

$$\begin{aligned}
a^2 &= \frac{\partial P}{\partial \rho} + \frac{p}{\rho^2} \frac{\partial P}{\partial e} \\
&= (\gamma - 1)e + \frac{P}{\rho}(\gamma - 1)
\end{aligned}$$

Recall:

$$P = (\gamma - 1)\rho e - \gamma P^\infty \tag{1}$$

We get:

$$(\gamma - 1)\rho e = P + \gamma P^\infty$$

Then, multiply both sides by  $\rho$

$$\begin{aligned}
a^2 \rho &= (\gamma - 1)\rho e + P(\gamma - 1) \\
a^2 \rho &= (P + \gamma P^\infty) + P(\gamma - 1) \\
a^2 &= \frac{\gamma(P + P^\infty)}{\rho} \\
a &= \sqrt{\frac{\gamma(P + P^\infty)}{\rho}}
\end{aligned}$$