Decision Analysis for Management Judgment

Third Edition

Paul Goodwin
The Management School, University of Bath

George Wright
Durham Business School, University of Durham

John Wiley & Sons, Ltd

Telephone (+44) 1243 779777

Email (for orders and customer service enquiries): cs-books@wiley.co.uk
Visit our Home Page on www.wileyeurope.com or www.wiley.com

Reprinted May 2004, September 2004, June 2005, September 2005, August 2006

Other Wiley Editorial Offices

John Wiley & Sons Inc., 111 River Street, Hoboken, NJ 07030, USA

Jossey-Bass, 989 Market Street, San Francisco, CA 94103-1741, USA

Wiley-VCH Verlag GmbH, Boschstr. 12, D-69469 Weinheim, Germany

John Wiley & Sons Australia Ltd, 33 Park Road, Milton, Queensland 4064, Australia

John Wiley & Sons (Asia) Pte Ltd, 2 Clementi Loop #02-01, Jin Xing Distripark, Singapore 129809

John Wiley & Sons Canada Ltd, 22 Worcester Road, Etobicoke, Ontario, Canada M9W 1L1

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic books.

Library of Congress Cataloging-in-Publication Data

Goodwin, Paul.
Decision analysis for management judgment / Paul Goodwin, George Wright. – 3rd ed.
p. cm.
Includes bibliographical references and index.
ISBN 0-470-86108-8 (pbk. : alk. paper)
1. Decision making. I. Wright, George, 1952– II. Title.

HD30.23.G66 2003
658.4′03 – dc22

2003064171

British Library Cataloguing in Publication Data

A catalogue record for this book is available from the British Library

ISBN 10: 0-470-86108-8 (PB) ISBN 13: 978-0-470-86108-0 (PB)

Typeset in 11/13pt Palatino by Laserwords Private Limited, Chennai, India
Printed and bound in Great Britain by TJ International, Padstow, Cornwall
This book is printed on acid-free paper responsibly manufactured from sustainable forestry in which at least two trees are planted for each one used for paper production.

5 Decision making under uncertainty

Introduction

In many decisions the consequences of the alternative courses of action cannot be predicted with certainty. A company which is considering the launch of a new product will be uncertain about how successful the product will be, while an investor in the stock market will generally be unsure about the returns which will be generated if a particular investment is chosen. In this chapter we will show how the ideas about probability, which we introduced in Chapter 4, can be applied to problems where a decision has to be made under conditions of uncertainty.

We will first outline a method which assumes that the decision maker is unable, or unwilling, to estimate probabilities for the outcomes of the decision and which, in consequence, makes extremely pessimistic assumptions about these outcomes. Then, assuming that probabilities can be assessed, we will consider an approach based on the expected value concept that we met in Chapter 4. Because an expected value can be regarded as an average outcome if a process is repeated a large number of times, this approach is arguably most relevant to situations where a decision is made repeatedly over a long period. A daily decision by a retailer on how many items to have available for sale might be an example of this sort of decision problem. In many situations, however, the decision is not made repeatedly, and the decision maker may only have one opportunity to choose the best course of action. If things go wrong then there will be no chance of recovering losses in future repetitions of the decision. In these circumstances some people might prefer the least risky course of action, and we will discuss how a decision maker's attitude to risk can be assessed and incorporated into a decision model.

Finally, we will broaden the discussion to consider problems which involve both uncertainty and more than one objective. As we saw in

Chapter 2, problems involving multiple objectives are often too large for a decision maker to comprehend in their entirety. We will therefore look at a method which is designed to allow the problem to be broken down into smaller parts so that the judgmental task of the decision maker is made more tractable.

The maximin criterion

Consider the following problem. Each morning a food manufacturer has to make a decision on the number of batches of a perishable product which should be produced. Each batch produced costs $800 while each batch sold earns revenue of $1000. Any batch which is unsold at the end of the day is worthless. The daily demand for the product is either one or two batches, but at the time of production the demand for the day is unknown and the food manufacturer feels unable to estimate probabilities for the two levels of demand. The manufacturer would like to determine the optimum number of batches which he should produce each morning.

Clearly the manufacturer has a dilemma. If he produces too many batches, he will have wasted money in producing food which has to be destroyed at the end of the day. If he produces too few, he will be forgoing potential profits. We can represent his problem in the form of a *decision table* (Table 5.1). The rows of this table represent the alternative courses of action which are open to the decision maker (i.e. produce one or two batches), while the columns represent the possible levels of demand which are, of course, outside the control of the decision maker. The monetary values in the table show the profits which would be earned per day for the different levels of production and demand. For example, if one batch is produced and one batch demanded, a profit of $1000 – $800 (i.e. $200) will be made. This profit would also apply if two batches were demanded, since a profit can only be made on the batch produced.

Table 5.1 – A decision table for the food manufacturer

(Daily profits)	**Demand (no. of batches)**	
Course of action	**1**	**2**
Produce 1 batch	$200	$200
Produce 2 batches	–$600	$400

Given these potential profits and losses, how should the manufacturer make his decision? (We will assume that he has only one objective, namely maximizing monetary gain so that other possible objectives, such as maximizing customer goodwill or market share, are of no concern.) According to the maximin criterion the manufacturer should first identify the worst possible outcome for each course of action and then choose the alternative yielding the best of these worst outcomes. If the manufacturer produces one batch, he will make the same profit whatever the demand, so the worst possible outcome is a profit of $200. If he decides to produce two batches the worst possible outcome is a loss of $600. As shown below, the best of these worst possible outcomes (the MAXImum of the MINimum possible profits) is associated with the production of one batch per day so, according to maximin, this should be the manufacturer's decision.

Course of action	*Worst possible profit*
Produce 1 batch	$200 – best of the worst possible outcomes
Produce 2 batches	–$600

Note that if the outcomes had been expressed in terms of costs, rather than profits, we would have listed the highest possible costs of each option and selected the option for which the highest possible costs were lowest. Because we would have been selecting the option with the minimum of the maximum possible costs our decision criterion would have been referred to as *minimax*.

The main problem with the maximin criterion is its inherent pessimism. Each option is assessed only on its worst possible outcome so that all other possible outcomes are ignored. The implicit assumption is that the worst is bound to happen while, in reality, the chances of this outcome occurring may be extremely small. For example, suppose that you were offered the choice of receiving $1 for certain or taking a gamble which had a 0.9999 probability of yielding $1 million and only a 0.0001 probability of losing you $1. The maximin criterion would suggest that you should not take the risk of engaging in the gamble because it would assume, despite the probabilities, that you would lose. This is unlikely to be a sensible representation of most decision makers' preferences. Nevertheless, the extreme risk aversion which is implied by the maximin criterion may be appropriate where decisions involve public safety or possible irreversible damage to the environment. A new cheaper form of food processing which had a one in ten thousand chance of killing the entire population would clearly be unacceptable to most people.

The expected monetary value (EMV) criterion

If the food manufacturer is able, and willing, to estimate probabilities for the two possible levels of demand, then it may be appropriate for him to choose the alternative which will lead to the highest *expected* daily profit. If he makes the decision on this basis then he is said to be using the *expected monetary value* or EMV criterion. Recall from Chapter 4 that an expected value can be regarded as an average result which is obtained if a process is repeated a large number of times. This may make the criterion particularly appropriate for the retailer who will be repeating his decision day after day. Table 5.2 shows the manufacturer's decision table again, but this time with the probabilities added.

As we showed in Chapter 4, an expected value is calculated by multiplying each outcome by its probability of occurrence and then summing the resulting products. The expected daily profits for the two production levels are therefore:

Produce one batch:

$$\text{expected daily profit} = (0.3 \times \$200) + (0.7 \times \$200) = \$200$$

Produce two batches:

$$\text{expected daily profit} = (0.3 \times -\$600) + (0.7 \times \$400) = \$100$$

These expected profits show that, in the long run, the highest average daily profit will be achieved by producing just one batch per day and, if the EMV criterion is acceptable to the food manufacturer, then this is what he should do.

Of course, the probabilities and profits used in this problem may only be rough estimates or, if they are based on reliable past data, they may be subject to change. We should therefore carry out sensitivity analysis to determine how large a change there would need to be in these values before the alternative course of action would be preferred. To illustrate

Table 5.2 – Another decision table for the food manufacturer

(Daily profits)		**Demand (no. of batches)**	
		1	**2**
Course of action	**Probability**	**0.3**	**0.7**
Produce 1 batch		\$200	\$200
Produce 2 batches		–\$600	\$400

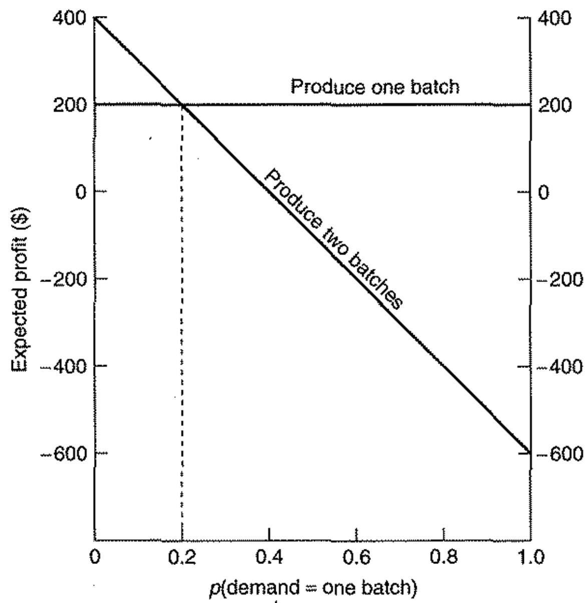

Figure 5.1 – A sensitivity analysis for the food manufacturer's problem

the process, Figure 5.1 shows the results of a sensitivity analysis on the probability that just one batch will be demanded. Producing one batch will always yield an expected profit of $200, whatever this probability is. However, if the probability of just one batch being demanded is zero, then the expected profit of producing two batches will be $400. At the other extreme, if the probability of just one batch being demanded is 1.0 then producing two batches will yield an expected profit of –$600. The line joining these points shows the expected profits for all the intermediate probabilities. It can be seen that producing one batch will continue to yield the highest expected profit as long as the probability of just one batch being demanded is greater than 0.2. Since currently this probability is estimated to be 0.3, it would take only a small change in the estimate for the alternative course of action to be preferred. Therefore in this case the probability needs to be estimated with care.

Limitations of the EMV criterion

The EMV criterion may have been appropriate for the food manufacturer because he was only concerned with monetary rewards, and his decision was repeated a large number of times so that a long-run average result

Table 5.3 – Returns and probabilities for the new component problem

	Outcome					
	Total failure		Partial success		Total success	
Course of action	Returns ($m)	Probability	Returns ($m)	Probability	Returns ($m)	Probability
Choose design 1	−1	0.1	0	0.1	03	0.8
Choose design 2	−6	0.3	1	0.1	10	0.6

would have been of relevance to him. Let us now consider a different decision problem.

Imagine that you own a high-technology company which has been given the task of developing a new component for a large engineering corporation. Two alternative, but untried, designs are being considered (for simplicity, we will refer to these as designs 1 and 2), and because of time and resource constraints only one design can be developed. Table 5.3 shows the estimated net returns which will accrue to your company if each design is developed. Note that these returns depend on how successful the design is. The estimated probabilities of failure, partial success and total success for each design are also shown in the table.

