

On the Period-Luminosity-Color Relation of Classical Cepheids

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Abstract. We present the comparison between theoretical and empirical PL relations in V and I bands for the Large Magellanic Cloud. We found that, within the current intrinsic dispersions, theoretical predictions are in remarkable agreement with observational data. We also discuss the PLC relations in (V,B-V) and (V,V-I) as well as the dependence of distance determinations on uncertainties in colors, in reddening corrections and in metal content.

1. Introduction

Classical Cepheids are the most popular primary distance indicators. The key role played by this group of radial variables in estimating the cosmic distance scale is soundly confirmed by the large number of both observational and theoretical investigations aimed at improving both accuracy and reliability of distance measurements. At least three are the main reasons for the widespread use of the Cepheid distance scale:

- Cepheids are bright intermediate-mass objects with visual magnitude range from $M_V = -3$ at short-periods ($\log P \approx 0.6$) to $M_V = -6$ at long-periods ($\log P \approx 2.0$). This feature and the luminosity variations over the pulsation cycle make these variables an appealing observational target, since they can be detected and measured in a large number of Local Group (LG) galaxies. This observational effort has reached the top thanks to the photometric data recently collected by the HST. In fact, the HST key-project succeeded in the identification of a reasonable number of Cepheids in two dozens galaxies belonging to the LG and to the Virgo cluster.
- The physical mechanisms which govern the pulsation instability of these objects have been firmly established. This notwithstanding, the pulsation behavior and the modal stability of Cepheids are often outlined by adopting naïve

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arguments. Moreover, up to now no general theoretical consensus on the dependence of the Cepheid luminosity on the metal abundance has been reached. In fact, theoretical predictions based on linear, nonadiabatic, radiative models suggest that the blue edge of the instability strip presents a negligible dependence on metal content (Saio & Gautschi 1998). On the other hand, recent theoretical predictions which account for both blue and red edges of the instability strip support the evidence that Cepheid luminosity depends on chemical composition. In particular, it turns out that metal-poor Cepheids are, at fixed period, brighter than metal-rich ones. This prediction is at odds with their empirical behavior, and indeed a trend opposite to this has been recently brought out in the literature (Sasselov et al. 1997; Kennicutt et al. 1998 and references therein).

- Some of the long-standing questions on distance determinations can be properly addressed within the Cepheid scenario. In fact, the intrinsic width of the instability strip substantially narrows when moving from optical to near infrared (NIR) bands. Metallicity dependence and reddening corrections give the same outcome, and indeed NIR PL relations are less affected by these uncertainties than optical PL relations. The calibration of the PL zero-points can be accomplished by adopting several independent methods such as the Baade-Wesselink method and its progeny (Krockenberger et al. 1997; Di Benedetto 1997), the trigonometric parallaxes (Feast & Catchpole 1997), and the main sequence fitting (Gieren & Fouqu   1993 and references therein). Therefore a straightforward analysis of both systematic and intrinsic errors affecting distance determinations may be undertaken.

Even though Cepheids are the most promising primary distance indicators for estimating extragalactic distances, the present scenario of the cosmic distance scale, and in turn of the evaluation of the Hubble constant $-H_0-$ has been perfectly accounted for by Trimble (1997) who noted that although over the years H_0 estimates decreased by one order of magnitude, the error bars have remained almost constant! This evidence strongly supports a comprehensive theoretical investigation of the deceptive errors which could affect distance measurements.

In section 2 we discuss the comparison between theoretical and empirical PL relations in V and I bands for Cepheids in the Large Magellanic Cloud (LMC), as well as the so called Wesenheit function. Theoretical prediction concerning the Period-Luminosity-Color (PLC) relations are presented in §3 together with a brief analysis of the uncertainties due to both metallicity and reddening.

2. PL relations for LMC Cepheids

LMC Cepheids are the backbone of cosmic distance derivation since the PL relations of the target galaxy are compared with the LMC PL relations when placing extragalactic distances on an absolute scale. Even though the Cepheid metallicity is not firmly constrained (see Luck et al. 1998) and the reddening estimates still present some uncertainties, LMC Cepheids cover a wide period range and are relatively close objects, thus enabling a proper sampling of the instability strip.

