BuildHeap and Heapsort

Goals

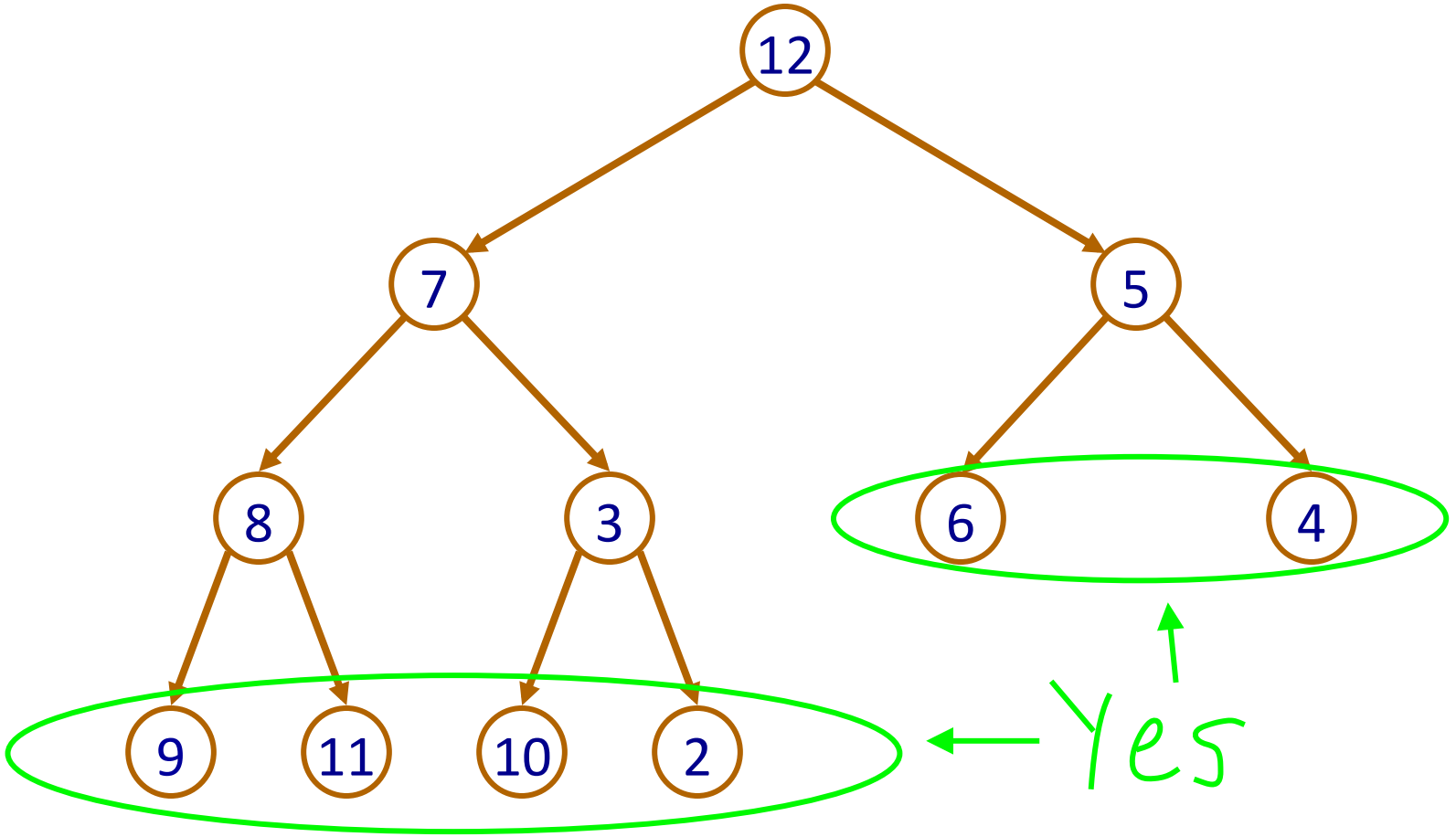
- Build a heap from an array of arbitrary values
- HeapSort algorithm
- Analysis of HeapSort

BuildHeap

- How do we build a heap from an arbitrary array of values???