The expected returns for design 1 are:

$$0.1 \times (-\$1\text{ m}) + 0.1 \times \$0 + 0.8 \times (\$3\text{ m}) = \$2.3\text{ m}$$

while for design 2 the expected returns are:

$$0.3 \times (-\$6\text{ m}) + 0.1 \times (\$1\text{ m}) + 0.6 \times (\$10\text{ m}) = \$4.3\text{ m}$$

Thus according to the EMV criterion you should develop design 2, but would this really be your preferred course of action? There is a 30% chance that design 2 will fail and lead to a loss of $6 million. If your company is a small one or facing financial problems then these sort of losses might put you out of business. Design 1 has a smaller chance of failure, and if failure does occur then the losses are also smaller. Remember that this is a one-off decision, and there is therefore no chance of recouping losses on subsequent repetitions of the decision. Clearly, the risks of design 2 would deter many people. The EMV criterion therefore fails to take into account the attitude to risk of the decision maker.

This can also be seen in the famous St Petersburg paradox described by Bernoulli. Imagine that you are offered the following gamble. A fair coin is to be tossed until a head appears for the first time. If the head appears on the first throw you will be paid \$2, if it appears on the second throw, \$4, if it appears on the third throw, \$8, and so on. How much would you be prepared to pay to have the chance of engaging in this gamble? The expected returns on the gamble are:

$$\$2 \times (0.5) + \$4 \times (0.25) + \$8 \times (0.125) + \ldots, \text{ etc.}$$

$$\text{which equals } 1 + 1 + 1 + \ldots \text{ to infinity}$$

so your expected returns will be infinitely large. On this basis, according to the EMV criterion, you should be prepared to pay a limitless sum of money to take part in the gamble. Given that there is a 50% chance that your return will be only \$2 (and an 87.5% chance that it will be \$8 or less), it is unlikely that many people would be prepared to pay anywhere near the amount prescribed by the EMV criterion!

It should also be noted that the EMV criterion assumes that the decision maker has a linear value function for money. An increase in returns from \$0 to \$1 million may be regarded by the decision maker as much more preferable than an increase from \$9 million to \$10 million, yet the EMV criterion assumes that both increases are equally desirable.

A further limitation of the EMV criterion is that it focuses on only one attribute: money. In choosing the design in the problem we considered above we may also wish to consider attributes such as the effect on company image of successfully developing a sophisticated new design, the spin-offs of enhanced skills and knowledge resulting from the development and the time it would take to develop the designs. All these attributes, like the monetary returns, would probably have some risk associated with them.

In the rest of this chapter we will address these limitations of the EMV criterion. First, we will look at how the concept of single-attribute utility can be used to take into account the decision maker's attitude to risk (or risk preference) in problems where there is just one attribute. The approach which we will adopt is based on the theory of utility which was developed by von Neumann and Morgenstern.[1] Then we will consider multi-attribute utility which can be applied to decision problems which involve both uncertainty and more than one attribute.

However, before we leave this section we should point out that the EMV criterion is very widely used in practice. Many people would argue that it is even appropriate to apply it to one-off decisions. Although an individual decision may be unique, over time a decision maker may make a large number of such decisions involving similar monetary sums so that returns should still be maximized by consistent application of the criterion. Moreover, large organizations may be able to sustain losses on projects that represent only a small part of their operations. In these circumstances it may be reasonable to assume that risk neutrality applies, in which case the EMV criterion will be appropriate.

Single-attribute utility

The attitude to risk of a decision maker can be assessed by eliciting a *utility function*. This is to be distinguished from the value functions we met in Chapter 3. Value functions are used in decisions where uncertainty is not a major concern, and therefore they do not involve any consideration of risk attitudes. (We will, however, have more to say about the distinction between utility and value from a practical point of view in a later section of this chapter.)

To illustrate how a utility function can be derived, consider the following problem. A business woman who is organizing a business equipment exhibition in a provincial town has to choose between two venues: the Luxuria Hotel and the Maxima Center. To simplify her problem, she decides to estimate her potential profit at these locations on the basis of two scenarios: high attendance and low attendance at the exhibition. If she chooses the Luxuria Hotel, she reckons that she has a 60% chance of achieving a high attendance and hence a profit of \$30 000 (after taking into account the costs of advertising, hiring the venue, etc.). There is, however, a 40% chance that attendance will be low, in which case her profit will be just \$11 000. If she chooses the Maxima Center, she reckons she has a 50% chance of high attendance, leading to a profit of \$60 000, and a 50% chance of low attendance leading to a loss of \$10 000.

We can represent the business woman's problem in the form of a diagram known as a decision tree (Figure 5.2). In this diagram a square represents a decision point; immediately beyond this, the decision maker can choose which route to follow. A circle represents a chance node. Immediately beyond this, chance determines, with the indicated probabilities, which route will be followed, so the choice of route is beyond the control of the decision maker. (We will consider decision trees in much more detail in Chapter 6.) The monetary values show the profits earned by the business woman if a given course of action is chosen and a given outcome occurs.

Now, if we apply the EMV criterion to the decision we find that the business woman's expected profit is \$22 400 (i.e. $0.6 \times \$30\,000 + 0.4 \times$

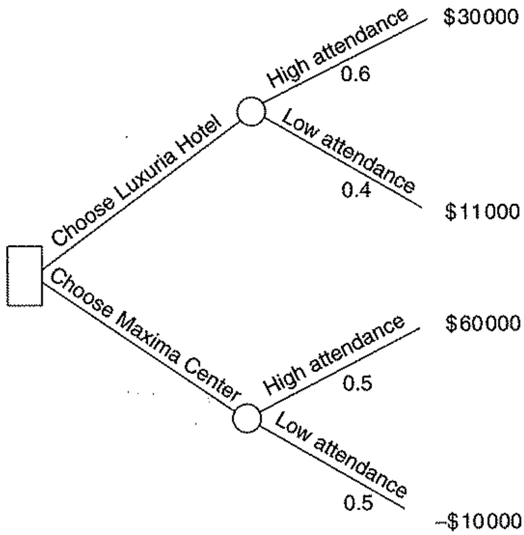

Figure 5.2 – A decision tree for the conference organizer's problem

\$11 000) if she chooses the Luxuria Hotel and \$25 000 if she chooses the Maxima Center. This suggests that she should choose the Maxima Center, but this is the riskiest option, offering high rewards if things go well but losses if things go badly.

Let us now try to derive a utility function to represent the business woman's attitude to risk. We will use the notation $u()$ to represent the utility of the sum of money which appears in the parentheses. First, we rank all the monetary returns which appear on the tree from best to worst and assign a utility of 1.0 to the best sum of money and 0 to the worst sum. As was the case with value functions in Chapter 3, any two numbers could have been assigned here as long as the best outcome is assigned the higher number. We used 0 and 100 for value functions, but the use of 0 and 1 here will enable us to interpret what utilities actually represent. (If other values were used they could easily be transformed to a scale ranging from 0 to 1 without affecting the decision maker's preference between the courses of action.) Thus so far we have:

Monetary sum	*Utility*
\$60 000	1.0
\$30 000	Not yet known
\$11 000	Not yet known
–\$10 000	0

We now need to determine the business woman's utilities for the intermediate sums of money. There are several approaches which can be adopted to elicit utilities. The most commonly used methods involve offering the decision maker a series of choices between receiving given sums of money for certain or entering hypothetical lotteries. The decision maker's utility function is then inferred from the choices that are made. The method which we will demonstrate here is an example of the *probability-equivalence* approach (an alternative elicitation procedure will be discussed in a later section).

To obtain the business woman's utility for \$30 000 using this approach we offer her a choice between receiving that sum for certain or entering a hypothetical lottery which will result in either the best outcome on the tree (i.e. a profit of \$60 000) or the worst (i.e. a loss of \$10 000) with specified probabilities. These probabilities are varied until the decision maker is indifferent between the certain money and the lottery. At this point, as we shall see, the utility can be calculated. A typical elicitation session might proceed as follows:

Question: Which of the following would you prefer?

A \$30 000 for certain; or
B A lottery ticket which will give you a 70% chance of \$60 000 and a 30% chance of −\$10 000?

Answer: A 30% chance of losing \$10 000 is too risky, I'll take the certain money.

We therefore need to make the lottery more attractive by increasing the probability of the best outcome.

Question: Which of the following would you prefer?

A \$30 000 for certain; or
B A lottery ticket which will give you a 90% chance of \$60 000 and a 10% chance of −\$10 000?

Answer: I now stand such a good chance of winning the lottery that I think I'll buy the lottery ticket.

The point of indifference between the certain money and the lottery should therefore lie somewhere between a 70% chance of winning \$60 000 (when the certain money was preferred) and a 90% chance (when the lottery ticket was preferred). Suppose that after trying several probabilities we pose the following question.

Question: Which of the following would you prefer?

A $30 000 for certain; or
B A lottery ticket which will give you an 85% chance of $60 000 and a 15% chance of –$10 000?

Answer: I am now indifferent between the certain money and the lottery ticket.

We are now in a position to calculate the utility of $30 000. Since the business woman is indifferent between options A and B the utility of $30 000 will be equal to the expected utility of the lottery. Thus:

$$u(\$30\,000) = 0.85\,u(\$60\,000) + 0.15\,u(-\$10\,000)$$

Since we have already allocated utilities of 1.0 and 0 to $60 000 and –$10 000, respectively, we have:

$$u(\$30\,000) = 0.85(1.0) + 0.15(0) = 0.85$$

Note that, once we have found the point of indifference, the utility of the certain money is simply equal to the probability of the best outcome in the lottery. Thus, if the decision maker had been indifferent between the options which we offered in the first question, her utility for $30 000 would have been 0.7.

We now need to determine the utility of $11 000. Suppose that after being asked a similar series of questions the business woman finally indicates that she would be indifferent between receiving $11 000 for certain and a lottery ticket offering a 60% chance of the best outcome ($60 000) and a 40% chance of the worst outcome (–$10 000). This implies that $u(\$11\,000) = 0.6$. We can now state the complete set of utilities and these are shown below:

Monetary sum	*Utility*
$60 000	1.0
$30 000	0.85
$11 000	0.60
–$10 000	0

These results are now applied to the decision tree by replacing the monetary values with their utilities (see Figure 5.3). By treating these

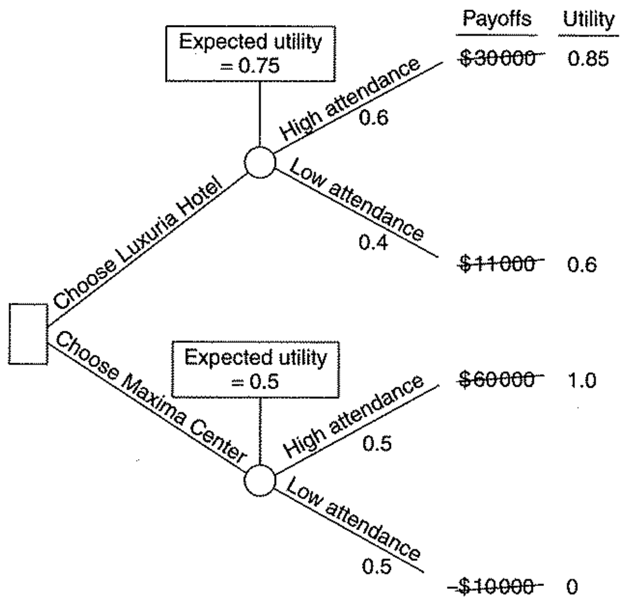

Figure 5.3 – The conference organizer's decision tree with utilities

utilities in the same way as the monetary values we are able to identify the course of action which leads to the highest expected utility.

Choosing the Luxuria Hotel gives an expected utility of:

$$0.6 \times 0.85 + 0.4 \times 0.6 = 0.75$$

Choosing the Maxima Center gives an expected utility of:

$$0.5 \times 1.0 + 0.5 \times 0 = 0.5$$

Thus the business woman should choose the Luxuria Hotel as the venue for her exhibition. Clearly, the Maxima Center would be too risky.