In a recent paper Tanvir (1997) performed a thorough analysis of the intrinsic and systematic uncertainties which affect Cepheid distance scale. In

particular, the author derived new LMC PL relations based on current available data in V and I bands, as well as a PL relation for the reddening free Wesenheit function, i.e. $W_{VI} = V - R[V - I]$ where $R \approx 2.45$ is the adopted extinction parameter (Cardelli et al. 1989). These empirical relations were calibrated by adopting an LMC distance modulus of 18.5 and a reddening of $E_{B-V} = 0.1$ mag, respectively. In order to supply a theoretical framework for a proper comparison with observational data we derived analytical PL relations by adopting several sequences of Cepheid models constructed with fixed chemical composition ($Y=0.25$ $Z=0.008$) and a wide range of stellar masses and effective temperatures (see Bono et al. 1999a,b for further details). The mass-luminosity relation adopted for fixing the luminosity of these models is based on evolutionary tracks which neglect the convective core overshooting during the hydrogen burning phase.

In deriving these relations we also nailed on the period at $\log P = 1.4$, but we did not restrict the period range to $\log P < 1.8$ (Tanvir 1997), since we are interested in testing theoretical predictions for long-period Cepheids. Due to the well known bending of the PL relation in the long-period range we performed a quadratic fit and the results we obtained are the following:

$$\begin{aligned} < M_V > = & -5.18 & -2.01 [\log P - 1.4] & +0.88 [\log P - 1.4]^2 & (\sigma_{rms} = 0.25) \\ < M_I > = & -6.09 & -2.43 [\log P - 1.4] & +0.67 [\log P - 1.4]^2 & (\sigma_{rms} = 0.18) \\ < W_{VI} > = & -7.41 & -3.02 [\log P - 1.4] & +0.38 [\log P - 1.4]^2 & (\sigma_{rms} = 0.08) \end{aligned}$$

Figure 1 shows the comparison between theoretical and empirical PL relations. From this comparison three interesting results emerge: 1) In the period range covered by empirical relations ($0.4 < \log P < 1.7$) the mean PL_V and PL_I relations are, within the intrinsic dispersions, in remarkable agreement with theoretical predictions. 2) The theoretical PL relations show a linear behavior up to $\log P \approx 1.6$ but toward longer periods they start to bend due to the shift of the instability strip toward redder colors. 3) Theoretical and empirical dispersions in the I band are, as expected, smaller than those of the V band, but they are still too large for constraining the distance modulus and the reddening values adopted for calibrating this sample. This notwithstanding, there is no plausible reason for assuming that the pulsation characteristics of LMC Cepheids are peculiar (Simon & Young 1997), thus supporting the use of these templates for estimating the absolute distance of target galaxies with similar metal contents.

The bottom panel of Figure 1 shows the comparison between theoretical and empirical PL_W relations based on Wesenheit magnitudes. The two relations are in agreement only marginally, and when moving from long to short-periods the discrepancy increases. Due to the agreement between theory and observations in the $\log P - M_V$ and in the $\log P - M_I$ plane, it is not clear whether this drift is caused by a systematic shift in the color-temperature relations we adopted or more likely is caused by a poor accuracy of (V-I) mean colors. In fact, in order to reduce the sampling of I light curves the V band light curves are transformed into I band light curves by adopting an empirical method (see Appendix A in Tanvir 1997). We note that the light curves are not characterized by a constant shape when moving from short to long-period Cepheids. In the short-period range they present a sawtoothed shape, then around $\log P \approx 1$ they show a bump along either the rising or the decreasing branch while in the long-period range the shape becomes more sinusoidal.

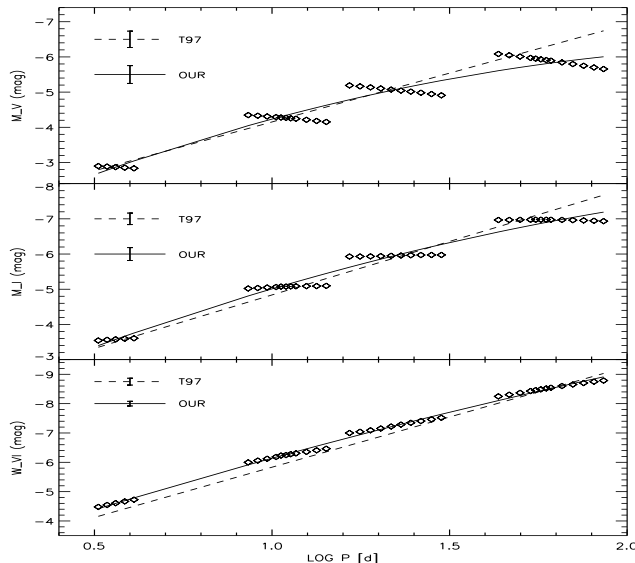


Figure 1. Comparison between our theoretical PL relations (solid lines) and the empirical relations (dashed lines) provided by Tanvir (1997). The error bars are referred to the intrinsic dispersions of analytical relations. From top to bottom the relation plotted in this figure are the PL_V , the PL_I and the Wesenheit function.

