Neural Network Methods for Application in Educational Measurement

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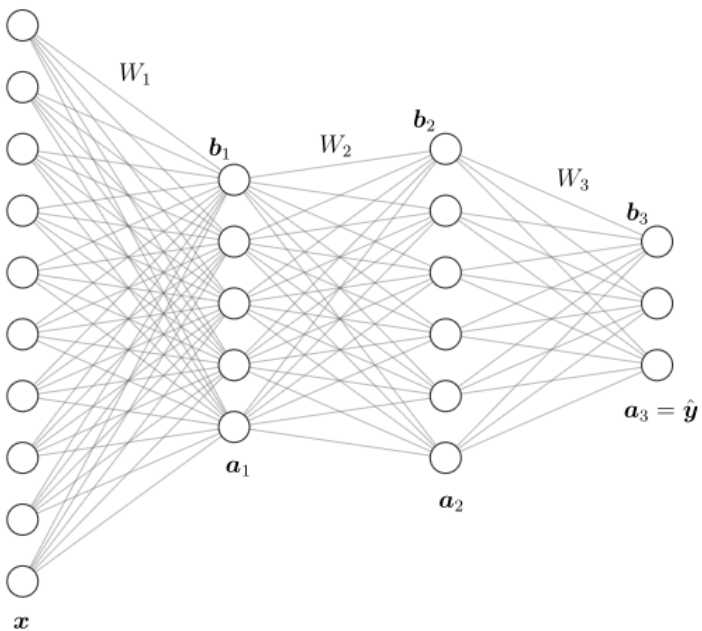
PhD Defense in Applied Mathematical and Computational Sciences

Overview

Develop machine learning algorithms to quantify student learning

Outline

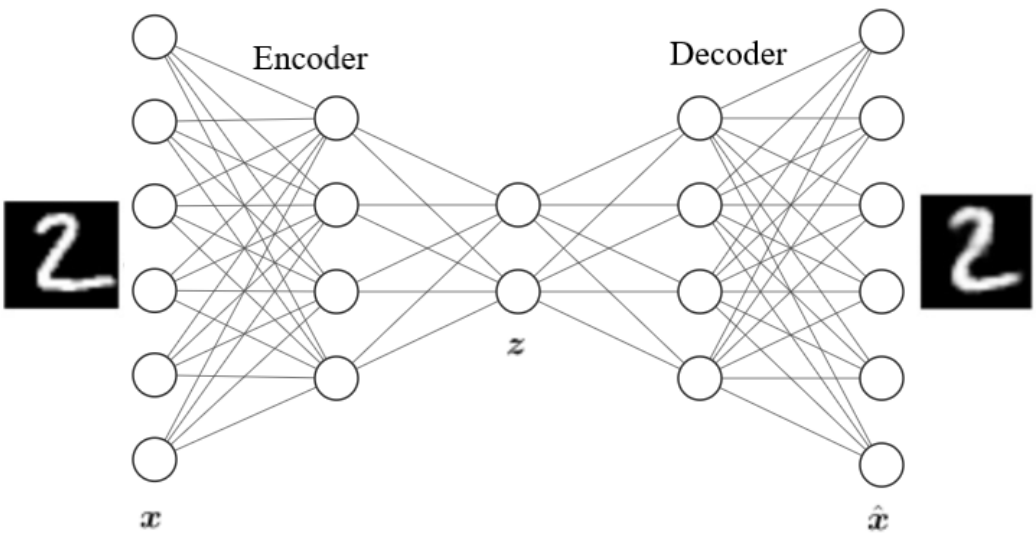
Artificial Neural Networks (ANN)



Input $\boldsymbol{x}$, approximate a true target $\boldsymbol{y}$ via a series of (learned) linear transformations and nonlinear re-scaling

$$\hat{\boldsymbol{y}} = \sigma(\, W_3\sigma(\, W_2\sigma(\, W_1\boldsymbol{x} + vectb_1) + \boldsymbol{b}_2) + \boldsymbol{b}_3)$$

Autoencoder (AE)



- Encode data into smaller dimension
- Reconstruct original input: $\mathcal{L} = ||\boldsymbol{x} - \hat{\boldsymbol{x}}||$

Variational Autoencoder (VAE)