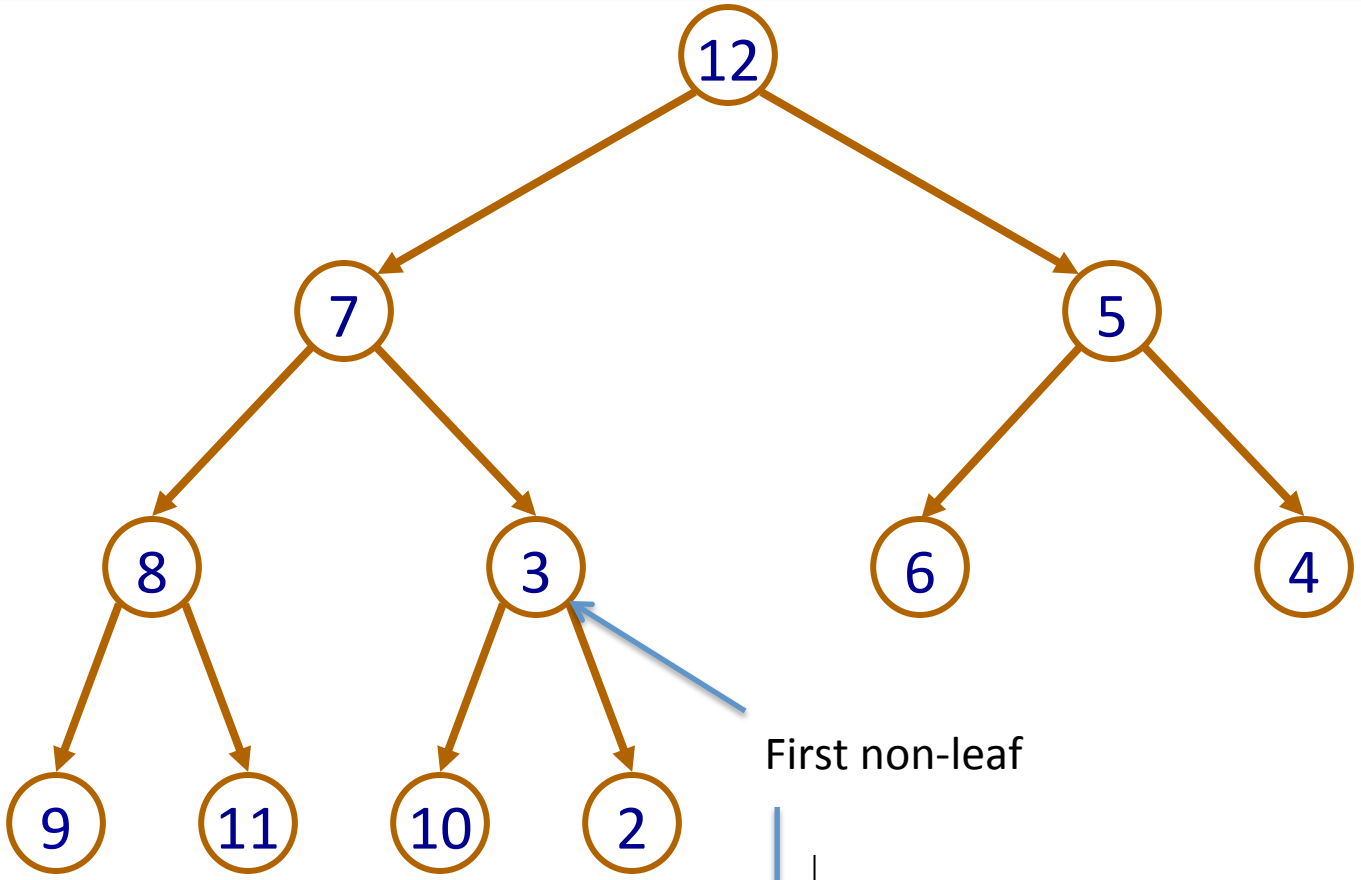
BuildHeap: Is this a proper heap?

0	1	2	3	4	5	6	7	8	9	10
12	7	5	8	3	6	4	9	11	10	2



Are any of the subtrees ***guaranteed*** to be proper heaps?

BuildHeap: Leaves are proper heaps



Size = 11

Size/2 – 1 = 4

0	1	2	3	4	5	6	7	8	9	10	11
12	7	5	8	3	6	4	9	11	10	2	

compute index of first non-left

BuildHeap

- How can we use this information to build a heap from a random array?
- _adjustHeap: takes a binary tree, rooted at a node, and percolates down from that node to ensure that the subtree is a proper heap

```
void _adjustHeap(struct DynArr *heap,int max, int pos)
```

Adjust up to (not inclusive)

Adjust from

BuildHeap

- Find the last non-leaf node,i, (going from left to right)
- adjust heap from it to max
- Decrement i and repeat until you process the root

HeapSort

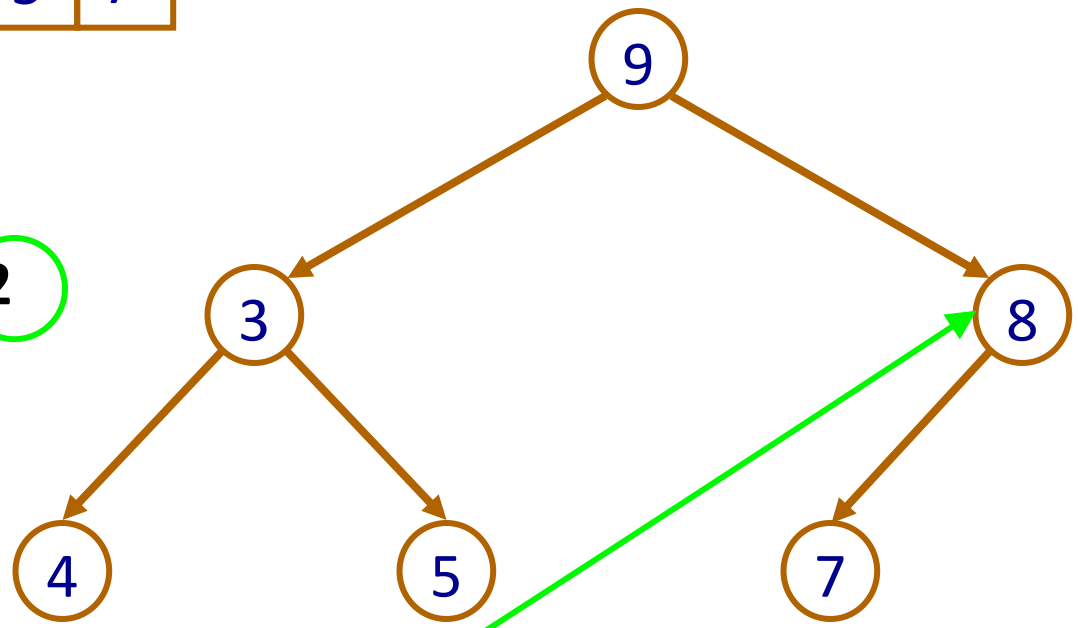
- BuildHeap and _adjustHeap are the keys to an efficient , in-place, sorting algorithm
- in-place means that we don't require any extra storage for the algorithm
- Any ideas???

