# CS261 Data Structures

## Trees

## Introduction and Applications

# Goals

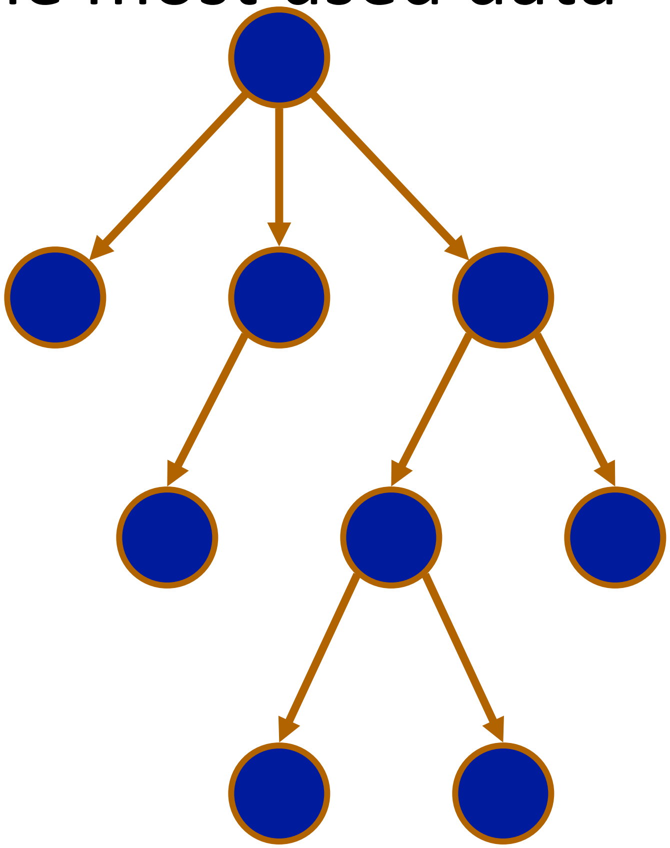
- Tree Terminology and Definitions
- Tree Representation
- Tree Application

# Trees

- Ubiquitous – they are everywhere in CS

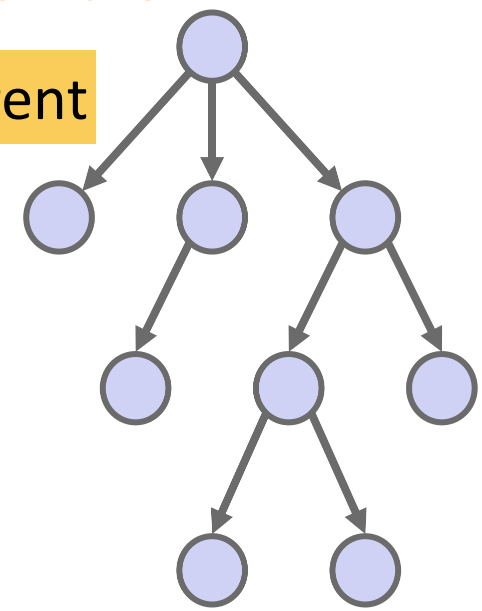
- Probably ranks third among the most used data structure:

  1. Arrays/Vectors
  2. Lists
  3. Trees



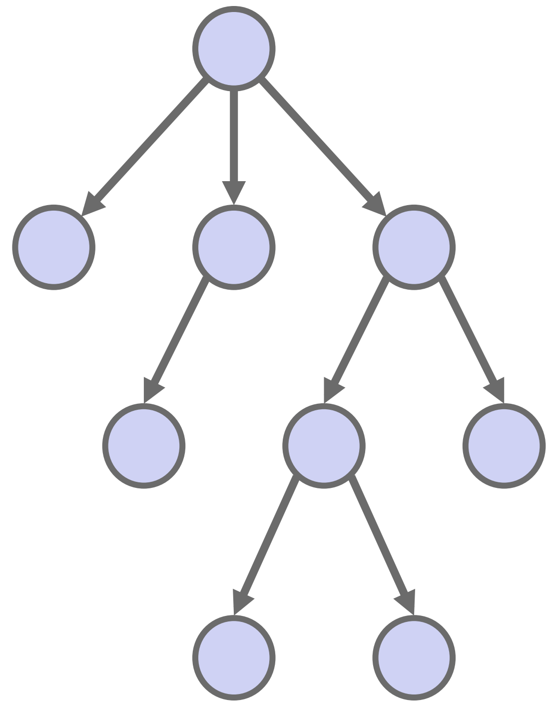
# Tree Characteristics

- A tree consists of a collection of nodes connected by directed arcs
- A tree has a single *root* node
  - By convention, the root node is usually drawn at the top
- A node that points to (one or more) other nodes is the *parent* of those nodes while the nodes pointed to are the *children*
- Every node (except the root) has exactly one parent
- Nodes with no children are *leaf* nodes
- Nodes with children are *interior* nodes

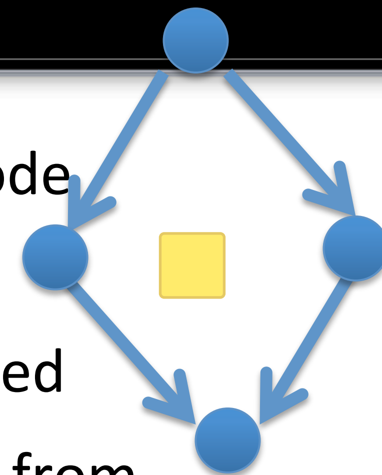


# Tree Characteristics (cont…)

- Nodes that have the same parent are *siblings*
- The *descendants* of a node consist of its children, and their children, and so on
  - All nodes in a tree are descendants of the root node (except, of course, the root node itself)
- Any node can be considered the root of a *subtree*
- A subtree rooted at a node consists of that node and all of its descendants



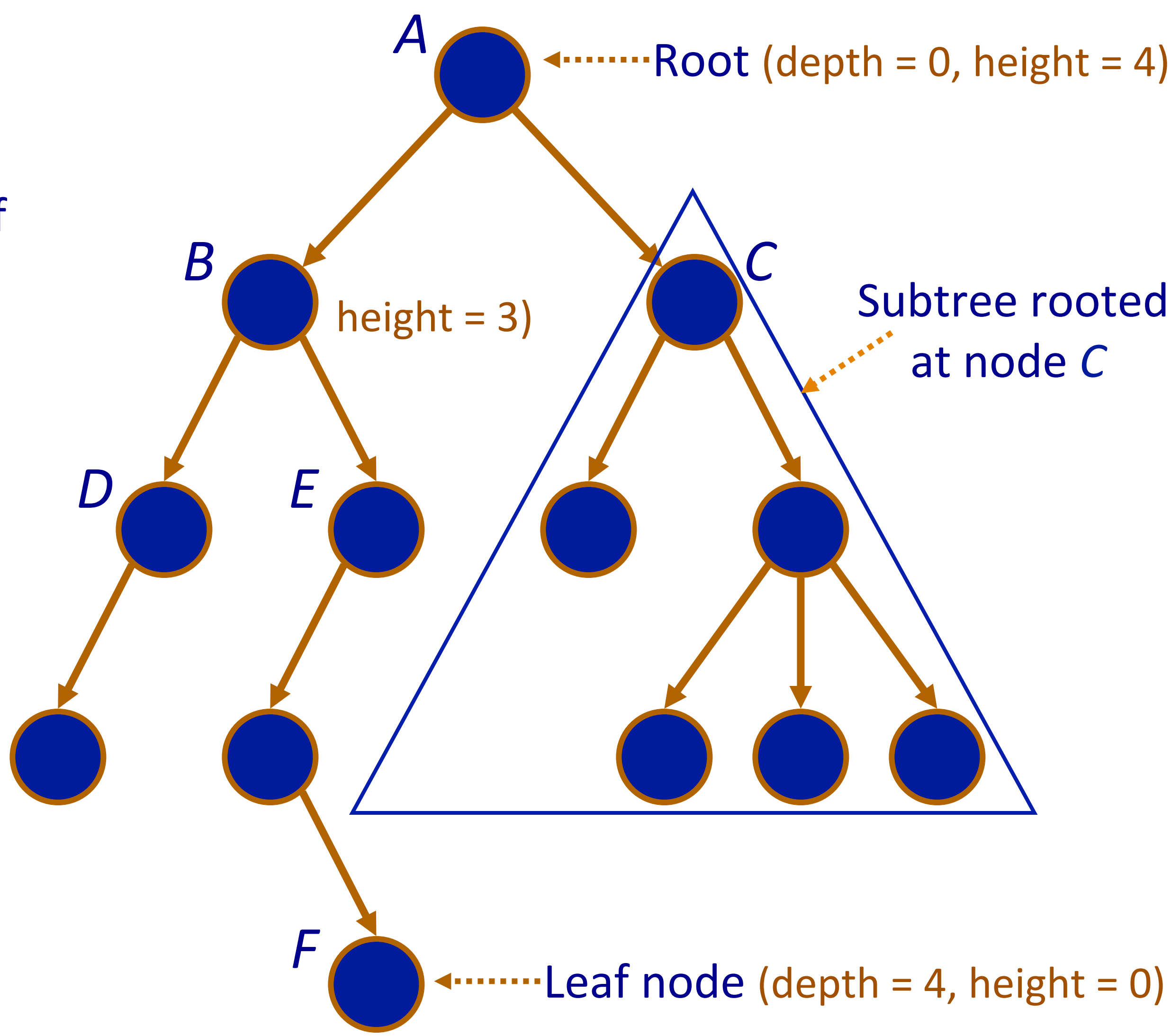
# Tree Characteristics (cont.)