- Commonly used as a generative neural network
- Learn a low-dimensional latent representation θ which is capable of generating original data x from some distribution

$$f(\boldsymbol{\theta}|\boldsymbol{x}) = \frac{P(\boldsymbol{X} = \boldsymbol{x}|\theta)f(\boldsymbol{\theta})}{P(\boldsymbol{X} = \boldsymbol{x})}$$

$$P(\boldsymbol{X} = \boldsymbol{x}) = \int P(\boldsymbol{X} = \boldsymbol{x}|\boldsymbol{\theta})f(\boldsymbol{\theta})\,d\boldsymbol{\theta},$$

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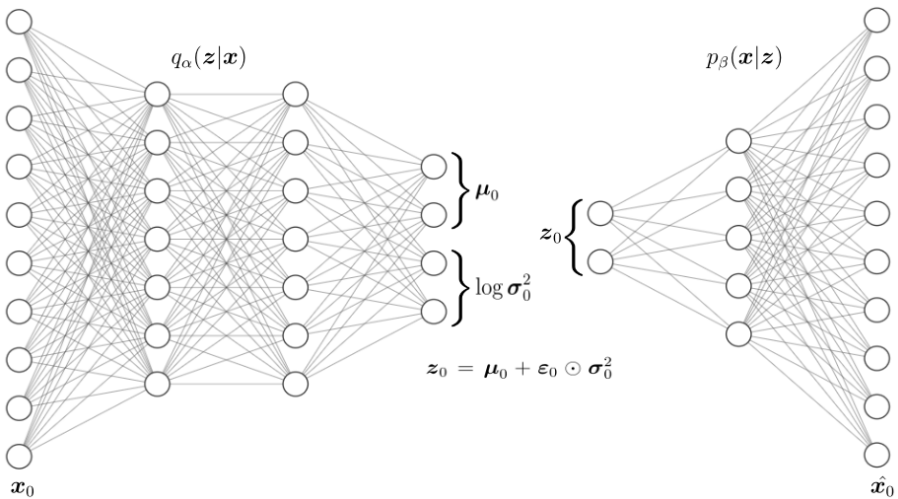
- Approximate $f(\boldsymbol{\theta}|\boldsymbol{x})$ with some $q(\boldsymbol{\theta}|\boldsymbol{x}) \Rightarrow$ minimize KL Divergence

VAE loss

TODO: derive VAE loss

Variational Autoencoder (VAE)



- Fit encoded space to a normal distribution
  - Loss funcion: $L(x) = L_0(x) + KL[q_\alpha(\theta|x)||\mathcal{N}(0, I)]$

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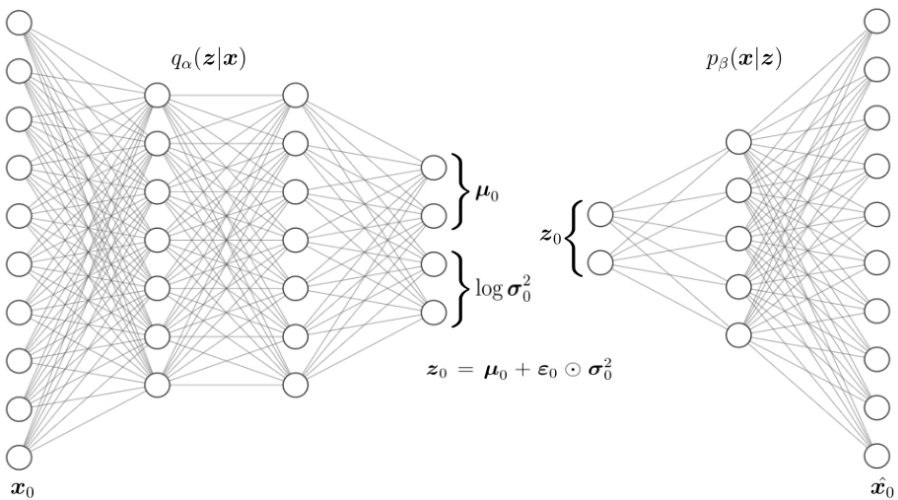

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  - Loss funcion: $L(x) = L_0(x) + KL[q_\alpha(\theta|x)||\mathcal{N}(0, I)]$
- Sample $\varepsilon \sim \mathcal{N}(0, 1)$, set $z = \mu + \sigma\varepsilon$

Variational Autoencoder (VAE)

mona lisa visual might be okay, but want to emphasize that encoder learns a posterior distribution

