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# LECTURE 19: LINGUISTICALLY EXPRESSIVE GRAMMARS

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Adapted from Julia Hockenmaier, NLP S2023 - course material  
<https://courses.grainger.illinois.edu/cs447/sp2023/>



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# PART 3: FEATURE STRUCTURE GRAMMARS

# WHY FEATURE STRUCTURES

Feature structures form the basis for many grammar formalisms used in computational linguistics.



Feature structure grammars (aka attribute-value grammars, or unification grammars) can be used as

a more compact way of  
representing rich CFGs

a way to represent more  
expressive grammars

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# SIMPLE GRAMMARS OVERGENERATE

$$\begin{aligned} S &\rightarrow NP \ VP \\ VP &\rightarrow Verb \ NP \\ NP &\rightarrow Det \ Noun \\ Det &\rightarrow the \mid a \mid these \\ Verb &\rightarrow eat \mid eats \\ Noun &\rightarrow cake \mid cakes \mid student \mid students \end{aligned}$$

This generates ungrammatical sentences like  
*“these student eats a cakes”*

We need to capture (number/person) agreement

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# REFINING THE NONTERMINALS

$$\begin{aligned} S &\rightarrow NP_{sg} VP_{sg} \\ S &\rightarrow NP_{pl} VP_{pl} \\ VP_{sg} &\rightarrow VerbSg NP \\ VP_{pl} &\rightarrow VerbPl NP \\ NP_{sg} &\rightarrow DetSg NounSg \\ DetSg &\rightarrow the \mid a \\ &\dots \quad \dots \quad \dots \end{aligned}$$

- This yields very large grammars.
- What about person, case, ...?
- Difficult to capture generalizations  
(Subject and verb have to have number agreement)
- $NP_{sg}$ ,  $NP_{pl}$  and  $NP$  are three distinct nonterminals

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# FEATURE STRUCTURES

- Replace atomic categories with feature structures:

CAT	NP	CAT	VP
NUM	SG	NUM	SG
PERS	3	PERS	3
CASE	NOM	VFORM	FINITE

A **feature structure** is a list of **features** (= attributes, e.g. CASE), and **values** (e.g. NOM).

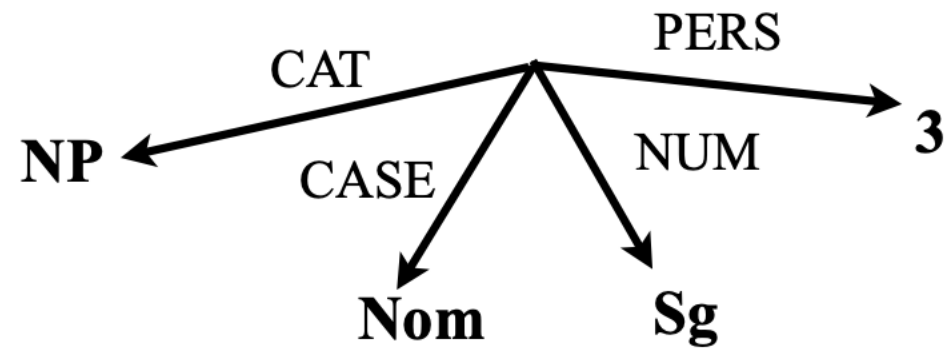
We often represent feature structures as **attribute value matrices (AVMs)**  
Usually, values are **typed** (to avoid CASE:SG)

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# FEATURE STRUCTURES AS DIRECTED GRAPHS

CAT	NP
NUM	SG
PERS	3
CASE	NOM

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# COMPLEX FEATURE STRUCTURES

- We distinguish between **atomic** and **complex** feature values.
- A complex value is a feature structure itself.
- This allows us to capture better generalizations.

Only atomic values:

$$\begin{bmatrix} \text{CAT} & \text{NP} \\ \text{NUM} & \text{SG} \\ \text{PERS} & 3 \\ \text{CASE} & \text{NOM} \end{bmatrix}$$

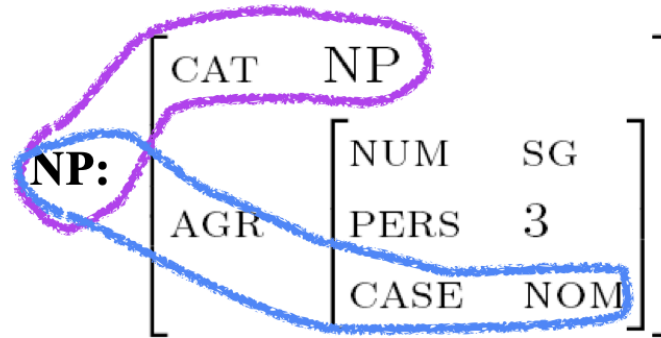
Complex values:

$$\begin{bmatrix} \text{CAT} & \text{NP} \\ & \begin{bmatrix} \text{NUM} & \text{SG} \\ \text{PERS} & 3 \\ \text{CASE} & \text{NOM} \end{bmatrix} \\ \text{AGR} & \end{bmatrix}$$



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# FEATURE PATHS



A **feature path** allows us to identify particular values in a feature structure:

$\langle \text{NP CAT} \rangle = \text{NP}$

$\langle \text{NP AGR CASE} \rangle = \text{NOM}$

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# UNIFICATION

- Two **feature structures A and B unify** ( $A \sqcup B$ ) if they can be merged into one consistent feature structure C:

$$\begin{bmatrix} \text{CAT} & \text{NP} \\ \text{NUM} & \text{SG} \\ \text{CASE} & \text{NOM} \end{bmatrix} \sqcup \begin{bmatrix} \text{CAT} & \text{NP} \\ \text{PERS} & 3 \end{bmatrix} = \begin{bmatrix} \text{CAT} & \text{NP} \\ \text{NUM} & \text{SG} \\ \text{PERS} & 3 \\ \text{CASE} & \text{NOM} \end{bmatrix}$$

- Otherwise, unification **fails**:

$$\begin{bmatrix} \text{CAT} & \text{NP} \\ \text{NUM} & \text{SG} \\ \text{CASE} & \text{NOM} \end{bmatrix} \sqcup \begin{bmatrix} \text{CAT} & \text{NP} \\ \text{NUM} & \text{PL} \end{bmatrix} = \emptyset$$

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# PATR-II STYLE FEATURE STRUCTURES

- CFG rules are augmented with constraints:
  - $A_0 \rightarrow A_1 \dots A_n$   
    {set of constraints}
  - There are two kinds of constraints:
    - **Unification constraints:**
      - $A_i \text{ feature-path}[] = A_j \text{ feature-path}[]$
    - **Value constraints:**
      - $A_i \text{ feature-path}[] = \text{atomic value}$
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# A GRAMMAR WITH FEATURE STRUCTURES

<b>S</b>	<b>→ NP VP</b>	<b>Grammar rule</b>
	$\langle \text{NP NUM} \rangle$ $\langle \text{NP CASE} \rangle$	$= \langle \text{VP NUM} \rangle$ $= \text{nom}$ <b>Constraints</b>
<b>NP</b>	<b>→ DT NOUN</b>	<b>Grammar rule</b>
	$\langle \text{NP NUM} \rangle$ $\langle \text{NP CASE} \rangle$	$= \langle \text{NOUN NUM} \rangle$ $= \langle \text{NOUN CASE} \rangle$ <b>Constraints</b>
<b>NOUN</b>	<b>→ cake</b>	<b>Lexical entry</b>
	$\langle \text{NOUN NUM} \rangle$	$= \text{sg}$ <b>Constraints</b>

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# WITH COMPLEX FEATURE STRUCTURES

<b>S</b>	<b>→ NP VP</b>	<b>Grammar rule</b>
$\langle \text{NP AGR} \rangle$	= $\langle \text{VP AGR} \rangle$	<b>Constraints</b>
$\langle \text{NP CASE} \rangle$	= <i>nom</i>	

<b>NP</b>	<b>→ DT NOUN</b>	<b>Grammar rule</b>
$\langle \text{NP AGR} \rangle$	= $\langle \text{NOUN AGR} \rangle$	<b>Constraints</b>

<b>NOUN</b>	<b>→ <i>cake</i></b>	<b>Lexical entry</b>
$\langle \text{NOUN AGR NUM} \rangle$	= <i>sg</i>	<b>Constraints</b>

**Complex feature structures** can capture **better generalizations** (and hence require fewer constraints) — cf. the previous slide

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# THE HEAD FEATURE

- Instead of implicitly specifying heads for each rewrite rule, let us define a **head feature**.
- The head of a VP has the same agreement feature as the VP itself:

$$\left[ \begin{array}{l} \text{CAT} \\ \text{AGR} \\ \text{HEAD} \end{array} \begin{array}{l} \text{VP} \\ \left[ \begin{array}{ll} \text{NUM} & \text{SG} \\ \text{PERS} & 3 \end{array} \right] \\ \left[ \begin{array}{l} \text{AGR} \\ \left[ \begin{array}{ll} \text{NUM} & \text{SG} \\ \text{PERS} & 3 \end{array} \right] \end{array} \right] \end{array} \right]$$

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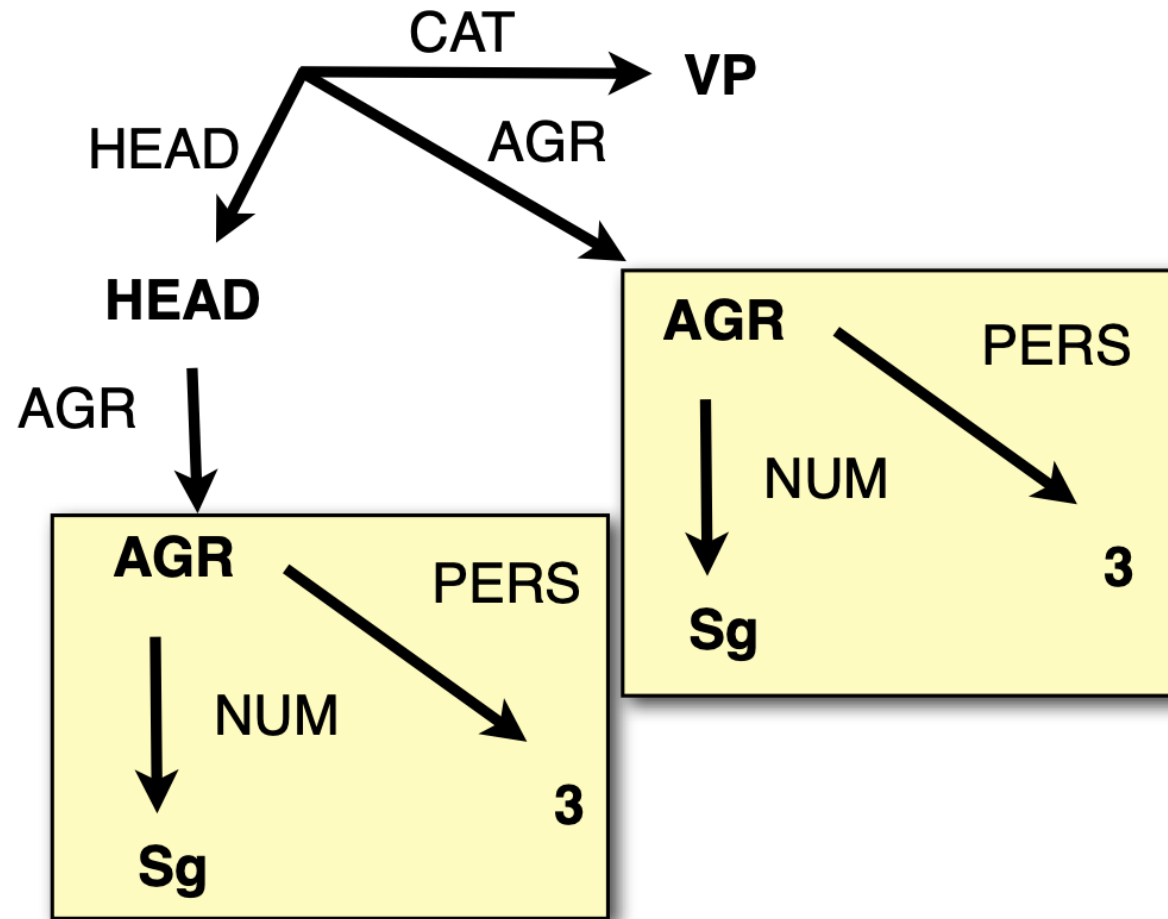
# RE-ENTRANCIES

- What we *really* want to say is that the agreement feature of the head is *identical* to that of the VP itself.
- This corresponds to a *re-entrancy* in the FS (indicated via coindexation **1** )

$$\left[ \begin{array}{cc} \text{CAT} & \text{VP} \\ \text{AGR} & \left[ \begin{array}{cc} \text{NUM} & \text{SG} \\ \text{PERS} & 3 \end{array} \right] \\ \text{HEAD} & \left[ \text{AGR} \right] \end{array} \right]$$

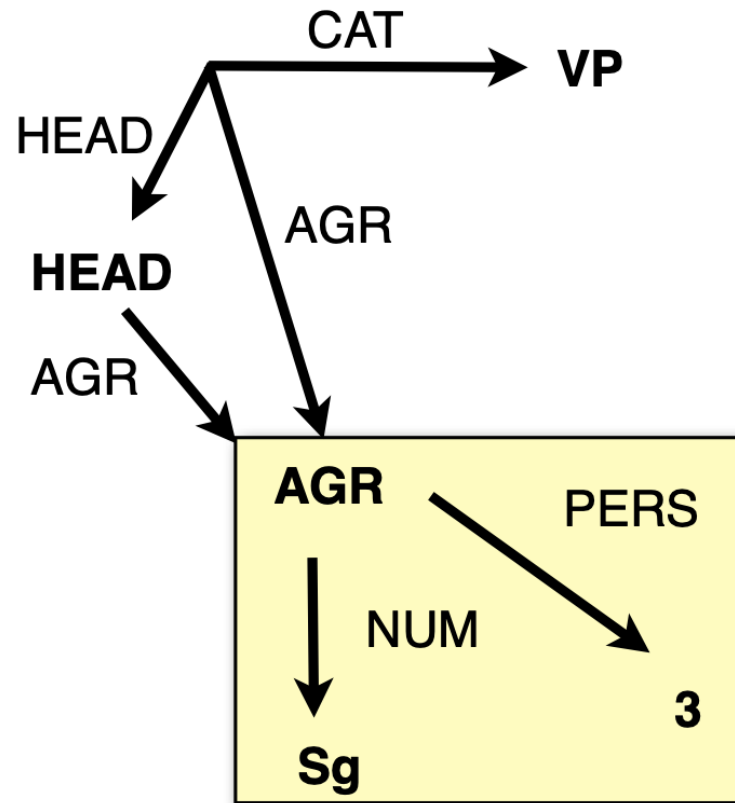
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## RE-ENTRANCIES — NOT LIKE THIS:



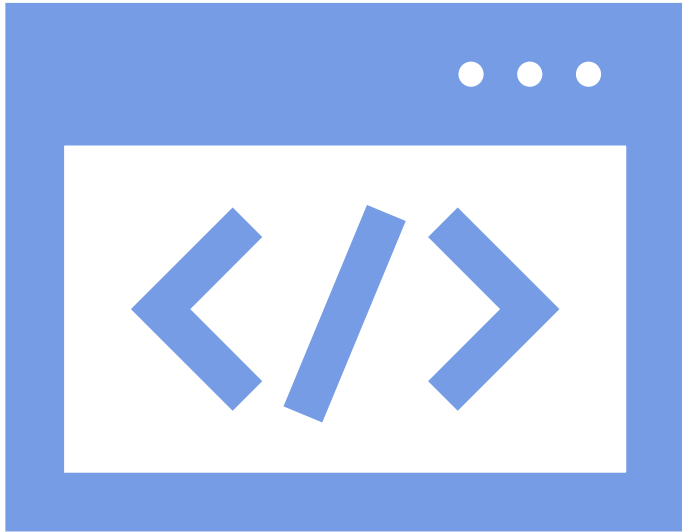


## RE-ENTRANCIES — BUT LIKE THIS:



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# ATTRIBUTE-VALUE GRAMMARS AND CFGS



- If every feature can only have a finite set of values, any attribute-value grammar can be compiled out into a (possibly huge) context-free grammar

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# GOING BEYOND CFGS

- The **power-of-2 language**:  $L_2 = \{w_i \mid i \text{ is a power of } 2\}$
- $L_2$  is a (fully) context-sensitive language.  
(*Mildly* context-sensitive languages have the **constant growth property** (the length of words always increases by a constant factor  $c$ ))

Here is a feature grammar which generates  $L_2$ :

$$A \rightarrow a$$

$$\langle A \ F \rangle = 1$$

$$A \rightarrow A_1 \ A_2$$

$$\langle A \ F \rangle = \langle A_1 \rangle$$

$$\langle A \ F \rangle = \langle A_2 \rangle$$

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# PART 4: TREE- ADJOINING GRAMMAR

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# (LEXICALIZED) TREE-ADJOINING GRAMMAR

TAG is a tree-rewriting formalism:

TAG defines operations (**substitution**, **adjunction**) on trees. The **elementary objects** in TAG are trees (not strings)

TAG is lexicalized:

Each elementary tree is **anchored** to a lexical item (word)

**“Extended domain of locality”**:

The elementary tree contains all arguments of the anchor.

TAG requires a linguistic theory which specifies the shape of these elementary trees.

TAG is mildly context-sensitive:

can capture Dutch cross-serial dependencies but is still efficiently parseable

AK Joshi and Y Schabes (1996) Tree Adjoining Grammars. In G. Rosenberg and A. Salomaa, Eds., Handbook of Formal

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# MILDLY CONTEXT- SENSITIVE GRAMMARS



Contain all context-free grammars/languages



Can be **parsed in polynomial time** (TAG/CCG:  $O(n^6)$ )



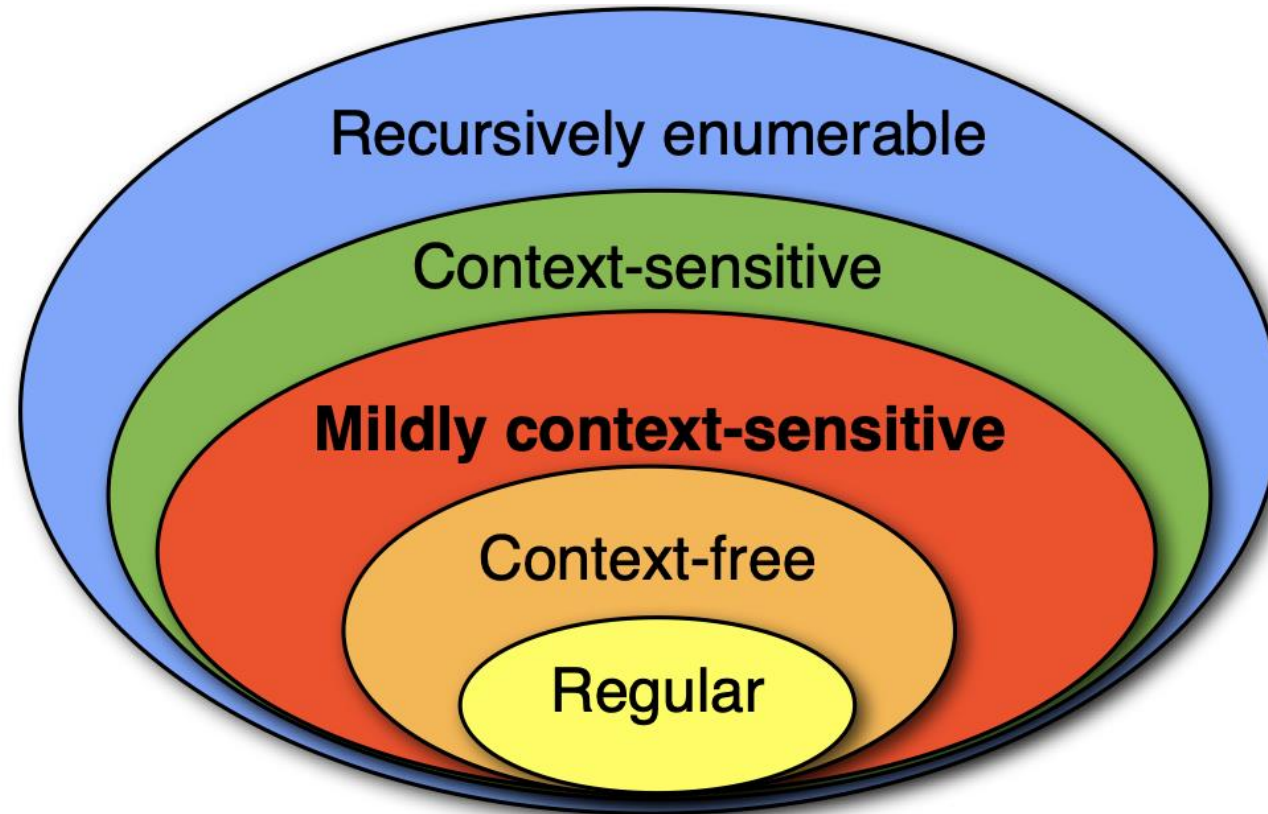
(*Strong* generative capacity) capture certain kinds of dependencies: **nested** (like CFGs) and **cross-serial** (like the Dutch example), but not the MIX language: MIX: the set of strings  $w \in \{a, b, c\}^*$  that contain equal numbers of *as*, *bs* and *cs*



Have the **constant growth** property:  
the length of strings grows in a linear way  
The power-of-2 language  $\{a^{2n}\}$  does not have the constant growth property.

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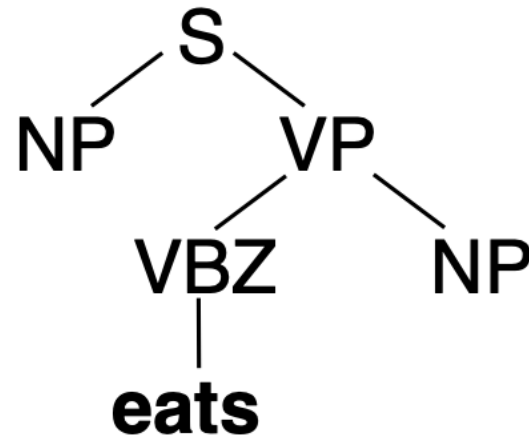
# THE CHOMSKY HIERARCHY



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# EXTENDED DOMAIN OF LOCALITY

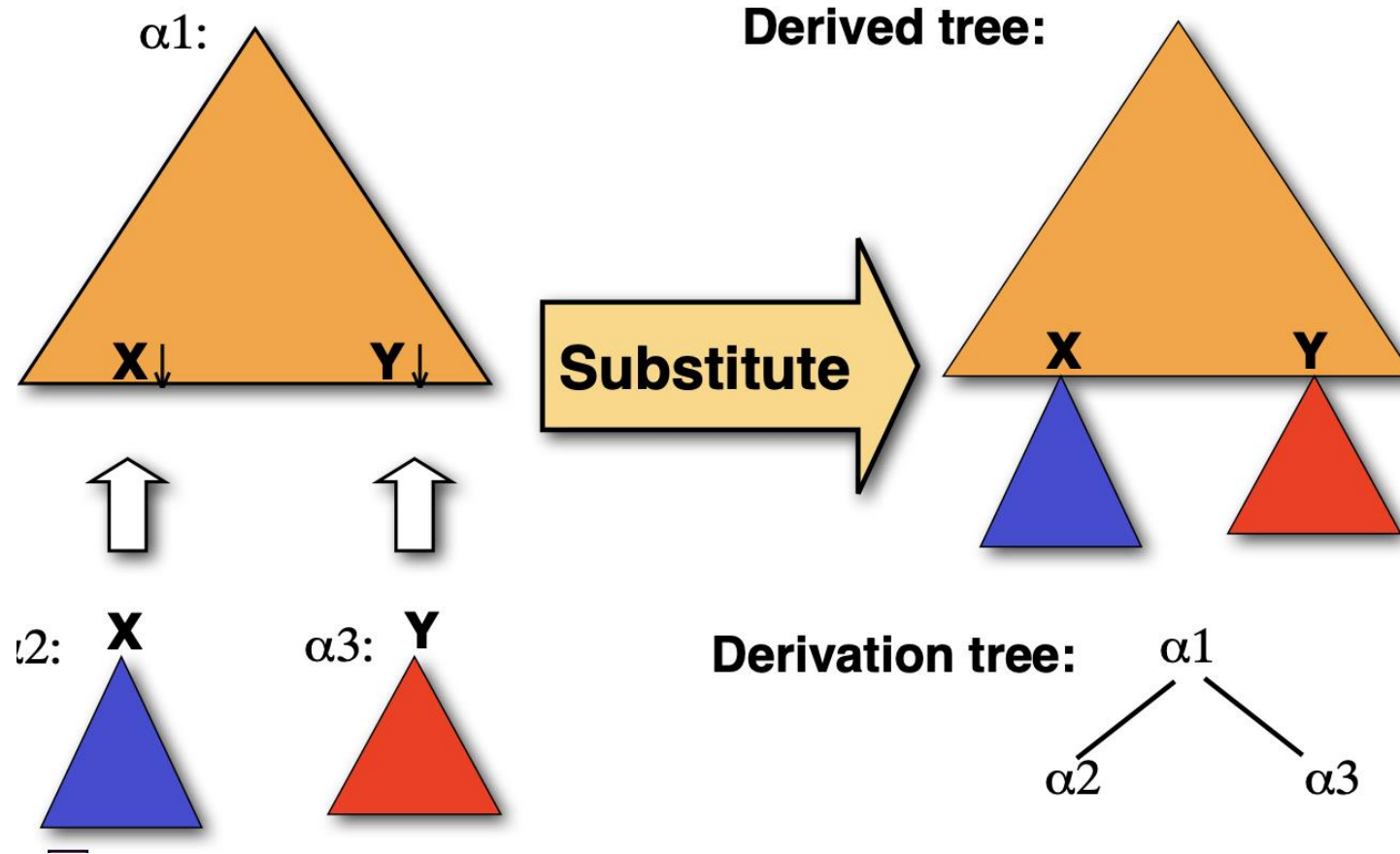
- We want to capture **all arguments of a word** in a **single elementary object**.
- We also want to retain certain syntactic structures (e.g. VPs).
- Our elementary objects are **tree fragments**:





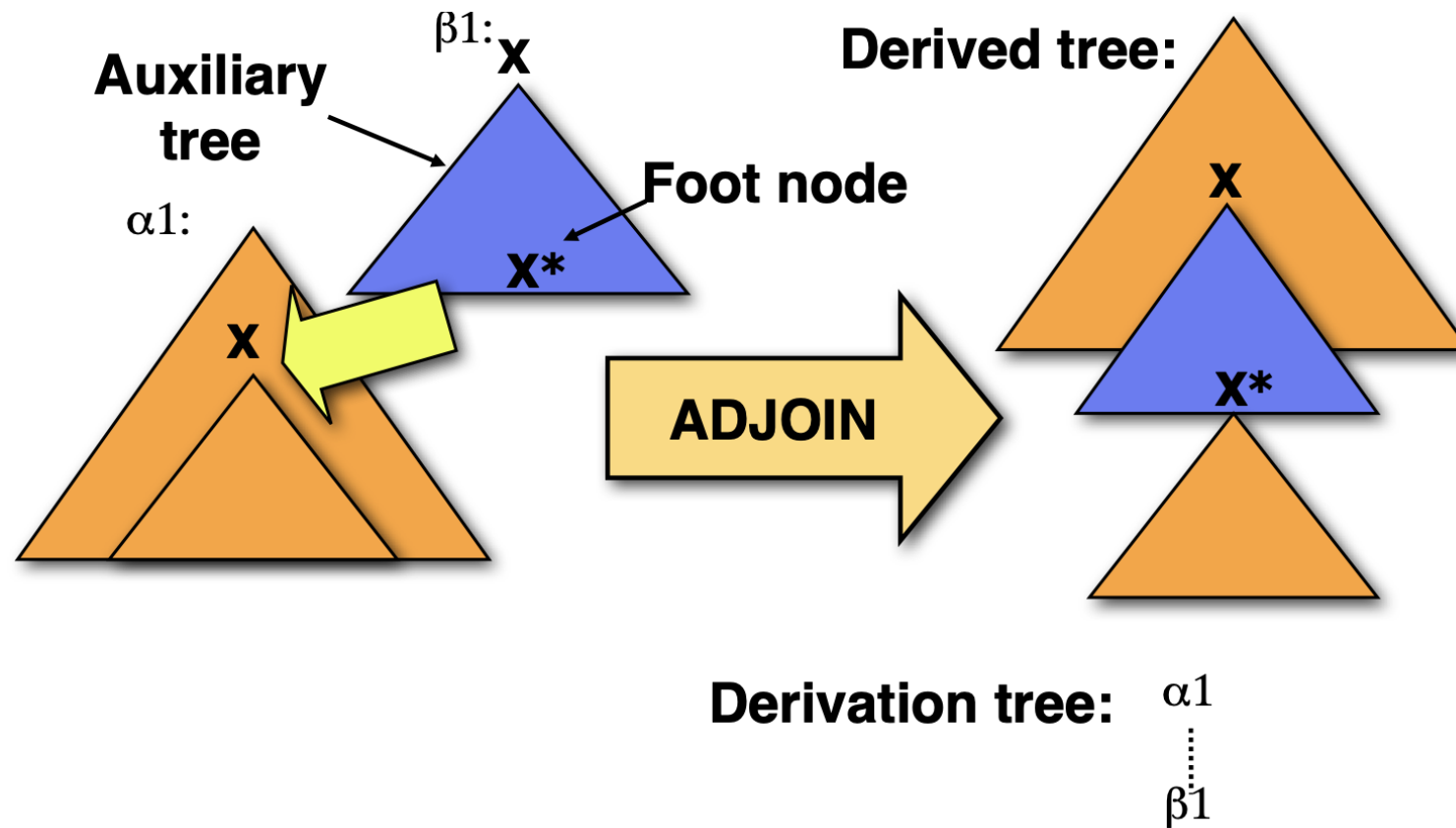
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# TAG SUBSTITUTION (ARGUMENTS)



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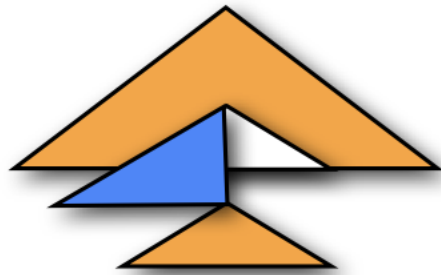
# TAG ADJUNCTION



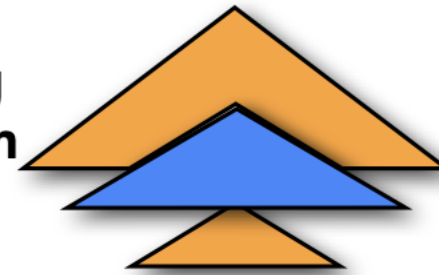
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# THE EFFECT OF ADJUNCTION

**TIG:**  
**sister**  
**adjunction**



**TAG:**  
**wrapping**  
**adjunction**



- No adjunction: TSG (Tree substitution grammar)  
TSG is context-free
  - Sister adjunction: TIG (Tree insertion grammar)  
TIG is also context-free, but has a linguistically more adequate treatment of modifiers
  - Wrapping adjunction: TAG (Tree-adjoining grammar)  
TAG is mildly context-sensitive
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# A SMALL TAG LEXICON

$\alpha_2$ :

NP  
|  
**John**

$\alpha_3$ :

NP  
|  
**tapas**

$\alpha_1$ :

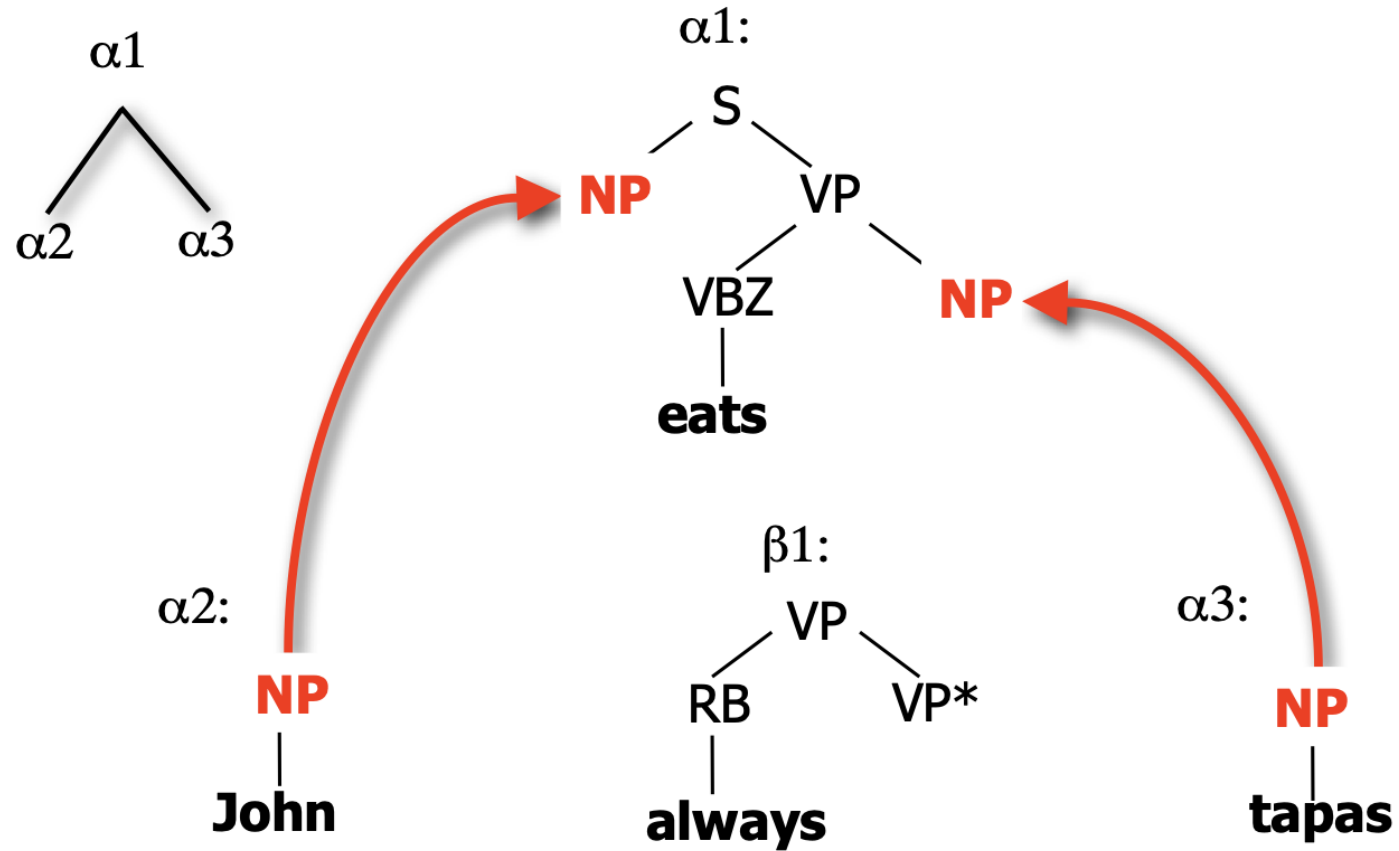
S  
/ \  
NP VP  
/ \  
VBZ NP  
|  
**eats**

$\beta_1$ :

VP  
/ \  
RB VP\*  
|  
**always**

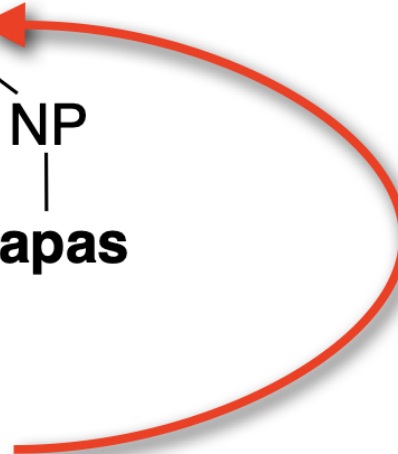
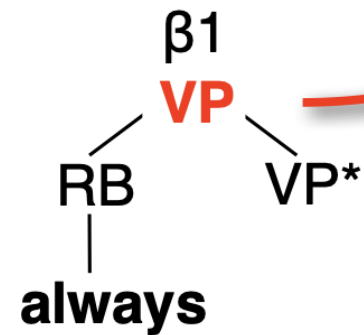
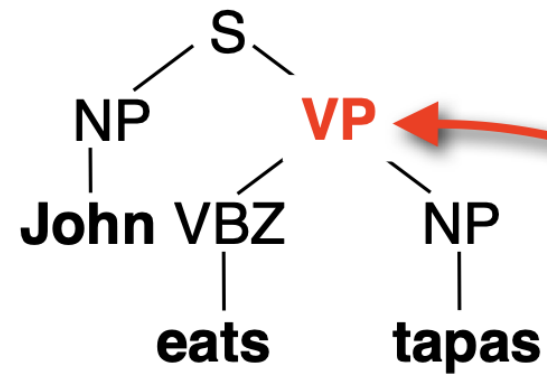
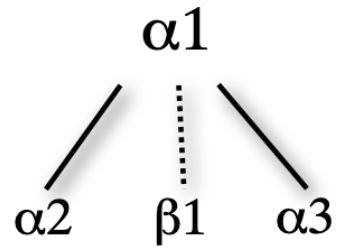
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# A TAG DERIVATION



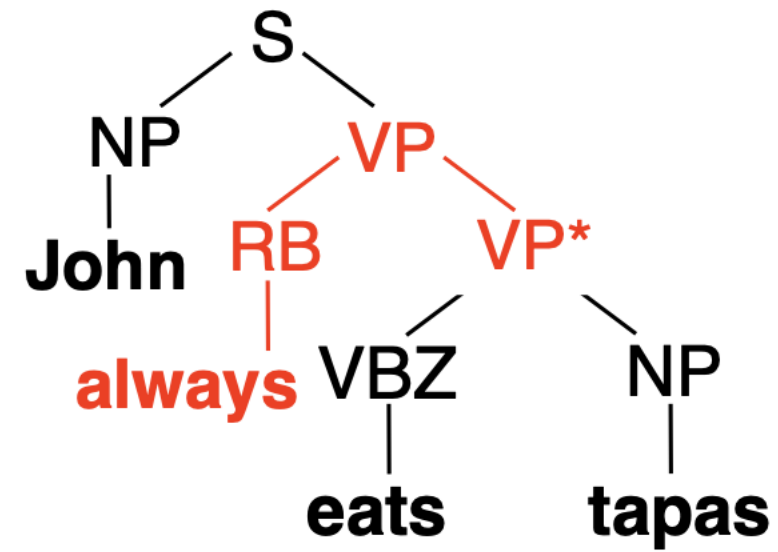
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# A TAG DERIVATION



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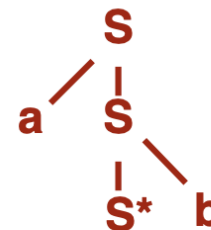
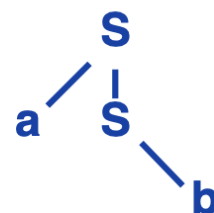
# A TAG DERIVATION



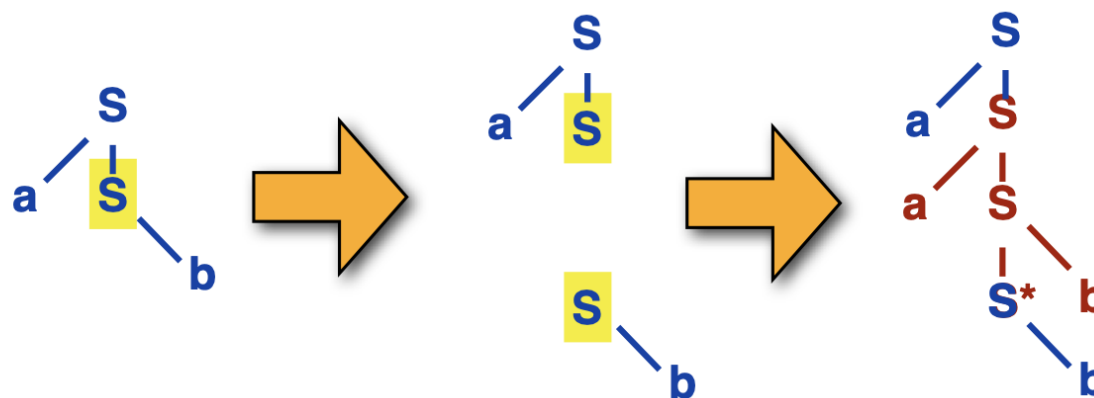
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# $A^N B^N$ : CROSS-SERIAL DEPENDENCIES

Elementary trees:



Deriving **aabb**





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# PART 5: (COMBINATORY) CATEGORIAL GRAMMAR

# CCG: THE MACHINERY

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Categories:  
specify subcat lists of words/constituents.

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Combinatory rules:  
specify how constituents can combine.

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The lexicon:  
specifies which categories a word can have.

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Derivations:  
spell out process of combining constituents.

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# CCG CATEGORIES

**Simple (atomic) categories:** NP, S, PP

**Complex categories** (functions):

Return a **result** when combined with an **argument**

VP, intransitive verb	$S \backslash NP$
Transitive verb	$(S \backslash NP) / NP$
Adverb	$(S \backslash NP) \backslash (S \backslash NP)$
Prepositions	$((S \backslash NP) \backslash (S \backslash NP)) / NP$ $(NP \backslash NP) / NP$ $PP / NP$

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# CCG CATEGORIES ARE FUNCTIONS

CCG has **a few atomic categories**, e.g

**S, NP, PP**

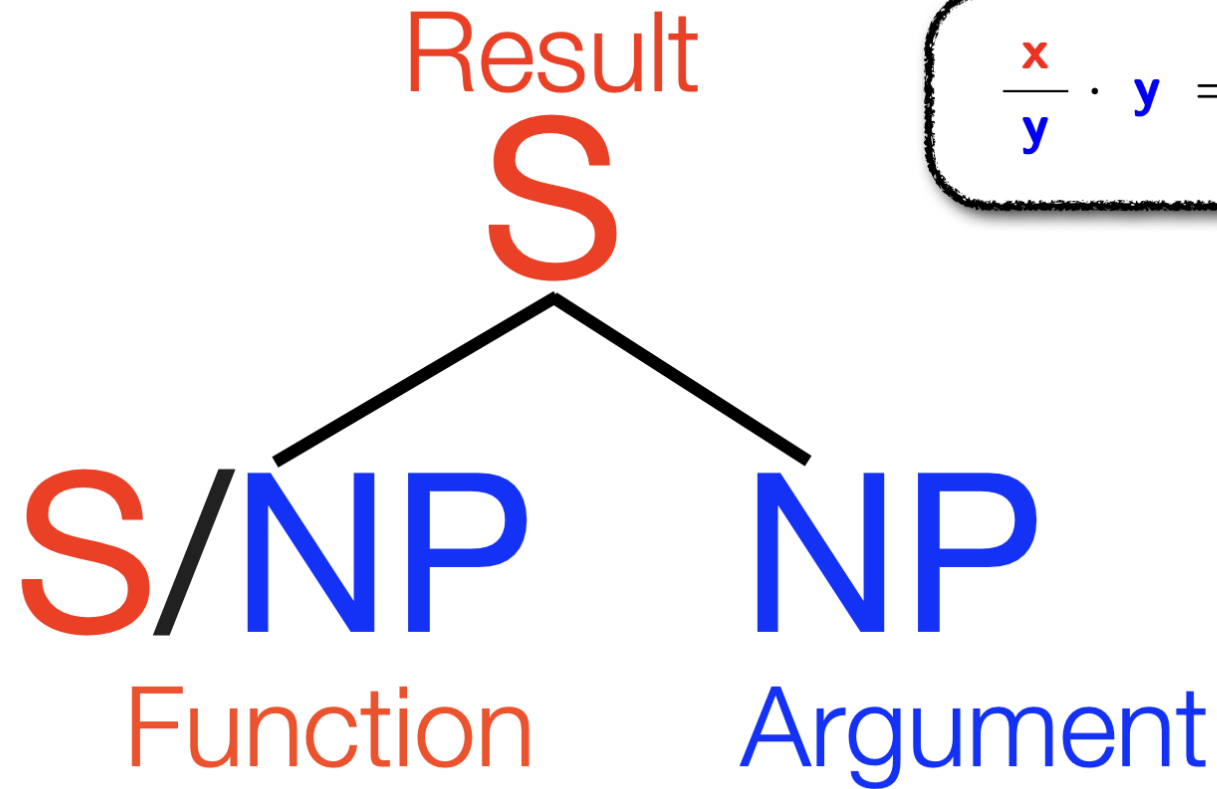
All other CCG categories are **functions**:

**S** / **NP**  
Result Dir. Argument

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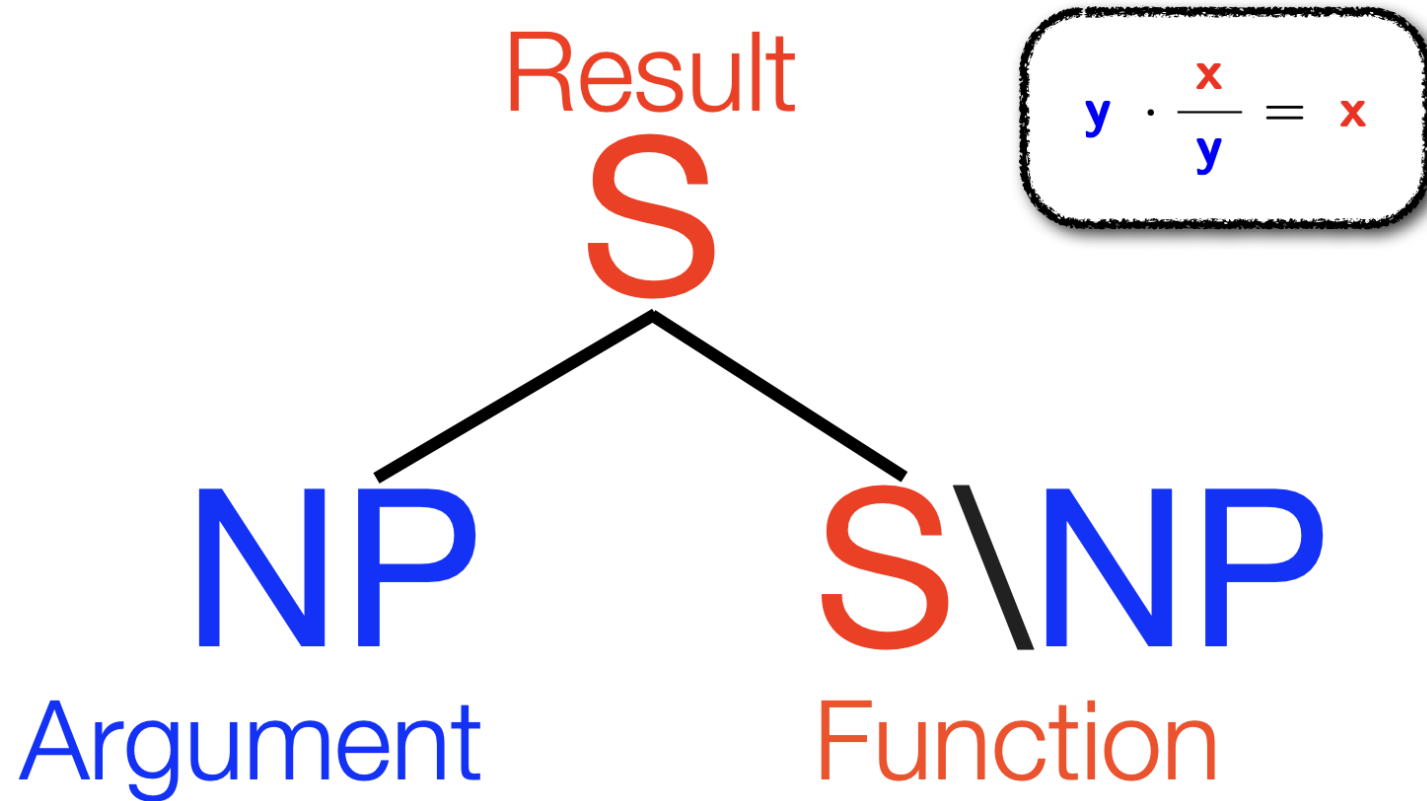
# RULES: FUNCTION APPLICATION



$$\frac{x}{y} \cdot y = x$$

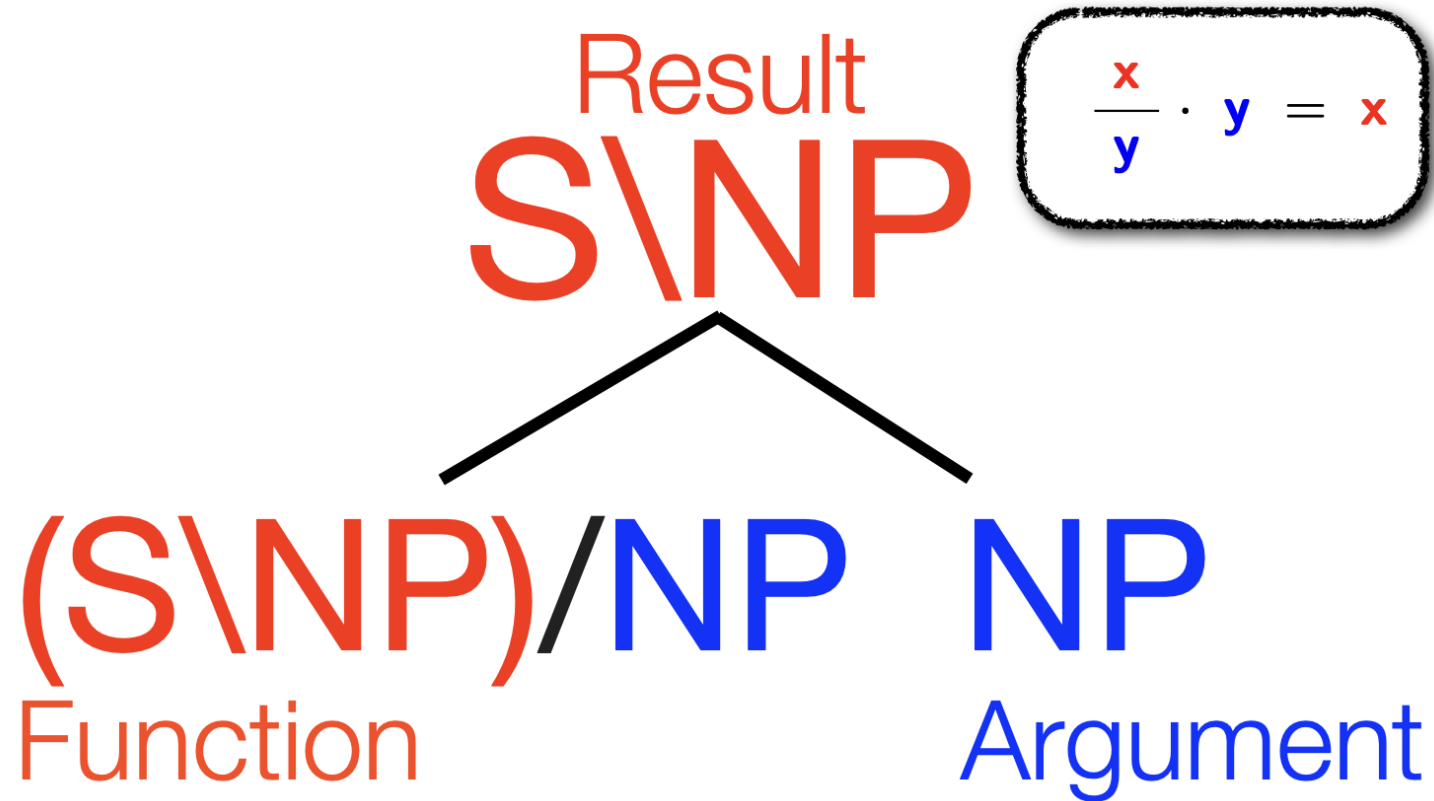
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# RULES: FUNCTION APPLICATION



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# RULES: FUNCTION APPLICATION



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# FUNCTION APPLICATION

**Forward application ( $\Rightarrow$ ):**

<b>(S\NP)/NP</b>	<b>NP</b>	$\Rightarrow_{>}$	<b>S\NP</b>
eats	tapas		eats tapas

**Backward application ( $\Rightarrow_{<}$ ):**

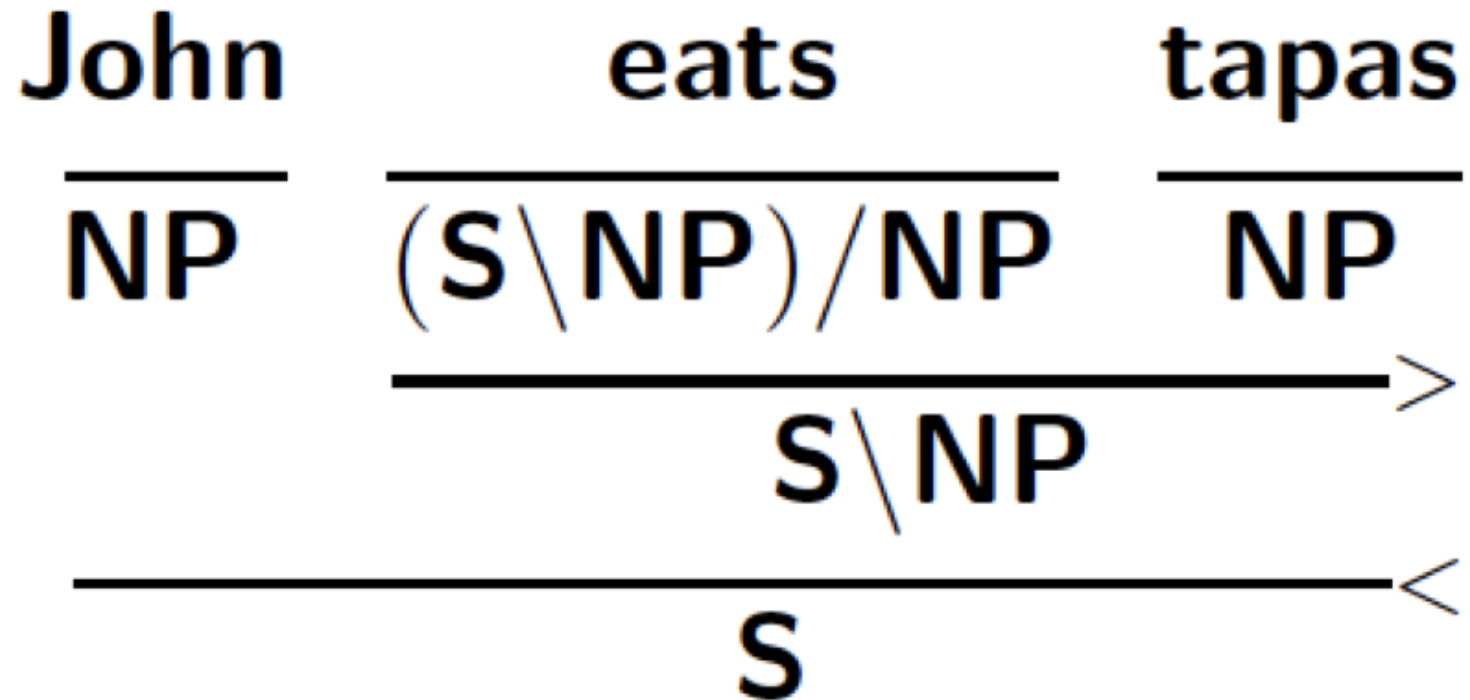
<b>NP</b>	<b>S\NP</b>	$\Rightarrow_{<}$	<b>S</b>
John	eats tapas		John eats tapas

**Combines function  $X/Y$  or  $X\backslash Y$  with argument  $Y$  to yield result  $X$**   
Used in all variants of categorial grammar

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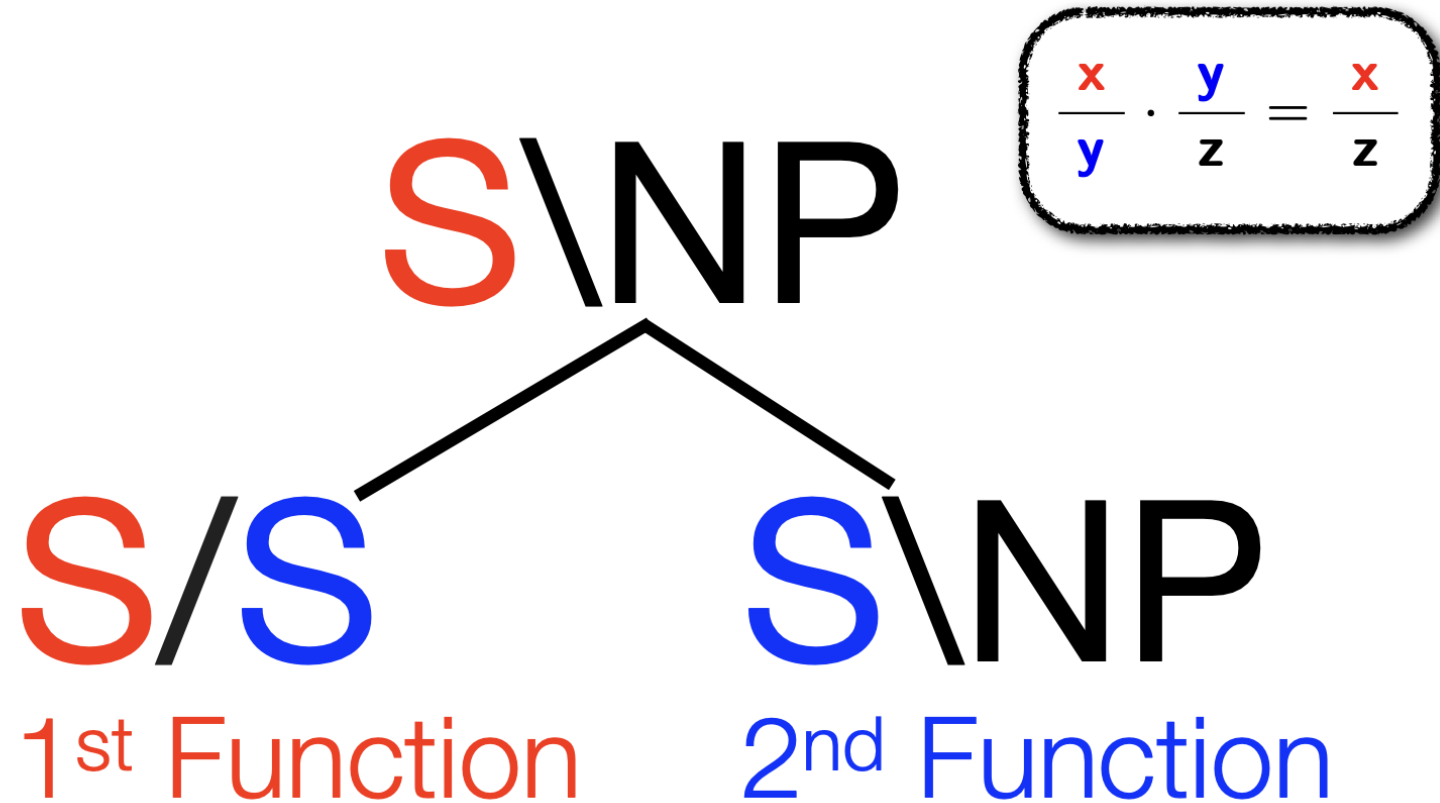


# A (C)CG DERIVATION



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# RULES: FUNCTION COMPOSITION



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## RULES: TYPE-RAISING

$S/(S \backslash NP)$

$\downarrow$   
 $NP$

$$y = \frac{x}{x} \cdot y = \frac{x}{\left(\frac{x}{y}\right)}$$

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# TYPE-RAISING AND COMPOSITION



**Type-raising:**  $X \rightarrow T/(T \backslash X)$



**Turns an argument into a function.**



$NP \rightarrow S/(S \backslash NP)$  (subject)    $NP \rightarrow (S \backslash NP) \backslash ((S \backslash NP)/NP)$  (object)



**Harmonic composition:**  $X/Y \ Y/Z \rightarrow X/Z$



**Composes two functions (complex categories),** same slashes  
 $(S \backslash NP)/PP \ PP/NP \rightarrow (S \backslash NP)/NP$   
 $S/(S \backslash NP) \ (S \backslash NP)/NP \rightarrow S/NP$



**Crossing composition:**  $X/Y \ Y \backslash Z \rightarrow X \backslash Z$

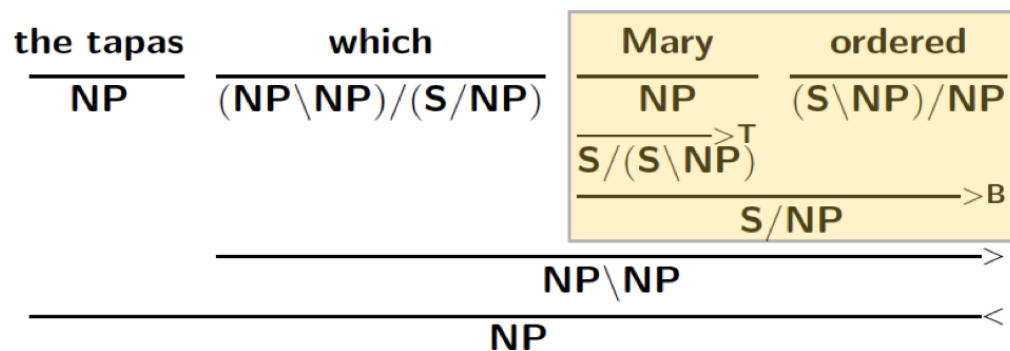


**Composes two functions (complex categories),** different slashes  
 $(S \backslash NP)/S \ S \backslash NP \rightarrow (S \backslash NP) \backslash NP$

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# TYPE-RAISING AND COMPOSITION

Wh-movement (relative clause):



Right-node raising:

