5.5. *The model*

We now have a symmetric hyperbolic non-equilibrium compressible two-phase flow model with heat and mass exchanges:

$$\frac{\partial \alpha_1}{\partial t} + \boldsymbol{u} \cdot \nabla \alpha_1 = K \operatorname{div}(\boldsymbol{u}) + \frac{\alpha_1 \alpha_2}{\alpha_2 \rho_1 c_1^2 + \alpha_1 \rho_2 c_2^2} \left(\frac{\Gamma_1}{\alpha_1} + \frac{\Gamma_2}{\alpha_2} \right) Q_1 + \frac{\dfrac{\rho_1 c_1^2}{\alpha_1} + \dfrac{\rho_2 c_2^2}{\alpha_2}}{\dfrac{c_1^2}{\alpha_1} + \dfrac{c_2^2}{\alpha_1}} \rho \dot{Y}_1, \tag{5.11a}$$

$$\frac{\partial \alpha_1 \rho_1}{\partial t} + \operatorname{div}(\alpha_1 \rho_1 \boldsymbol{u}) = \rho \dot{Y}_1, \tag{5.11b}$$

$$\frac{\partial \alpha_2 \rho_2}{\partial t} + \operatorname{div}(\alpha_2 \rho_2 \boldsymbol{u}) = -\rho \dot{Y}_1, \tag{5.11c}$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \operatorname{div}(\rho \boldsymbol{u} \otimes \boldsymbol{u}) + \nabla p = 0, \tag{5.11d}$$

$$\frac{\partial \rho E}{\partial t} + \operatorname{div}(\boldsymbol{u}(\rho E + p)) = 0, \tag{5.11e}$$

where

$$\begin{aligned} K &= \frac{\alpha_1 \alpha_2 \left(\rho_2 c_2^2 - \rho_1 c_1^2 \right)}{\alpha_2 \rho_1 c_1^2 + \alpha_1 \rho_2 c_2^2}, \\ \dot{Y}_1 &= \nu (\overline{g}_2 - \overline{g}_1), \\ Q_1 &= H (T_2 - T_1). \end{aligned}$$

The mixture pressure is given by (4.4):

$$p(\rho, e, \alpha_1, \alpha_2, Y_1, Y_2) = \frac{\rho(e - Y_1 q_1 - Y_2 q_2) - \left(\dfrac{\alpha_1 \gamma_1 p_{\infty,1}}{\gamma_1 - 1} + \dfrac{\alpha_2 \gamma_2 p_{\infty,2}}{\gamma_2 - 1} \right)}{\dfrac{\alpha_1}{\gamma_1 - 1} + \dfrac{\alpha_2}{\gamma_2 - 1}}.$$

6.1. *Hyperbolic solver*

The hyperbolic system (5.11) without heat and mass transfer is

$$\left.\begin{aligned}
&\frac{\partial \alpha_1}{\partial t} + \boldsymbol{u}\cdot\nabla\alpha_1 = K(\alpha_1, \rho_1, \rho_2, p)\,\mathrm{div}\,(\boldsymbol{u}),\\
&\frac{\partial \alpha_1\rho_1}{\partial t} + \mathrm{div}\,(\alpha_1\rho_1\boldsymbol{u}) = 0,\\
&\frac{\partial \alpha_2\rho_2}{\partial t} + \mathrm{div}\,(\alpha_2\rho_2\boldsymbol{u}) = 0,\\
&\frac{\partial \rho\boldsymbol{u}}{\partial t} + \mathrm{div}\,(\rho\boldsymbol{u}\otimes\boldsymbol{u}) + \nabla p = 0,\\
&\frac{\partial \rho E}{\partial t} + \mathrm{div}\,[\boldsymbol{u}(\rho E + p)] = 0.
\end{aligned}\right\} \tag{6.1}$$

$$\left.\begin{aligned}
&\frac{\partial \alpha_1}{\partial t} = \frac{\alpha_1\alpha_2}{\alpha_2\rho_1 c_1^2 + \alpha_1\rho_2 c_2^2}\left(\frac{\Gamma_1}{\alpha_1} + \frac{\Gamma_2}{\alpha_2}\right)Q_1 + \frac{\dfrac{\rho_1 c_1^2}{\alpha_1} + \dfrac{\rho_2 c_2^2}{\alpha_2}}{\dfrac{c_1^2}{\alpha_1} + \dfrac{c_2^2}{\alpha_1}}\rho\dot{Y}_1 = S_{\alpha_1},\\
&\frac{\partial \alpha_1\rho_1}{\partial t} = \rho\dot{Y}_1 = S_{Y_1}, \quad \frac{\partial \alpha_2\rho_2}{\partial t} = -\rho\dot{Y}_1,\\
&\frac{\partial \rho\boldsymbol{u}}{\partial t} = 0, \quad \frac{\partial \rho E}{\partial t} = 0.
\end{aligned}\right\} \tag{6.3}$$

$$\frac{\partial \Delta T}{\partial t} = AQ_1 + B\dot{Y}_1, \quad \frac{\partial \Delta g}{\partial t} = A'Q_1 + B'\dot{Y}_1, \tag{6.4}$$

where A, B, A', B' are functions of all flow variables. Their expressions are detailed in Appendix B.

The simplest numerical approximation of these equations is used. Let n and $n+1$ denote two successive time steps. The variables at time t^n are taken equal to those resulting from the numerical integration of system (6.1). The variables at time t^{n+1} denote the end of the integration process, including both hydrodynamic effects and source terms (6.3). The simplest numerical approximation of this ODE system is

$$\left.\begin{aligned} \frac{(\Delta T)^{n+1} - (\Delta T)^n}{\Delta t} &= A^n Q_1^n + B^n \dot{Y}_1^n, \\ \frac{(\Delta g)^{n+1} - (\Delta g)^n}{\Delta t} &= A'^n Q_1^n + B'^n \dot{Y}_1^n. \end{aligned}\right\} \tag{6.5}$$

By imposing that thermodynamic equilibrium is reached at the end of the time step we have $(\Delta T)^{n+1} = 0$ and $(\Delta g)^{n+1} = 0$. The corresponding heat and mass transfer terms are given by

$$\left.\begin{aligned} Q_1 &= -\frac{B'}{AB' - A'B}\frac{(\Delta T)^n}{\Delta t} + \frac{B}{AB' - A'B}\frac{(\Delta g)^n}{\Delta t}, \\ \dot{Y}_1 &= \frac{A'}{AB' - A'B}\frac{(\Delta T)^n}{\Delta t} - \frac{A}{AB' - A'B}\frac{(\Delta g)^n}{\Delta t}. \end{aligned}\right\} \tag{6.6}$$