sin(u) \frac{\partial}{\partial y} (x+y) =$$

$$-\sin(x+y) \frac{\partial}{\partial y} (x+y)$$

$$= \frac{1}{x} (-\sin(x+y) \cdot 1) = -\frac{\sin(x+y)}{x}$$

$$\mathbf{R// Falso} \quad -\frac{\sin(x+y)}{x} \neq \frac{\sin(x+y)}{x}$$

D) $F_{yx} = F_{xy}$; Si $f(x, y) = \frac{\sin(x+y)}{x}$

$$\frac{\partial}{\partial y} \left(\frac{\sin(x+y)}{x} \right) = \frac{\cos(x+y)}{x}$$

$$\frac{\partial}{\partial x} \left(\frac{\cos(x+y)}{x} \right) = \frac{\frac{\partial}{\partial x} (\cos(x+y))x - \frac{\partial}{\partial x} (x) \cos(x+y)}{x^2}$$

$$\frac{\partial}{\partial x} (\cos(x+y)) =$$

$$-\sin(x+y) \frac{\partial}{\partial x} (x+y) = -\sin(x+y) \frac{\partial}{\partial x} (x+y) =$$

$$-\sin(x+y) \cdot 1 = -\sin(x+y)$$

$$\frac{\partial}{\partial x} (x) = 1$$

$$\frac{(-\sin(x+y))x - 1 \cdot \cos(x+y)}{x^2} = \frac{-x \sin(x+y) - \cos(x+y)}{x^2}$$

$$\frac{\partial}{\partial x} \left(\frac{\sin(x+y)}{x} \right) = \frac{\frac{\partial}{\partial x} (\sin(x+y))x - \frac{\partial}{\partial x} (x) \sin(x+y)}{x^2}$$

$$\frac{\partial}{\partial x} (\sin(x+y)) = \cos(x+y) \frac{\partial}{\partial x} (x+y)$$

$$\cos(x+y) \cdot 1 = \cos(x+y)$$

$$\frac{\partial}{\partial x} (x) = 1$$

$$= \frac{\cos(x+y)x - 1 \cdot \sin(x+y)}{x^2} = \frac{x \cos(x+y) - \sin(x+y)}{x^2}$$

$$\frac{\partial}{\partial y} \left(\frac{x \cos(x+y) - \sin(x+y)}{x^2} \right) = \frac{1}{x^2} \frac{\partial}{\partial y} (x \cos(x+y) - \sin(x+y)) =$$

$$\frac{1}{x^2} \left(\frac{\partial}{\partial y} (x \cos(x+y)) - \frac{\partial}{\partial y} (\sin(x+y)) \right) =$$

$$\frac{\partial}{\partial y} (x \cos(x+y)) = x \frac{\partial}{\partial y} (\cos(x+y)) =$$

$$- \sin(x+y) \frac{\partial}{\partial y} (x+y) = x (-\sin(x+y) \cdot 1) = -x \sin(x+y)$$

$$\frac{\partial}{\partial y} (\sin(x+y)) = \cos(x+y) \frac{\partial}{\partial y} (x+y) =$$

$$\cos(x+y) \cdot 1 = \cos(x+y)$$

$$= \frac{1}{x^2} (-x \sin(x+y) - \cos(x+y)) = \frac{-x \sin(x+y) - \cos(x+y)}{x^2}$$

$$\mathbf{R} // \text{ Verdadero } \frac{-x \sin(x+y) - \cos(x+y)}{x^2} = \frac{-x \sin(x+y) - \cos(x+y)}{x^2}$$

2 Halle los extremos relativos de:

A) $F_{y,x} = -x^3 + 4xy - 2y^2 + 1$

Puntos criticos

$$\frac{\partial}{\partial x} (-x^3 + 4xy - 2y^2 + 1) = -\frac{\partial}{\partial x} (x^3) + \frac{\partial}{\partial x} (4xy) - \frac{\partial}{\partial x} (2y^2) + \frac{\partial}{\partial x} (1) = 4y - 3x^2$$

$$\frac{\partial}{\partial y} (-x^3 + 4xy - 2y^2 + 1) = -\frac{\partial}{\partial y} (x^3) + \frac{\partial}{\partial y} (4xy) - \frac{\partial}{\partial y} (2y^2) + \frac{\partial}{\partial y} (1) = 4x - 4y$$

$$\nabla f(x, y) (4y - 3x^2, 4x - 4y)$$

$$\begin{bmatrix} 4y - 3x^2 = 0 \\ 4x - 4y = 0 \end{bmatrix}$$

$$4y - 3x^2 + (4x - 4y) = 0 + 0$$

$$-3x^2 + 4x = 0$$

$$x_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4(-3) \cdot 0}}{2(-3)} \quad (x = 0, x = \frac{4}{3})$$