- There is a single, unique path from the root to any node
  - Arcs don't join together
- A path's *length* is equal to the number of arcs traversed
- A node's *height* is equal to the maximum path length from that node to a leaf node:
  - A leaf node has a height of 0
  - The height of a tree is equal to the height of the root
- A node's *depth* is equal to the path length from the root to that node:
  - The root node has a depth of 0
  - A tree's depth is the maximum depth of all its leaf nodes (which, of course, is equal to the tree's height)

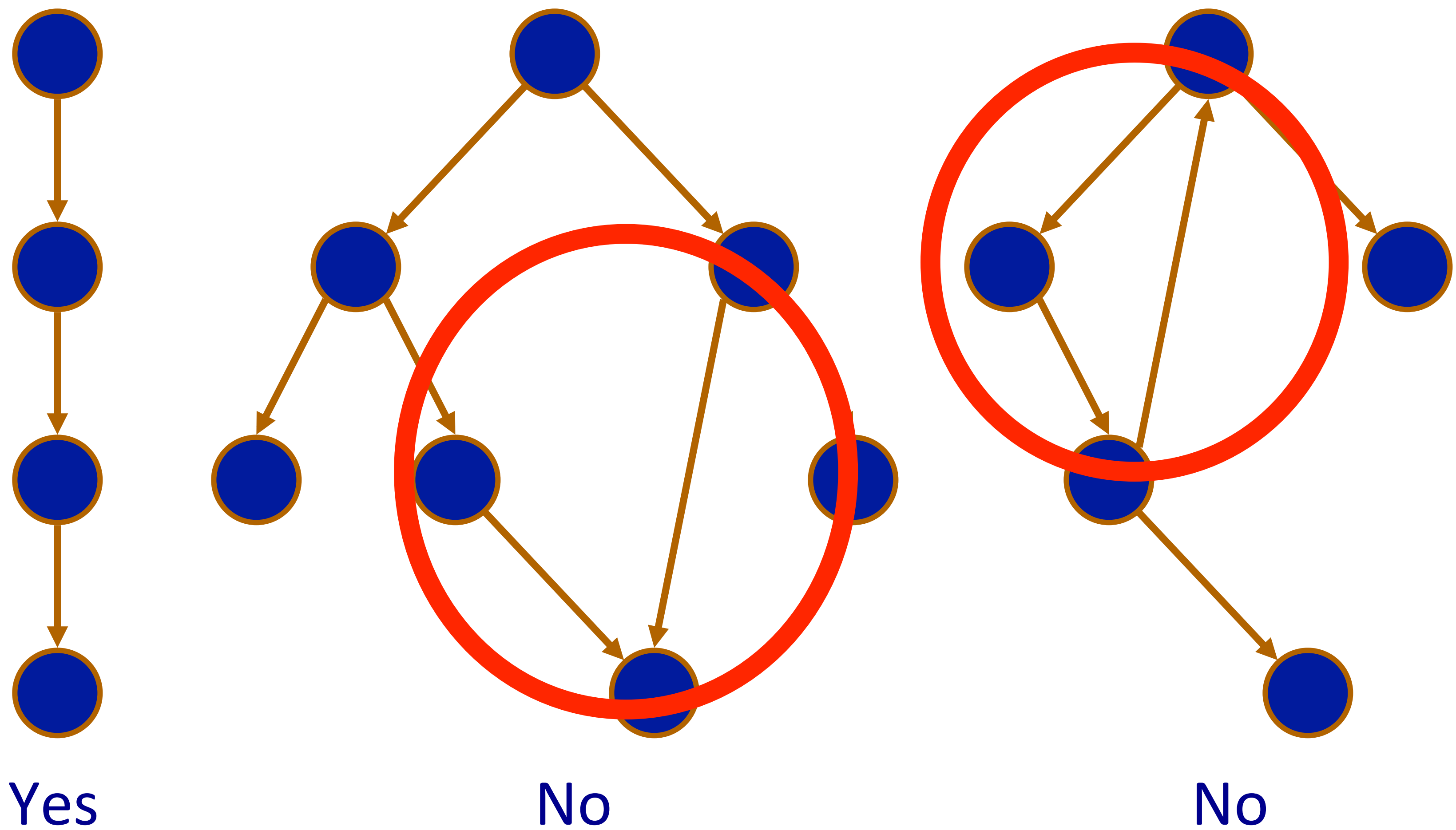
# Tree Characteristics (cont.)

- Nodes *D* and *E* are children of node *B*
- Node *B* is the parent of nodes *D* and *E*
- Nodes *B*, *D*, and *E* are descendents of node *A* (as are all other nodes in the tree…except *A*)
- *E* is an interior node
- *F* is a leaf node



# Tree Characteristics (cont.)

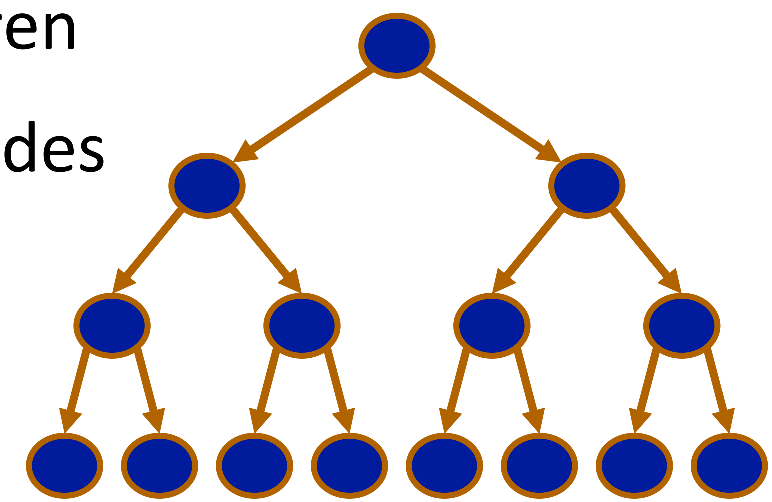
Are these trees?



