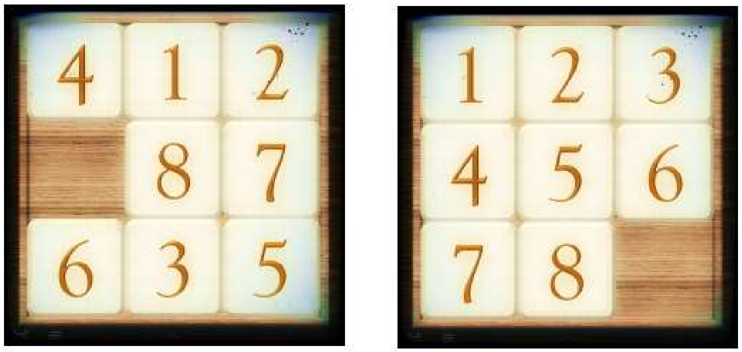
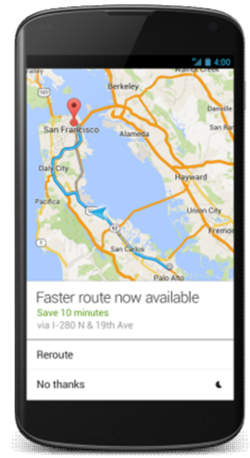
Artificial Intelligence: State Space Search

- Russell & Norvig – chap. 3 & section 4.1.1
- Many slides from: robotics.stanford.edu/~latombe/cs121/2003/home.htm

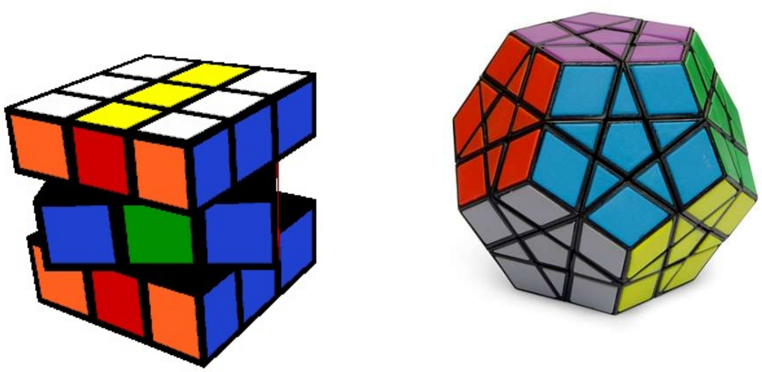
Motivation



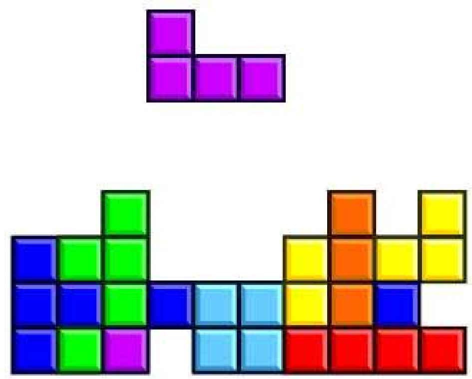
8-puzzle



Google Maps



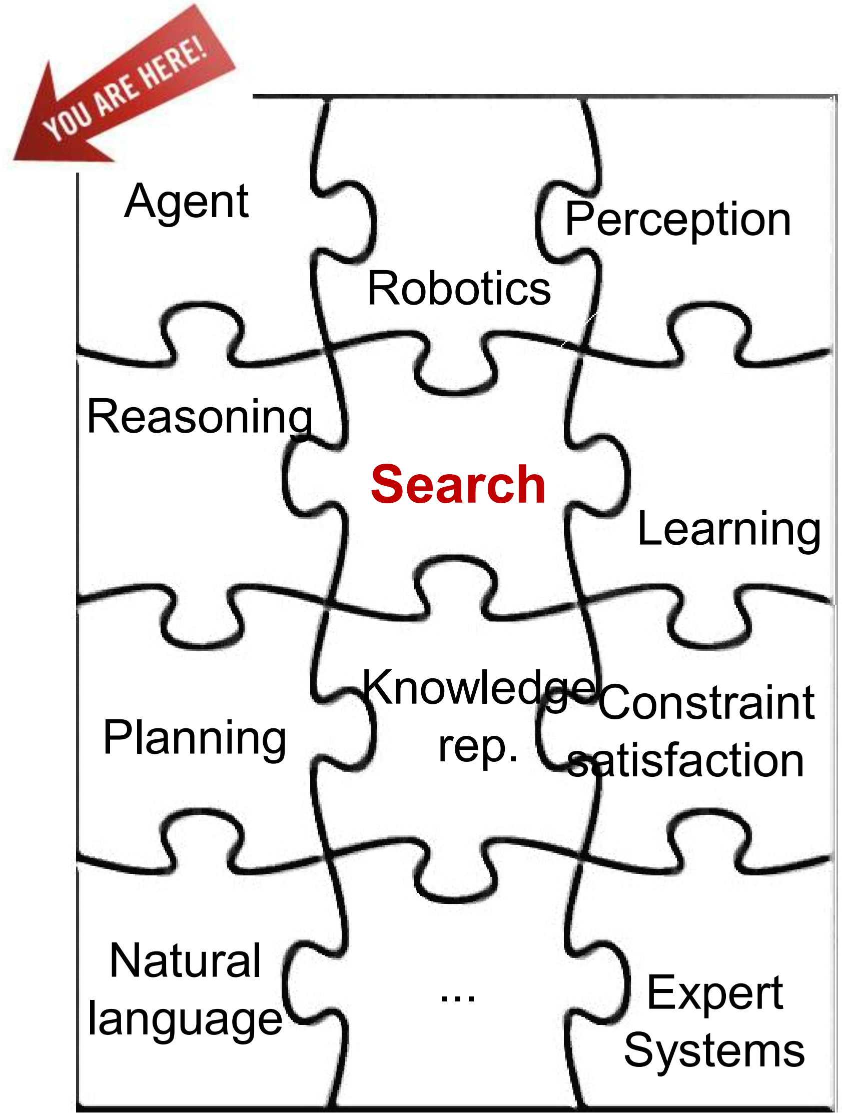
Rubik's cube



Tetris

Today

- State Space Representation
- State Space Search
  - Uninformed search
    - Breadth-first and Depth-first
    - Depth-limited Search
    - Iterative Deepening
    - Uniform Cost
  - Informed search
    - Hill climbing
    - Best-First
    - (Designing Heuristics)
    - A*
- Summary



Example: 8-Puzzle

State: Any arrangement of 8 numbered tiles and an empty tile on a 3x3 board

8	2	
3	4	7
5	1	6

Initial state

1	2	3
4	5	6
7	8	

Goal state

there are several standard goals states for the 8-puzzle

1	2	3
4	5	6
7	8	

1	2	3
8		4
7	6	5

...

(n^2-1)-puzzle

8	2	
3	4	7
5	1	6

8-puzzle

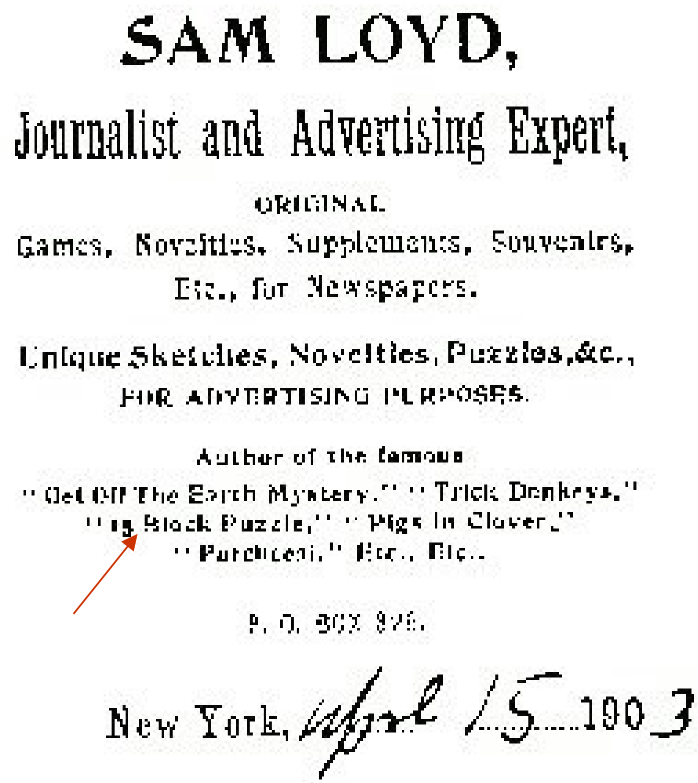
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

15-puzzle

....

