COMP 472: Artificial Intelligence

State Space Search

Solutions

Question 1 Once upon a time a farmer went to the market and purchased a fox, a goose, and a bag of beans. On his way home, the farmer came to the bank of a river and rented a boat. But in crossing the river by boat, the farmer could carry only himself and a single one of his purchases - the fox, the goose, or the bag of the beans.

If left alone, the fox would eat the goose, and the goose would eat the beans. The farmer's challenge was to carry himself and his purchases to the far bank of the river, leaving each purchase intact.

Represent this problem as a search problem. Choose a representation for the problem's states and:

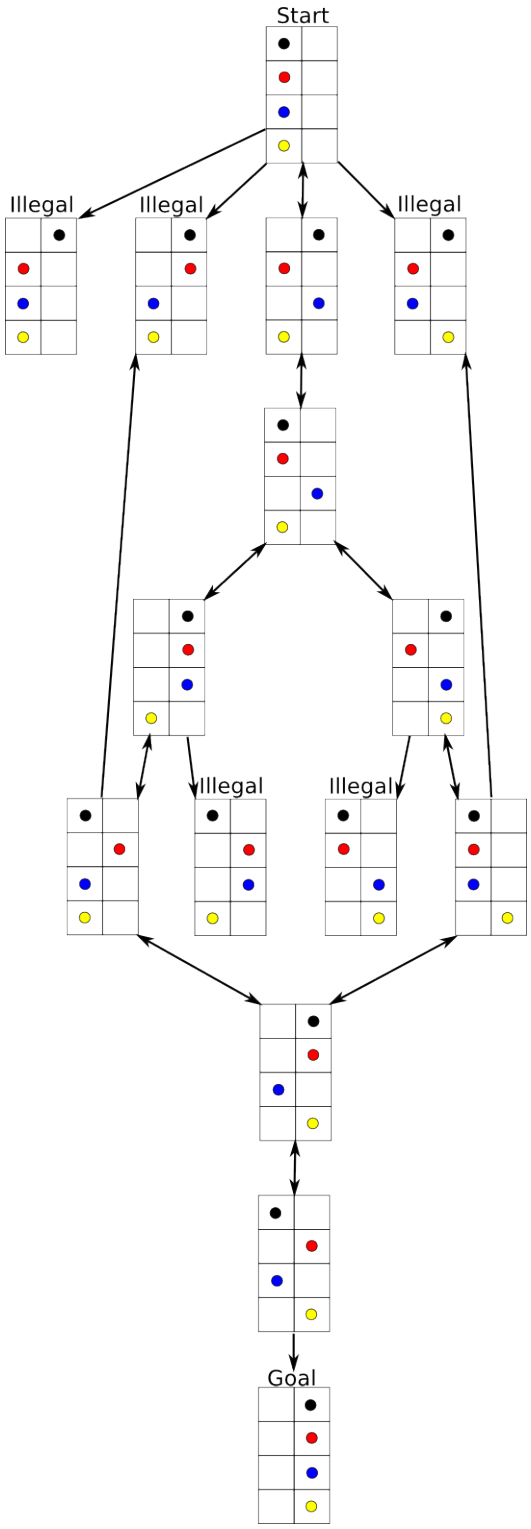
(a) Write down the initial state
Let position(farmer, fox, goose, beans) represent the position of the farmer, the fox, the goose and the beans with respect to the river. The possible positions are o *for "original bank" and* f *for "far bank".*
Initially, the state is: position(o,o,o,o)

(b) Write down the goal state
position(f,f,f,f)

(c) Write down all illegal states
positions(f,o,o,o), positions(f,o,o,f), positions(o,f,f,o), positions(o,f,f,f), positions(f,o,o,o), positions(f,f,o,o), positions(o,o,f,f), positions(o,f,f,f).