# Full Binary Tree

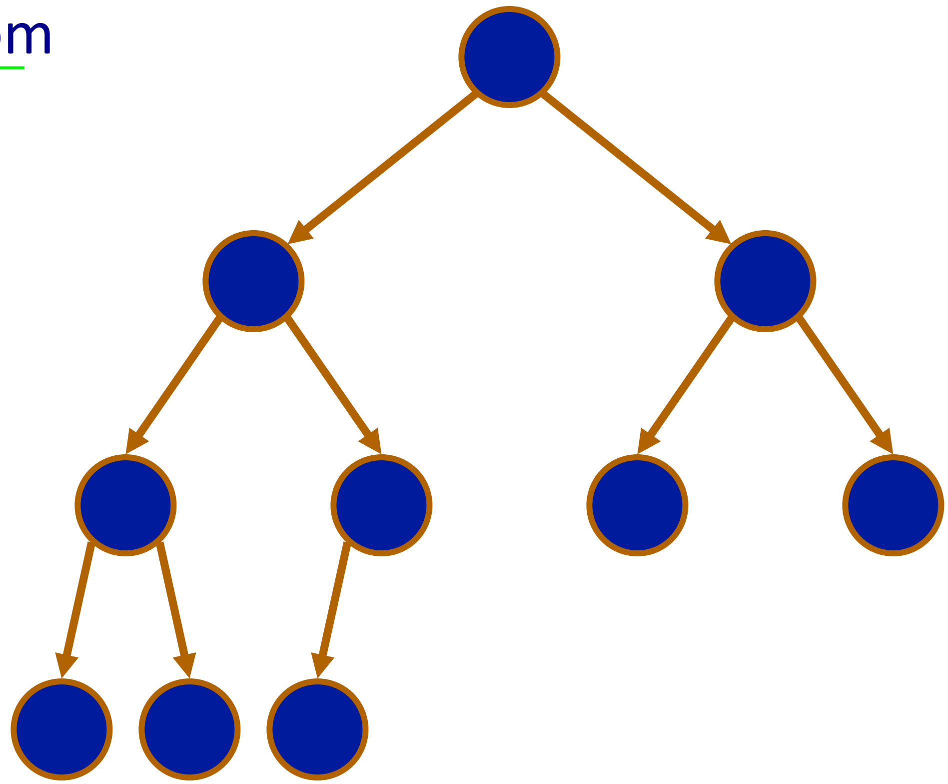
- Nodes have no more than two children:
  - Children are generally referred to as "left" and "right"
- Full Binary Tree:
  - every leaf is at the same depth
  - Every internal node has 2 children
  - Height of $h$ will have $2^{h+1} - 1$ nodes
  - Height of $h$ will have $2^h$ leaves

$d = 3$



# Complete Binary Tree

- Complete Binary Tree:
  full except for the bottom level which is filled from left to right