It may be useful at this point to establish what expected utilities actually represent. Indeed, given that we have just applied the concept to a one-off decision, why do we use the term *expected* utility? To see what we have done, consider Figure 5.4(a). Here we have the business woman's decision tree with the original monetary sums replaced by the lotteries which she regarded as being equally attractive. For example, receiving \$30 000 was considered to be equivalent to a lottery offering a 0.85 probability of \$60 000 and a 0.15 probability of –\$10 000. Obviously, receiving \$60 000 is equivalent to a lottery ticket offering \$60 000 for certain. You will see that every payoff in the tree is now expressed in

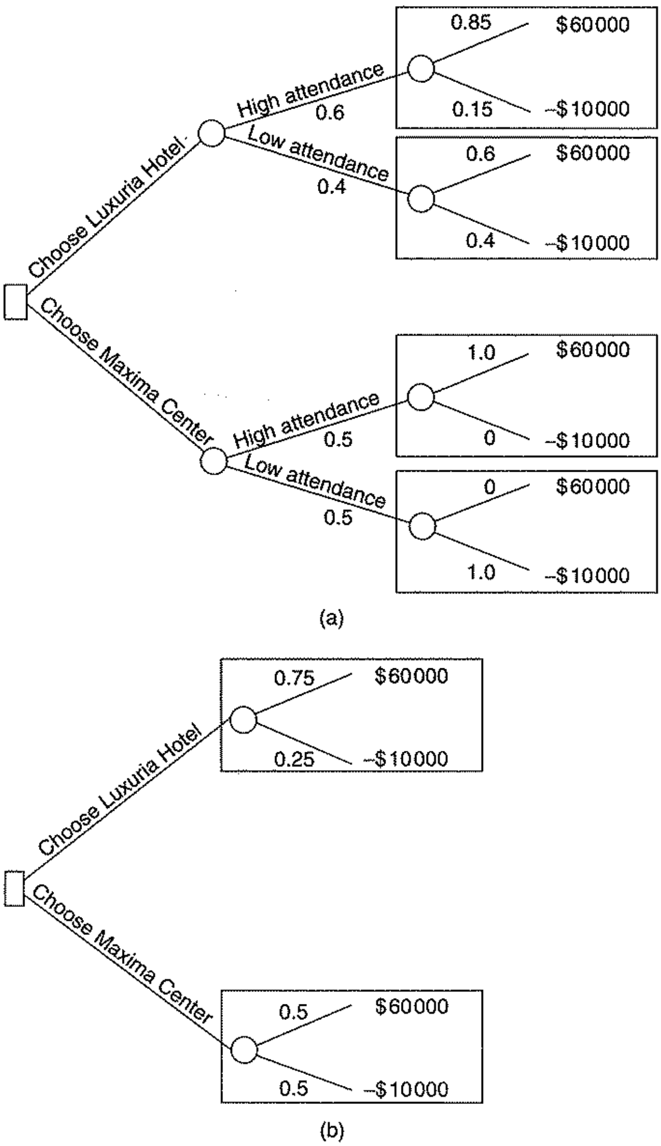

Figure 5.4 – A demonstration of how expected utility reduces the decision to a simple choice between lotteries

terms of a probability of obtaining either the best outcome ($60 000) or the worst outcome (–$10 000).

Now, if the business woman chooses the Luxuria Hotel she will have a 0.6 probability of finishing with a profit which she perceives to be equivalent to a lottery ticket offering a 0.85 probability of $60 000 and a

0.15 probability of $-\$10\,000$. Similarly, she will have a 0.4 probability of a profit, which is equivalent to a lottery ticket offering a 0.6 probability of \$60 000 and a 0.4 chance of $-\$10\,000$. Therefore the Luxuria Hotel offers her the equivalent of a $0.6 \times 0.85 + 0.4 \times 0.6 = 0.75$ probability of the best outcome (and a 0.25 probability of the worst outcome). Note that 0.75 is the expected utility of choosing the Luxuria Hotel.

Obviously, choosing the Maxima Center offers her the equivalent of only a 0.5 probability of the best outcome on the tree (and a 0.5 probability of the worst outcome). Thus, as shown in Figure 5.4(b), utility allows us to express the returns of all the courses of action in terms of simple lotteries all offering the same prizes, namely the best and worst outcomes, but with different probabilities. This makes the alternatives easy to compare. The probability of winning the best outcome in these lotteries is the expected utility. It therefore seems reasonable that we should select the option offering the highest expected utility.

Note that the use here of the term 'expected' utility is therefore somewhat misleading. It is used because the procedure for calculating expected utilities is arithmetically the same as that for calculating expected values in statistics. It does *not*, however, necessarily refer to an average result which would be obtained from a large number of repetitions of a course of action, nor does it mean a result or consequence which should be 'expected'. In decision theory, an 'expected utility' is only a 'certainty equivalent', that is, a single 'certain' figure that is equivalent in preference to the uncertain situations.

Interpreting utility functions

The business woman's utility function has been plotted on a graph in Figure 5.5. If we selected any two points on this curve and drew a straight line between them then it can be seen that the curve would always be above the line. Utility functions having this *concave* shape provide evidence of *risk aversion* (which is consistent with the business woman's avoidance of the riskiest option).

This is easily demonstrated. Consider Figure 5.6, which shows a utility function with a similar shape, and suppose that the decision maker, from whom this function has been elicited, has assets of \$1000. He is then offered a gamble which will give him a 50% chance of doubling his money to \$2000 and a 50% chance of losing it all, so that he finishes with \$0. The expected monetary value of the gamble is \$1000 (i.e.

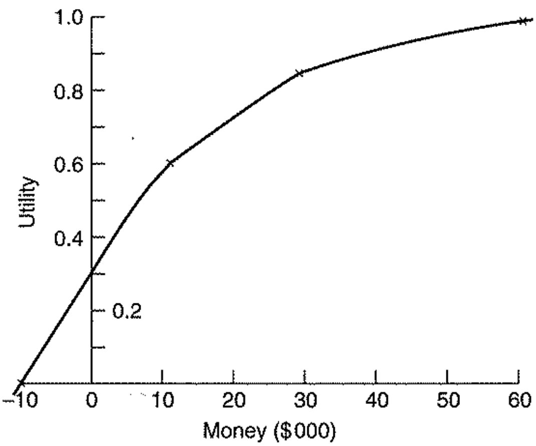

Figure 5.5 – A utility function for the conference organizer

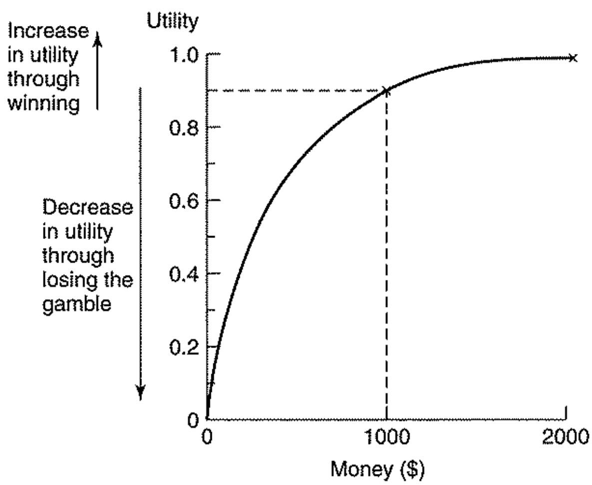

Figure 5.6 – A utility function demonstrating risk aversion

$0.5 \times \$2000 + 0.5 \times \$0$), so according to the EMV criterion he should be indifferent between keeping his money and gambling. However, when we apply the utility function to the decision we see that currently the decision maker has assets with a utility of 0.9. If he gambles he has a 50% chance of increasing his assets so that their utility would increase to 1.0 and a 50% chance of ending with assets with a utility of 0. Hence the expected utility of the gamble is $0.5 \times 1 + 0.5 \times 0$, which equals 0.5. Clearly, the certain money is more attractive than the risky option of gambling. In simple terms, even though the potential wins and losses

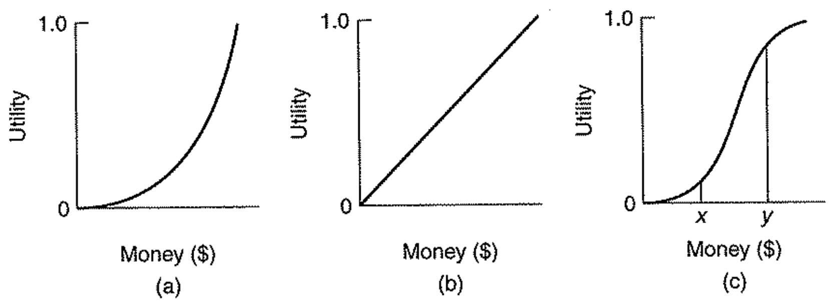

Figure 5.7 – Interpreting the shape of a utility function. (a) A risk-seeking attitude; (b) risk neutrality, which means that the EMV criterion would represent the decision maker's preferences; (c) both a risk-seeking attitude and risk aversion

are the same in monetary terms and even though he has the same chance of winning as he does of losing, the increase in utility which will occur if the decision maker wins the gamble is far less than the loss in utility he will suffer if he loses. He therefore stands to lose much more than he stands to gain, so he will not be prepared to take the risk.

Figure 5.7 illustrates other typical utility functions. Figure 5.7(a) shows a utility function which indicates a risk-seeking attitude (or risk proneness). A person with a utility function like this would have accepted the gamble which we offered above. The linear utility function in Figure 5.7(b) demonstrates a risk-neutral attitude. If a person's utility function looks like this then the EMV criterion will represent their preferences. Finally, the utility function in Figure 5.7(c) indicates both a risk-seeking attitude and risk aversion. If the decision maker currently has assets of $\$y$ then he will be averse to taking a risk. The reverse is true if currently he has assets of only $\$x$. It is important to note that individual's utility functions do not remain constant over time. They may vary from day to day, especially if the person's asset position changes. If you win a large sum of money tomorrow then you may be more willing to take a risk than you are today.

Utility functions for non-monetary attributes

Utility functions can be derived for attributes other than money. Consider the problem which is represented by the decision tree in Figure 5.8. This relates to a drug company which is hoping to develop a new product. If the company proceeds with its existing research methods it estimates

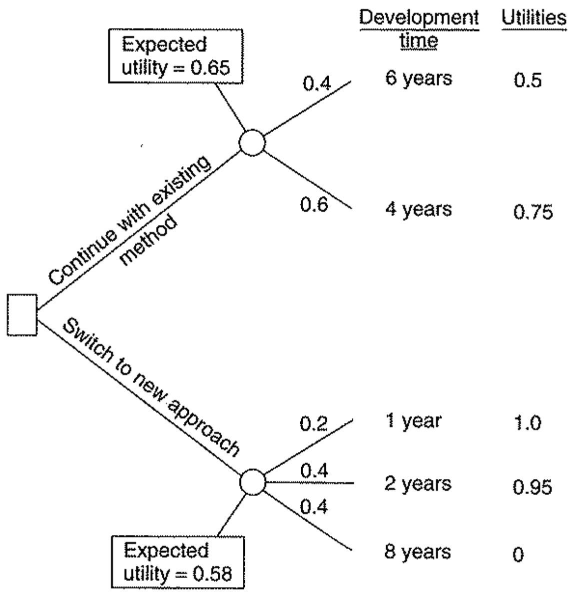

Figure 5.8 – A decision tree for the drug company research department problem

that there is a 0.4 probability that the drug will take 6 years to develop and a 0.6 probability that development will take 4 years. However, recently a 'short-cut' method has been proposed which might lead to significant reductions in the development time, and the company, which has limited resources available for research, has to decide whether to take a risk and switch completely to the proposed new method. The head of research estimates that, if the new approach is adopted, there is a 0.2 probability that development will take a year, a 0.4 probability that it will take 2 years and a 0.4 probability that the approach will not work and, because of the time wasted, it will take 8 years to develop the product.