Item Response Theory (IRT)

TODO: clean this up

- Goal: Explain relationship between student ability and exam performance
- Assume each subject has a latent "ability" value θ

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  - θ is not directly observable
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  - Naive solution: accuracy (percent correct)
- For an assessment with n items taken by N subjects, what is the probability that student j answers item i correctly?

$$P(u_{ij} = 1|\theta_j) = f(\theta_j; V_i)$$

  - θ_j = latent ability of subject j
  - V_i = set of parameters associated with item i

Rasch Model

- Define $\delta_i > 0$ as the difficulty of item i, and $\eta_j > 0$ the ability of subject j.
- Rasch: Probability of success depends on ratio $\frac{\delta_i}{\eta_j}$

$$P(u_{ij} = 1|\eta_j, \delta_i) = \frac{1}{1 + \delta_i/\eta_j} = \frac{\eta_j}{\eta_j + \delta_i}$$

- Logarithmic transformation: $\theta_j = \log \eta_j$ and $\beta_i = \log \delta_i$
- Rasch Model :

$$P(u_{ij} = 1|\theta_j, \beta_i) = \frac{1}{1 + e^{\beta_i - \theta_j}}$$

2-Parameter Logistic Model (2PL)

- Probability of a correct response follows the logistic equation:

$$P(u_{ij} = 1|\theta_j; a_i, b_i) = \frac{1}{1 + e^{-a_i(\theta_j - b_i)}}$$

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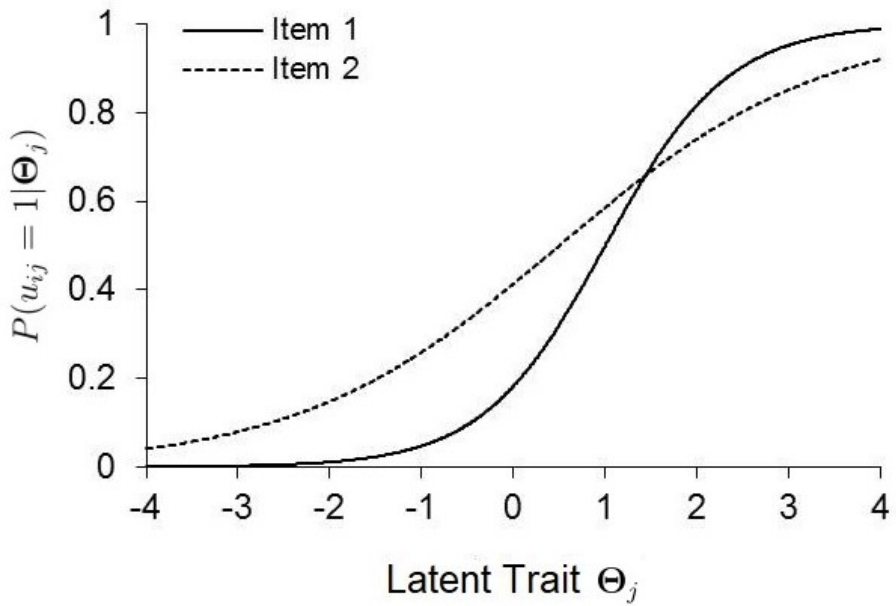
  - a_i = discrimination parameter
  - b_i = difficulty parameter

Item Characteristic Curve (ICC)



Multidimensional IRT

- Now assume that an assessment is testing K skills
  - For example, a math exam can test skills add, subtract, multiply, divide
  - Each student has a vector of skills $\Theta_j = (\theta_{j1}, .., \theta_{jK})^T$
  - Multiple skills can be assessed by a single item

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  - Each student has a vector of skills $\Theta_j = (\theta_{j1}, .., \theta_{jK})^T$
  - Multiple skills can be assessed by a single item
- Binary Q-matrix defines relationship between items and skills
  - $Q \in \mathbb{R}^{n \times K}$,

$$q_{ik} = \begin{cases} 1 & \text{if item } i \text{ requires skill } k \\ 0 & \text{otherwise} \end{cases}$$

Multidimensional Logistic 2-Parameter (ML2P) Model

- Probability of correct response given by:

$$P(u_{ij} = 1|\Theta_j) = \frac{1}{1 + \exp[-\sum_{k=1}^{K} a_{ik}\theta_{jk} + b_i]}$$