(d) Write down the possible actions
moveFar(farmer, empty), moveFar(farmer, fox), moveFar(farmer, goose), moveFar(farmer, beans), moveBack(farmer, empty), moveBack(farmer, fox), moveBack(farmer, goose), moveBack(farmer, beans)

(e) Draw the state space for this problem

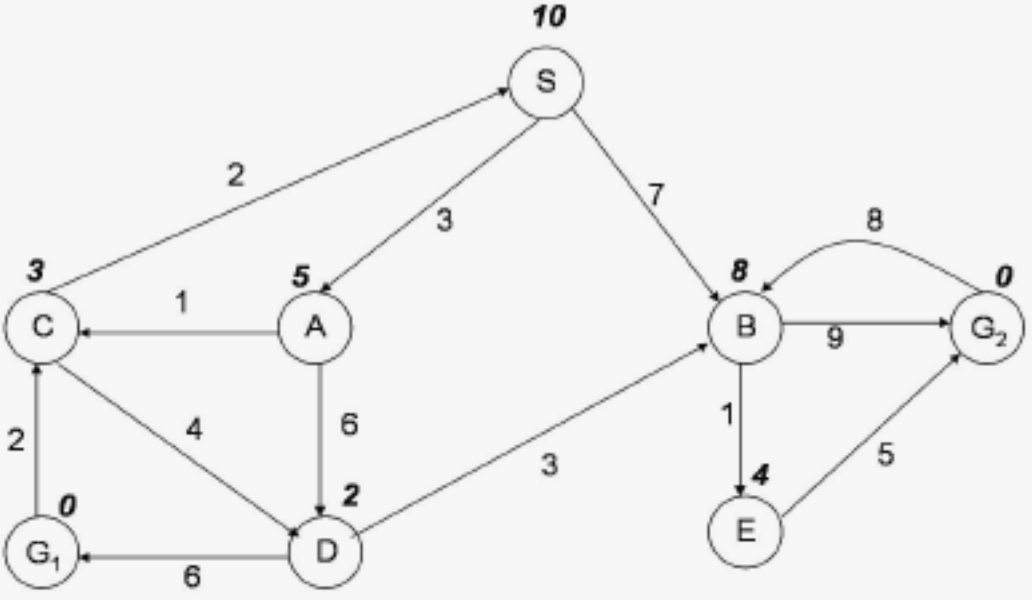


(f) Find a series of moves to solve this problem
moveFar(farmer, goose), moveBack(farmer, empty), moveFar(farmer, fox), moveBack(farmer, goose), moveFar(farmer, beans), moveBack(farmer, empty), moveBack(farmer, goose).

OR

moveFar(farmer, goose), moveBack(farmer, empty), moveFar(farmer, beans), moveBack(farmer, goose), moveFar(farmer, fox), moveBack(farmer, empty), moveBack(farmer, goose).

Question 2 Assume that S is the initial state and G_1 and G_2 are the goal states. The possible actions between states are indicated by arrows. The number labelling each arc is the actual cost of the action. For example, the cost of going from S to A is 3. The number in bold italic near each state is the value of the heuristic function h at that state. For example, the value of h at state C is 3. When all else is equal, expand states in alphabetical order.



For the following search strategies, show the states visited, along with the open and closed lists at each step (where it applies).

(a) Breadth-first search

Step	Visited State	Closed List	Open List
1			S
2	S	S	A, B
3	A	S, A	B, C, D
4	B	S, A, B	C, D, E, G2
5	C	S, A, B, C	D, E, G2
6	D	S, A, B, C, D	E, G2, G1
7	E	S, A, B, C, D, E	G2, G1
8	G2	S, A, B, C, D, E, G2	G1

(b) Depth-first search

Step	Visited State	Closed List	Open List
1			S
2	S	S	A, B
3	A	S, A	C, D, B
4	C	S, A, C	D, B
5	D	S, A, C, D	G1, B
6	G1	S, A, C, D, G1	B

(c) Iterative deepening depth-first search *Note: Depth of each state in the open list is in parenthesis.*

Depth 1:

Step	Visited State	Closed List	Open List
1			S(1)
2	S	S	

Depth 2:

Step	Visited State	Closed List	Open List
1			S(1)
2	S	S	A(2), B(2)
3	A	S, A	B(2)
4	B	S, A, B	