HeapSort

1. BuildHeap – turn arbitrary array into a heap
2. Swap first and ***last*** elements
3. Adjust Heap (from 0 to the ***last***…not inclusive!)
4. Repeat 2-3 but decrement ***last*** each time through

HeapSort Simulation: BuildHeap

0	1	2	3	4	5
9	3	8	4	5	7



Max = 6

i=Max/2 – 1 = 2

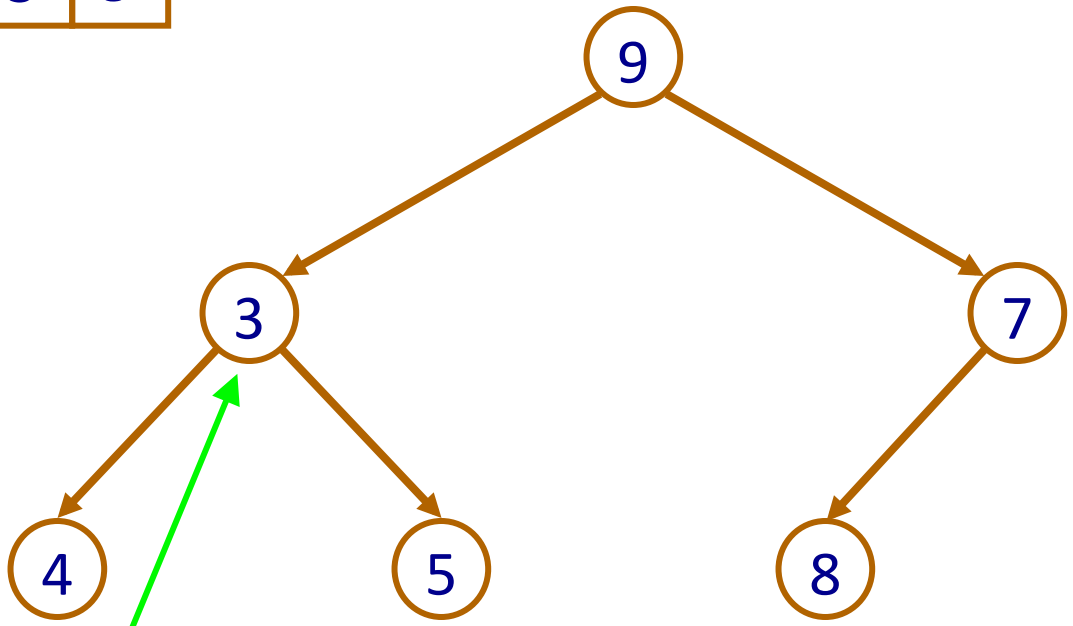
find first subtree that might be out of heap order

i=2

_adjustHeap(heap,6,2)

HeapSort Simulation: BuildHeap

0	1	2	3	4	5
9	3	7	4	5	8

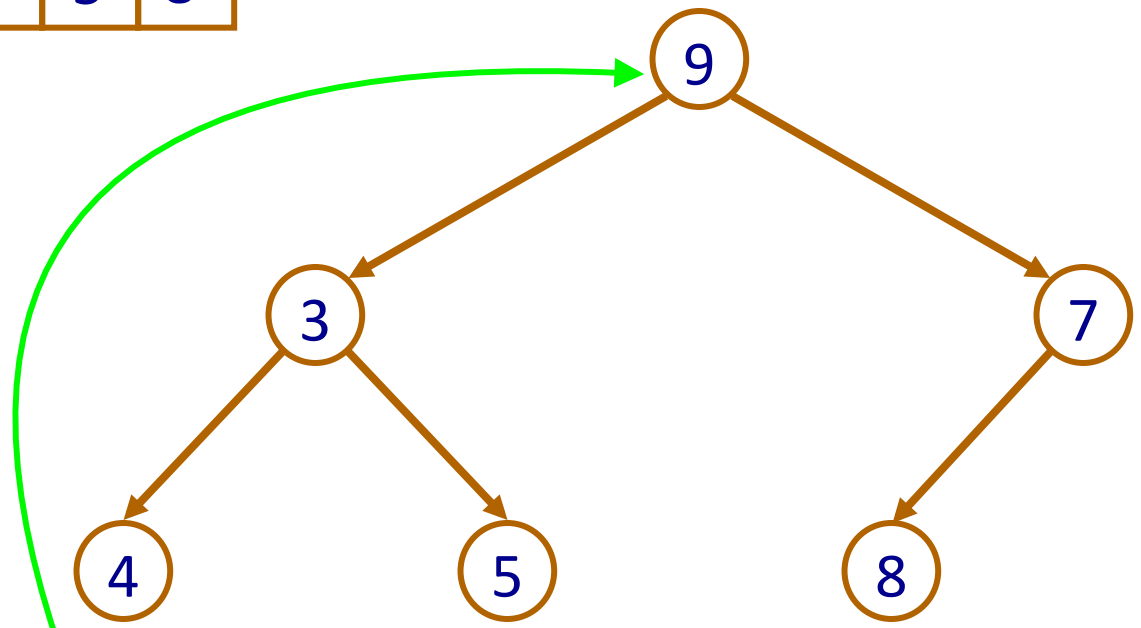


Max = 6
i=1

i=1
_adjustHeap(heap,6,1)