Clearly, adopting the new approach is risky, so we need to derive utilities for the development times. The worst development time is 8 years, so $u(8\text{ years}) = 0$ and the best time is 1 year, so $u(1\text{ year}) = 1.0$. After being asked a series of questions, based on the variable probability method, the head of research is able to say that she is indifferent between a development time of 2 years and engaging in a lottery which will give her a 0.95 probability of a 1-year development and a 0.05 probability of an 8-year development time. Thus:

$$\begin{aligned} u(2\text{ years}) &= 0.95\,u(1\text{ year}) + 0.05\,u(8\text{ years}) \\ &= 0.95(1.0) + 0.05(0) = 0.95 \end{aligned}$$

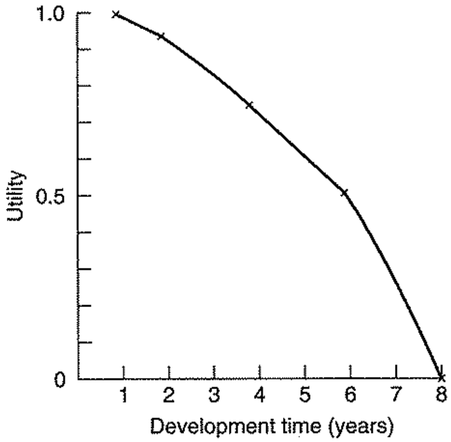

Figure 5.9 – A utility function for product development time

By a similar process we find that u(4 years) = 0.75 and u(6 years) = 0.5. The utilities are shown on the decision tree in Figure 5.8, where it can be seen that continuing with the existing method gives the highest expected utility. Note, however, that the two results are close, and a sensitivity analysis might reveal that minor changes in the probabilities or utilities would lead to the other alternative being selected. The utility function is shown in Figure 5.9. This has a concave shape indicating risk aversion.

It is also possible to derive utility functions for attributes which are not easily measured in numerical terms. For example, consider the choice of design for a chemical plant. Design A may have a small probability of failure which may lead to pollution of the local environment. An alternative, design B, may also carry a small probability of failure which would not lead to pollution but would cause damage to some expensive equipment. If a decision maker ranks the possible outcomes from best to worst as: (i) no failure, (ii) equipment damage and (iii) pollution then, clearly, u(no failure) = 1 and u(pollution) = 0. The value of u(equipment damage) could then be determined by posing questions such as which would you prefer:

(1) A design which was certain at some stage to fail, causing equipment damage; or
(2) A design which had a 90% chance of not failing and a 10% chance of failing and causing pollution?

Once a point of indifference was established, u(equipment damage) could be derived.

Ronen *et al.*[2] describe a similar application in the electronics industry, where the decision relates to designs of electronic circuits for cardiac pacemakers. The designs carry a risk of particular malfunctions and the utilities relate to outcomes such as 'pacemaker not functioning at all', 'pacemaker working too fast', 'pacemaker working too slow' and 'pacemaker functioning OK'.

The axioms of utility

In the last few sections we have suggested that a rational decision maker should select the course of action which maximizes expected utility. This will be true if the decision maker's preferences conform to the following axioms:

Axiom 1: The complete ordering axiom
To satisfy this axiom the decision maker must be able to place all lotteries in order of preference. For example, if he is offered a choice between two lotteries, the decision maker must be able to say which he prefers or whether he is indifferent between them. (For the purposes of this discussion we will also regard a certain chance of winning a reward as a lottery.)
Axiom 2: The transitivity axiom
If the decision maker prefers lottery A to lottery B and lottery B to lottery C then, if he conforms to this axiom, he must also prefer lottery A to lottery C (i.e. his preferences must be transitive).
Axiom 3: The continuity axiom
Suppose that we offer the decision maker a choice between the two lotteries shown in Figure 5.10. This shows that lottery 1 offers a reward of B for certain while lottery 2 offers a reward of A, with probability p and a reward of C with probability $1 - p$. Reward A is preferable to reward B, and B in turn is preferred to reward C. The continuity axiom states that there must be some value of p at which the decision maker will be indifferent between the two lotteries. We obviously assumed that this axiom applied when we elicited the conference organizer's utility for \$30 000 earlier in the chapter.
Axiom 4: The substitution axiom
Suppose that a decision maker indicates that he is indifferent between the lotteries shown in Figure 5.11, where X, Y and Z are rewards and p is a probability. According to the substitution axiom, if reward X

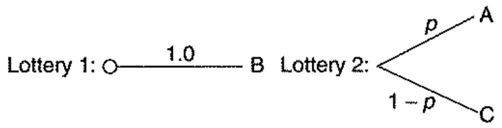

Figure 5.10

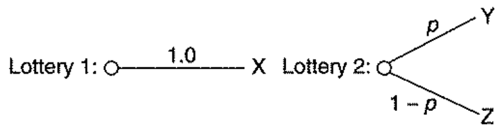

Figure 5.11

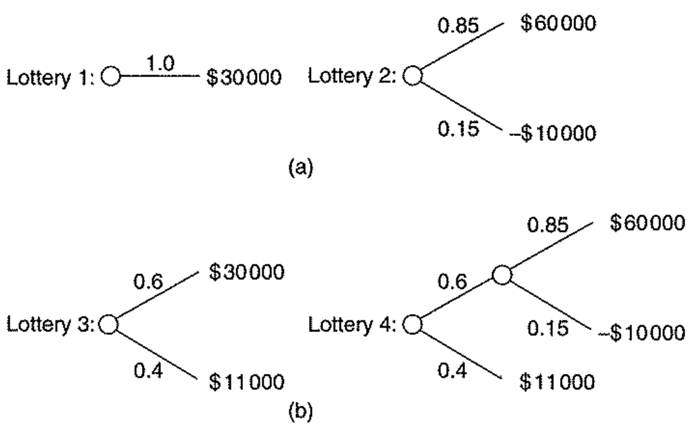

Figure 5.12

appears as a reward in another lottery it can always be substituted by lottery 2 because the decision maker regards X and lottery 2 as being equally preferable. For example, the conference organizer indicated that she was indifferent between the lotteries shown in Figure 5.12(a). If the substitution axiom applies, she will also be indifferent between lotteries 3 and 4, which are shown in Figure 5.12(b). Note that these lotteries are identical, except that in lottery 4 we have substituted lottery 2 for the $30 000. Lottery 4 offers a 0.6 chance of winning a ticket in another lottery and is therefore referred to as a *compound lottery*.

Axiom 5: Unequal probability axiom

Suppose that a decision maker prefers reward A to reward B. Then, according to this axiom, if he is offered two lotteries which only offer rewards A and B as possible outcomes he will prefer the lottery offering the highest probability of reward A. We used this axiom in our explanation of utility earlier, where we reduced the conference organizer's decision to a comparison of the two lotteries shown in Figure 5.13. Clearly, if the conference organizer's preferences conform to this axiom then she will prefer lottery 1.

Axiom 6: Compound lottery axiom

If this axiom applies then a decision maker will be indifferent between a compound lottery and a simple lottery which offers the same rewards with the same probabilities. For example, suppose that the conference organizer is offered the compound lottery shown in Figure 5.14(a). Note that this lottery offers a 0.28 (i.e. 0.4×0.7) probability of \$60 000 and a 0.72 (i.e. $0.4 \times 0.3 + 0.6$) probability of −\$10 000. According to this axiom she will also be indifferent between the compound lottery and the simple lottery shown in Figure 5.14(b).

It can be shown (see, for example, French[3]) that if the decision maker accepts these six axioms then a utility function exists which represents his preferences. Moreover, if the decision maker behaves in a manner which is consistent with the axioms (i.e. rationally), then he will choose

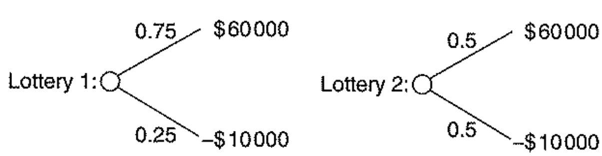

Figure 5.13

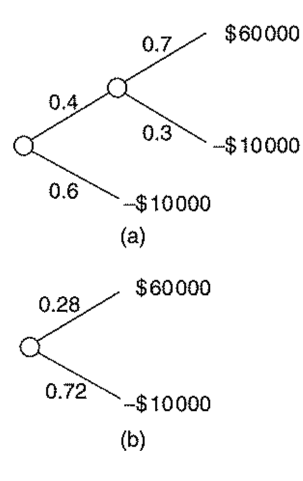

Figure 5.14

the course of action which has the highest expected utility. Of course, it may be possible to demonstrate that a particular decision maker does not act according to the axioms of utility theory. However, this does not necessarily imply that the theory is inappropriate in his case. All that is required is that he *wishes* to behave consistently according to the axioms. Applying decision analysis *helps* a decision maker to formulate preferences, assess uncertainty and make judgments in a coherent fashion. Thus coherence is the *result* of decision analysis, not a prerequisite.

More on utility elicitation

So far, we have only considered utility assessment based on the probability-equivalence approach. A disadvantage of this approach is that the decision maker may have difficulty in thinking in terms of probabilities like 0.90 or 0.95. Because of this, a number of alternative approaches have been developed (for example, Farquahar[4] reviews 24 different methods). Perhaps the most widely used of these is the *certainty-equivalence approach*, which, in its most common form, only requires the decision maker to think in terms of 50:50 gambles.

To illustrate the approach, let us suppose that we wish to elicit a decision maker's utility function for monetary values in the range \$0–40 000 (so that $u(\$0) = 0$ and $u(\$40\,000) = 1$). An elicitation session might proceed as follows:

Analyst: If I offered you a hypothetical lottery ticket which gave a 50% chance of \$0 and a 50% chance of \$40 000, how much would you be prepared to pay for it? Obviously, its expected monetary value is \$20 000, but I want to know the minimum amount of money you would just be willing to pay for the ticket.
Decision maker: (after some thought) \$10 000.

$$\text{Hence } u(\$10\,000) = 0.5\,u(\$0) + 0.5\,u(\$40\,000)$$
$$= 0.5(0) \quad + 0.5(1) \qquad = 0.5$$

The analyst would now use the \$10 000 as the worst payoff in a new hypothetical lottery.

Analyst: If I now offered you a hypothetical lottery ticket which gave you a 50% chance of \$40 000 and a 50% chance of \$10 000 how much would you be prepared to pay for it?

Decision maker: About \$18 000.

$$\text{Hence } u(\$18\,000) = 0.5\,u(\$10\,000) + 0.5\,u(\$40\,000)$$
$$= 0.5(0.5) \quad + 0.5(1) \quad = 0.75$$

The \$10 000 is also used as the best payoff in a lottery which will also offer a chance of \$0.

Analyst: What would you be prepared to pay for a ticket offering a 50% chance of \$10 000 and a 50% chance of \$0?
Decision maker: \$3000.

$$\text{Thus } u(\$3000) = 0.5\,u(\$0) + 0.5\,u(\$10\,000)$$
$$= 0.5(0) \quad + 0.5(0.5) \quad = 0.25$$

It can be seen that the effect of this procedure is to elicit the monetary values which have utilities of 0, 0.25, 0.5, 0.75 and 1. Thus we have:

Monetary value:	\$0	\$3000	\$10 000	\$18 000	\$40 000
Utility	0	0.25	0.5	0.75	1.0

If we plotted this utility function on a graph it would be seen that the decision maker is risk averse for this range of monetary values. The curve could, of course, also be used to estimate the utilities of other sums of money.