Depth 3:

Step	Visited State	Closed List	Open List
1			S(1)
2	S	S	A(2), B(2)
3	A	S, A	C(3), D(3), B(2)
4	C	S, A, C	D(3), B(2)
5	D	S, A, C, D	B(2)
6	B	S, A, C, D, B	E(3), G2(3)
7	E	S, A, C, D, B, E	G2(3)
8	G2	S, A, C, D, B, E, G2	

(d) Uniform cost search

Step	Visited State	Closed List	Open List
1			S(0)
2	S	S	A(3), B(7)
3	A	S, A	C(4), B(7), D(9)
4	C	S, A, C	B(7), D(8); lower cost to D replaces previous cost
5	B	S, A, C, B	D(8), E(8), G2(16)
6	D	S, A, C, B, D	E(8), G1(14), G2(16)
7	E	S, A, C, B, D, E	G2(13), G1(14); G2(13) replaces G2(16)
8	G2	S, A, C, B, D, E, G2	

(e) Hill climbing *Note: Hill climbing does not use open and closed lists, instead the first neighboring state with a better heuristic than the current state is visited.*

Step	Visited State	Evaluations
1	S	A(5) < S(10)
2	A	C(3) < A(5)
3	C	D(2) < C(3)
4	D	B(8) $\not<$ D(2); G1(0) < D(2)
5	G1	

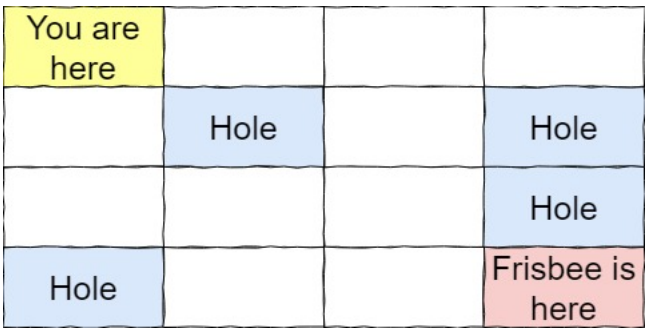
(f) Best-first search

Step	Visited State	Closed List	Open List
1			S(10)
2	S	S	A(5), B(8)
3	A	S, A	D(2), C(3), B(8)
4	D	S, A, D	G1(0), C(3), B(8)
5	G1	S, A, D, G1	C(3), B(8)

(g) Algorithm A

Step	Visited State	Closed List	Open List
1			S(10)
2	S	S	A(3+5=8), B(7+8=15)
3	A	S, A	C(3+1+3=7), D(3+6+2=11), B(15)
4	C	S, A, C	D(3+1+4+2=10), B(15)
5	D	S, A, C, D	G1(3+1+4+6+0=14), B(15)
6	G1	S, A, C, D, G1	

Question 3 *Exercise from OpenAI*[1]*:* Winter is here. You and your friends were tossing around a Frisbee at the park when you made a wild throw that left the Frisbee out in the middle of the lake. The water is mostly frozen, but there are a few holes where the ice has melted. If you step into one of those holes, you'll fall into the freezing water. At this time, there's an international Frisbee shortage, so it's absolutely imperative that you navigate across the lake and retrieve the disc as soon as you can. The surface is described using a rectangular grid like the figure below:



(a) What are the initial and goal state?
Let a 4×4 *matrix represent the above grid. The position of each cell in the grid can then be addressed by the indices of the elements of the matrix. Hence we have:*

- *Initial state:* $(1, 1)$
- *Goal state:* $(4, 4)$

(b) What are the illegal states?
The illegal states are the holes in the grid with the following positions:

Illegal State 1: $(2, 2)$

Illegal State 2: $(2, 4)$

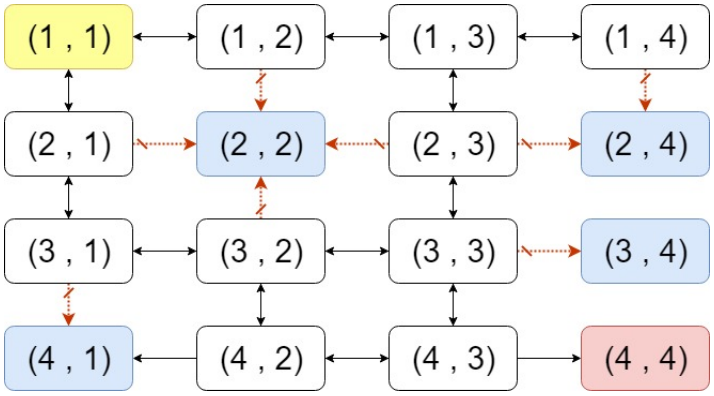
Illegal State 3: $(3, 4)$

Illegal State 4: $(4, 1)$

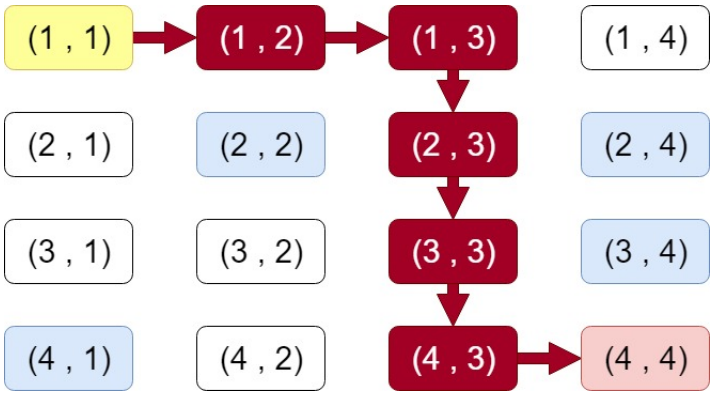
(c) What are the possible actions and what should be their costs?
The possible actions in a grid world are moving left, right, up, and down. Since we would like to reach the goal state as soon as possible (i.e. minimizing the number of actions), then we can assign a constant uniform cost for each action, for example a cost of 1*. The algorithm aims to minimize the cost while reaching the goal state.*

(d) Draw the state space for this problem.
The state space is shown in the figure below, where the initial state is colored yellow, the goal state is colored red, and illegal states are colored blue.

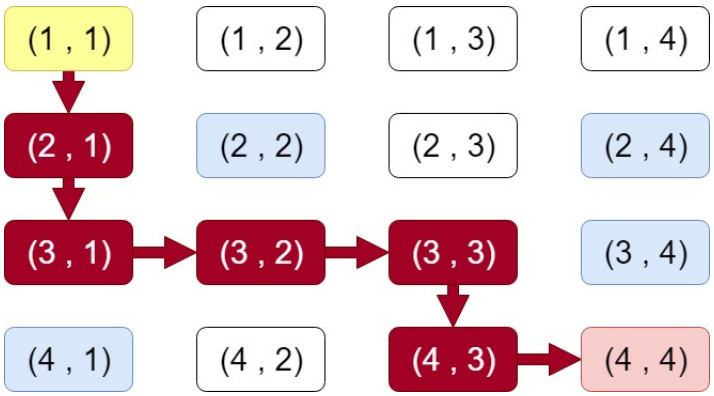
[1] `https://gym.openai.com/envs/FrozenLake-v0/`



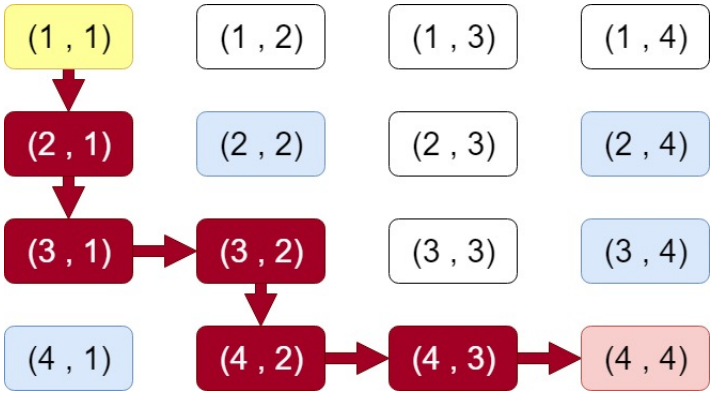
(e) Find a series of moves to solve this problem.
Any of the following series of moves constitute a possible solution for this problem:
Path 1: $(1,1) \to (1,2) \to (1,3) \to (2,3) \to (3,3) \to (4,3) \to (4,4)$