HeapSort Simulation: BuildHeap

0	1	2	3	4	5
9	3	7	4	5	8



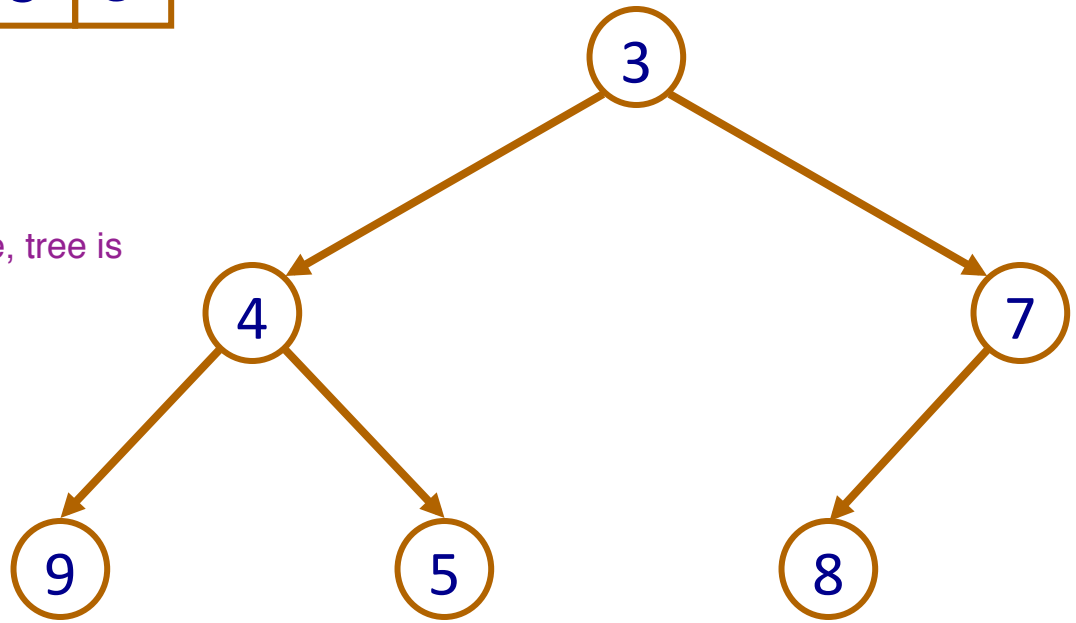
Max = 6
i= 0

i=0
_adjustHeap(heap,6,0)

HeapSort Simulation: BuildHeap

0	1	2	3	4	5
3	4	7	9	5	8

after buildHeap() is done, tree is complete



i=-1

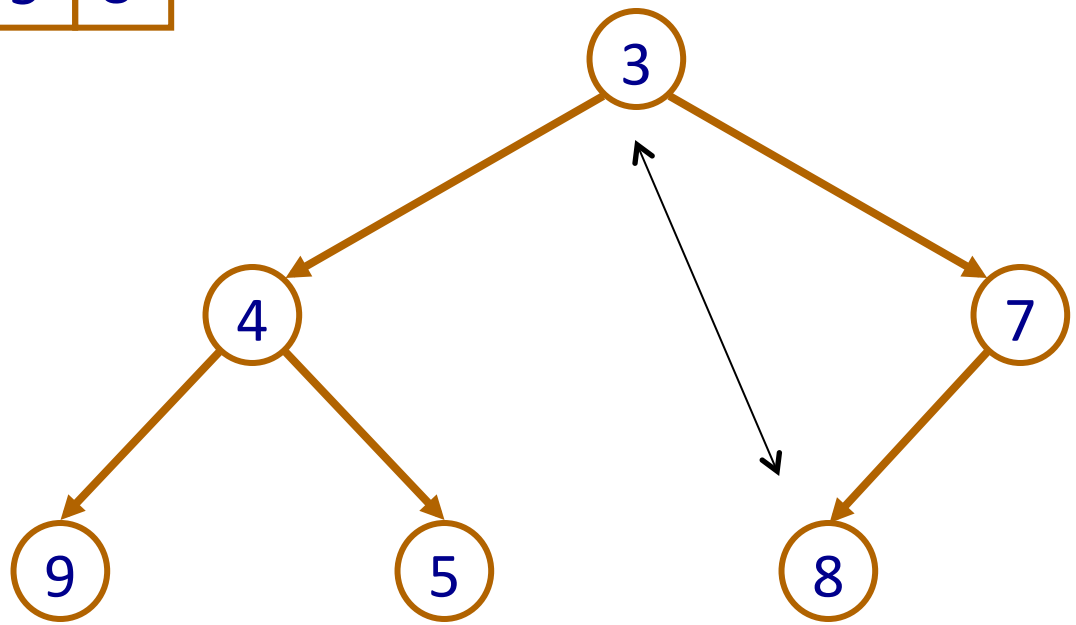
Done…with BuildHeap ….now let's sort it!

HeapSort

1. BuildHeap
2. Swap first and last
3. Adjust Heap (from 0 to the last)
4. Repeat 2-3 but decrement last

HeapSort Simulation: Sort in Place

0	1	2	3	4	5
3	4	7	9	5	8



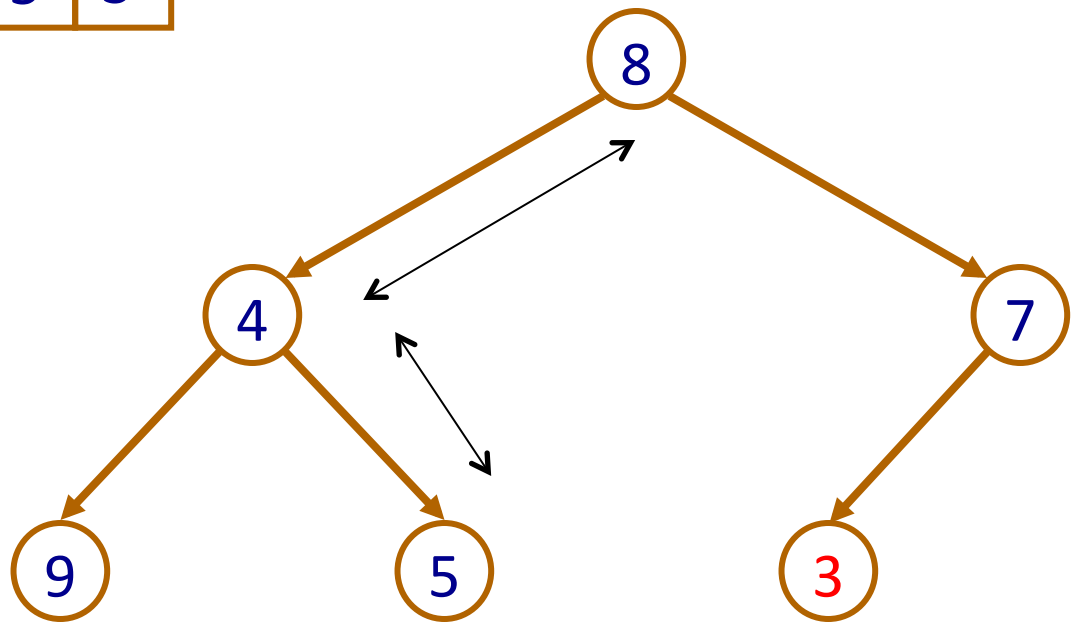
i=5

Swap(v, 0, i)