15-Puzzle

Invented in 1874 by Noyes Palmer Chapman
… but Sam Loyd claimed he invented it!



SAM LOYD,

Journalist and Advertising Expert,

ORIGINAL

Games, Novelties, Supplements, Souvenirs, Etc., for Newspapers.

Unique Sketches, Novelties, Puzzles, &c., FOR ADVERTISING PURPOSES.

Author of the famous
"Get Off The Earth Mystery," "Trick Donkeys,"
"15 Block Puzzle," "Pigs In Clover,"
"Parcheesi," Etc., Etc.

P. O. BOX 826.

New York, April 15 1903

15-Puzzle

Sam Loyd even offered $1,000 of his own money to the first person who would solve the following problem:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

$\xrightarrow{?}$

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

But no one ever won the prize…

State Space

- Many AI problems, can be expressed in terms of going from an initial state to a goal state
  - *Ex: to solve a puzzle, to drive from home to Concordia…*

- Often, there is no direct way to find a solution to a problem
- but we can list the possibilities and search through them

1. Brute force search:
   - generate and search <u>all</u> possibilities (but inefficient)
2. Heuristic search:
   - only try the possibilities that you *think* (based on your current best guess) are more likely to lead to good solutions

State Space

- Problem is represented by:
  1. Initial State
     - starting state
     - ex. unsolved puzzle, being at home
  2. Set of operators
     - actions responsible for transition between states
  3. Goal test function
     - Applied to a state to determine if it is a goal state
     - ex. solved puzzle, being at Concordia
  4. Path cost function
     - Assigns a cost to a path to tell if a path is preferable to another
- Search space: the set of all states that can be reached from the initial state by any sequence of action
- Search algorithm: how the search space is visited

Example: The 8-puzzle

8	2	
3	4	7
5	1	6

Initial state

1	2	3
4	5	6
7	8	

Goal state

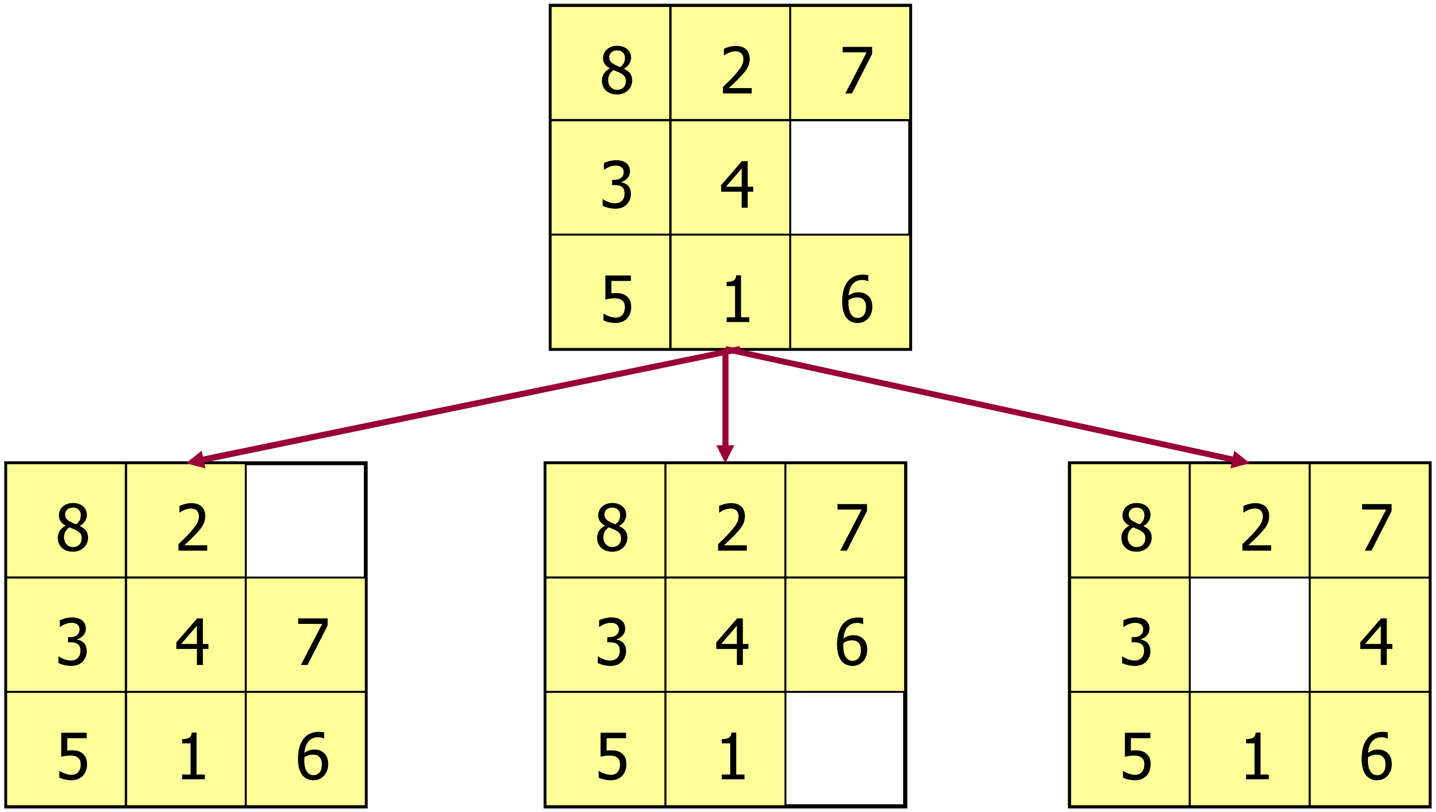
Set of operators:
blank moves up, blank moves down, blank moves left, blank moves right

Goal test function:
state matches the goal state

Path cost function:
each movement costs 1
so the path cost is the length of the path (the number of moves)

source: G. Luger (2005)