It has been often pointed out that the use of these magnitudes causes a substantial decrease in the dispersion of the PL relation. This is mainly due to the fact that the Wesenheit function in two arbitrary bands (β , ξ) is the projection of the PLC (β , $\beta - \xi$) relation onto the $\log P - M_\beta$ plane. As a consequence, in this plane the dispersion is much smaller since the location of each individual Cepheid inside the instability strip is being fixed according to both period and color.

Is therefore the Wesenheit function equivalent to using a PLC relation for estimating distances?

On general grounds the answer is no, because the Wesenheit function is only mimicking the behavior of the equivalent PLC relation. Due to an unaccountable reason the value of the color coefficient in the optical PLC (V, B-V) relation is quite similar (see Bono et al. 1999b) to the extinction parameter, and thus in these bands the Wesenheit function and the PLC relation present the same behavior. However, the Wesenheit function (V,I) is derived by adopting $R \approx 2.45$ (Cardelli et al. 1989) and therefore it is quite different from the color coefficient of the PLC (V, V-I) relation (see *infra*).

On the basis of the above results three main conclusions concerning the use of the $\log P - W_{VI}$ relation can be drawn: 1) in these bands the Wesenheit function does not mimic the behavior of the PLC (V, V-I) relation and therefore it does not properly account for the intrinsic width of the instability strip. 2) This relation is sensitive to the photometric accuracy of mean colors and therefore for a sound empirical evaluation a good sampling of both V and I band light curves is needed. This means that if accurate mean colors are available the use of the PLC (V, V-I) relation for estimating distances is more physically plau-

sible than the Wesenheit function. The interesting feature that the Wesenheit function supplies reddening free magnitudes will be discussed in a forthcoming paper (Caputo et al. 1999).

3. Discussion and Conclusions

The above discussion has been focused on the PL relations at fixed chemical composition. In order to account for the dependence of distance determinations on metallicity we adopt the PLC relations, since in this parameter space the Cepheid location is not ambiguous. We derived two analytical relations by taking into account sequences of models constructed by adopting three different chemical compositions, namely $Y=0.25$, $Z=0.004/[Fe/H]=-0.7$; $Y=0.25$, $Z=0.008/[Fe/H]=-0.4$; $Y=0.25$, $Z=0.02/[Fe/H]=0.0$. The results of this fit for (V, B-V) and (V, V-I) PLC relations are the following:

$$\begin{aligned} \langle M_V \rangle = & \begin{array}{cccc} -2.83 & -3.57 \log P & +2.92[\langle B \rangle - \langle V \rangle] & -0.35[Fe/H] \\ \pm 0.05 & \pm 0.03 & \pm 0.05 & \pm 0.03 \end{array} \\ \\ \langle M_V \rangle = & \begin{array}{cccc} -3.57 & -3.59 \log P & +3.80[\langle V \rangle - \langle I \rangle] & +0.03[Fe/H] \\ \pm 0.03 & \pm 0.02 & \pm 0.04 & \pm 0.02 \end{array} \end{aligned}$$

where the symbols have their usual meaning. The standard deviations of these two relations are $\sigma_{rms} = 0.05$ and 0.03 respectively. As already pointed out by Bono et al. (1999b) the PLC (V, B-V) relation suggests that metal-rich Cepheids are, at fixed period and color, brighter than metal-poor ones. On the other hand, the PLC (V, V-I) relation discloses a marginal dependence on metallicity, and thus leads strong support to the use of this relation for estimating distances of target galaxies for which the metal content is poorly known.