_adjustHeap(v, i, 0);

HeapSort Simulation: Sort in Place

0	1	2	3	4	5
8	4	7	9	5	3

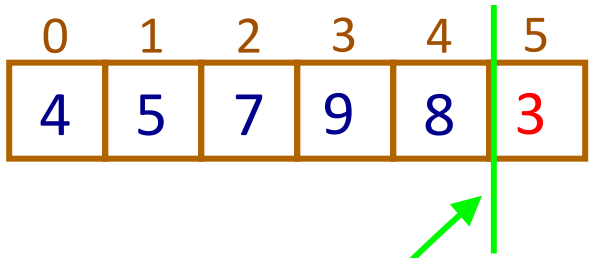


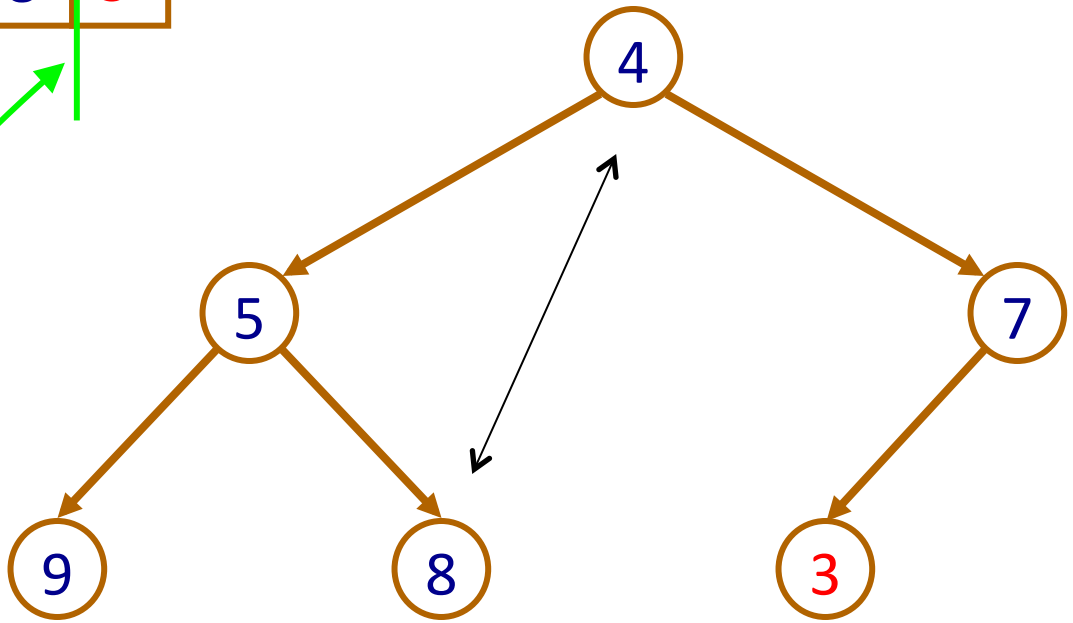
i=5

Swap(v, 0, i)

_adjustHeap(v, i, 0);

HeapSort Simulation: Sort in Place





decrement last

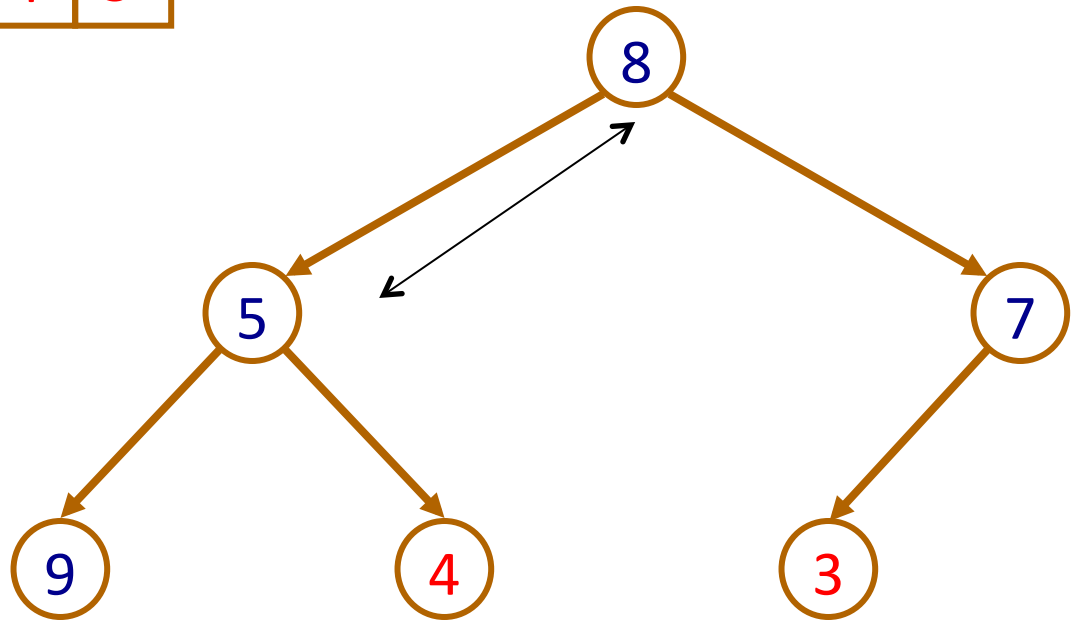
i=4

Swap(v, 0, i)