While the certainty-equivalence method we have just demonstrated frees the decision maker from the need to think about awkward probabilities it is not without its dangers. You will have noted that the decision maker's first response (\$10 000) was used by the analyst in subsequent lotteries, both as a best and worst outcome. This process is known as chaining, and the effect of this can be to propagate earlier judgmental errors.

The obvious question is, do these two approaches to utility elicitation produce consistent responses? Unfortunately, the evidence is that they do not. Indeed, utilities appear to be extremely sensitive to the elicitation method which is adopted. For example, Hershey *et al.*[5] identified a number of sources of inconsistency. Certainty-equivalence methods were found to yield greater risk seeking than probability-equivalence methods. The payoffs and probabilities used in the lotteries and, in particular, whether or not they included possible losses also led to different utility functions. Moreover, it was found that responses differed depending upon whether the choice offered involved risk being assumed or transferred away. For example, in the certainty-equivalence method we could either ask the decision maker how much he would be prepared to pay to *buy* the lottery ticket or, assuming that he already owns the ticket, how much he would accept to *sell* it. Research suggests that people tend to offer a lower price to buy the ticket than they would accept to sell it. There is thus a propensity to prefer the status quo, so that people are generally happier to retain a given risk than to take the same risk on (see also Thaler[6]). Finally, the context in which the questions were framed was found to have an effect on responses. For example, Hershey *et al.*[5] refer to an earlier experiment when the same choice was posed in different ways, the first involving an insurance decision and the second a gamble as shown below:

Insurance formulation
Situation A: You stand a one out of a thousand chance of losing $1000.
Situation B: You can buy insurance for $10 to protect you from this loss.
Gamble formulation
Situation A: You stand a one out of a thousand chance of losing $1000.
Situation B: You will lose $10 with certainty.

It was found that 81% of subjects preferred B in the insurance formulation, while only 56% preferred B in the gamble formulation.

Tversky and Kahneman[7] provide further evidence that the way in which the choice is framed affects the decision maker's response. They found that choices involving statements about gains tend to produce risk-averse responses, while those involving losses are often risk seeking. For example, in an experiment subjects were asked to choose a program to combat a disease which was otherwise expected to kill 600 people. One group was told that Program A would certainly save 200 lives while Program B offered a 1/3 probability of saving all 600 people and a 2/3 probability of saving nobody. Most subjects preferred A. A second group were offered the equivalent choice, but this time the statements referred to the number of deaths, rather than lives saved. They were therefore told that the first program would lead to 400 deaths while the second would offer a 1/3 probability of no deaths and a 2/3 probability of 600 deaths. Most subjects in this group preferred the second program, which clearly carries the higher risk. Further experimental evidence that different assessment methods lead to different utilities can be found in a paper by Johnson and Schkade.[8]

What are the implications of this research for utility assessment? First, it is clear that utility assessment requires effort and commitment from the decision maker. This suggests that, before the actual elicitation takes place, there should be a pre-analysis phase in which the importance of the task is explained to the decision maker so that he will feel motivated to think carefully about his responses to the questions posed.

Second, the fact that different elicitation methods are likely to generate different assessments means that the use of several methods is advisable. By posing questions in new ways the consistency of the original utilities can be checked and any inconsistencies between the assessments can be explored and reconciled.

Third, since the utility assessments appear to be very sensitive to both the values used and the context in which the questions are framed it is a good idea to phrase the actual utility questions in terms which are closely related to the values which appear in the original decision problem. For example, if there is no chance of losses being incurred in the original problem then the lotteries used in the utility elicitation should not involve the chances of incurring a loss. Similarly, if the decision problem involves only very high or low probabilities then the use of lotteries involving 50:50 chances should be avoided.

How useful is utility in practice?

We have seen that utility theory is designed to provide guidance on how to choose between alternative courses of action under conditions of uncertainty, but how useful is utility in practice? It is really worth going to the trouble of asking the decision maker a series of potentially difficult questions about imaginary lotteries given that, as we have just seen, there are likely to be errors in the resulting assessments? Interestingly, in a survey of published decision analysis applications over a 20-year period, Corner and Corner[9] found that 2/3 of applications used expected values as the decision criterion and reported no assessment of attitudes to risk. We will summarize here arguments both for and against the application of utility and then present our own views at the end of the section.

First, let us restate that the *raison d'être* of utility is that it allows the attitude to risk of the decision maker to be taken into account in the decision model. Consider again the drug research problem which we discussed earlier. We might have approached this in three different ways. First, we could have simply taken the course of action which led to the shortest expected development time. These expected times would have been calculated as follows:

Expected development time of continuing with the existing method

$$= 0.4 \times 6 + 0.6 \times 4 = 4.8 \text{ years}$$

Expected development time of switching to new research approach

$$= 0.2 \times 1 + 0.4 \times 2 + 0.4 \times 8 = 4.2 \text{ years}$$

The adoption of this criterion would therefore suggest that we should switch to the new research approach. However, this criterion ignores two factors. First, it assumes that each extra year of development time is perceived as being equally bad by the decision maker, whereas it is possible, for example, that an increase in time from 1 to 2 years is much less serious than an increase from 7 to 8 years. This factor could be captured by a *value function*. We could therefore have used one of the methods introduced in Chapter 3 to attach numbers on a scale from 0 to 100 to the different development times in order to represent the decision maker's relative preference for them. These *values* would then have replaced the actual development times in the calculations above and the course of action leading to the highest expected value could be selected. You will recall, however, from Chapter 3 that the derivation of a value function does not involve any considerations about probability, and it therefore will not capture the second omission from the above analysis, which is, of course, the attitude to risk of the decision maker. A utility function is therefore designed to allow *both* of these factors to be taken into account.

Despite this, there are a number of arguments against the use of utility. Perhaps the most persuasive relates to the problems of measuring utility. As Tocher[10] has argued, the elicitation of utilities takes the decision maker away from the real world of the decision to a world of hypothetical lotteries. Because these lotteries are only imaginary, the decision maker's judgments about the relative attractiveness of the lotteries may not reflect what he would really do. It is easy to say that you are prepared to accept a 10% risk of losing \$10 000 in a hypothetical lottery, but would you take the risk if you were really facing this decision? Others (e.g. von Winterfeldt and Edwards[11]) argue that if utilities can only be measured approximately then it may not always be worth taking the trouble to assess them since a value function, which is more easily assessed, would offer a good enough approximation. Indeed, even Howard Raiffa,[12] a leading proponent of the utility approach, argues:

> Many analysts assume that a value scoring system – designed for tradeoffs under certainty – can also be used for probabilistic choice (using expected values). Such an assumption is wrong theoretically, but as I become more experienced I gain more tolerance for these analytical simplifications. This is, I believe, a relatively benign mistake in practice.

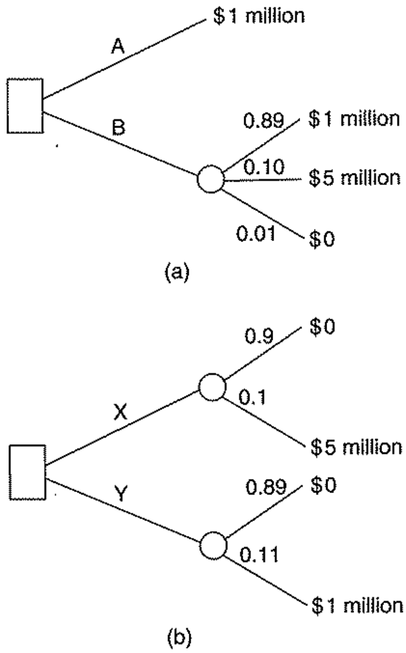

Figure 5.15 – Allais's paradox

Another criticism of utility relates to what is known as Allais's paradox. To illustrate this, suppose that you were offered the choice of options A and B as shown in Figure 5.15(a). Which would you choose? Experiments suggest that most people would choose A (e.g. see Slovic and Tversky[13]). After all, $1 million for certain is extremely attractive while option B offers only a small probability of $5 million and a chance of receiving $0.

Now consider the two options X and Y which are shown in Figure 5.15(b). Which of these would you choose? The most popular choice in experiments is X. With both X and Y, the chances of winning are almost the same, so it would seem to make sense to go for the option offering the biggest prize.

However, if you did choose options A and X your judgments are in conflict with utility theory, as we will now show. If we let $u(\$5\text{ m}) = 1$ and $u(\$0) = 0$, then selecting option A suggests that:

$$u(\$1\text{ m}) \text{ is greater than } 0.89\, u(\$1\text{ m}) + 0.1\, u(\$5\text{ m}) + 0.01\, u(\$0\text{ m})$$

$$\text{i.e. } u(\$1\text{ m}) \text{ exceeds } 0.89\, u(\$1\text{ m}) + 0.1 \text{ which implies:}$$

$$u(\$1\text{ m}) \textit{ exceeds } 0.1/0.11$$

However, choosing X implies that:

$$0.9\ u(\$0) + 0.1\ u(\$5\text{ m}) \text{ exceeds } 0.89\ u(\$0) + 0.11\ u(\$1\text{ m})$$

$$\text{i.e. } 0.1 \text{ exceeds } 0.11\ u(\$1\text{ m})$$

$$\text{so that: } u(\$1\text{ m}) \text{ is } \textit{less than } 0.1/0.11$$

This paradox has stimulated much debate[14] since it was put forward in 1953. However, we should emphasize that utility theory does not attempt to describe the way in which people make decisions like those posed above. It is intended as a normative theory, which indicates what a rational decision maker should do *if* he accepts the axioms of the theory. The fact that people make inconsistent judgments does not by itself invalidate the theory. Nevertheless, it seems sensible to take a relaxed view of the problem. Remember that utility theory is designed as simply an aid to decision making, and if a decision maker wants to ignore its indications then that is his prerogative.

Having summarized some of the main arguments, what are our views on the practical usefulness of utility? First, we have doubts about the practice adopted by some analysts of applying utility to decisions where risk and uncertainty are not central to the decision maker's concerns. Introducing questions about lotteries and probabilities to these sorts of problems seems to us to be unnecessary. In these circumstances the problem of trading off conflicting objectives is likely to be the main concern, and we would therefore recommend the approach of Chapter 3. In important problems which do involve a high level of uncertainty and risk we do feel that utility has a valuable role to play as long as the decision maker is familiar with the concept of probability, and has the time and patience to devote the necessary effort and thought to the questions required by the elicitation procedure. In these circumstances the derivation of utilities may lead to valuable insights into the decision problem. In view of the problems associated with utility assessment, we should not regard the utilities as perfect measures and automatically follow the course of action they prescribe. Instead, it is more sensible to think of the utility function as a useful tool for gaining a greater understanding of the problem.

If the decision maker does not have the characteristics outlined above or only requires rough guidance on a problem then it may not be worth eliciting utilities. Given the errors which are likely to occur in utility assessment, the derivation of values (as opposed to utilities) and the identification of the course of action yielding the highest expected value may offer a robust enough approach. (Indeed, there is evidence that linear utility functions are extremely robust approximations.) Sensitivity analysis would, of course, reveal just how precise the judgments needed to be (e.g. see Kirkwood[15]).

In the final section of this chapter we extend the application of utility to problems involving more than one attribute. We should point out that multi-attribute utility analysis can be rather complex and the number of people applying it is not large. In the light of this, and the points made in our discussion above, we have decided to give only an introduction to this area so that a general appreciation can be gained of the type of judgments required.