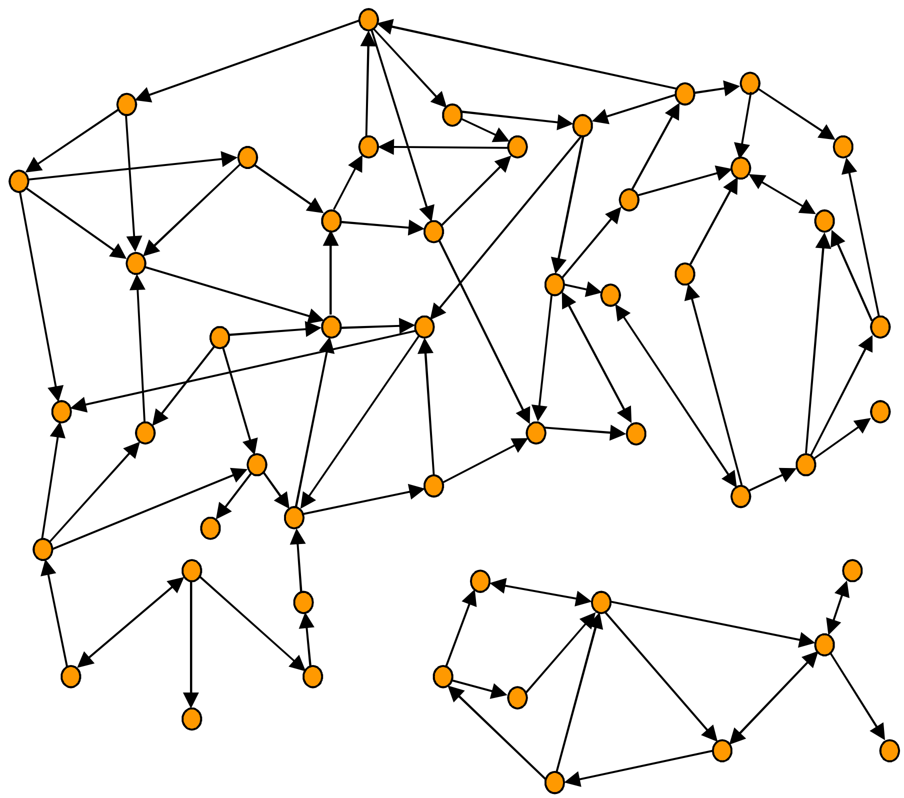
8-Puzzle: Successor Function



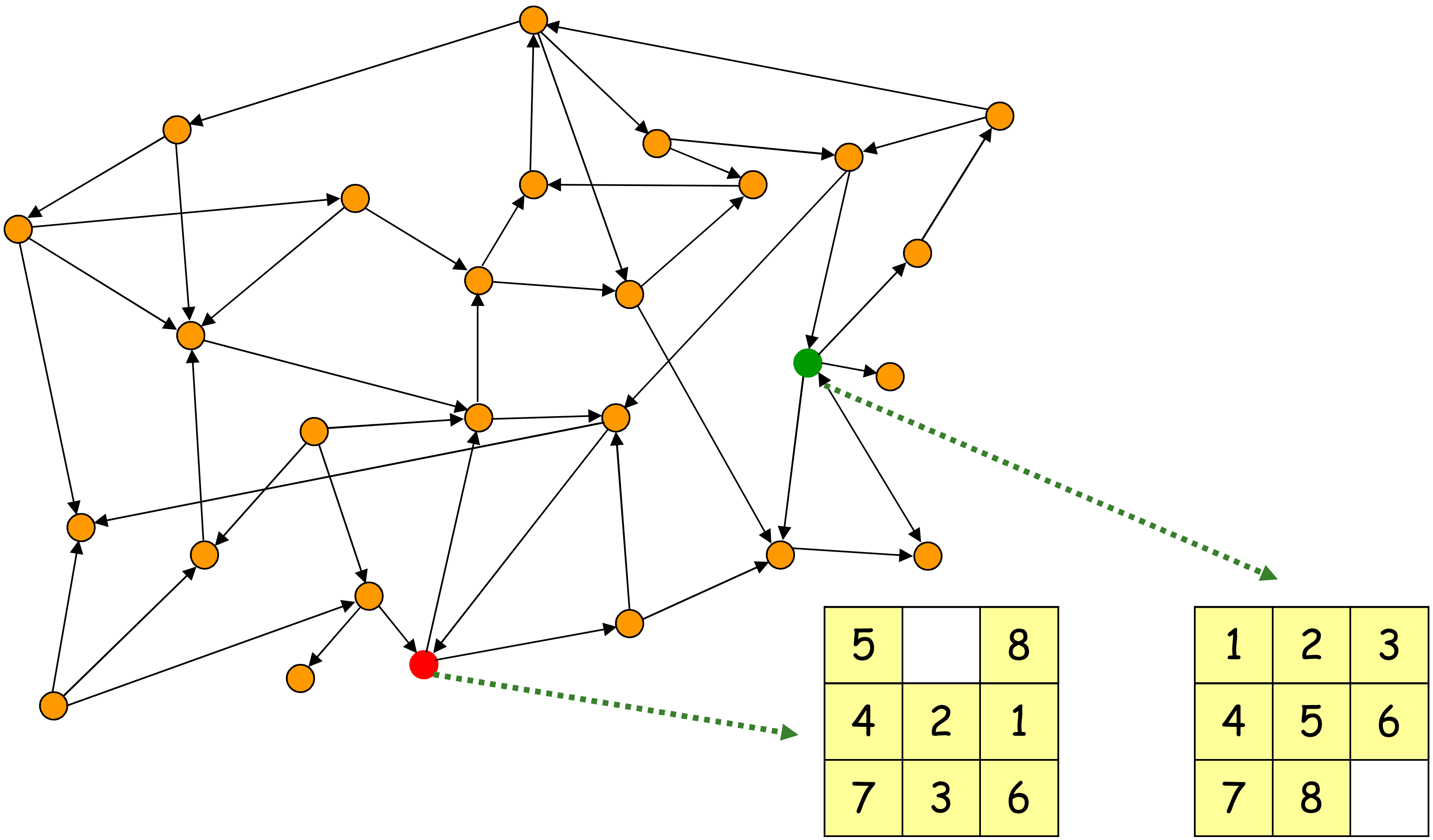
Search is about the exploration of alternatives

State Graph

- Each state is represented by a distinct node
- An arc (or edge) connects a node s to a node s' if s' ∈ SUCCESSOR(s)
- The state graph may contain more than one connected component