_adjustHeap(v, i, 0);

HeapSort Simulation: Sort in Place

0	1	2	3	4	5
8	5	7	9	4	3



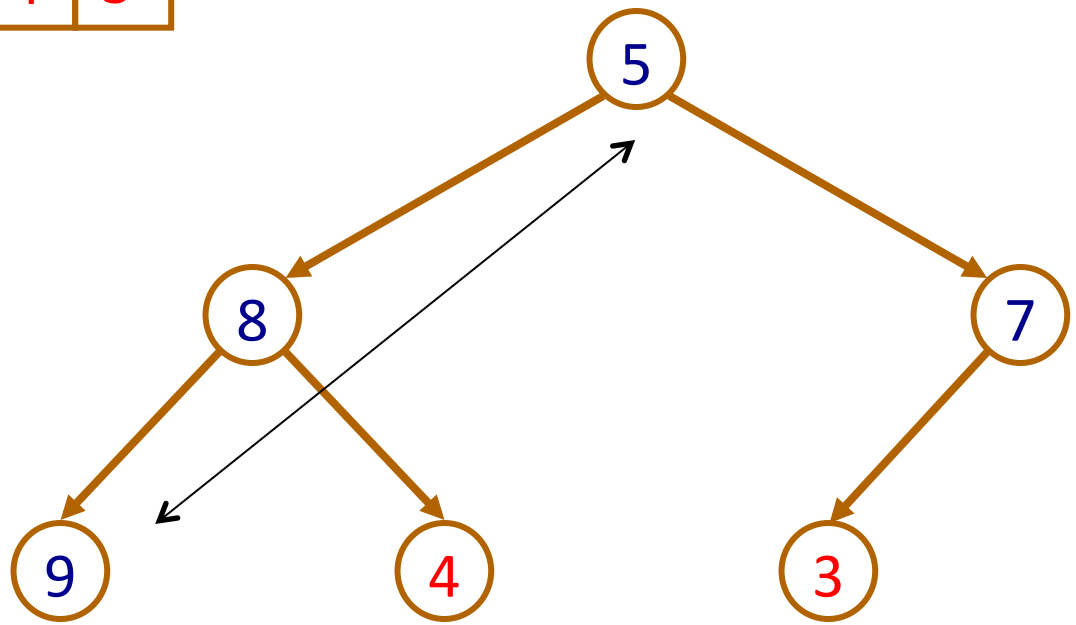
i=4

Swap(v, 0, i)

_adjustHeap(v, i, 0);

HeapSort Simulation: Sort in Place

0	1	2	3	4	5
5	8	7	9	4	3



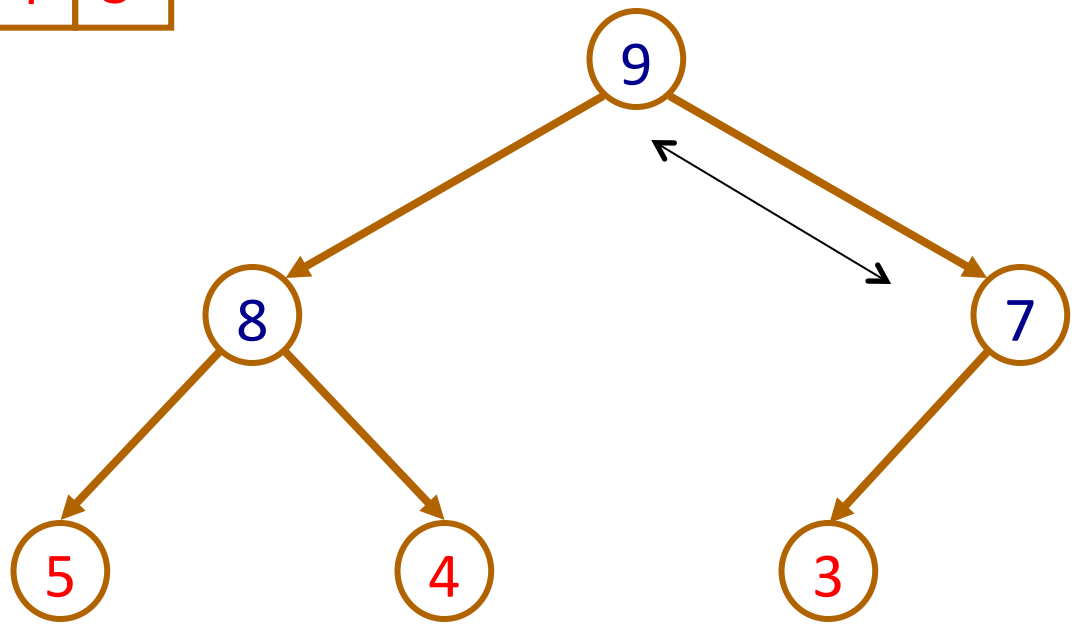
i=3

Swap(v, 0, i)

_adjustHeap(v, i, 0);