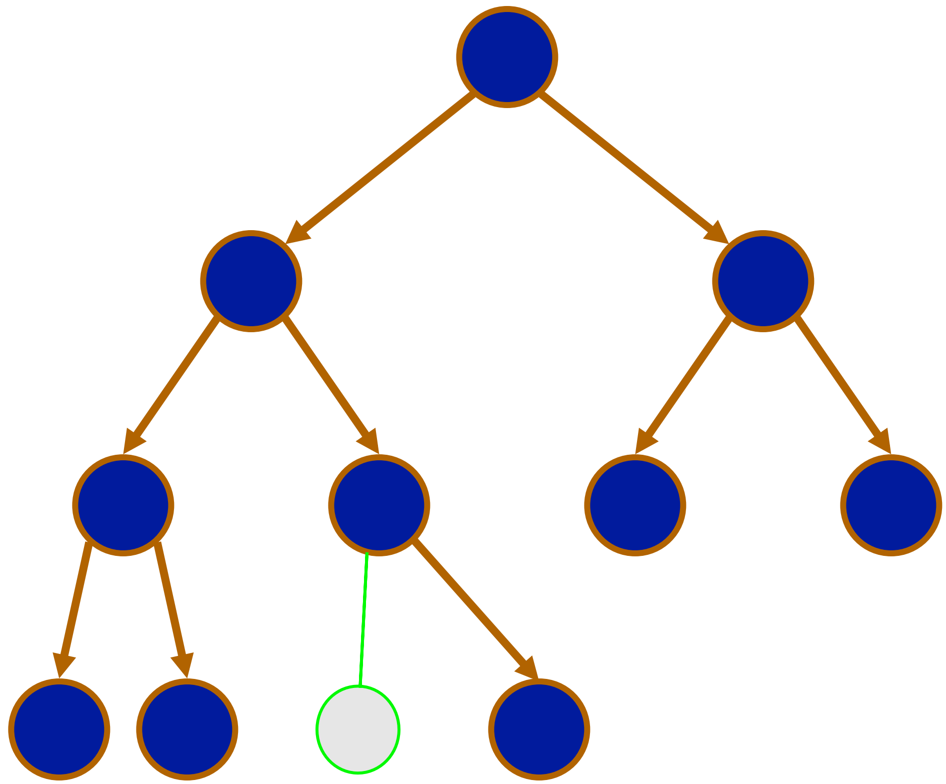


# Binary Tree

- Is this a complete binary tree?

No

Complete if:



- MUCH more on these later!

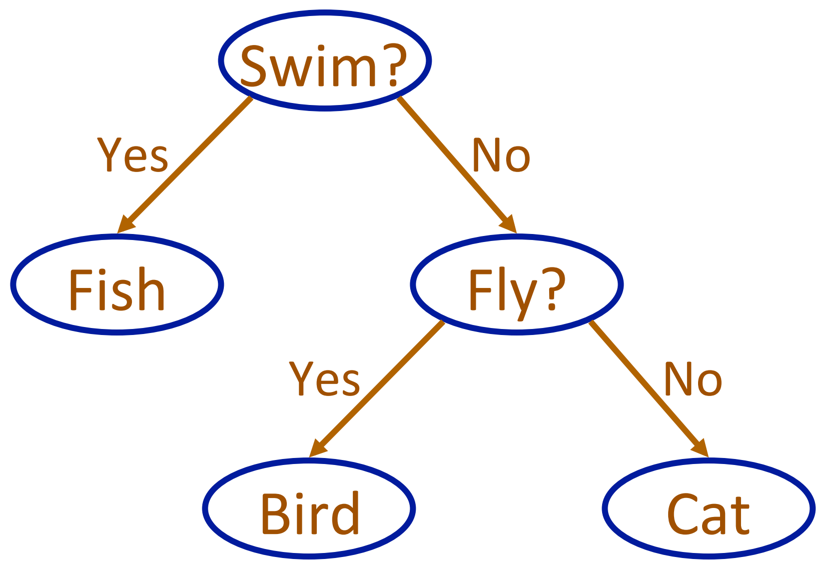
# Dynamic Memory Implementation

```
struct Node {
   TYPE           val;
   struct Node *left;      /* Left   child. */
   struct Node *rght;      /* Right child. */
};
```

Like the **Link** structure in a linked list: we will use this structure in several data structures

# Binary Tree Application: Animal Game

- Purpose: guess an animal using a sequence of questions
  - Internal nodes contain yes/no questions
  - Leaf nodes are animals
- How do we build it?