Multi-attribute utility

So far in this chapter we have focused on decision problems which involve uncertainty and only one attribute. We next examine how problems involving uncertainty and multiple attributes can be handled. In essence, the problem of deriving a multi-attribute utility function is analogous to that of deriving a multi-attribute value function, which we discussed in Chapter 3. Again, the 'divide and conquer' philosophy applies. As we argued before, large multi-faceted problems are often difficult to grasp in their entirety. By dividing the problem into small parts and allowing the decision maker to focus on each small part separately we aim to simplify his judgmental task. Thus if certain conditions apply, we can derive a single-attribute utility function for each attribute using the methods of earlier sections and then combine these to obtain a multi-attribute utility function. A number of methods have been proposed for performing this analysis, but the approach we will discuss is associated with Keeney and Raiffa.[16] This approach has been applied to decision problems ranging from the expansion of Mexico City Airport (de Neufville and Keeney[17]) to the selection of sites for nuclear power plants (Kirkwood[18]).

The Decanal Engineering Corporation

To illustrate the approach let us consider the following problem which involves just two attributes. The Decanal Engineering Corporation has recently signed a contract to carry out a major overhaul of a company's equipment. Ideally, the customer would like the overhaul to be completed in 12 weeks and, if Decanal meet the target or do not exceed it by a significant amount of time, they are likely to gain a substantial amount of goodwill from the customer and an enhanced reputation throughout the industry. However, to increase the chances of meeting the target, Decanal would have to hire extra labor and operate some 24-hour working, which would increase their costs. Thus the company has two conflicting objectives: (1) minimize the time that the project overruns the target date and (2) minimize the cost of the project.

For simplicity, we will assume that Decanal's project manager has two options: (1) work normally or (2) hire extra labor and work 24-hour shifts. His estimates of the probabilities that the project will overrun the target date by a certain number of weeks are shown on the decision tree in Figure 5.16. The costs of the project for the two options and for different project durations are also shown on the tree. (Note that, once a given option is chosen, the longer the project takes to complete, the greater will be the costs because labor, equipment, etc. will be employed on the project for a longer period.)

To analyze this problem we need to derive a multi-attribute utility function which will enable the project manager to compare the two options. This process is simplified if certain assumptions can be made. The most important of these is that of mutual utility independence.

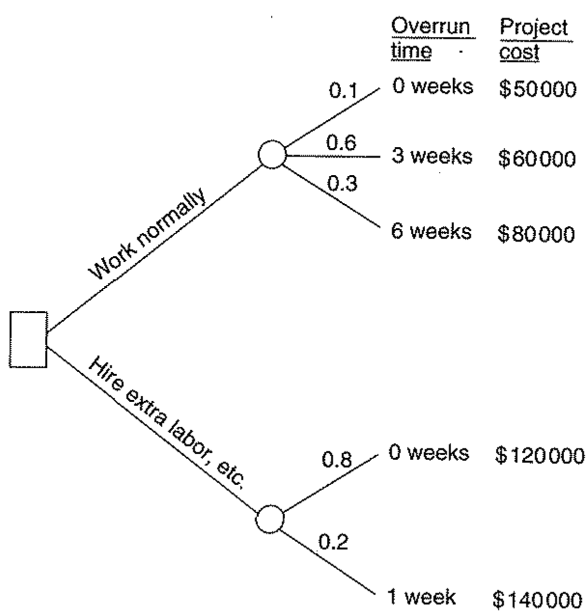

Figure 5.16 – A decision tree for the project manager's problem

Mutual utility independence

Suppose that the project manager is indifferent between the following alternatives:

A: A project which will certainly overrun by 2 weeks and which will certainly cost \$50 000; and
B: A gamble which will give him a 50% chance of a project which overruns by 0 weeks (i.e. it meets the target) and which will cost \$50 000 and a 50% chance of a project which will overrun by 6 weeks and cost \$50 000.

These alternatives are shown in Figure 5.17(a) (note that all the costs are the same).

Suppose that we now offer the project manager the same two options, but with the project costs increased to \$140 000, as shown in Figure 5.17(b). If the project manager is still indifferent between the options then clearly his preference between the overrun times is unaffected by the change in costs. If this is the case for all possible costs then overrun time is said to be *utility independent* of project cost. Putting this in more general terms: attribute A is utility independent of attribute B if the decision maker's preferences between gambles involving different levels of A, but the same level of B, do not depend on the level of attribute B.

It can be seen that utility independence is analogous to preference independence, which we discussed in Chapter 3, except that we are now

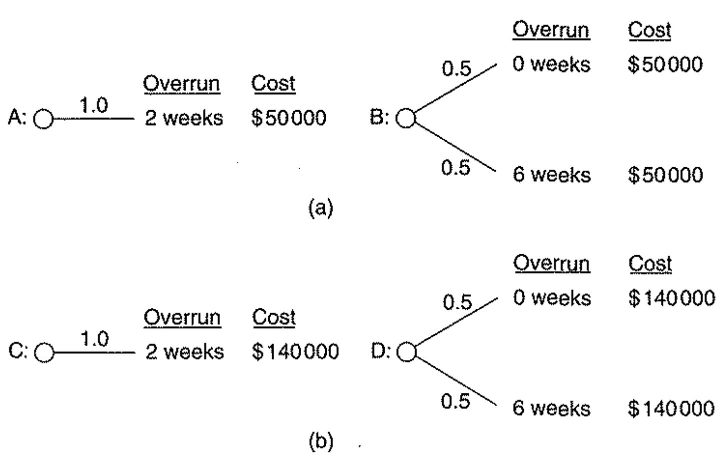

Figure 5.17

considering problems which involve uncertainty. If project cost is also utility independent of overrun time (this will not automatically be the case) then we can say that overrun time and project cost are *mutually utility independent*.

The great advantage of mutual utility independence, if it exists, is that it enables the decision maker to concentrate initially on deriving utility function for one attribute at a time without the need to worry about the other attributes. If this independence does not exist then the analysis can be extremely complex (see Keeney and Raiffa[16]), but in very many practical situations it is usually possible to define the attributes in such a way that they do have the required independence.

Deriving the multi-attribute utility function

Assuming that mutual utility independence does exist, we now derive the multi-attribute utility function as follows.

Stage 1: Derive single-attribute utility functions for overrun time and project cost.

Stage 2: Combine the single-attribute functions to obtain a multi-attribute utility function so that we can compare the alternative courses of action in terms of their performance over both attributes.

Stage 3: Perform consistency checks, to see if the multi-attribute utility function really does represent the decision maker's preferences, and sensitivity analysis to examine the effect of changes in the figures supplied by the decision maker.

Stage 1

First we need to derive a utility function for project overrun. Using the approach which we discussed earlier in the context of single-attribute utility, we give the best overrun (0 weeks) a utility of 1.0 and the worst (6 weeks) a utility of 0. We then attempt to find the utility of the intermediate values, starting with an overrun of 3 weeks. After being asked a series of questions, the project manager indicates that he is indifferent between:

A: A project which will certainly overrun by 3 weeks; and

B: A gamble offering a 60% chance of a project with 0 weeks overrun and a 40% chance of a 6 week overrun.

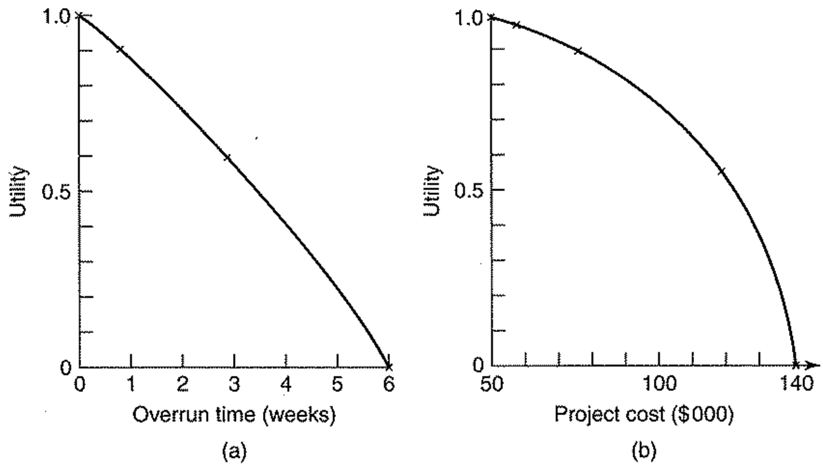

Figure 5.18 – Utility functions for overrun time and project cost

Table 5.4 – The project manager's utilities for overrun and cost

No. of weeks project overruns target	Utility	Cost of project ($)	Utility
0	1.0	50 000	1.00
1	0.9	60 000	0.96
3	0.6	80 000	0.90
6	0.0	120 000	0.55
		140 000	0.00

This implies that u(3 weeks overrun) = 0.6. By a similar process, the manager indicates that u(1 week overrun) = 0.9. The resulting utility function is shown in Figure 5.18(a).

We then repeat the elicitation process to obtain a utility function for project cost. The function obtained from the manager is shown in Figure 5.18(b). Table 5.4 summarizes the utilities which have been elicited for overrun and cost.

Stage 2

We now need to combine these utility functions to obtain the multi-attribute utility function. If the two attributes are mutually utility independent then it can be shown that the multi-attribute utility function will have the following form:

$$u(x_1, x_2) = k_1u(x_1) + k_2u(x_2) + k_3u(x_1)u(x_2)$$

where

$$x_1 = \text{the level of attribute 1,}$$

$$x_2 = \text{the level of attribute 2,}$$

$$u(x_1, x_2) = \text{the multi-attribute utility if attribute 1 has a level } x_1 \text{ and attribute 2 has a level } x_2,$$

$$u(x_1) = \text{the single-attribute utility if attribute 1 has a level } x_1,$$

$$u(x_2) = \text{the single-attribute utility if attribute 2 has a level } x_2$$

and k_1, k_2, and k_3 are numbers which are used to 'weight' the single-attribute utilities.

In stage 1 we derived $u(x_1)$ and $u(x_2)$, so we now need to find the values of k_1, k_2 and k_3. We note that k_1 is the weight attached to the utility for overrun time. In order to find its value we offer the project manager a choice between the following alternatives:

A: A project where overrun is certain to be at its best level (i.e. 0 weeks), but where the cost is certain to be at its worst level (i.e. $140 000); or
B: A lottery which offers a probability of k_1 that both cost and overrun will be at their best levels (i.e. 0 weeks and $50 000) and a $1 - k_1$ probability that they will both be at their worst levels (i.e. 6 weeks and $140 000, respectively).

These options are shown in Figure 5.19. Note that because we are finding k_1 it is attribute 1 (i.e. overrun) which appears at its best level in the certain outcome.

The decision maker is now asked what value the probability k_1 must have to make him indifferent between the certain outcome and the

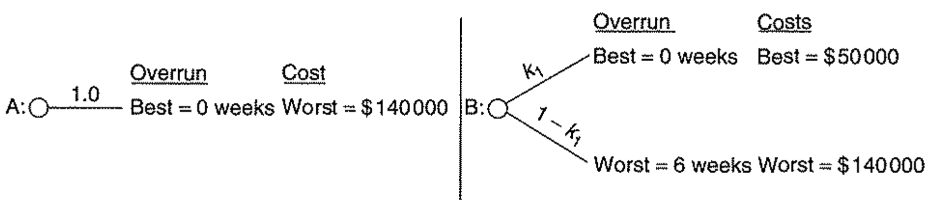

Figure 5.19

lottery. After some thought, he indicates that this probability is 0.8, so $k_1 = 0.8$. This suggests that the 'swing' from the worst to the best overrun time is seen by the project manager to be significant relative to project cost. If he hardly cared whether the overrun was 0 or 6 weeks, it would have taken only a small value of k_1 to have made him indifferent to a gamble where overrun time might turn out to be at its worst level.