HeapSort Simulation: Sort in Place

0	1	2	3	4	5
9	8	7	5	4	3



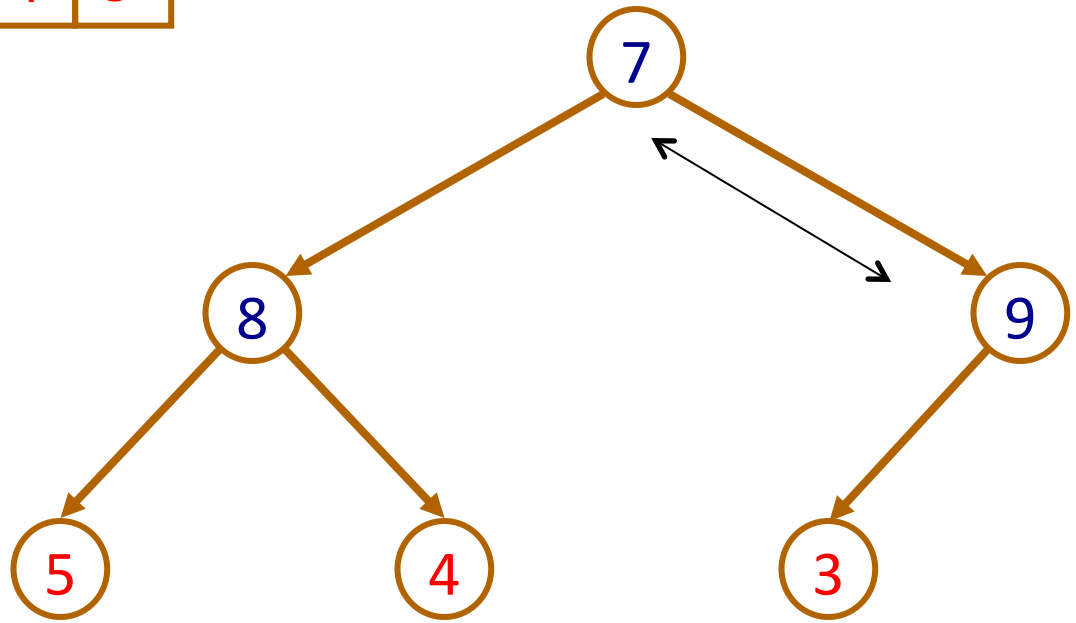
i=3

Swap(v, 0, i)

_adjustHeap(v, i, 0);

HeapSort Simulation: Sort in Place

0	1	2	3	4	5
7	8	9	5	4	3



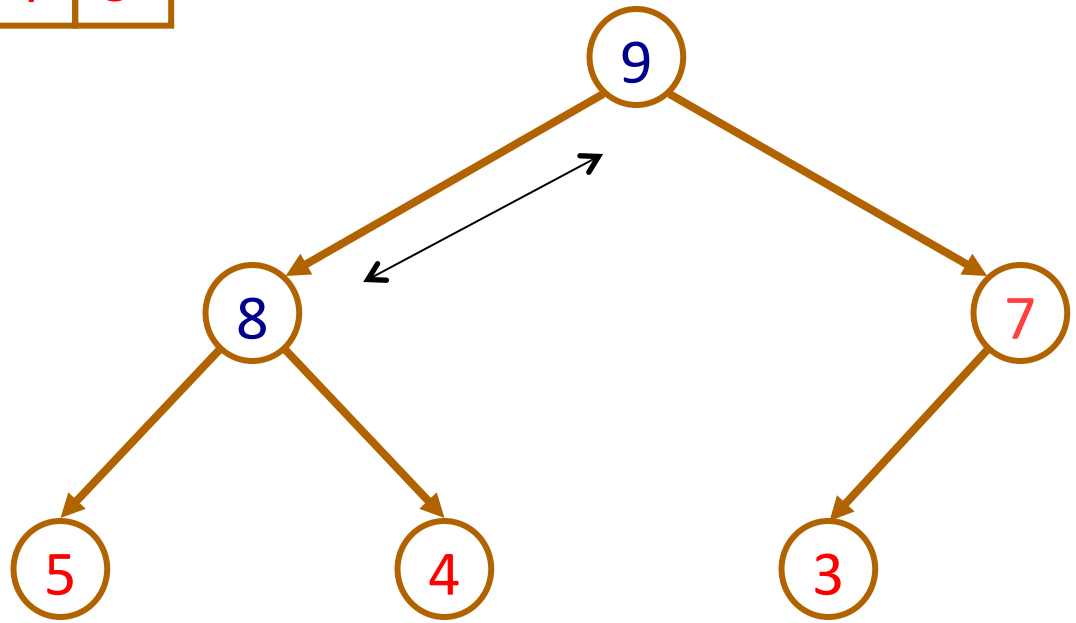
i=2

Swap(v, 0, i)

_adjustHeap(v, i, 0);

HeapSort Simulation: Sort in Place

0	1	2	3	4	5
9	8	7	5	4	3



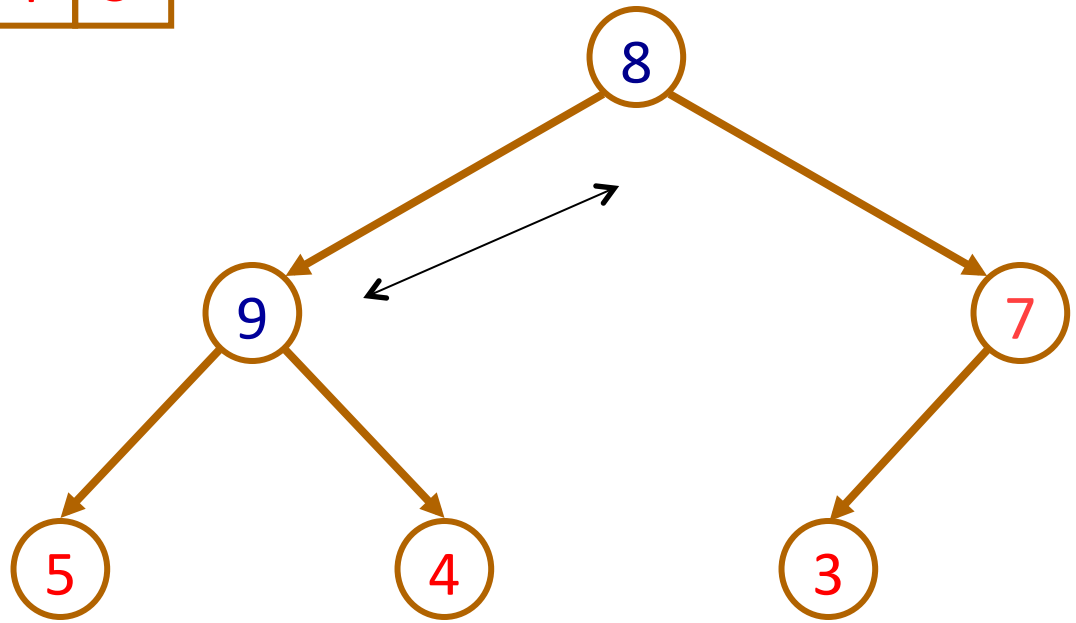
i=2

Swap(v, 0, i)