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- N students and n items, build $n \times N$ response matrix $[u_{ij}]$ and ability vector $\Theta = (\theta_1, ..., \theta_N)^T$
- Probability of item responses:

$$P(U|\Theta) = \prod_{j=1}^{N} \prod_{i=1}^{n} P_{ij}^{u_{ij}} (1 - P_{ij})^{1-u_{ij}}$$

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- Maximize log-likelihood:

$$L = \log P(U|\Theta) = \sum_{j=1}^{N} \sum_{i=1}^{n} u_{ij} \log P_{ij} + (1 - u_{ij}) \log(1 - P_{ij})$$

Marginal Maximum Likelihood (MMLE)

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- Assume that θ follows some distribution $g(\theta)$
- Maximize the mariginal likelihood for each student

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- Posterior probability

$$P(\theta_j|\, U_j) = \frac{P(U_j|\theta_j)\, g(\theta_j)}{P(U_j)} = \frac{P(U_j|\theta_j)\, g(\theta_j)}{\int P(U_j|\theta)\, g(\theta)\, d\theta}$$

Difficulties of IRT Parameter Estimation

TODO: summarize the problems with high-dim theta

Combining IRT and ANN

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- Key similarities:
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  - IRT and VAE assume normally distributed latent space
  - ML2P model and sigmoidal activation function:

$$P(u_{ij} = 1|\Theta_j) = \frac{1}{1 + \exp[-\sum_{k=1}^{K} a_{ik}\theta_{jk} + b_i]}$$

$$\sigma(z) = \sigma(\vec{w}^T\vec{a} + b) = \frac{1}{1 + \exp[-\sum_{k=1} w_k a_k - b]}$$

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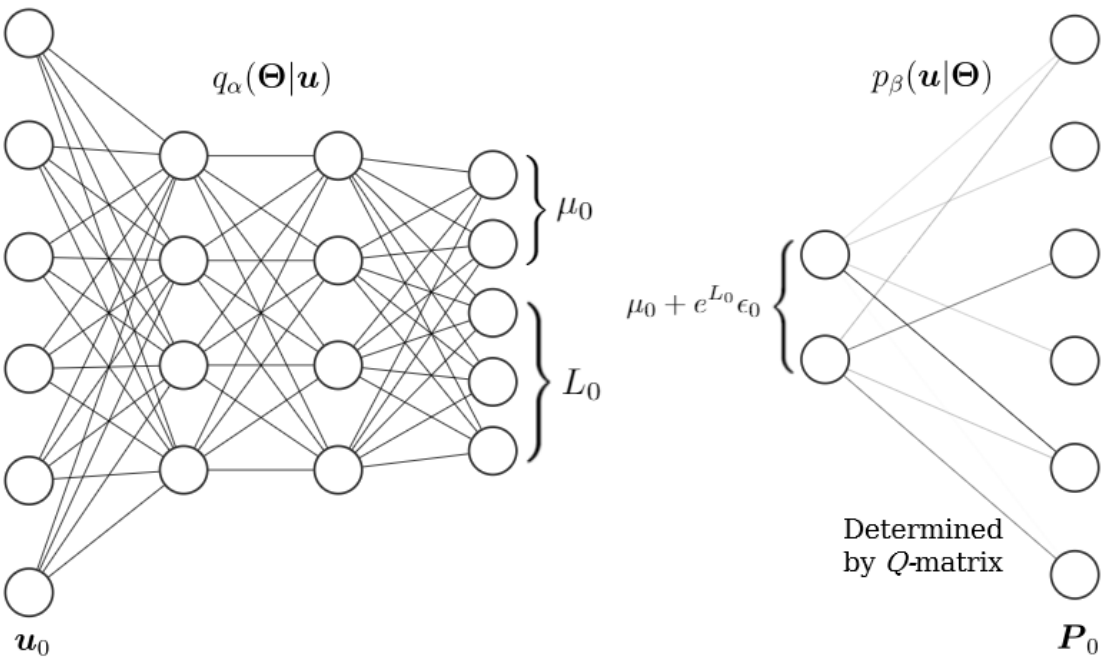
- Sigmoidal activation function in output layer
- Require decoder weights to be nonnegative
- Fit VAE latent space to $\mathcal{N}(0, I)$
- Decoder interpreted as the ML2P model
  - Activation of nodes in learned distribution $\Rightarrow$ latent skills
  - Weights in decoder $\Rightarrow$ discrimination parameters
  - Bias of output nodes $\Rightarrow$ difficulty parameters

ML2P-VAE

TODO: this image is for correlated latent traits



VAE vs AE Comparison

TODO: clean up and be better

- Guo, Cutumisu, and Cui proposed using AE in skill estimation
- Directly compare neural networks in ML2P application
  - Parameter recovery
  - Skill estimation
- How useful is prior $p(\boldsymbol{\Theta})$?
- What is the effect of the KL-Divergence term in the loss function?