To obtain k_2, the weight for project cost, we offer the project manager a similar pair of options. However, in the certain outcome, project cost is now at its best level and the other attribute at its worst level. The probability of the best outcome in the lottery is now k_2. These two options are shown in Figure 5.20.

We now ask the project manager what value k_2 would need to be to make him indifferent between the two options. He judges this probability to be 0.6, so $k_2 = 0.6$. The fact that k_2 is less than k_1 suggests that the project manager sees the swing from the worst to the best cost as being less significant than the swing from the worst to the best overrun time. Having been offered a project which is certain to incur the lowest cost, he requires a smaller probability to tempt him to the lottery, where he might gain a project where overrun is also at its best level but where there is also a risk of a project with costs at their worst level.

Finally, we need to find k_3. This is a simple calculation and it can be shown that:

$$k_1 + k_2 + k_3 = 1, \text{ so } k_3 = 1 - k_1 - k_2$$

Thus, in our case $k_3 = 1 - 0.8 - 0.6 = -0.4$. The project manager's multi-attribute utility function is therefore:

$$u(x_1, x_2) = 0.8\, u(x_1) + 0.6\, u(x_2) - 0.4\, u(x_1)u(x_2)$$

We can now use the multi-attribute utility function to determine the utilities of the different outcomes in the decision tree. For example, to find the utility of a project which overruns by 3 weeks and costs $60 000 we proceed as follows. From the single-attribute functions we know that

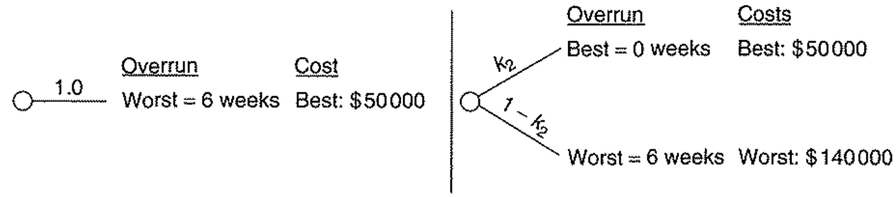

Figure 5.20

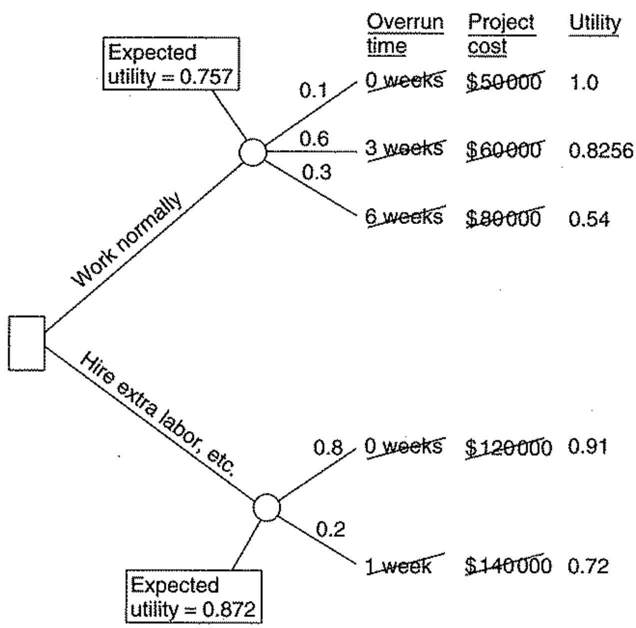

Figure 5.21 – The project manager's decision tree with utilities

u(3 weeks overrun) = 0.6 and u($60 000 cost) = 0.96. Therefore:

$$\begin{aligned} &u(\text{3 weeks overrun, \$60 000 cost}) \\ &\quad = 0.8\,u(\text{3 weeks overrun}) + 0.6\,u(\text{\$60 000 cost}) \\ &\qquad - 0.4\,u(\text{3 weeks overrun})u(\text{\$60 000 cost}) \\ &\quad = 0.8(0.6) + 0.6(0.96) - 0.4(0.6)(0.96) = 0.8256 \end{aligned}$$

Figure 5.21 shows the decision tree again with the multi-attribute utilities replacing the original attribute values. By multiplying the probabilities of the outcomes by their utilities we obtain the expected utility of each option. The results shown on the tree indicate that the project manager should hire the extra labor and operate 24-hour working, as this yields the highest expected utility.

Stage 3

It is important that we should check that the results of the analysis have faithfully represented the project manager's preferences. This can involve tracking back through the analysis and explaining why one

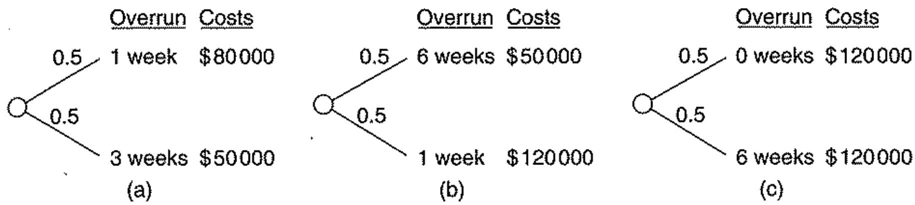

Figure 5.22

option has performed well and another has performed badly. If the decision maker does not feel that the explanations are consistent with his preferences then the analysis may need to be repeated. In fact, it is likely that several iterations will be necessary before a consistent representation is achieved and, as the decision maker gains a greater understanding of his problem, he may wish to revise his earlier responses.

Another way of checking consistency is to offer the decision maker a new set of lotteries and to ask him to rank them in order of preference. For example, we could offer the project manager the three lotteries shown in Figure 5.22. The expected utilities of these lotteries are A: 0.726, B: 0.888 and C: 0.620, so if he is consistent then he should rank them in the order B, A, C. We should also carry out sensitivity analysis on the probabilities and utilities by, for example, examining the effect of changes in the values of k_1 and k_2.

Interpreting multi-attribute utilities

In the analysis above we derived an expected utility of 0.872 for the 'hire extra labor ...' option, but what does this figure actually represent? We demonstrated earlier that we could use the concept of utility to convert a decision problem to a simple choice between lotteries which the decision maker regarded as being equivalent to the original outcomes. Each of these lotteries would result in either the best or worst possible outcome, but with different probabilities. The same is true for multi-attribute utility. This time the lotteries will result in either the best outcome on both attributes (i.e. the best/best outcome) or the worst possible outcome on both attributes (i.e. worst/worst). Thus the expected utility of 0.872 for the 'hire extra labor ...' option implies that the decision maker regards this option as being equivalent to a lottery offering a 0.872 chance of the best/best outcome (and a complementary probability of the worst/worst outcome). It therefore seems reasonable that he should prefer this option to the 'work normally' alternative, which is regarded as being equivalent to a lottery offering only a 0.757 chance of the best/best outcome.

Further points on multi-attribute utility

The principles which we applied to the two-attribute problem above can be extended to any number of attributes (see, for example, Bunn[19] who discusses a problem involving four attributes), though the form of the multi-attribute utility function becomes more complex as the number of attributes increases. Models have also been developed which can handle situations where mutual utility independence does not exist (see Keeney and Raiffa[16]), but the complexities of these models have meant that they have proved to be of little practical value. In any case, as we mentioned earlier, if mutual utility independence does not exist it is likely that by redefining the attributes a new set can be found which does exhibit the required independence (we discussed the analogous problem in Chapter 3 when looking at multi-attribute value functions).

The approach to multi-attribute utility which we discussed above clearly requires a major commitment of time and effort from the decision maker and, since the method lacks the 'transparency' of the SMART procedure, which we met in Chapter 3, a non-mathematical person may be suspicious of its results. In all models a balance has to be struck between the accuracy with which the model represents the real problem and the effort required to formulate the model. If a problem is of major importance, and if the decision maker is happy to make the necessary judgments, then what Watson and Buede[20] refer to as the 'deep soul searching' engendered by Keeney and Raiffa's approach may lead to valuable insights into the decision problem. In other circumstances, where the decision maker only requires outline guidance from the model, a less sophisticated approach based, for example, on values rather than utilities may suffice. Sensitivity analysis will provide useful guidance on the robustness of any approximations which are used.

Summary

In this chapter we have considered a number of methods which enable a decision maker to make rational decisions when the outcomes of courses of action are not known for certain. The approach based on expected monetary value was relatively simple, but if the decision maker does not have a neutral attitude to risk, then the adoption of this criterion may lead to the most-preferred course of action not being chosen. We therefore introduced the concept of expected utility to show how the decision maker's attitude to risk can be incorporated into the decision model. Finally, we showed how the application of utility can be extended to decision problems involving more than one attribute.

Exercises

(1) An entertainment company is organizing a pop concert in London. The company has to decide how much it should spend on publicizing the event and three options have been identified:
Option 1: Advertise only in the music press;
Option 2: As option 1 but also advertise in the national press;
Option 3: As options 1 and 2 but also advertise on commercial radio.
For simplicity, the demand for tickets is categorized as low, medium or high. The payoff table below shows how the profit which the company will earn for each option depends on the level of demand.

		Demand		
Option	*Low*	*Medium*	*High*	*Profits* ($000s)
1	−20	−20	100	
2	−60	−20	60	
3	−100	−60	20	

It is estimated that if option 1 is adopted the probabilities of low, medium and high demand are 0.4, 0.5 and 0.1, respectively. For option 2 the respective probabilities are 0.1, 0.3 and 0.6 while for option 3 they are 0.05, 0.15 and 0.8. Determine the option which will lead to the highest expected profit. Would you have any reservations about recommending this option to the company?

(2) A speculator is considering the purchase of a commodity which he reckons has a 60% chance of increasing in value over the next month. If he purchases the commodity and it does increase in value the speculator will make a profit of about $200 000, otherwise he will lose $60 000.

(a) Assuming that the expected monetary value criterion is applicable, determine whether the speculator should purchase the commodity.
(b) Perform a sensitivity analysis on the speculator's estimate of the probability of a price increase and interpret your result.
(c) What reservations would you have about applying the expected monetary value criterion in this context?

(3) A team of scientists is due to spend six months in Antarctica carrying out research. One major piece of equipment they will be taking is subject to breakdowns caused by the sudden failure of a particular component. Because a failed component cannot be repaired the team intend to carry a stock of spare units of the component, but it will cost them roughly \$3000 for each spare unit they take with them. However, if the equipment breaks down and a spare is not available a new unit will have to be specially flown in and the team will incur a total cost of \$4000 for each unit that is delivered in this way. An engineer who will be traveling with the team has estimated that the number of spares that will be required during the six months follows the probability distribution shown below:

No. of spares required	0	1	2	3
Probability	0.2	0.3	0.4	0.1

Determine the number of spares that the team should carry if their objective is to minimize expected costs.

(4) You are a contestant on a television game show and you have won \$5000 so far. You are now offered a choice: either you can keep the money and leave or you can continue into the next round, where you have a 70% chance of increasing your winnings to \$10 000 and a 30% chance of losing the \$5000 and finishing the game with nothing.
(a) Which option would you choose?
(b) How does your choice compare with that which would be prescribed by the expected monetary value criterion?

(5) A building contractor is submitting an estimate to a potential customer for carrying out some construction work at the customer's premises. The builder reckons that if he offers to carry out the work for \$150 000 there is a 0.2 probability that the customer will agree to the price, a 0.5 probability that a price of \$120 000 would eventually be agreed and a 0.3 probability that the customer will simply refuse

the offer and give the work to another builder. If the builder offers to carry out the work for $100 000 he reckons that there is a 0.3 probability that the customer will accept this price, a 0.6 probability that the customer will bargain so that a price of $80 000 will eventually be agreed and a 0.1 probability that the customer will refuse the offer and take the work elsewhere.