Since these relations are not affected by systematic errors such as reddening corrections, photometric accuracy of both magnitude and colors, and the metal content, we can estimate how plausible errors within these parameters may affect distance determinations. At first, to account for photometric and zero-point calibration uncertainties we assumed, by following Tanvir (1997), that $\sigma_B \approx \sigma_V \approx \sigma_I \approx 0.04$ mag. These uncertainties imply errors on (B-V) and (V-I) colors of the order of 0.06 mag, and in turn an uncertainty in distance of 8% and 10%. At the same time, a systematic error of the order of 0.03 mag in the reddening correction $-E_{B-V}$ implies errors on (B-V) and (V-I) colors of 0.03 and 0.04 mag respectively. These uncertainties imply errors in distance equal to 4% and 7%. However, if we simultaneously account for the two quoted uncertainties, the errors on the colors are equal to 0.19 and 0.26 mag, while in distances the errors are 9% and 12% respectively.

In order to account for the dependence of distance determinations on chemical composition we assumed an uncertainty on metal abundance of the order of 0.4 dex. This error takes into account not only the intrinsic errors in metallicity measurements but also the spread in metallicity of Cepheids in external galaxies. In fact, Monteverde et al. (1997) by adopting spectroscopic data of B-type supergiants in M33 estimated that the iron abundance gradient in this spiral galaxy is of the order $-0.20(\pm 0.05)$ dex kpc^{-1} . The quoted uncertainty implies

errors in distance modulus of 0.14 (V,B-V) and of 0.012 (V,V-I) mag and thus errors of 7% and 1% respectively.

The main outcomes of this leading term error analysis are the following: 1) PLC relations based on different photometric bands present pros and cons, and indeed the PLC (V,B-V) relation, in comparison with the PLC (V,V-I) relation, is less affected by errors on both reddening corrections and colors but is more affected by a spread in metallicity. 2) The PLC (V,V-I) relation seems a promising distance indicator for target galaxies with accurate reddening and color measurements since it is marginally affected by metallicity.

Obviously a comprehensive analysis of the error budget of Cepheid distance scale based on PLC relations is not a trivial effort, since in reality it is a mixture of all previous uncertainties. A thorough analysis can be undertaken only by means of Montecarlo simulations of Cepheids inside the instability strip, which can simultaneously account for all plausible errors on observables.

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References

- Bono, G., Caputo, F., Castellani, V., & Marconi, M. 1999b, *ApJ*, 512, February
Bono, G., Marconi, M., & Stellingwerf, R. F. 1999a, *ApJS*, 120, January
Caputo, F. et al. 1999, in preparation
Cardelli, J. A., Clayton, G. C. & Mathis, J. S. 1989, *ApJ*, 345, 245
Di Benedetto, G. P. 1997, *ApJ*, 486, 60
Feast, M. W. 1984, in *IAU Symp. 108, Structure and Evolution of the Magellanic Clouds*, S. van der Bergh & K.S. de Boer, Dordrecht: Kluwer, 157
Feast, M. W. 1991, in *Observational Tests of Cosmological Inflation*, T. Shanks et al., Dordrecht: Kluwer, C348, 147
Feast, M. W. 1995, in *IAU Colloq. 155, Astrophysical Applications of Stellar Pulsation*, R.S. Stobie & P.A. Whitelock San Francisco: ASP, 209
Feast, M. W., & Catchpole, R. M. 1997, *MNRAS*, 286, L1
Gieren, W. P., Fouqu  , P. 1993, *AJ*, 106, 734
Kennicutt, R. C., et al. 1998, *ApJ*, 498, 181
Krockenberger, M., Sasselov, D. D., & Noyes, R. W. 1997, *ApJ*, 479, 875
Luck, R. E., Moffett, T. J., Barnes, T. G., & Gieren, W. P. 1998, *AJ*, 115, 605
Monteverde, M. I., Herrero, A., Lennon, D. J. & Kudritzki, R.P. 1997, *ApJ*, 474, L107
Saio, H. & Gautschy, A. 1998, *ApJ*, 1998, 498, 360
Sasselov, et al. 1997, *A&A*, 324, 471
Simon, N. R. & Young, T.S. 1997, *MNRAS*, 288, 267
Tanvir, N. R. 1997, in *The Extragalactic Distance Scale*, M. Livio, M. Donahue & N. Panagia, Cambridge: Cambridge Univ. Press, 91
Trimble, V. 1997, in *The Extragalactic Distance Scale*, M. Livio, M. Donahue & N. Panagia, Cambridge: Cambridge Univ. Press, 313