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- Results presented at Artificial Intelligence in Education (AIED) 2019

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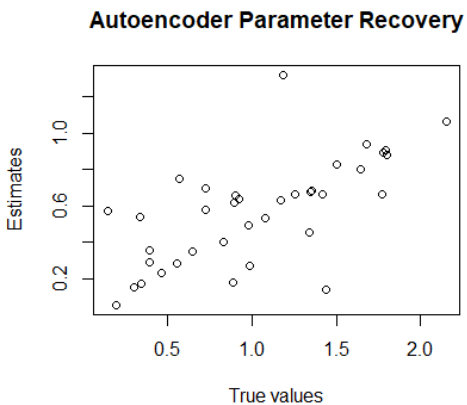

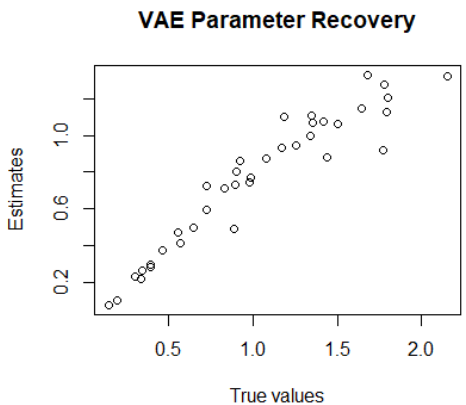

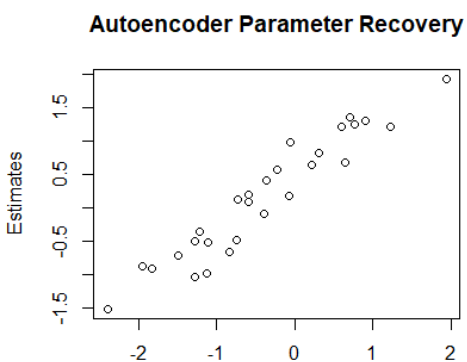

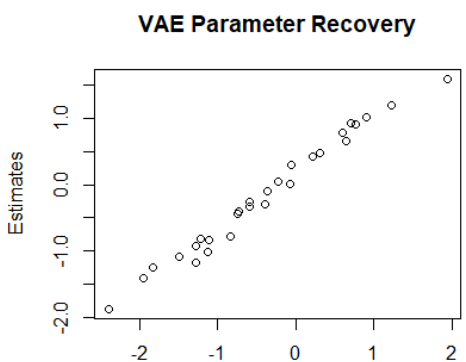

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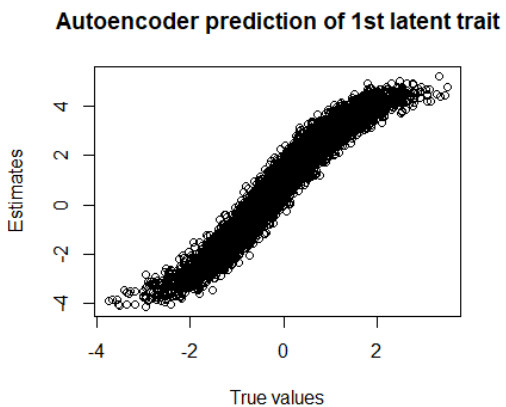

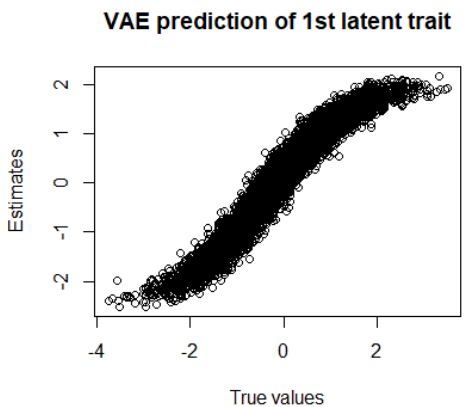

- Similar skill estimate correlation, but on different scale
- VAE much more accurate parameter recovery

