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SM4轮函数F:

$$F(X_0, X_1, X_2, X_3, rk) = X_0 \oplus T(X_1 \oplus X_2 \oplus X_3 \oplus rk)$$

其中, (X_0, X_1, X_2, X_3) 为输入, rk 为轮密钥, T 为S盒中T变换及线性变换

加密:

$$X_{i+4} = F(X_i, X_{i+1}, X_{i+2}, X_{i+3}, rk_i), i = 0, 1, \dots, 3, X_i \text{ 为明文}$$

$$\text{反序: } (Y_0, Y_1, Y_2, Y_3) = (X_{35}, X_{34}, X_{33}, X_{32}), Y_i \text{ 为密文}$$

$$\text{解密: } X_i = F(X_{i+4}, X_{i+3}, X_{i+2}, X_{i+1}, rk_i), Y_i \text{ 为密文}$$

$$\text{反序: } (X_0, X_2, X_1, X_3) = (M_0, M_1, M_2, M_3), X_i \text{ 为明文}$$

密钥扩展:

$$\text{密钥 } MK = (MK_0, MK_1, MK_2, MK_3)$$

$$\text{中间数据: } (k_0, k_1, k_2, k_3) = (MK_0 \oplus FK_0, MK_1 \oplus FK_1, MK_2 \oplus FK_2, MK_3 \oplus FK_3)$$

$$\text{轮密钥 } rk_i = k_{i+4} = k_i \oplus T'(k_{i+1} \oplus k_{i+2} \oplus k_{i+3} \oplus Ck_i)$$

$$\text{其中 } T' \text{ 与 } T \text{ 变换基本一致, 只需将 } L \text{ 变换改为 } L'(b) = b \oplus (b \lll b) \oplus (b \lll c_{23})$$

证明: 轮函数 $F_i = G_i \circ E$

$$\text{其中, } G_i(X_i, X_{i+1}, X_{i+2}, X_{i+3}, rk_i) = (X_i \oplus T(X_{i+1}, X_{i+2}, X_{i+3}, rk_i), X_{i+1}, X_{i+2}, X_{i+3})$$

$$E(X_{i+4}, (X_{i+1}, X_{i+2}, X_{i+3})) = ((X_{i+1}, X_{i+2}, X_{i+3}), X_{i+4})$$

$$(G_i)^2 = G_i(X_i \oplus T(X_{i+1}, X_{i+2}, X_{i+3}, rk_i), X_{i+1}, X_{i+2}, X_{i+3}, rk_i)$$

$$= (X_i \oplus T(X_{i+1}, X_{i+2}, X_{i+3}, rk_i) \oplus T(X_{i+1}, X_{i+2}, X_{i+3}, rk_i), X_{i+1}, X_{i+2}, X_{i+3}, rk_i)$$

$$= (X_i, X_{i+1}, X_{i+2}, X_{i+3}, rk_i)$$

$$= I$$

$\Rightarrow G$ 为对合函数

$$L = 2 \times \begin{pmatrix} X_{i+4} \\ X_{i+1} \\ X_{i+2} \\ X_{i+3} \end{pmatrix} = I$$

$\Rightarrow E$ 为对合函数

\Rightarrow SM4 对合

再由

SM4加密过程:

$$(X_i, X_{i+1}, X_{i+2}, X_{i+3}) \rightarrow (X_{i+1}, X_{i+2}, X_{i+3}, X_{i+4})$$

⋮
若干轮后, 反序

SM4解密过程:

$$(X_{i+3}, X_{i+2}, X_{i+1}, X_{i+0}) \rightarrow (X_{i+2}, X_{i+1}, X_i, X_{i-1})$$

⋮
若干轮后反序

由于轮函数 F_i 对合, SM4过程对合.

$$F_i^{-1} = F_i \Rightarrow SM4^{-1}(SM4(X_0, X_1, X_2, X_3)) = (X_0, X_1, X_2, X_3)$$

\Rightarrow SM4 可逆.