Analysis of Algorithms

Homework I

Due January 28th 2017

1. (25 Points) Bounding summations

   Give tight bounds on the following summations. Assume that $r \geq 0$ and $s \geq 0$ are constants.

   (a) $\sum_{k=1}^{n} k^r$

   (b) $\sum_{k=1}^{n} \lg^s k$

   (c) $\sum_{k=0}^{\lfloor \lg n \rfloor} \left\lceil \frac{n}{2^k} \right\rceil$

2. (25 Points) Given the Tree Sort Algorithm (https://en.wikipedia.org/wiki/Tree_sort)

   (a) Please give me the count of each step in the Tree Sort.

   (b) Provide the best case, the average and worst case for Tree Sort.

   (c) Give me the correctness of the algorithm using induction.

3. (25 Points) From chapter 2 and 3 in Cormen's book do

   (a) Exercise 2.3-3

   (b) Exercise 3.1-7

4. (25 Points) Give asymptotic upper and lower bounds for $T(n)$ in:

   (a) $T(n) = 9T\left(\frac{n}{81}\right) + \log n$.

   (b) $T(n) = T(n-1) + n^c$,where $c \leq 1$ is a constant.

   (c) $T(n) = T(n^{\frac{1}{10}}) + 1$.