$$4y - 3 \cdot 0^2 = 0$$

$$y = 0$$

$$4y - 3 \left(\frac{4}{3} \right)^2 = 0$$

$$y = \frac{4}{3}$$

$$\left(\begin{matrix} x = 0, & y = 0 \\ x = \frac{4}{3}, & y = \frac{4}{3} \end{matrix} \right) (0, 0), \left(\frac{4}{3}, \frac{4}{3} \right)$$

$$\frac{\partial^2}{\partial x^2} (-x^3 + 4xy - 2y^2 + 1) = -\frac{\partial}{\partial x} (x^3) + \frac{\partial}{\partial x} (4xy) - \frac{\partial}{\partial x} (2y^2) + \frac{\partial}{\partial x} (1) =$$

$$-3x^2 + 4y - 0 + 0$$

$$\frac{\partial}{\partial x} (4y - 3x^2) = -6x$$

$$\frac{\partial^2}{\partial y^2} (-x^3 + 4xy - 2y^2 + 1) = -\frac{\partial}{\partial y} (x^3) + \frac{\partial}{\partial y} (4xy) - \frac{\partial}{\partial y} (2y^2) + \frac{\partial}{\partial y} (1) =$$

$$-0 + 4x - 4y + 0$$

$$\frac{\partial}{\partial y} (4x - 4y) = -4$$

$$\frac{\partial^2}{\partial x \partial y} (-x^3 + 4xy - 2y^2 + 1)$$

$$\frac{\partial}{\partial x} (-x^3 + 4xy - 2y^2 + 1) = -3x^2 + 4y - 0 + 0 = 4y - 3x^2$$

$$\frac{\partial}{\partial y} (4y - 3x^2) = 4$$

$$D(x, y) = (-6x)(-4) - (4)^2 = 24x - 16$$

$$D(x, y) = 24x - 16 \text{ en } (0, 0) : \text{ Negativo}$$

$$\text{Silla } (0, 0)$$

$$D(x, y) = 24x - 16 \text{ en } \left(\frac{4}{3}, \frac{4}{3} \right) : \text{ Positivo}$$

Máximo $(\frac{4}{3}, \frac{4}{3})$

B) $F_{y,x} = x^2 - y^2 - x - y$

Puntos criticos

$$f = x^2 - y^2 - x - y$$

$$\frac{\partial}{\partial x} (x^2 - y^2 - x - y) = \frac{\partial}{\partial x} (x^2) - \frac{\partial}{\partial x} (y^2) - \frac{\partial}{\partial x} (x) - \frac{\partial}{\partial x} (y) = 2x - 1$$

$$\frac{\partial}{\partial y} (x^2 - y^2 - x - y) = \frac{\partial}{\partial y} (x^2) - \frac{\partial}{\partial y} (y^2) - \frac{\partial}{\partial y} (x) - \frac{\partial}{\partial y} (y) = -2y - 1$$

$$\nabla f(x, y) = (2x - 1, -2y - 1)$$

$$\begin{bmatrix} 4y - 3x^2 = 0 \\ 4x - 4y = 0 \end{bmatrix}; 4y - 3x^2 + (4x - 4y) = 0 + 0 = -3x^2 + 4x = 0$$

$$x_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4(-3) \cdot 0}}{2(-3)} \quad (x = 0, x = \frac{4}{3})$$

$$2x - 1 = 0 \quad x = \frac{1}{2}; \quad 2y - 1 = 0 \quad y = -\frac{1}{2}$$

$$\frac{\partial^2}{\partial y^2} (x^2 - y^2 - x - y) = \frac{\partial}{\partial y} (x^2) - \frac{\partial}{\partial y} (y^2) - \frac{\partial}{\partial y} (x) - \frac{\partial}{\partial y} (y) = -2y - 1$$

$$\frac{\partial}{\partial y} (-2y - 1) = -\frac{\partial}{\partial y} (2y) - \frac{\partial}{\partial y} (1) = -2$$

$$\frac{\partial^2}{\partial x^2} (x^2 - y^2 - x - y) = \frac{\partial}{\partial x} (2x - 1) = 2$$

$$\frac{\partial^2}{\partial y^2} (x^2 - y^2 - x - y) = \frac{\partial}{\partial y} (-2y - 1) = -2$$

$$\frac{\partial^2}{\partial x \partial y} (x^2 - y^2 - x - y)$$

$$\frac{\partial}{\partial x} (x^2 - y^2 - x - y) = 2x - 1$$

$$\frac{\partial}{\partial y} (2x - 1) = 0$$

$$D(x, y) = 2(-2) - (0)^2 = -4$$

$$D(x, y) = -4 \text{ en } (\frac{1}{2}, -\frac{1}{2}) : \text{ Negativo}$$

$$D < 0 \text{ en } (\frac{1}{2}, -\frac{1}{2}), \text{ Silla } (\frac{1}{2}, -\frac{1}{2})$$