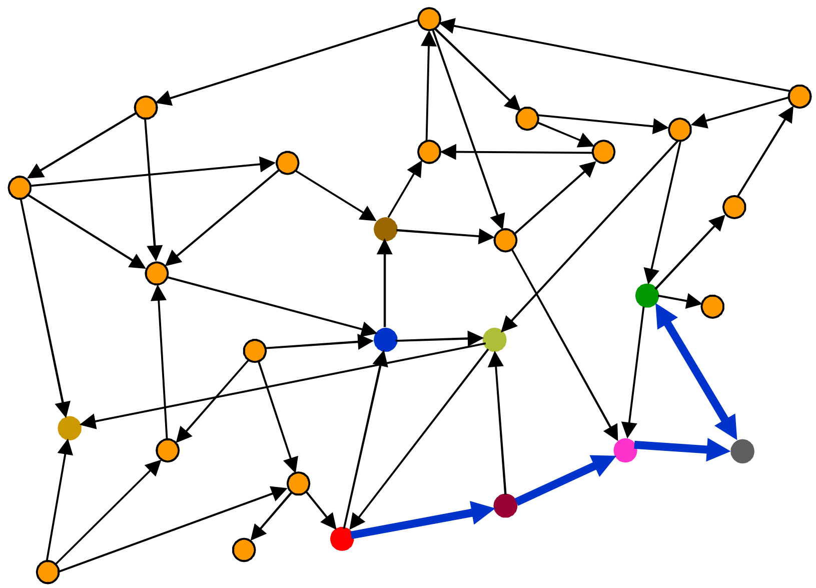
Just to make sure we're clear…

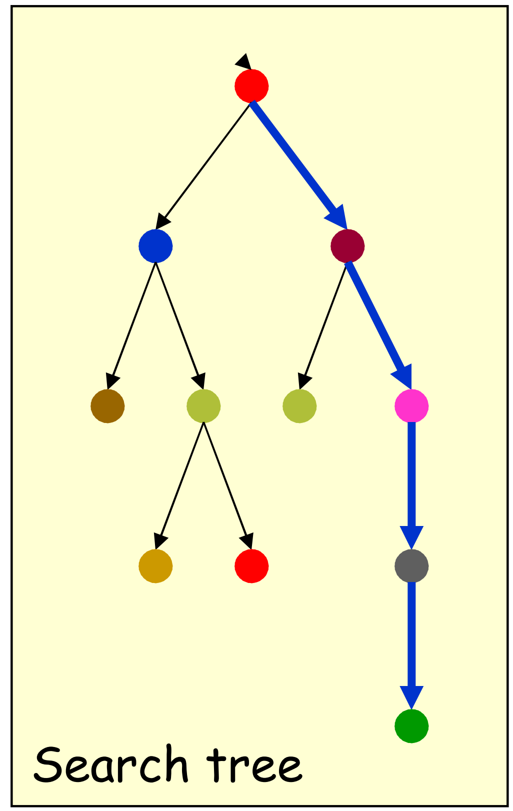


Initial state

Goal state

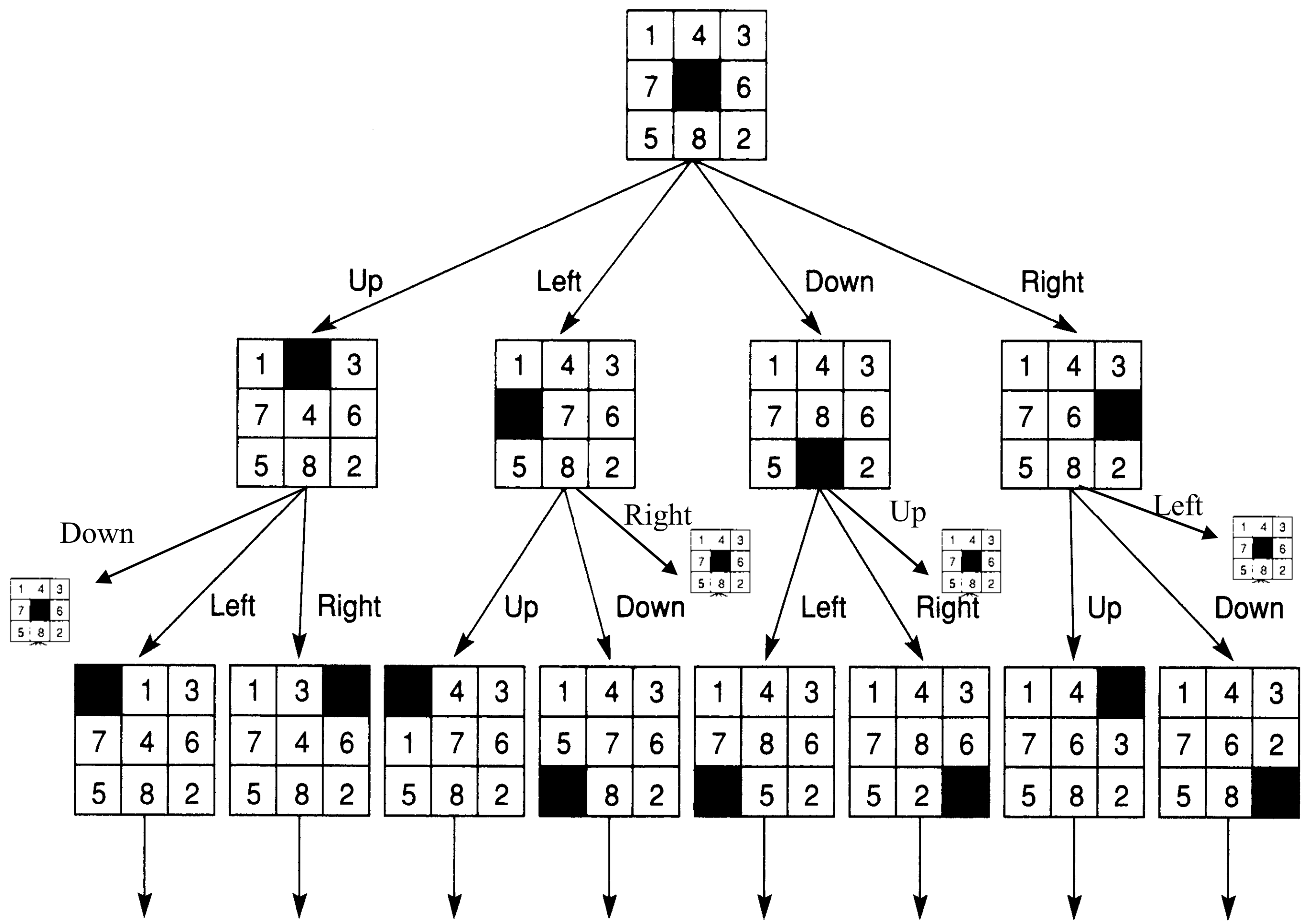
State Space as a Search Tree





- In graph representation, cycles can prevent termination
  - Blind search without cycle check may never terminate
- Use a tree representation, and check for cycles

State Space for the 8-puzzle



source: G. Luger (2005)

How large is the state space of the (n^2-1)-puzzle?

- Nb of states:
  - 8-puzzle --> 9! = 362,880 states
  - 15-puzzle --> 16! ~ 2.09×10^{13} states
  - 24-puzzle --> 25! ~ 10^{25} states
- At 100 millions states/sec:
  - 8-puzzle --> 0.036 sec
  - 15-puzzle --> ~ 55 hours
  - 24-puzzle --> > 10^9 years

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  - Uninformed search
    - Breadth-first and Depth-first
    - Depth-limited Search
    - Iterative Deepening
    - Uniform Cost
  - Informed search
    - Hill climbing
    - Best-First
    - (Designing Heuristics)
    - A*
- Summary

Uninformed VS Informed Search

- Uninformed search
  - We systematically explore the alternatives
  - aka: systematic/exhaustive/blind/brute force search
    - Breadth-first
    - Depth-first
    - Uniform-cost
    - Depth-limited search
    - Iterative deepening search
    - Bidirectional search
    - …
- Informed search (heuristic search)
  - We try to choose smartly
    - Hill climbing
    - Best-First
    - A*
    - …

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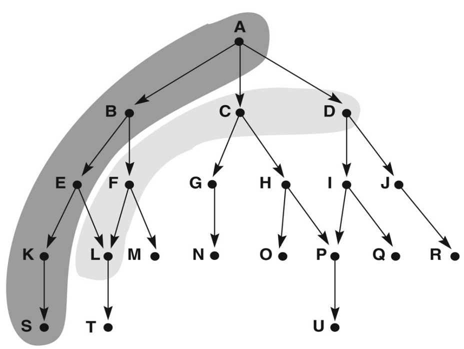
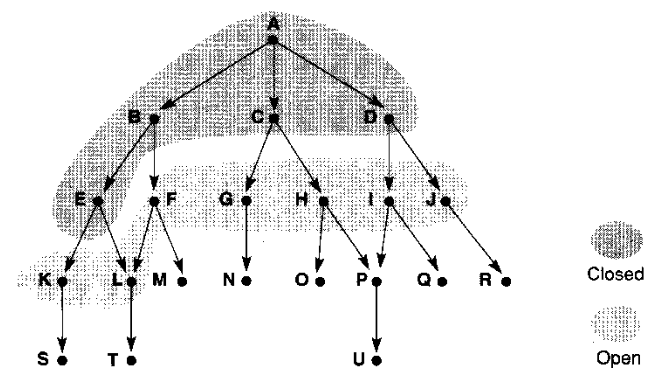
Breadth-first vs Depth-first Search

- Determine order for examining states
  - Depth-first:
    - visit successors before siblings
  - Breadth-first:
    - visit siblings before successors
    - ie. visit level-by-level



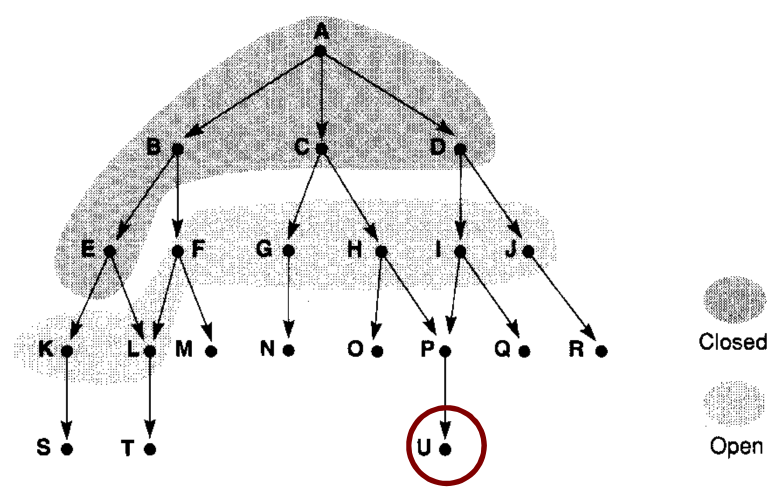


source: *G. Luger (2005)*

Data Structures

- In all search strategies, you need:
  - ***open list*** (aka the frontier)
    - lists generated nodes not yet expanded
    - order of nodes controls order of search
  - ***closed list*** (aka the explored set)
    - stores all the nodes that have already been visited (to avoid cycles).
- ex:

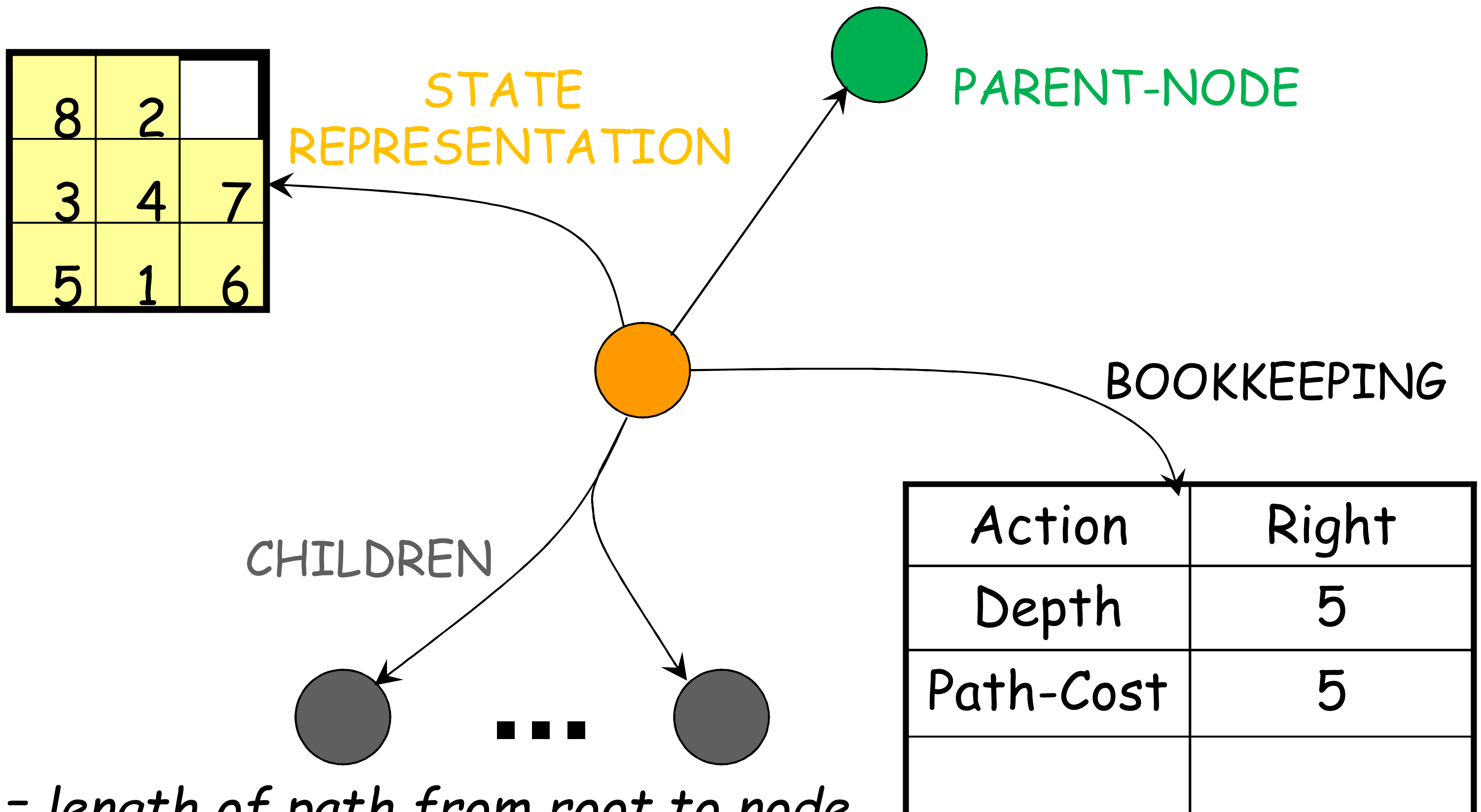
Closed = [A, B, C, D, E]
Open = [F, G, H, I, J, K, L]



source: G. Luger (2005)

Data Structures

- To trace back the entire path of the solution after the search, each node in the lists contain:



Action	Right
Depth	5
Path-Cost	5

Depth = length of path from root to node

Generic Search Algorithm

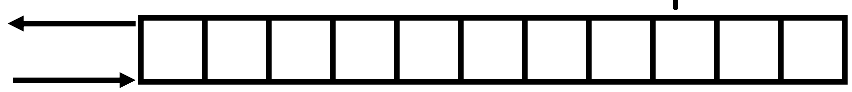
1. Initialize the open list with the initial node s_o (top node)
2. Initialize the closed list to empty
3. Repeat
   a) If the open list is empty, then exit with failure.
   b) Else, take the first node s from the open list.
   c) If s is a goal state, exit with success. Extract the solution path from s to s_o
   d) Else, insert s in the closed list (s has been visited /expanded)
   e) Insert the successors of s in the open list in a certain order if they are not already in the closed and/or open lists (to avoid cycles)

Notes:

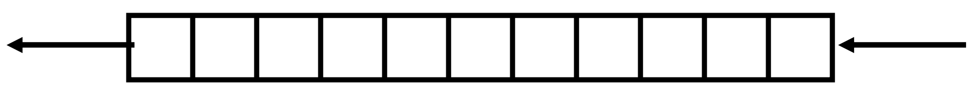
- The order of the nodes in the open list depends on the search strategy

DFS and BFS

- DFS and BFS differ only in the way they order nodes in the open list:
  - DFS uses a stack:
    - nodes are added on the top of the list.



  - BFS uses a queue:
    - nodes are added at the end of the list.



Breadth-First Search

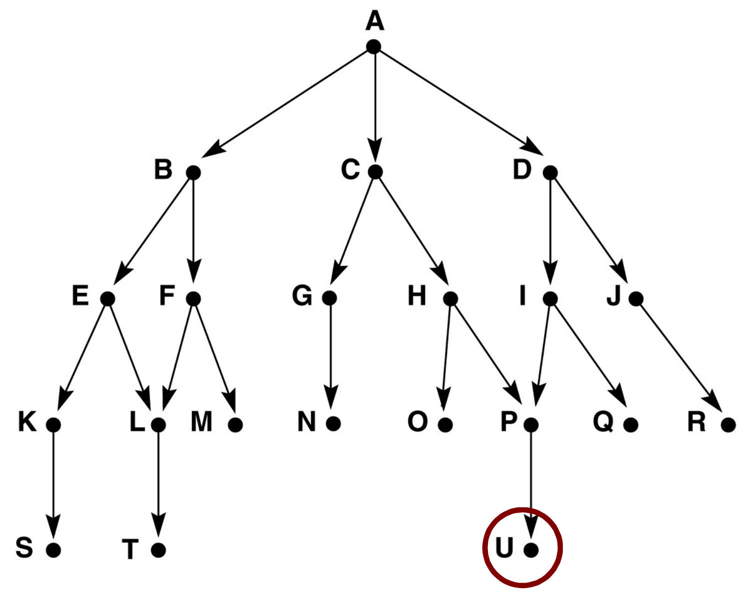
```
begin
  open := [Start];                                                % initialize
  closed := [ ];
  while open ≠ [ ] do                                             % states remain
    begin
      remove leftmost state from open, call it X;
        if X is a goal then return SUCCESS                        % goal found
          else begin
            generate children of X;
            put X on closed;
            discard children of X if already on open or closed;  % loop check
            put remaining children on right end of open           % queue
          end
    end
  return FAIL                                                     % no states left
end.
```

source: G. Luger (2005)

Breadth-First Search Example

- BFS: (open is a queue)