Path 2: $(1,1) \to (2,1) \to (3,1) \to (3,2) \to (3,3) \to (4,3) \to (4,4)$



Path 3: $(1,1) \to (2,1) \to (3,1) \to (3,2) \to (4,2) \to (4,3) \to (4,4)$



(f) Perform the following search strategies, show the states visited, along with the open and closed lists at each step.

I Breadth-first search

The detailed execution of breadth-first search is listed below. A pictorial representation of the progress of this algorithm can be seen in Figure [1].

Step 1
- *Visited:*
- *open:* $\{(1,1)\}$
- *closed:* $\emptyset$

Step 2
- *Visited:* $(1,1)$
- *open:* $\{(1,2),(2,1)\}$
- *closed:* $\{(1,1)\}$

Step 3
- *Visited:* $(1,2)$
- *open:* $\{(2,1),(1,3)\}$
- *closed:* $\{(1,2),(1,1)\}$

Step 4
- *Visited:* $(2,1)$
- *open:* $\{(1,3),(3,1)\}$
- *closed:* $\{(2,1),(1,2),(1,1)\}$

Step 5
- *Visited:* $(1,3)$
- *open:* $\{(3,1),(1,4),(2,3)\}$
- *closed:* $\{(1,3),(2,1),(1,2),(1,1)\}$

Step 6
- *Visited:* $(3,1)$
- *open:* $\{(1,4),(2,3),(3,2)\}$
- *closed:* $\{(3,1),(1,3),(2,1),(1,2),(1,1)\}$

Step 7
- *Visited:* $(1,4)$
- *open:* $\{(2,3),(3,2)\}$
- *closed:* $\{(1,4),(3,1),(1,3),(2,1),(1,2),(1,1)\}$

Step 8
- *Visited:* $(2,3)$
- *open:* $\{(3,2),(3,3)\}$
- *closed:* $\{(2,3),(1,4),(3,1),(1,3),(2,1),(1,2),(1,1)\}$

Step 9
- *Visited:* $(3, 2)$
- *open:* $\{(3, 3), (4, 2)\}$
- *closed:* $\{(3, 2), (2, 3), (1, 4), (3, 1), (1, 3), (2, 1), (1, 2), (1, 1)\}$

Step 10
- *Visited:* $(3, 3)$
- *open:* $\{(4, 2), (4, 3)\}$
- *closed:* $\{(3, 3), (3, 2), (2, 3), (1, 4), (3, 1), (1, 3), (2, 1), (1, 2), (1, 1)\}$

Step 11
- *Visited:* $(4, 2)$
- *open:* $\{(4, 3)\}$
- *closed:* $\{(4, 2), (3, 3), (3, 2), (2, 3), (1, 4), (3, 1), (1, 3), (2, 1), (1, 2), (1, 1)\}$

Step 12
- *Visited:* $(4, 3)$
- *open:* $\{(4, 4)\}$
- *closed:* $\{(4, 3), (4, 2), (3, 3), (3, 2), (2, 3), (1, 4), (3, 1), (1, 3), (2, 1), (1, 2), (1, 1)\}$

Step 13
- *Visited:* $(4, 4)$
- *open:* $\{\}$
- *closed:*

$$\{(4, 4), (4, 3), (4, 2), (3, 3), (3, 2), (2, 3), (1, 4), (3, 1), (1, 3), (2, 1), (1, 2), (1, 1)\}$$

- *The goal state is observed at this point and the search has terminated.*

II Uniform cost search

Since we have assumed that all step cost are equal, then the uniform-cost search and breadth-first search will have the same result.