C) $F_{y,x} = x^2 - y^2 - 3xy$

Puntos criticos

$$f = x^2 - y^2 - 3xy$$

$$\frac{\partial}{\partial x} (x^2 - y^2 - 3xy) = \frac{\partial}{\partial x} (x^2) - \frac{\partial}{\partial x} (y^2) - \frac{\partial}{\partial x} (3xy) = 2x - 3y$$

$$\frac{\partial}{\partial y} (x^2 - y^2 - 3xy) = \frac{\partial}{\partial y} (x^2) - \frac{\partial}{\partial y} (y^2) - \frac{\partial}{\partial y} (3xy) = -3x - 2y$$

$$\nabla f(x, y) = (2x - 3y, -3x - 2y)$$

$$\begin{bmatrix} 2x - 3y = 0 \\ -3x - 2y = 0 \end{bmatrix}$$

$$2x - 3y = 0 \Rightarrow x = \frac{3y}{2}$$

$$-3 \cdot \frac{3y}{2} - 2y = 0 \quad -\frac{13y}{2} = 0 \quad x = 0, y = 0$$

$$\frac{\partial^2}{\partial y^2} (x^2 - y^2 - 3xy)$$

$$\frac{\partial}{\partial y} (x^2 - y^2 - 3xy) = \frac{\partial}{\partial y} (x^2) - \frac{\partial}{\partial y} (y^2) - \frac{\partial}{\partial y} (3xy) = -3x - 2y$$

$$\frac{\partial}{\partial y}(-3x - 2y) = -\frac{\partial}{\partial y}(3x) - \frac{\partial}{\partial y}(2y) = -0 - 2 = -2$$

$$f(x, y) = x^2 - y^2 - 3xy$$

$$\frac{\partial^2}{\partial x^2}(x^2 - y^2 - 3xy) = \frac{\partial}{\partial x}(2x - 3y) = 2$$

$$\frac{\partial^2}{\partial y^2}(x^2 - y^2 - 3xy) = \frac{\partial}{\partial y}(-3x - 2y) = -2$$

$$\frac{\partial^2}{\partial x \partial y}(x^2 - y^2 - 3xy) = \frac{\partial}{\partial y}(2x - 3y) = -3$$

$$D(x, y) = 2(-2) - ((-3))^2 = -13$$

$D(x, y) = -13$ en $(0, 0)$: Negativo

$D(x, y) < 0$, Silla $(0, 0)$

- 3 Grafique todas las funciones del ejercicio anterior en el software de su preferencia. Luego compruebe que los resultados obtenidos son correctos:

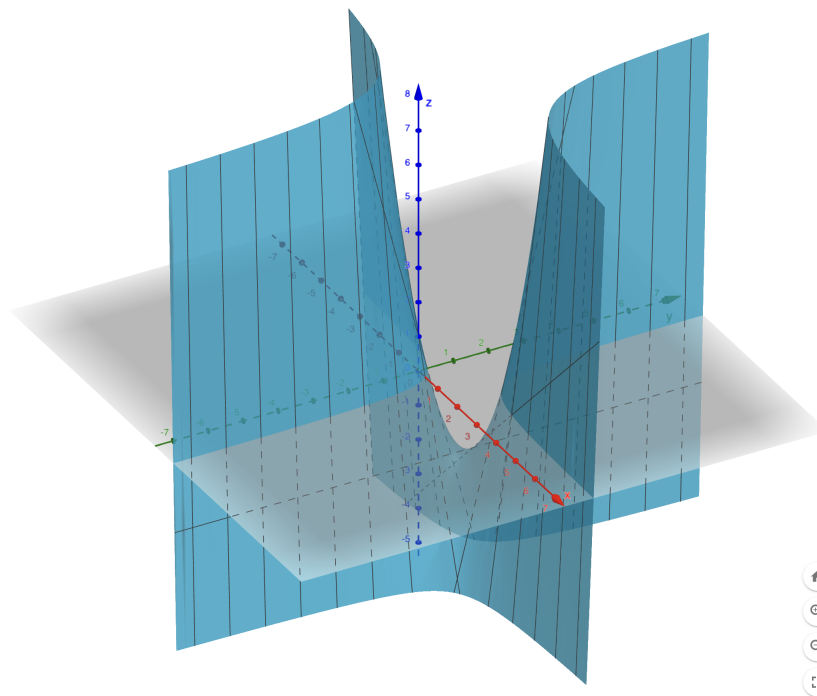


Figura 0.1: $F_{y,x} = -x^3 + 4xy - 2y^2 + 1$ Resultados coinciden

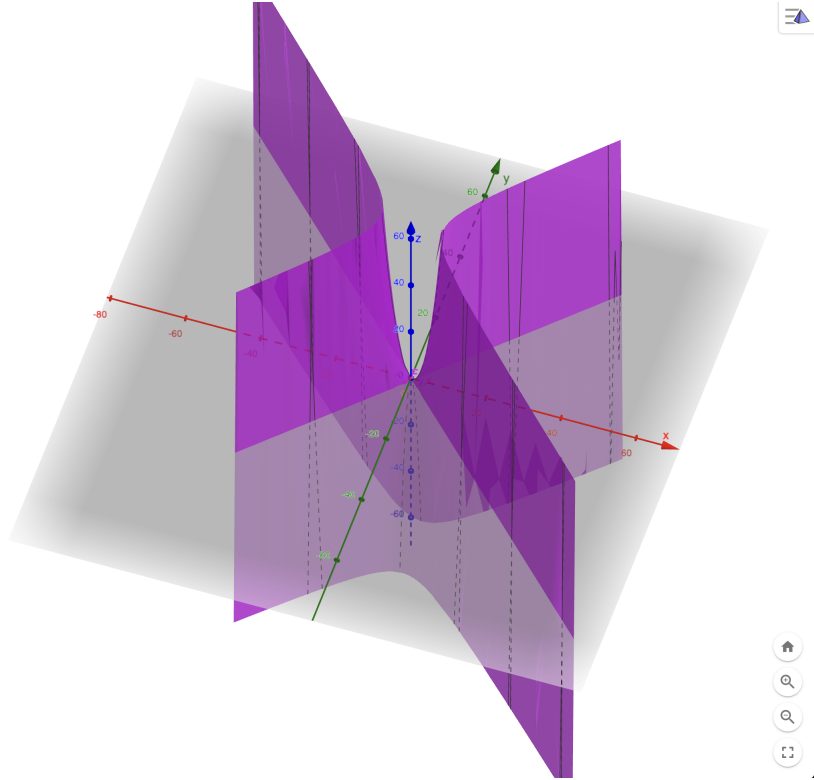


Figura 0.2: $F_{y,x} = x^2 - y^2 - x - y$ Resultados coinciden

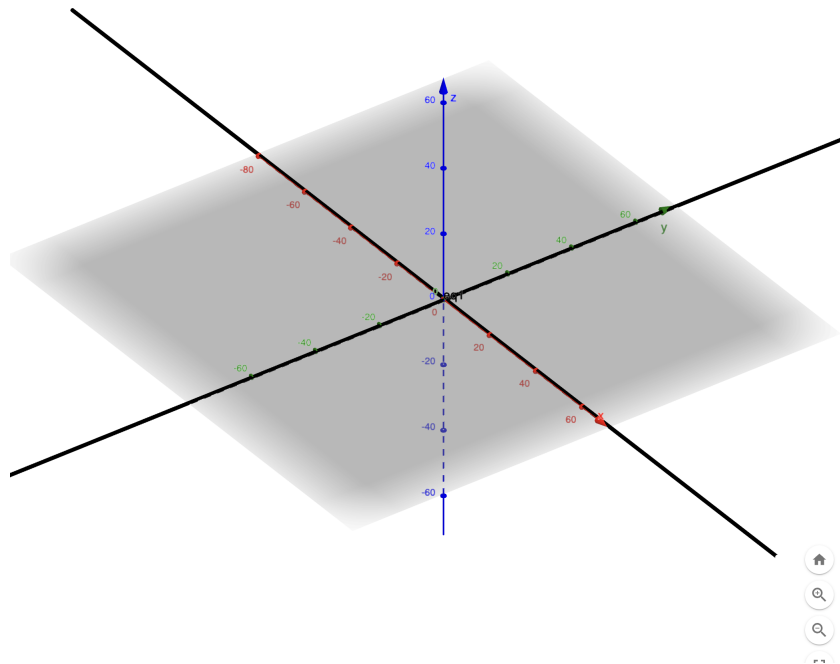


Figura 0.3: $F_{y,x} = x^2 - y^2 - 3xy$ Resultados coinciden

- 4 ¿Cómo se calcula el plano tangente a una superficie? Busque y copie su definición. Luego copie un ejemplo del libro donde se calcule un plano tangente (sección 14,4 del libro). Por último, calcule y grafique el plano tangente a: