

ML2P-VAE

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`library(ML2Pvae)`

Introduction

Overview of everything.

Multidimensional Logistic 2-Parameter (ML2P) Model

In Item Response Theory, the goal is often to quantify the relation between the underlying ability (or abilities) of students and their performance on an assessment. Formally, we assume that every student j possesses some latent ability value Θ_j , which is not directly observable. Often, we assume that Θ_j is distributed normally across the population of N students. In a practical setting, our data for each student may be a binary vector $u_j \in \mathbb{R}^n$ where a 1 (resp. 0) corresponds to a correct (resp. incorrect) answer, to an exam consisting of n questions (items). Given a student j 's assessment results, how can we infer the latent value Θ_j ?

A naive approach is to simply use the raw exam score, or accuracy. But if two students both score 80% on an exam while missing different items (so $u_1 - u_2 \neq 0_n$), it is unlikely that they have the exact same ability value. Not all exam items are equal - they often vary in difficulty and in the skills required to answer them correctly.

Instead, theory has been developed [1] that states that the probability of student j answering item i correctly is a function of Θ_j , along with parameters Λ_i associated with item i :

$$P(u_{ij} = 1 | \Theta_j) = f(\Theta_j; \Lambda_i)$$

In this work, we are concerned with the case where $\Theta_j = (\theta_{j1}, \dots, \theta_{jK})^T \in \mathbb{R}^K$, so that an exam is designed to assess multiple skills. For example, an elementary math exam may test the abilities “add,” “subtract,” “multiply,” and “divide.” In order to keep track of which items assess which abilities, we can develop a binary matrix $Q \in \mathbb{R}^{K \times n}$ where

$$Q_{ki} = \begin{cases} 1 & \text{if item } i \text{ requires skill } k \\ 0 & \text{otherwise} \end{cases}$$

It is important to note that multiple items can require multiple skills.

The model that this package focuses on is the Multidimensional Logistic 2-Parameter (ML2P) model. The probability of a correct response is given by

$$P(u_{ij} = 1 | \Theta_j) = \frac{1}{1 + \exp[-\tilde{a}_i^T \Theta_j + b_i]} = \frac{1}{1 + \exp[-\sum_{k=1}^K a_{ik} \theta_{jk} + b_i]}$$

where the discrimination parameters $\tilde{a}_i^T = (a_{ik})_{1 \leq k \leq K}$ determine *how much* of skill k is required to answer item i correctly, and the difficulty parameter b_i quantifies the difficulty of item i .

In application, we often only have response vectors for a set of students - so we need to estimate the ability parameters Θ_j for each student, along with the discrimination and difficulty parameters for each item. While there are many parameter estimation methods, including the Expectation-Maximization algorithm and various MCMC methods, many of them become infeasible as the number of latent traits increases beyond $\dim(\Theta_j) = 7$.

Variational Autoencoders

How a VAE works.

ML2P-VAE

Basic model description

List of package functions

Proof of implementation of full covariance

Show why my shit works Note that typical LaTeX equations work $GG^T = \Sigma$

$$y = 2x^2$$

$$\begin{aligned} y &= x \\ &\leq x \end{aligned}$$

Code Examples of how to use the package

References

[1] Wainer and Thissen, D. “Test Scoring”. Erlbaum Associates, Publishers, 2001.