III Depth-first search

The detailed execution of depth-first search is listed below. A pictorial representation of the progress of this algorithm can be seen in Figure [2].

Step 1
- *Visited:*
- *open:* $\{(1, 1)\}$
- *closed:* $\emptyset$

Step 2
- *Visited:* $(1, 1)$
- *open:* $\{(1, 2), (2, 1)\}$
- *closed:* $\{(1, 1)\}$

Step 3
- *Visited:* $(1, 2)$
- *open:* $\{(1, 3), (2, 1)\}$
- *closed:* $\{(1, 2), (1, 1)\}$

Step 4
- *Visited:* $(1, 3)$
- *open:* $\{(1, 4), (2, 3), (2, 1)\}$

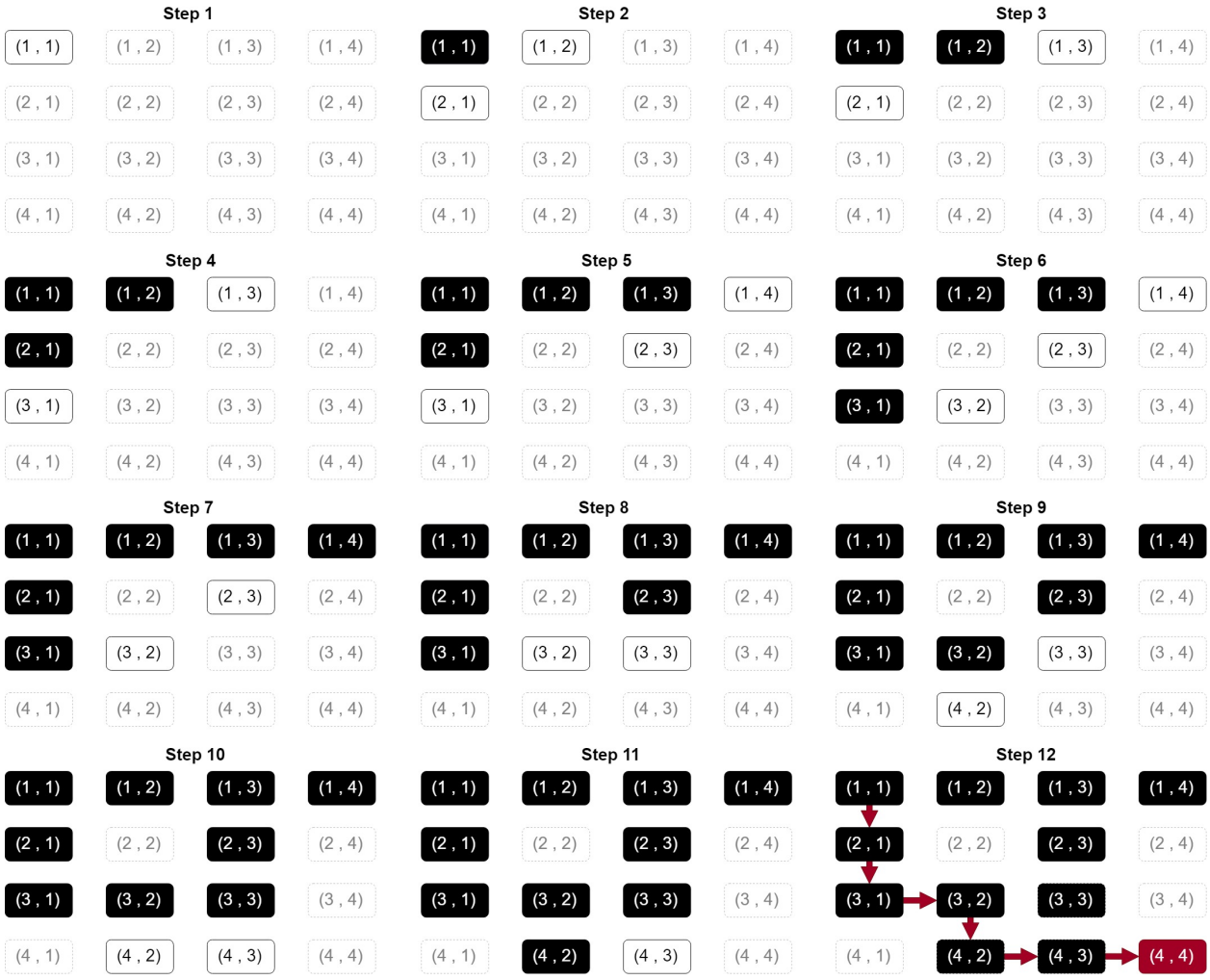


Figure 1: Progress of breadth-first search algorithm is depicted above. The cells with solid white background represent the open set. The cells with solid black background represent the closed set and the cell with solid red background is the goal state.

- *closed:* $\{(1,3),(1,2),(1,1)\}$

Step 5
- *Visited:* $(1,4)$
- *open:* $\{(2,3),(2,1)\}$
- *closed:* $\{(1,4),(1,3),(1,2),(1,1)\}$

Step 6
- *Visited:* $(2,3)$
- *open:* $\{(3,3),(2,1)\}$
- *closed:* $\{(2,3),(1,4),(1,3),(1,2),(1,1)\}$

Step 7
- *Visited:* $(3,3)$
- *open:* $\{(3,2),(4,3),(2,1)\}$
- *closed:* $\{(3,3),(2,3),(1,4),(1,3),(1,2),(1,1)\}$

Step 8
- *Visited:* $(3,2)$
- *open:* $\{(3,1),(4,2),(4,3),(2,1)\}$
- *closed:* $\{(3,2),(3,3),(2,3),(1,4),(1,3),(1,2),(1,1)\}$

Step 9
- *Visited:* $(3,1)$
- *open:* $\{(4,2),(4,3),(2,1)\}$
- *closed:* $\{(3,1),(3,2),(3,3),(2,3),(1,4),(1,3),(1,2),(1,1)\}$

Step 10
- *Visited:* $(4,2)$
- *open:* $\{(4,3),(2,1)\}$
- *closed:* $\{(4,2),(3,1),(3,2),(3,3),(2,3),(1,4),(1,3),(1,2),(1,1)\}$

Step 11
- *Visited:* $(4,3)$
- *open:* $\{(4,4),(2,1)\}$
- *closed:* $\{(4,3),(4,2),(3,1),(3,2),(3,3),(2,3),(1,4),(1,3),(1,2),(1,1)\}$

Step 12
- *Visited:* $(4,4)$
- *open:* $\{(2,1)\}$
- *closed:* $\{(4,4),(4,3),(4,2),(3,1),(3,2),(3,3),(2,3),(1,4),(1,3),(1,2),(1,1)\}$
- *The goal state is observed at this point and the search has terminated.*

