ASP 5203, Estrellas Variables

2019-1

Profesor: Márcio Catelan

Tarea 3

06/06/2019

Fecha de entrega: 21/06/2019

Problem 1. Check the literature for up-to-date estimates of the radius and mass of the star δ Cep. Next, assume that the radius of the star changes by 12% during a pulsation cycle. Evaluate the relative range of variation in the effective temperature of the star during its pulsation cycle. If you wish, you may assume that the oscillations can be approximated, to first order, as adiabatic.

Problem 2. In the course webpage you will find a FORTRAN program called PULS.FOR, which integrates the LAWE numerically. PULS uses the output from ZAMS.FOR (also available on this webpage), which in turn computes the detailed structure of a star on the zero-age main sequence by integrating the basic equations of stellar structure. More information on how these programs work can be found in the accompanying readme.pdf file.

i) Start by computing an equilibrium model on the zero-age main sequence. In this calculation, use a stellar mass, chemical composition and thermodynamic properties obtained on the basis of Tables 1.1 and 1.2 in the readme.pdf file. (You should **very carefully** carry out extrapolations based on the values in that table, if necessary. If you need to extrapolate, do it in multiple stages, taking very small intermediate steps, until the solution for the desired mass value is reached – otherwise, you may have difficulty converging a model, or even obtain spurious results.) The mass value should be the same as used in Problem 3 below.

ii) Assuming $\ell = 0$ (i.e., radial pulsations), find the periods of at least the fundamental, first, and second overtone modes. How do the period ratios compare with those seen in class?

iii) For these modes, prepare plots showing the run of $\delta r/r$, $\delta\rho/\rho$, and $\delta T/T$ at maximum pulsation amplitude, all as a function of the radial distance from the center. Discuss your results.

iv) Verify if the eigenfunctions you obtain are indeed orthogonal, as expected, to within the numerical errors of your calculations.

Problem 3. The following is Problem 14.13 in Carroll & Ostlie's book, *"An Introduction to Modern Astrophysics"* (2nd ed.). It describes the calculation of radial pulsation in a very simplified, so-called *one-zone model*, in the *nonlinear case*. For the mass of the Cepheid, you should adopt the following values: Giannina, 4.0 $M_\odot$; Nicolás Castro, 4.5 $M_\odot$; Carolina, 5.0 $M_\odot$; Felipe, 5.5 $M_\odot$; Katerine, 6.0 $M_\odot$; Constanza, 6.5 $M_\odot$; Camila, 7.0 $M_\odot$; Carlos, 7.5 $M_\odot$; Pablo, 8.0 $M_\odot$; Nicolás Rodríguez, 8.5 $M_\odot$; Pascal, 9.0 $M_\odot$; Álvaro, 9.5 $M_\odot$.

In this problem you will carry out a nonlinear calculation of the radial pulsation of the one-zone model described in Example 14.3.1. The equations that describe the oscillation of this model star are Newton's second law for the forces on the shell,

$$m\frac{dv}{dt} = -\frac{GMm}{R^2} + 4\pi R^2 P, \tag{14.20}$$

and the definition of the velocity, v, of the mass shell,

$$v = \frac{dR}{dt}. \tag{14.21}$$

As in Example 14.3.1, we assume that the expansion and contraction of the gas are adiabatic:

$$P_i V_i^\gamma = P_f V_f^\gamma, \tag{14.22}$$

where the "initial" and "final" subscripts refer to any two instants during the pulsation cycle.

(a) Explain in words the meaning of each term in Eq. (14.20).

(b) Use Eq. (14.22) to show that

$$P_i R_i^{3\gamma} = P_f R_f^{3\gamma}. \tag{14.23}$$

(c) You will not be taking derivatives. Instead, you will take the difference between the initial and final values of the radius R and radial velocity v of the shell divided by the time interval Δt separating the initial and final values. That is, you will use $(v_f - v_i)/\Delta t$ instead of dv/dt, and $(R_f - R_i)/\Delta t$ instead of dR/dt in Eqs. (14.20) and (14.21). A careful analysis shows that you should use $R = R_i$ and $P = P_i$ on the right-hand side of Eq. (14.20), and use $v = v_f$ on the left-hand side of Eq. (14.21). Make these substitutions in Eqs. (14.20) and (14.21), and show that you can write

$$v_f = v_i + \left(\frac{4\pi R_i^2 P_i}{m} - \frac{GM}{R_i^2}\right)\Delta t \tag{14.24}$$

and

$$R_f = R_i + v_f \Delta t. \tag{14.25}$$

(d) Now you are ready to calculate the oscillation of the model star. The mass of a typical classical Cepheid is $M = 1 \times 10^{31}$ kg (5 $M_\odot$), and the mass of the surface layers may be arbitrarily assigned $m = 1 \times 10^{26}$ kg. For starting values at time $t = 0$, take

$$R_i = 1.7 \times 10^{10} \text{ m}$$

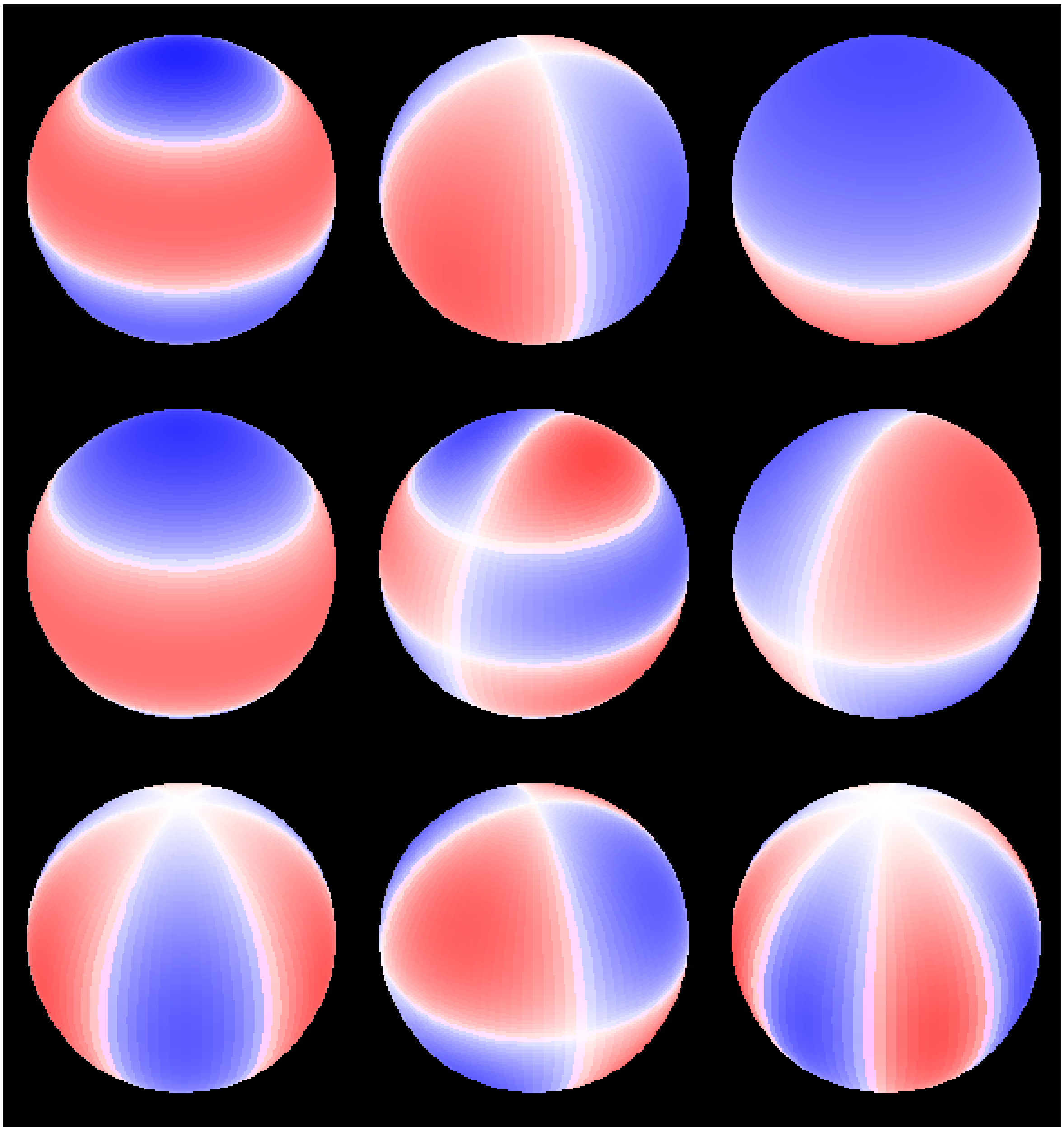
$$v_i = 0 \text{ m s}^{-1}$$

$$P_i = 5.6 \times 10^4 \text{ N m}^{-2}$$

and use a time interval of $\Delta t = 10^4$ s. Take the ratio of specific heats to be $\gamma = 5/3$ for an ideal monatomic gas. Use Eq. (14.24) to calculate the final velocity v_f at the end of one time interval (at time $t = 1 \times 10^4$ s); then use Eq. (14.25) to calculate the final radius R_f and Eq. (14.23) to calculate the final pressure P_f. Now take these final values to be your new initial values, and find new values for R, v, and P after two time intervals (at time $t = 2 \times 10^4$ s). Continue to find R, v, and P for 150 time intervals, until $t = 1.5 \times 10^6$ s. Make three graphs of your results: R vs. t, v vs. t, and P vs. t. Plot the time on the horizontal axis.

(e) From your graphs, measure the period Π of the oscillation (both in seconds and in days) and the equilibrium radius, R_0, of the model star. Compare this value of the period with that obtained from Eq. (14.14). Also compare your results with the period and radial velocity observed for δ Cephei.

Problem 4. En las siguientes figuras, se indican los comportamientos superficiales de distintos modos no radiales de oscilación de estrellas. Cada figura corresponde a un modo con valores de ℓ, *m* distintos. Se pide indicar los valores de ℓ, *m* correspondientes a cada caso, explicando los criterios utilizados en su elección. ¿Qué se puede decir, en base a esas figuras, con respecto al orden ***radial*** de las pulsaciones, en cada caso?



Problem 5. La siguiente figura, extraída de Christensen-Dalsgaard (2002, Rev. Mod. Phys., 74, 1073; acceso aquí) indica los resultados de los cálculos de frecuencias de oscilaciones no radiales para el modelo solar estándar. Los valores que aparecen en la parte superior izquierda corresponden al orden radial *n*; modos *p* están indicados por valores de *n* positivos, y modos *g* por valores de *n* negativos.

a) Se pide intentar reproducir algunas de las frecuencias ahí indicadas, para al menos un modo *p* y un modo *g*, en base al mismo código PULS.FOR utilizado en un problema anterior, a través del cálculo de un modelo de 1.0 $M_\odot$ y composición química adecuada para el Sol. Discuta sus resultados.

b) Confeccione gráficos indicando el comportamiento radial de los desplazamientos radiales relativos, para al menos un modo *p* y un modo *g*. Discuta sus resultados.