(a) Determine which price the builder should quote in order to maximize the expected payment he receives from the customer.
(b) Suppose that, after some questioning, the builder is able to make the following statements:

'I am indifferent between receiving $120 000 for certain or entering a lottery that will give me a 0.9 probability of $150 000 and a 0.1 probability of winning $0.'

'I am indifferent between receiving $100 000 for certain or entering a lottery that will give me a 0.85 probability of winning $150 000 and a 0.15 probability of winning $0.'

'I am indifferent between receiving $80 000 for certain or entering a lottery that will give me a 0.75 probability of winning $150 000 and a 0.25 probability of winning $0.'

(i) Sketch the builder's utility function and comment on what it shows.
(ii) In the light of the above statements which price should the builder now quote to the customer and why?

(6) (a) Use the following questions to assess your own utility function for money values between $0 and $5000. You should assume that all sums of money referred to will be received immediately.
(i) You are offered either a sum of money for certain or a lottery ticket that will give you a 50% chance of winning $5000 and a 50% chance of winning $0. Write down below the certain sum of money which would make you indifferent between whether you received it or the lottery ticket.
$ . . . . (we will now refer to this sum of money as X)
The utility of X is 0.5.
(ii) You are now offered a lottery ticket which offers you a 50% chance of $ . . . . (enter X here) and a 50% chance of $0. Alternatively, you will receive a sum of money for certain. Write down below the certain sum of money which would make you indifferent between whether you received it or the lottery ticket.

$ . . . .
The utility of this sum of money is 0.25.

(iii) Finally, you are offered a sum of money for certain or a lottery ticket which will give you a 50% chance of $5000 and a 50% chance of $. . . . . (enter *X* here). Write down below the certain sum of money which would make you indifferent between whether you received it or the lottery ticket.
$. . . . .
The utility of this sum of money is 0.75.

(b) Plot your utility function and discuss what it reveals.

(c) Discuss the strengths and limitations of the assessment procedure which was used in (a).

(7) A company is planning to re-equip one of its major production plants and one of two types of machine, the Zeta and the Precision II, is to be purchased. The prices of the two machines are very similar so the choice of machine is to be based on two factors: running costs and reliability. It is agreed that these two factors can be represented by the variables: average weekly operating costs and number of breakdowns in the first year of operation. The company's production manager estimates that the following probability distributions apply to the two machines. It can be assumed that the probability distributions for operating costs and number of breakdowns are independent.

Zeta

Average weekly operating costs ($)	*Prob.*	*No. of breakdowns*	*Prob.*
20 000	0.6	0	0.15
30 000	0.4	1	0.85

Precision II

Average weekly operating costs ($)	*Prob.*	*No. of breakdowns*	*Prob.*
15 000	0.5	0	0.2
35 000	0.5	1	0.7
		2	0.1

Details of the manager's utility functions for operating costs and number of breakdowns are shown below:

Average weekly operating costs ($)	*Utility*	*No. of breakdowns*	*Utility*
15 000	1.0	0	1.0
20 000	0.8	1	0.9
30 000	0.3	2	0
35 000	0		

(a) The production manager's responses to questions reveal that, for him, the two attributes are mutually utility independent. Explain what this means.
(b) The production manager also indicates that for him $k_1 = 0.7$ (where attribute 1 = operating costs) and $k_2 = 0.5$. Discuss how these values could have been determined.
(c) Which machine has the highest expected utility for the production manager?

(8) The managers of the Lightning Cycle Company are hoping to develop a new bicycle braking system. Two alternative systems have been proposed and, although the mechanics of the two systems are similar, one design will use mainly plastic components while the other will use mainly metal ones. Ideally, the design chosen would be the lightest and the most durable but, because some of the technology involved is new, there is some uncertainty about what the characteristics of the resulting product would be.

The leader of Lightning's research and development team has estimated that if the plastic design is developed there is a 60% chance that the resulting system would add 130 grams to a bicycle's weight and would have a guaranteed lifetime of one year. He also reckons that there is a 40% chance that a product with a 2-year lifetime could be developed, but this would weigh 180 grams.

Alternatively, if the metal design was developed the team leader estimates that there is a 70% chance that a product with a 2-year guaranteed life and weighing 250 grams could be developed. However, he estimates that there is a 30% chance that the resulting product would have a guaranteed lifetime of 3 years and would weigh 290 grams.

It was established that, for the team leader, weight and guaranteed lifetime were mutually utility independent. The following utilities were then elicited from him:

Weight (grams)	*Utility*	*Guaranteed lifetime* (years)	*Utility*
130	1.0	3	1.0
180	0.9	2	0.6
250	0.6	1	0
290	0		

After further questioning the team leader indicated that he would be indifferent between the following alternatives:

A: A product which was certain to weigh 130 grams, but which had a guaranteed lifetime of only one year; or

B: A gamble which offered a 0.7 probability of a product with a weight of 130 grams and a guaranteed lifetime of 3 years and a 0.3 probability of a product with a weight of 290 grams and a guaranteed lifetime of 1 year.

Finally, the team leader said that he would be indifferent between alternatives C and D below:

C: A product which was certain to weigh 290 grams, but which had a guaranteed lifetime of 3 years;

D: A gamble which offered a 0.9 probability of a product with a weight of 130 grams and a guaranteed lifetime of 3 years and a 0.1 probability of a product with a weight of 290 grams and a guaranteed lifetime of 1 year.

(a) What do the team leader's responses indicate about his attitude to risk and the relative weight which he attaches to the two attributes of the proposed design?

(b) Which design should the team leader choose, given the above responses?

(c) What further analysis should be conducted before a firm recommendation can be made to the team leader?

(9) To celebrate the centenary of the local football club, a pottery manufacturer has decided to produce a commemorative plate. A choice has to be made on whether to have a large-scale production run or a small-scale run. For technical reasons only a large- or small-scale run is possible and it would not be economical, because of other commitments, to have more than one production run. The manufacturer has two objectives: (i) to maximize the profits earned from the plate and (ii) to minimize the number of customers who will be disappointed because insufficient plates were produced (a large number of disappointed customers would not be good for customer goodwill). For simplicity, the potential demand for the plates has

been classified as either high or low and it is estimated that there is a 70% chance that demand will be high.

If the manufacturer opts for a large-scale production run and demand is high then an estimated profit of \$40 000 will be made, but it is also estimated that 2000 customers who wished to buy the plate would still be disappointed (production capacity constraints mean that it would be impossible to meet all the potential demand). If demand is low then the company would just break even, but no customers would be disappointed.

If the manufacturer opts for a small-scale production run and demand is high then an estimated profit of \$30 000 will be made but around 5000 customers would be disappointed. Low demand would still yield profits of \$10 000 and no customers would be disappointed. It has been established that 'profit' and 'number of disappointed customers' are mutually utility independent.

(a) Draw a decision tree to represent the manufacturer's problem.

(b) The manufacturer's utility function for profit can be approximated by the function:

$$U(x) = 0.4x - 0.0375x^2$$

where: x = profit in *tens of thousands of dollars* (this function is valid for profits from \$0 to \$40 000, i.e. x values from 0 to 4). The manufacturer's utility function for the number of disappointed customers is given below:

Number of customers disappointed	*Utility*
0	1.0
2000	0.3
5000	0

Plot these two utility functions on separate graphs and explain what they show.

(c) After much questioning the manufacturer is able to say that he is indifferent between alternatives A and B below.

A: A production run which will yield a certain profit of \$40 000, but which will certainly disappoint 5000 customers.

B: A production run which will have a 0.8 probability of a profit of \$40 000 with no customers disappointed and a 0.2 probability of a profit of \$0 with 5000 customers disappointed.

After further questioning he is also able to say that he is indifferent between options C and D below:

C: A production run which will certainly yield a profit of \$0 but which is also certain to disappoint no customers.

D: A production run which will have a 0.6 probability of a profit of \$40 000 with no customers disappointed and a 0.4 probability of a profit of \$0 with 5000 customers disappointed.

(i) Determine whether the manufacturer should choose the large- or small-scale production run in order to maximize his expected utility.

(ii) Interpret the expected utilities which you obtained in part (i).

(d) Would the Simple Multi-attribute Rating Technique (SMART) have offered a better way of tackling the pottery manufacturer's problem?

References

1. Von Neumann, J. and Morgenstern, O. (1944) *The Theory of Games and Economic Behavior*, Princeton University Press.
2. Ronen, B., Pliskin, J. S. and Feldman, S. (1984) Balancing the Failure Modes in the Electronic Circuit of a Cardiac Pacemaker: A Decision Analysis, *Journal of the Operational Research Society*, **35**, No. 5, 379–387.
3. French, S. (1988) *Decision Theory: An Introduction to the Mathematics of Rationality*, Ellis Horwood, Chichester.
4. Farquahar, P. H. (1984) Utility Assessment Methods, *Management Science*, **30**, No. 11, 1283–1300.
5. Hershey, J. C., Kunreuther, H. C. and Schoemaker, P. J. H. (1982) Sources of Bias in Assessment Procedures for Utility Functions, *Management Science*, **28**, No. 8, 936–954.
6. Thaler, R. H. (1983) Illusions and Mirages in Public Policy, in H. R. Arkes and K. R. Hammond (eds) *Judgment and Decision Making*, Cambridge University Press, Cambridge.
7. Tversky, A. and Kahneman, D. (1981) The Framing of Decisions and the Psychology of Choice, *Science*, **211**, 30 January, 453–458.
8. Johnson, E. J. and Schkade, D. A. (1989) Bias in Utility Assessments: Further Evidence and Explanations, *Management Science*, **35**, No. 4, 406–424.
9. Corner, J. L. and Corner, P. D. (1995) Characteristics of Decisions in Decision Analysis Practice, *Journal of the Operational Research Society*, **46**, 304–314.
10. Tocher, K. D. (1977) Planning Systems, *Philosophical Transactions of the Royal Society of London*, **A287**, 425–441.

11. Von Winterfeldt, D. and Edwards, W. (1986) *Decision Analysis and Behavioral Research*, Cambridge University Press, Cambridge.
12. Raiffa, H. (1982) *The Art and Science of Negotiation*, Harvard University Press, Cambridge, MA.
13. Slovic, P. and Tversky A. (1974) Who Accepts Savage's Axiom? *Behavioral Science*, **19**, 368–373.
14. Indeed, a whole debate has been organized around paradoxical choice and the psychological acceptability of the axioms of expected utility theory and its variants. See, for example, Stigum, B. P. and Wenstop, F. (eds) (1983) *Foundations of Risk and Utility Theory with Applications*, Reidel, Dordrecht.
15. Kirkwood, C. W. (in press) Approximating Risk Aversion in Decision Analysis Applications, *Decision Analysis*.
16. Keeney, R. L. and Raiffa, H. (1976) *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*, Wiley, New York.
17. De Neufville, R. and Keeney, R. L. (1972) Systems Evaluation through Decision Analysis: Mexico City Airport, *Journal of Systems Engineering*, **3**, No. 1, 34–50.
18. Kirkwood, C. W. (1982) A Case History of Nuclear Power Plant Site Selection, *Journal of the Operational Research Society*, **33**, 353–363.
19. Bunn, D. W. (1982) *Analysis for Optimal Decisions*, Wiley, Chichester.
20. Watson, S. R. and Buede, D. M. (1987) *Decision Synthesis*, Cambridge University Press, Cambridge.