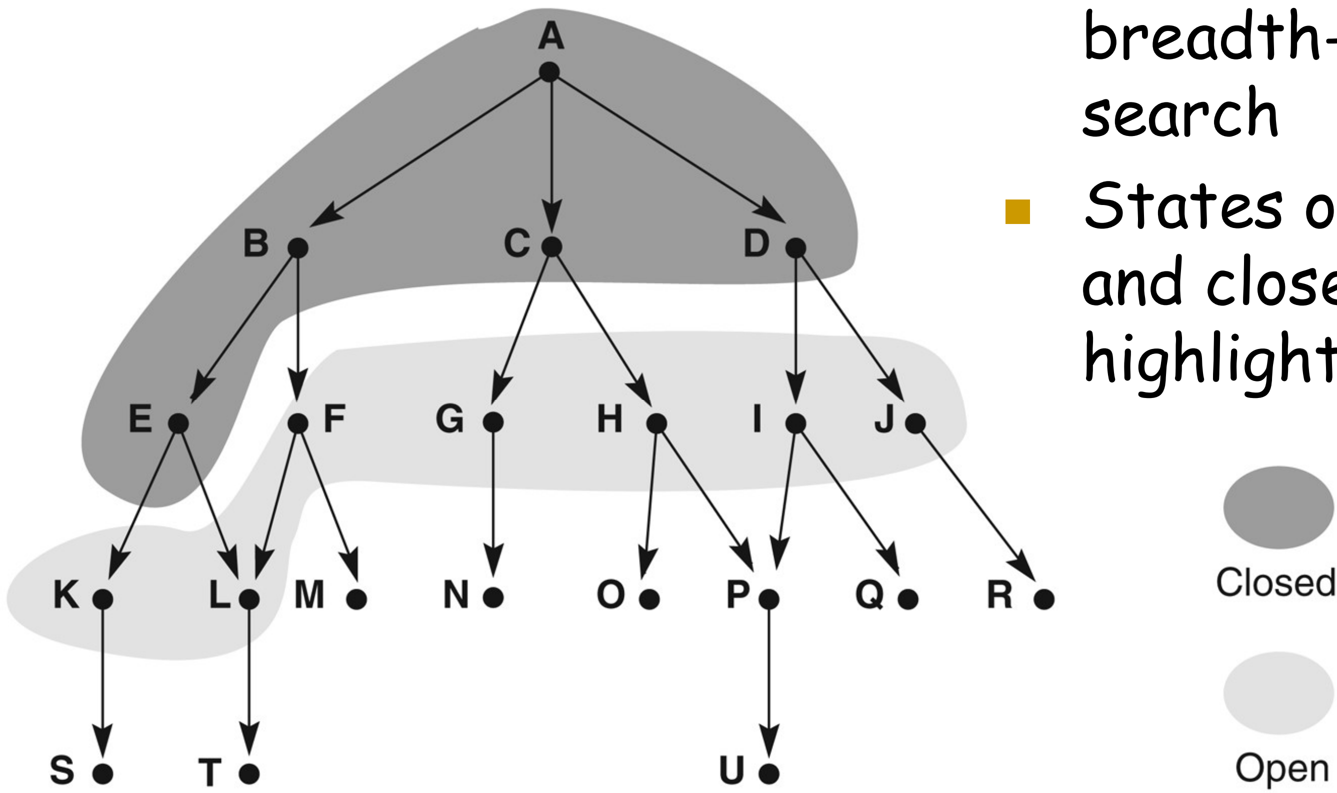
Assume U is goal state



1. open = [A-null] closed = []
2. open = [B-A C-A D-A] closed [A]
3. open = [C-A D-A E-B F-B] closed = [B A]
4. open = [D-A E-B F-B G-C H-C] closed = [C B A]
5. open = [E-B F-B G-C H-C I-D J-D] closed = [D C B A]
6. open = [F-B G-C H-C I-D J-D K-E L-E] closed = [E D C B A]
7. open = [G-C H-C I-D J-D K-E L-E M-F] *as L is already in open* closed = [F E D C B A]
8. and so on until either U is found or open = []

source: G. Luger (2005)

Snapshot of BFS



- Search graph at iteration 6 of breadth-first search
- States on open and closed are highlighted

source: *G. Luger (2005)*

Function Depth-First Search

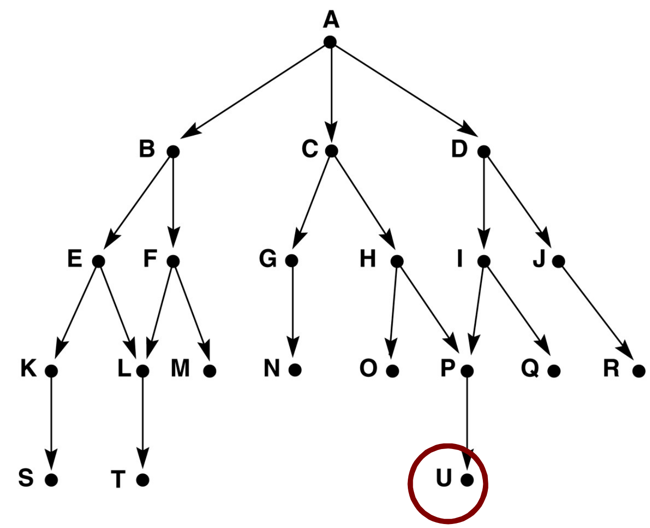
```
begin
   open := [Start];                                                   % initialize
   closed := [ ];
   while open ≠ [ ] do                                                % states remain
      begin
         remove leftmost state from open, call it X;
         if X is a goal then return SUCCESS                           % goal found
            else begin
               generate children of X;
               put X on closed;
               discard children of X if already on open or closed;    % loop check
               put remaining children on left end of open             % stack
            end
      end;
   return FAIL                                                        % no states left
end.
```

source: *G. Luger (2005)*

Depth-First Search Example

- DFS: (open is a stack)

Assume U is goal state



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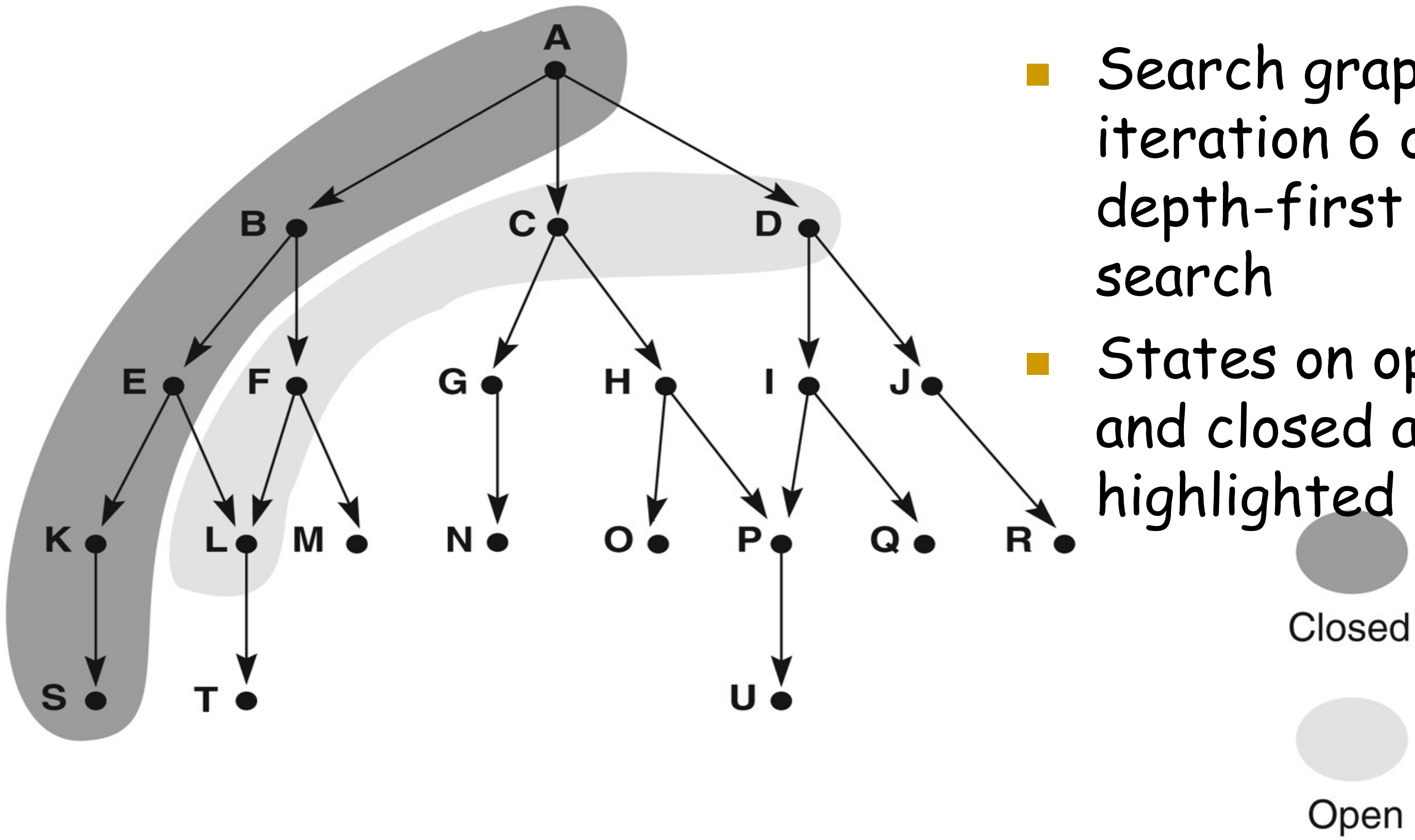
3. open = [E-B F-B C-A D-A] closed = [B A]
4. open = [K-E L-E F-B C-A D-A] closed = [E B A]
5. open = [S-K L-E F-B C-A D-A] closed = [K E B A]
6. open = [L-E F-B C-A D-A] closed = [S K E B A]
7. open = [T-L F-B C-A D-A] closed = [L S K E B A]
8. open = [F-B C-A D-A] closed = [T L S K E B A]
9. open = [M-F C-A D-A] *as L is already on closed* closed = [F T L S K E B A]
10. open = [C-A D-A] closed = [M F T L S K E B A]
11. open = [G-C H-C D-A] closed = [C M F T L S K E B A]

source: G. Luger (2005)

Snapshot of DFS



- Search graph at iteration 6 of depth-first search
- States on open and closed are highlighted

source: G. Luger (2005)

Depth-first vs. Breadth-first

- Breadth-first:
  - Optimal: will always finds shortest path

  But:
  - inefficient if branching factor **B** is very high
  - memory requirements high -- exponential space for states required: $\mathbf{B}^n$

- Depth-first:
  - Not optimal (no guarantee to find the shortest path)

  But:
  - Requires less memory

- But both search are impractical in real applications because search space is too large!

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Depth-Limited Search

Compromise for DFS :

- Do depth-first but with depth cutoff k (depth at which nodes are not expanded)

- Three possible outcomes:
  - Solution
  - Failure (no solution)
  - Cutoff (no solution within cutoff)

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Iterative Deepening

Compromise between BFS and DFS:

- use depth-first search, but
- with a maximum depth before going to next level

- Repeats depth first search with gradually increasing depth limits
  - Requires little memory (fundamentally, it's a depth first)
  - Finds the shortest path (limited depth)

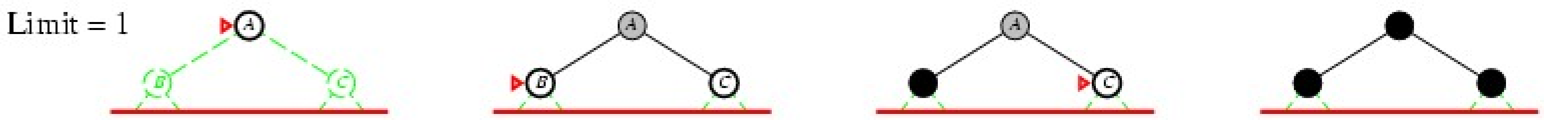
- Preferred search method when there is a large search space and the depth of the solution is unknown

Iterative Deepening: Example

Limit = 0

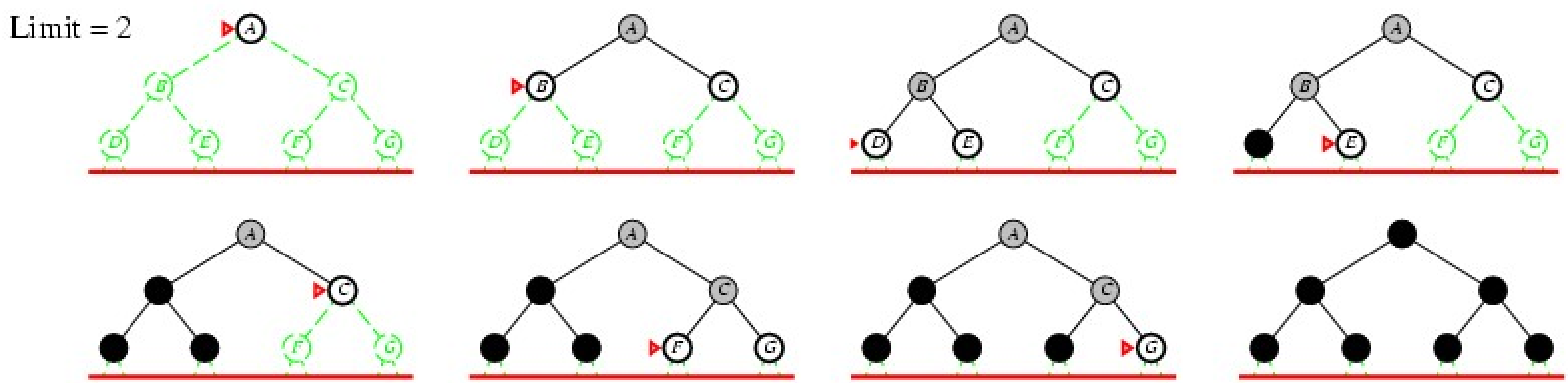
source: Russel & Norvig (2003)

Iterative Deepening: Example



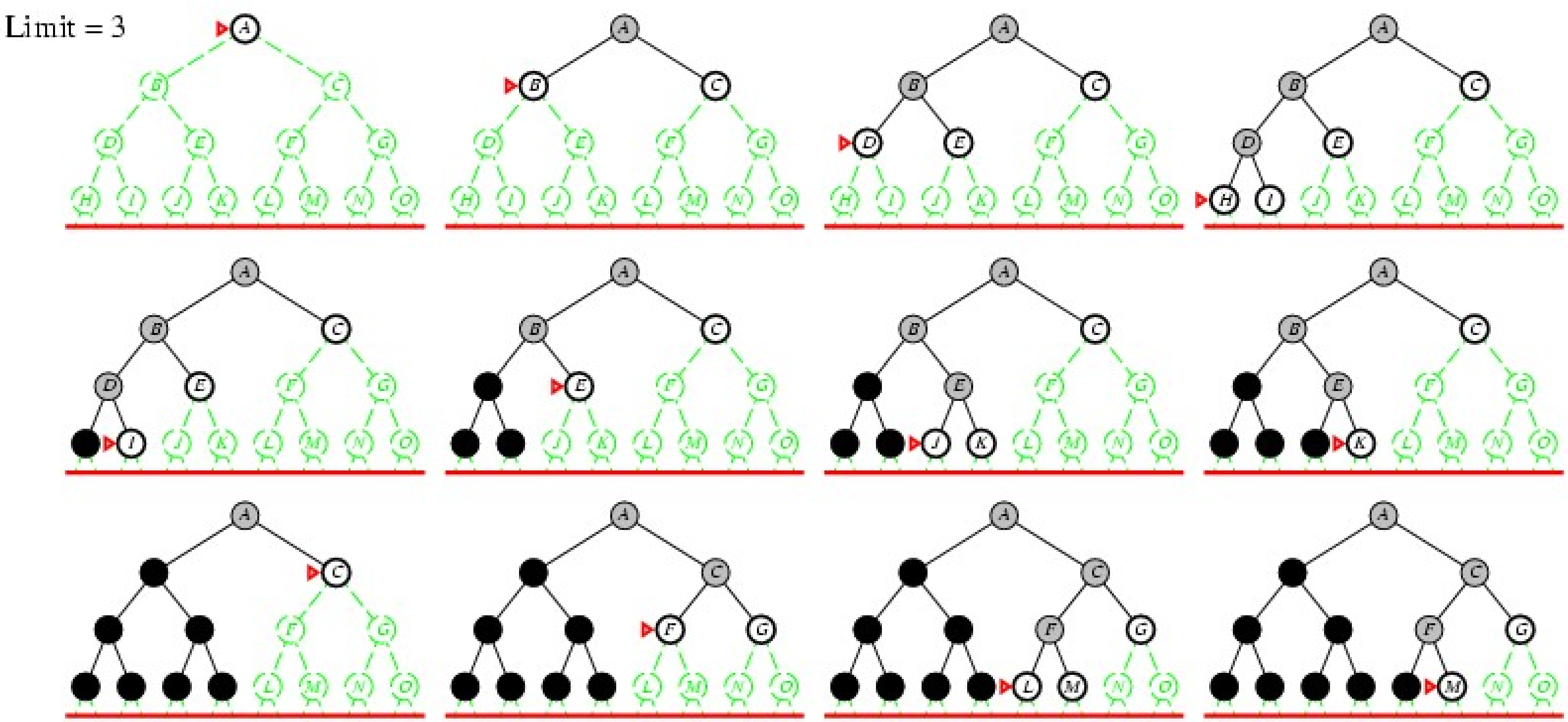
source: Russel & Norvig (2003)