Correlated Latent Traits in IRT

- In real applications, independent skills are not realistic: $\boldsymbol{\Theta} \sim \mathcal{N}(\boldsymbol{\mu}, \Sigma)$, not $\mathcal{N}(0, I)$.
  - Example: students who are good at adding are also good at subtracting

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  - Example: students who are good at adding are also good at subtracting
- Covariance matrix is symmetric, positive definite matrix

$$\Sigma = \begin{bmatrix} \sigma_1^2 & c_{12} & \cdots & c_{1k} \\ c_{21} & \sigma_2^2 & \cdots & c_{2k} \\ \vdots & & \ddots & \vdots \\ c_{k1} & \cdots & c_{k(k-1)} & \sigma_k^2 \end{bmatrix}$$

- With variances σ_i^2 and covariances $c_{ij} = c_{ji}$

Correlated Latent Code in VAE

- In most VAE applications, it is convenient to assume latent code $\boldsymbol{z}$ is *independent*
  - Forces each dimension of $\boldsymbol{z}$ to measure different features
  - $\boldsymbol{z}$ is *abstract*, with **no real-world understanding**
- For ML2P-VAE, we know that latent code $\boldsymbol{z}$ approximates latent traits $\boldsymbol{\Theta}$
  - We may have **domain knowledge** of the distribution of $\boldsymbol{\Theta}$

KL Divergence for correlated latent code

- Calculate KL Divergence between two k-dimensional multivariate normal distributions:

$$\mathcal{D}_{KL}\left[\mathcal{N}(\mu_0, \Sigma_0)||\mathcal{N}(\mu_1, \Sigma_1)\right] =$$
$$\frac{1}{2}\left(\operatorname{tr}(\Sigma_1^{-1}\Sigma_0) + (\mu_1 - \mu_0)^T\Sigma_1^{-1}(\mu_1 - \mu_0) - k + \ln\left(\frac{\det\Sigma_1}{\det\Sigma_0}\right)\right)$$

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- When fitting a VAE, $\mathcal{N}(\mu_1, \Sigma_1)$ is known, so μ_1 and Σ_1 are constant
- μ_0 and Σ_0 obtained from feeding one sample through the encoder

Requirements for Correlated VAE

- 1. To sample from a multivariate normal distribution $\mathcal{N}(\mu_0, \Sigma_0)$:
  - Find a matrix G such that $GG^T = \Sigma_0$
  - Sample $\varepsilon = (\varepsilon_1, ..., \varepsilon_k)^T$ with each $\varepsilon_i \sim \mathcal{N}(0, 1)$
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- 2. KL Divergence calculation uses μ_0, Σ_0, and requires $\det \Sigma_0 > 0$

Correlated VAE Implementation

- Architecture: Encoder outputs $k + k(k+1)/2$ nodes
  - k nodes for μ_0, and $k(k+1)/2$ nodes for L_0 lower triangular

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- KL Divergence: Calculate $\Sigma_0 = G_0 G_0^T$
  - Note that

$$\begin{aligned}\det \Sigma_0 &= \det(e^{L_0}(e^{L_0})^T) = \det e^{L_0} \cdot \det(e^{L_0})^T \\ &= e^{\operatorname{tr} L_0} \cdot e^{\operatorname{tr} L_0{}^T} = \left(e^{\operatorname{tr} L_0}\right)^2 \\ &> 0\end{aligned}$$

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  - Claim: Σ_0 is symmetric positive definite

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Proof.