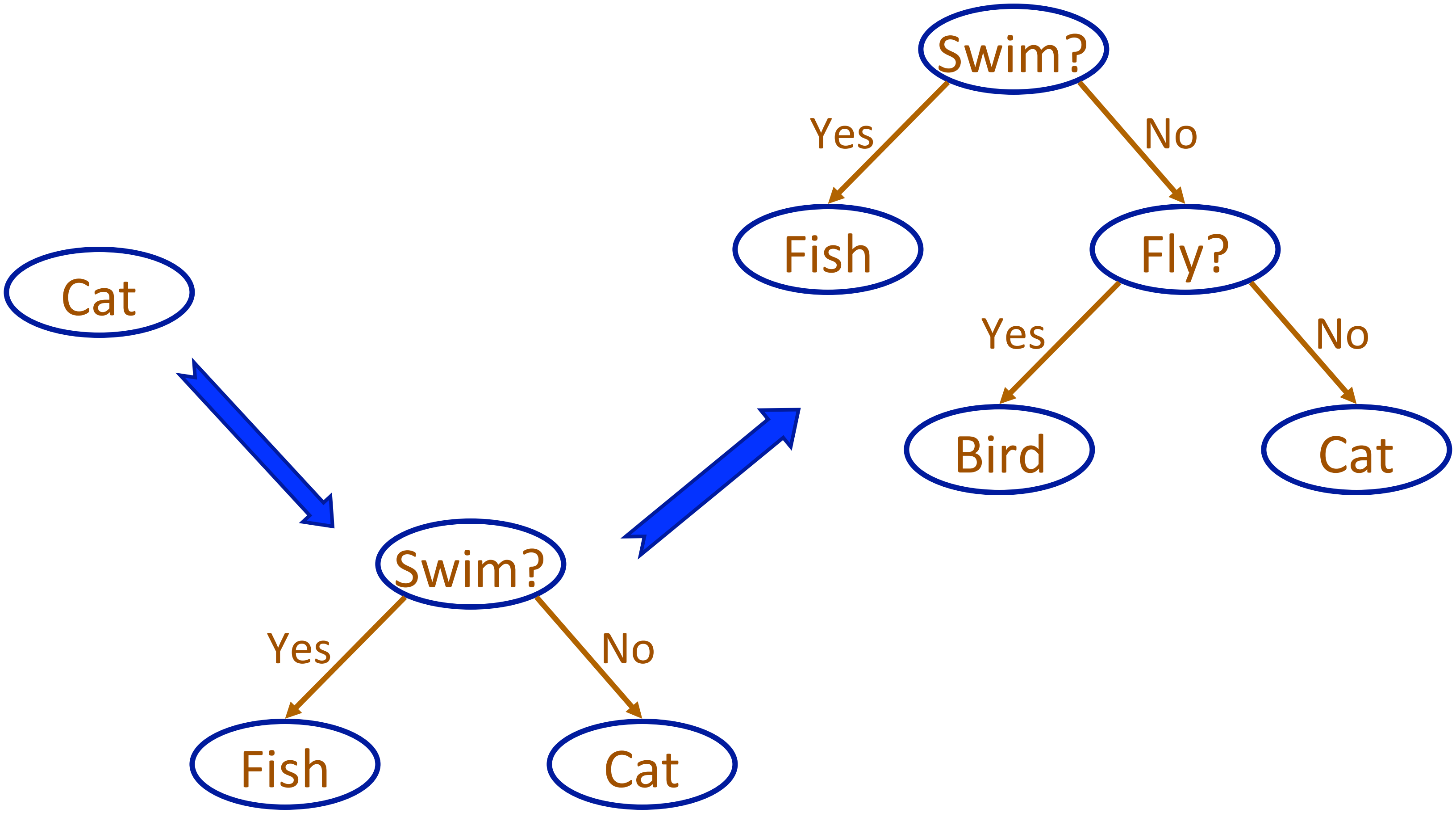
# Binary Tree Application: Animal Game

Initially, tree contains a single animal (e.g., a "cat") stored in the root node

Guessing….

1. Start at root.
2. If internal node → ask yes/no question
   - Yes → go to left child and repeat step 2
   - No → go to right child and repeat step 2
3. If leaf node → guess "I know. Is it a …":
   - If right → done
   - If wrong → "learn" new animal by ***asking*** for a yes/no question that distinguishes the new animal from the guess

# Binary Tree Application: Animal Game



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# Decision Tree

- If you can ask at most ***q*** questions, the number of possible answers we can distinguish between, ***n***, is the number of leaves in a binary tree with height at most q, which is at most $2^q$
- Taking logs on both sides: $\log(n) = \log(2^q)$
- $\log(n) = q$ : for n outcomes, we need q questions
- ***For 1,048,576 outcomes we need 20 questions***

# Still To Come...

- Implementation Concepts
- Implementation Code