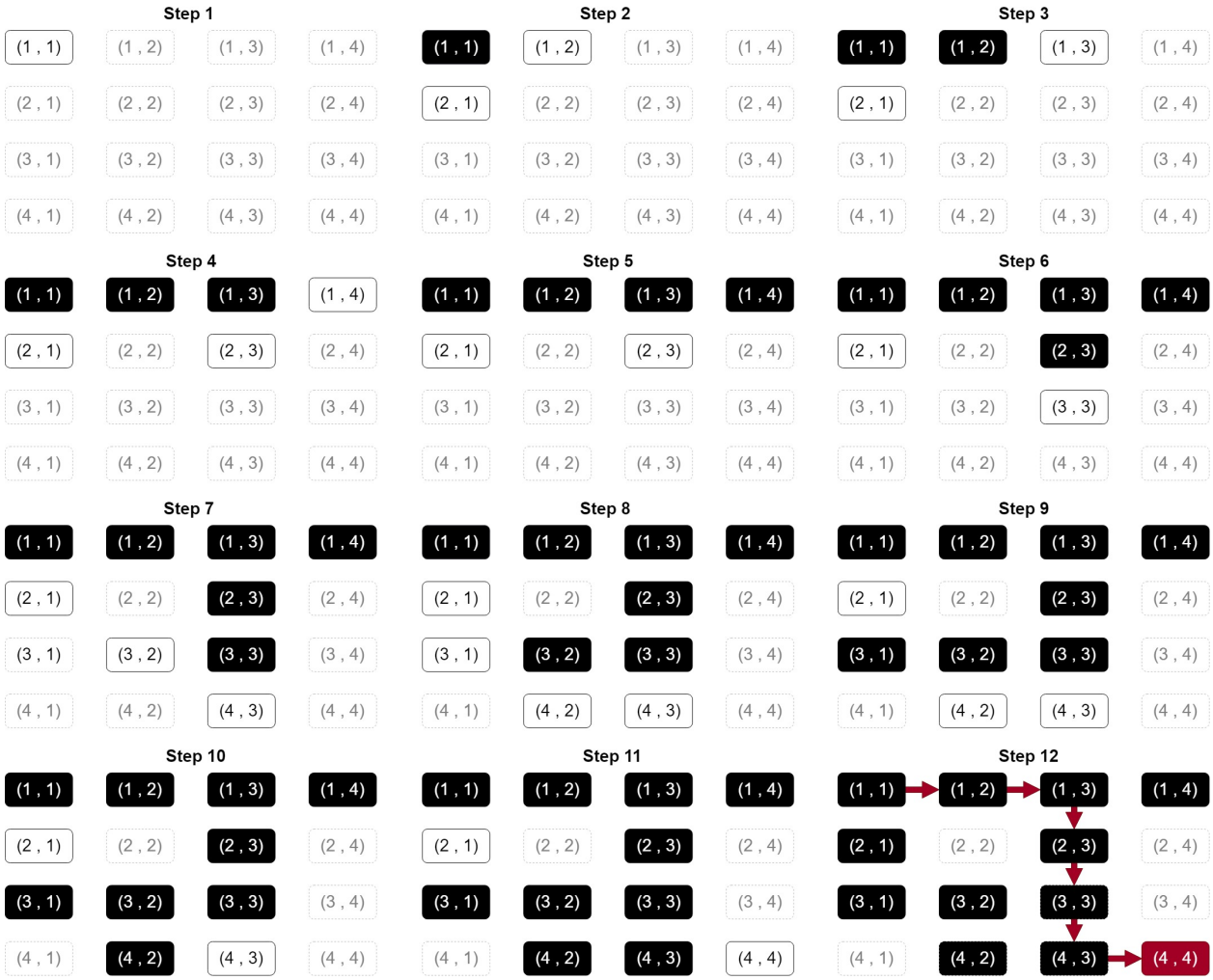


Figure 2: Progress of depth-first search algorithm is depicted above. The cells with solid white background represent the open set. The cells with solid black background represent the closed set and the cell with solid red background is the goal state.

Question 4 Consider the classical game of Tetris. The objective of the game is to move and rotate each falling block so as to create an entire row of block pieces without any gap. If such a row is created, then that row disappears, and all rows on top of it fall down.

For example, in the figure above, a 1x4 block is falling from the top. The player can move this block left and right and rotate it by 90 degree to have it upright. As the figure shows, the player has at least 5 choices to position the block. Out of these 5 choices, position 5 is preferable, because then the 4 bottom rows would be complete. These rows would then disappear; all rows above would fall down, leaving more space on top to place the next random block to fall. The game ends when too many blocks are stacked up (no new complete rows can be created) and no more blocks can be placed on the board.

(a) Formulate a simple heuristic to determine how to place a falling (random) block on an existing board.
A possible heuristic is to count the number of complete rows created (that will be deleted). In that case, h(option2) = 0 and h(option4) = 0.
Another heuristic is to count the number of "touching sides" i.e. how many sides of the falling block will be in contact with the Tetris pile. In that case, h(option2) = 5 (2 edges on the left + the bottom + 2 edges on the right) h(option4) = 8 (4 edges on the left + the bottom + 3 edges on the right)
There are plenty of other possible heuristics.

(b) Now apply your heuristic to evaluate option 2 and option 4 of the figure above.