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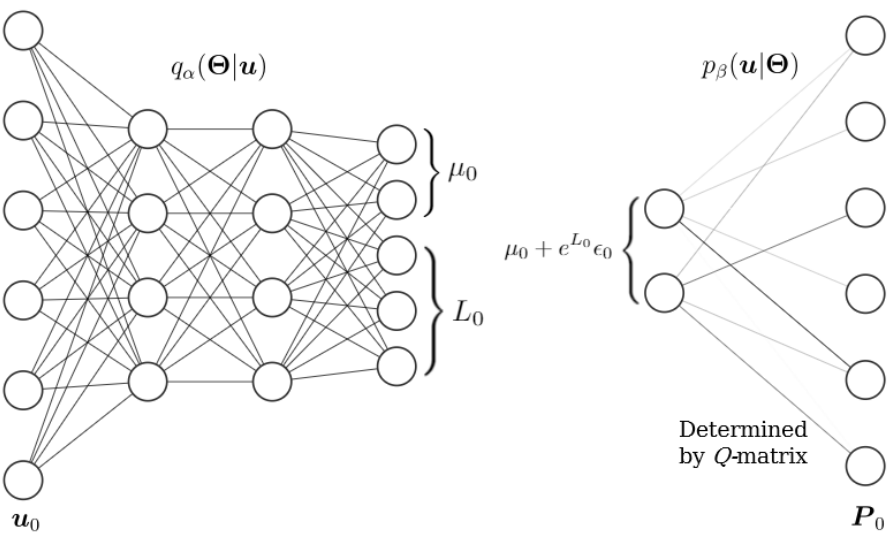
VAE architecture for correlated latent traits



Encoder structure for VAE learning $\mathcal{N}(0, I)$

Datasets

a table might be sufficient

Correlated ML2P-VAE Results

Sim 6 or Sim 4

Correlated ML2P-VAE Results

ECPE

Correlated ML2P-VAE Results

Sim 20

Correlated ML2P-VAE Results

big table

Scalability of ML2P-VAE

Plots of accuracy/time elapsed vs num training samples

Package in R

TODO: give better explanation of why this is worth mentioning

- Create software package in R with ML2P-VAE method
- Submit to CRAN for resarchers without tensorflow experience to use

Package in R

TODO: give better explanation of why this is worth mentioning

- Create software package in R with ML2P-VAE method
- Submit to CRAN for resarchers without tensorflow experience to use
- Package functions:
  - Construct ML2P-VAE model to desired architecture
  - Option for independent latent traits, or full covariance matrix
  - Sufficient documentation and working examples

ML2Pvae package

another frame for R package? not sure if I need two frames here

Bayesian Knowledge Tracing

TODO: background/motivation of KT

RNN / LSTM

RNN, LSTM

Attention-based networks

Transformer / Attention

Deep Knowledge Tracing

DKT

SAKT

SAKT

Other methods

might want to mention DKVMN or PFA

Do deep models actually "trace" knowledge?

motivation

IRT-inspired Knowledge Tracing

method description

IRT-inspired Knowledge Tracing

image of architecture

Datasets

datasets

Results

Table and theta trace plot

Recovery of IRT parameters

disc and theta recovery plots

Learning a Q-matrix

cor heatmap and clustering

Extending ML2P-VAE to other IRT models

3PL and Samejima

Other application areas

BDI and personality questionnaires

Utilizing more domain knowledge

use Q matrix in attn calculation missing responses with embedding of interactions

Summary

Summary

Thank you!

References

TODO: choose citations in the right way

[1] Wainer and Thissen, D. “Test Scoring”. Erlbaum Associates, Publishers, 2001.

[2] da Silva, Liu, Huggins-Manley, Bazan. “Incorporating the Q-matrix into Multidimensional Item Response Models.” Journal of Educational and Psychological Measurement, 2018.

[3] Baker and Kim. “Item Response Theory: Parameter Estimation Techniques”. CRC Press, 2004.

[4] Bock and Aitken. “Marginal Maximum Likelihood Estimation of Item Parameters: Application of an EM Algorithm”. Psychometrika, 1981.

[5] Nielsen, Michael. “Neural Networks and Deep Learning”. Determination Press, 2015.

[6] Curi, Converse, Hajewski, Oliveira. “Interpretable Variational Autoencoders for Cognitive Models.” In Proceddings of the International Joint Conference on Neural Networks (IJCNN), 2019.

[7] Q. Guo, M. Cutumisu, and Y. Cui. “A Neural Network Approach to Estimate Student Skill Mastery in Cognitive Diagnostic Assessments”. In: 10th International Conference on Educational Data Mining. 2017.

[8] Converse, Curi, Oliveira. “Autoencoders for Educational Assessment.” In Proceedings of the Conference on Artifical Intelligence in Education (AIED), 2019.

[9] Converse, Curi, Oliveira, and Arnold. “Variational Autoencoders for Baseball Player Evaluation.” In Proceedings of the Fuzzy Systems and Data Mining Conference (FSDM), 2019.

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