_adjustHeap(v, i, 0);

HeapSort Simulation: Sort in Place

0	1	2	3	4	5
8	9	7	5	4	3



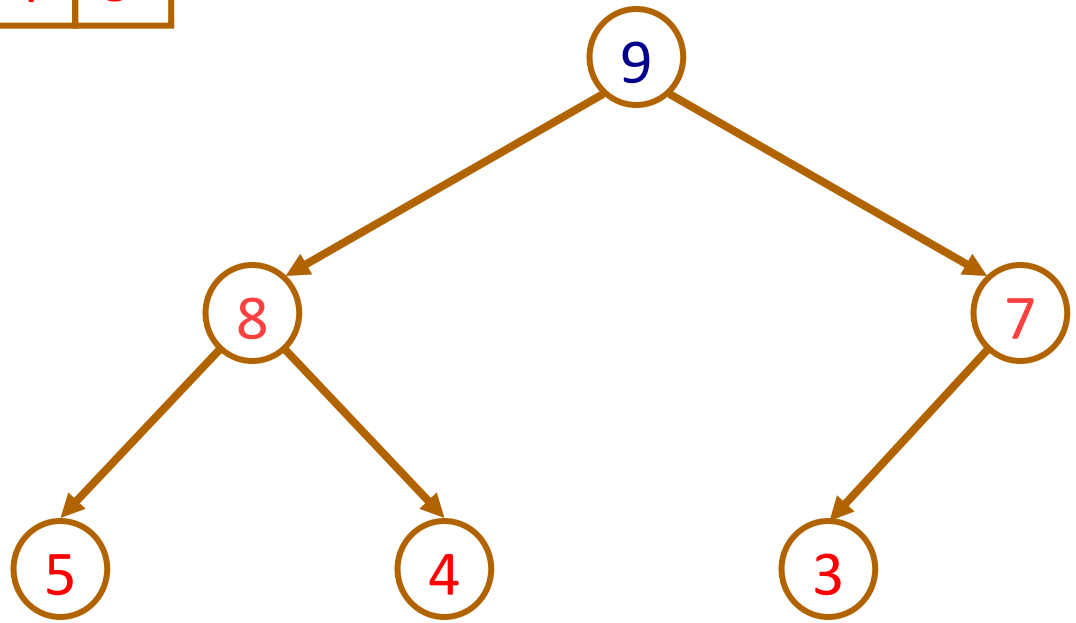
i=1

Swap(v, 0, i)

_adjustHeap(v, i, 0);

HeapSort Simulation: Sort in Place

0	1	2	3	4	5
9	8	7	5	4	3



i=1

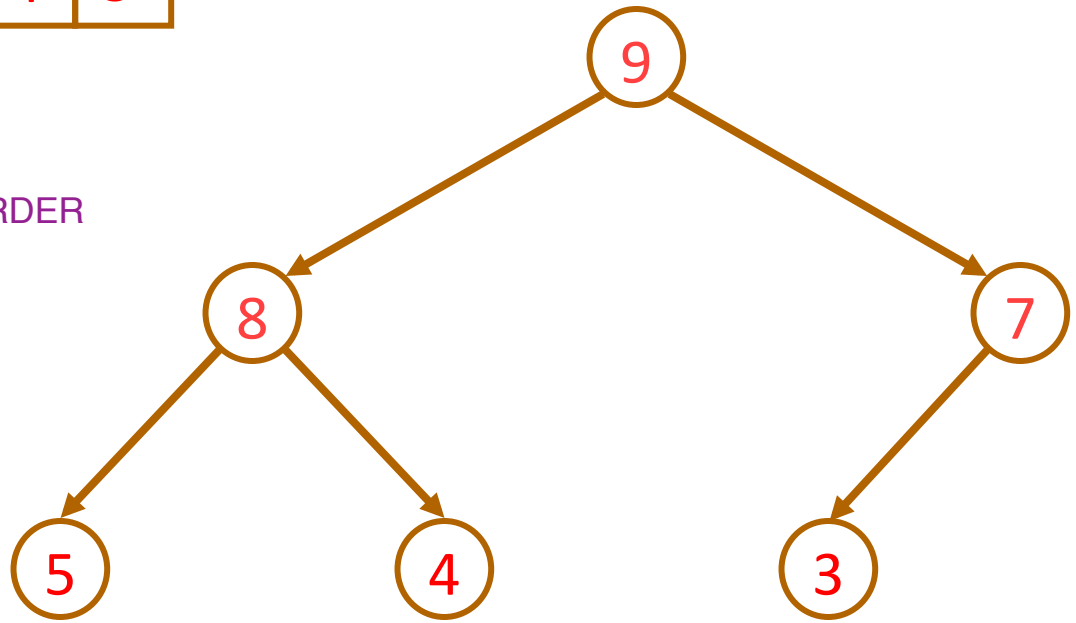
Swap(v, 0, i)

_adjustHeap(v, i, 0);

HeapSort Simulation: Sort in Place

0	1	2	3	4	5
9	8	7	5	4	3

REVERSE SORTED ORDER



i=0

DONE

HeapSort Performance

- Build Heap:

  n

  n/2 calls to _adjustHeap = O(n log n)

- HeapSort:

  n calls to swap and adjust = O(n log n)

- Total:

  O(n log n)

Your Turn

- Worksheet 34 – BuildHeap and Heapsort