Iterative Deepening: Example



source: Russel & Norvig (2003)

Iterative Deepening: Example



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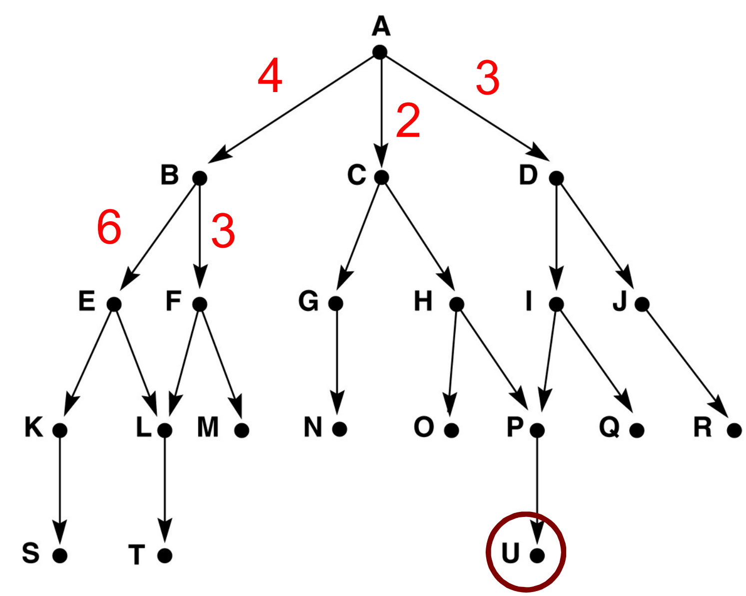
Uniform Cost Search

- Breadth First Search
  - Open is a priority queue sorted using the depth of the nodes from the root
  - guarantees to find the shortest solution path
- But what if all edges/moves do not have the same cost?
- Uniform Cost Search
  - uses a priority queue sorted using the cost from the root to node n – later called g(n)
  - guarantees to find the lowest cost solution path



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Informed Search (aka heuristic search)

- Most of the time, it is not feasible to do an exhaustive search, search space is too large
  - e.g. state space of all possible moves in chess = 10^{120}
    - 10^{75} = nb of molecules in the universe
    - 10^{26} = nb of nanoseconds since the "big bang"

- so far, all search algorithms have been uninformed (general search)
- so need an informed/heuristic search

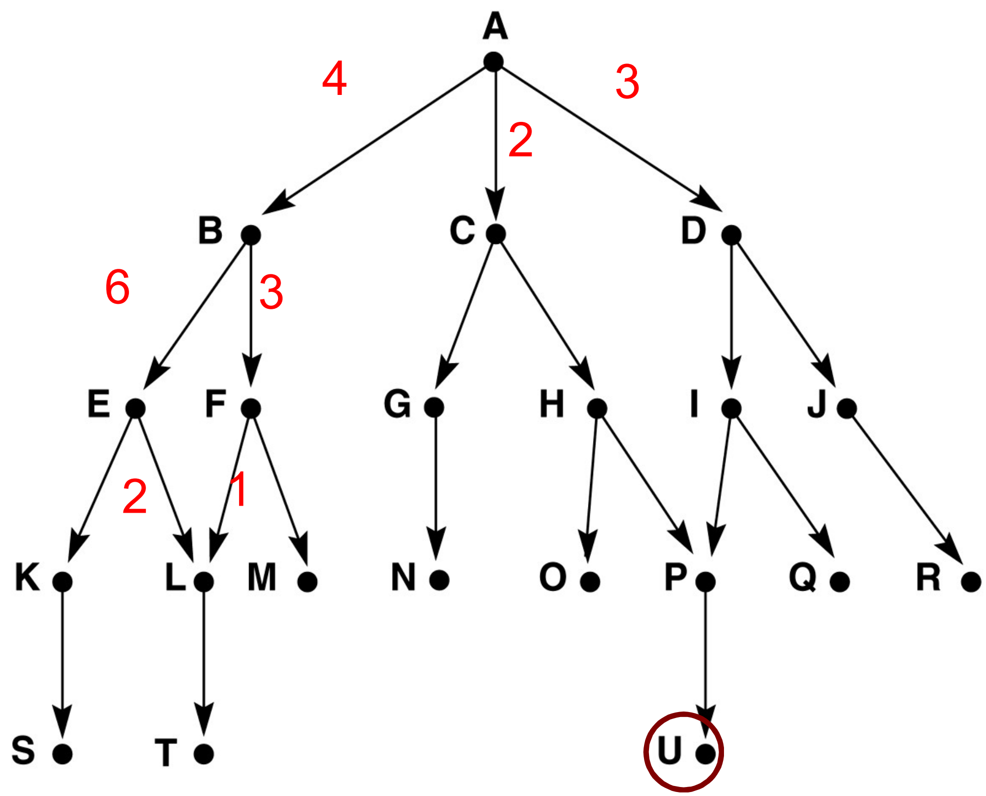
- Idea:
  - choose "best" next node to expand
  - according to a selection function (i.e. a heuristic function h(n))

- But: heuristic might fail

Heuristic - *Heureka!*

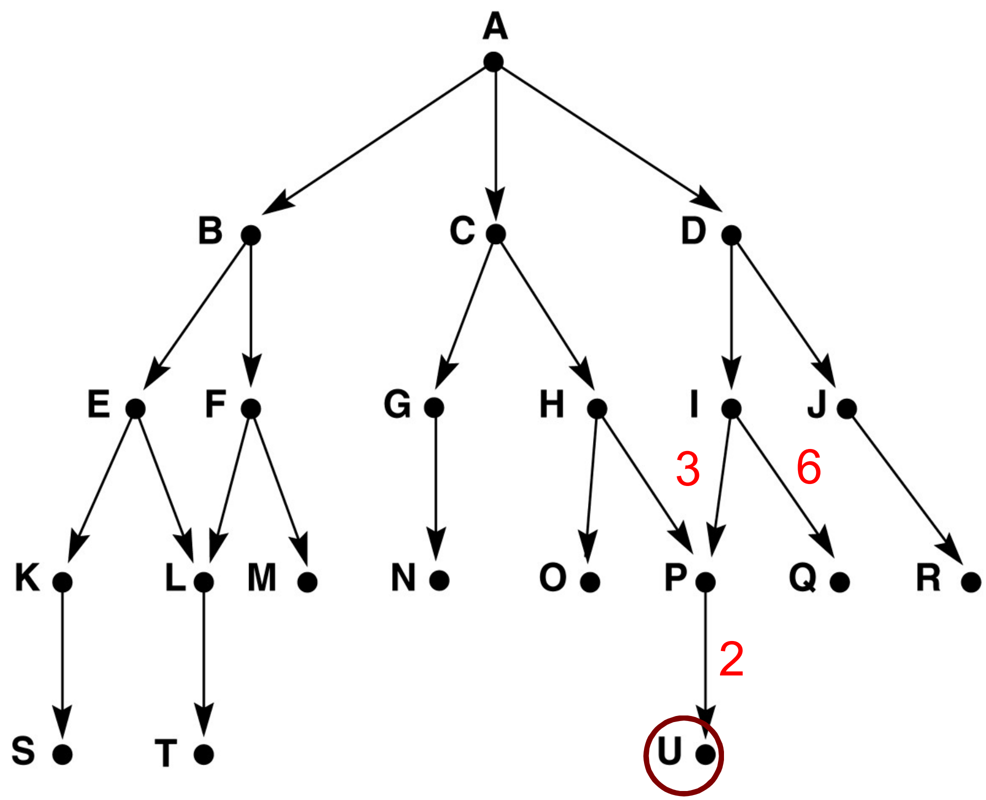
- Heuristic:
  - a rule of thumb, a good bet
  - but has no guarantee to be correct whatsoever!
- Heuristic search:
  - A technique that improves the efficiency of search, possibly sacrificing on completeness
  - Focus on paths that seem most promising according to some function
  - Need an evaluation function (heuristic function) to score a node in the search tree
- Heuristic function h(n) = an approximation of the lowest cost from node n to the goal

g(n)



- **g(n)** = cost of current path from start to node *n*

h(n)



- **h(n)** = estimate of the lowest cost from *n* to *goal*

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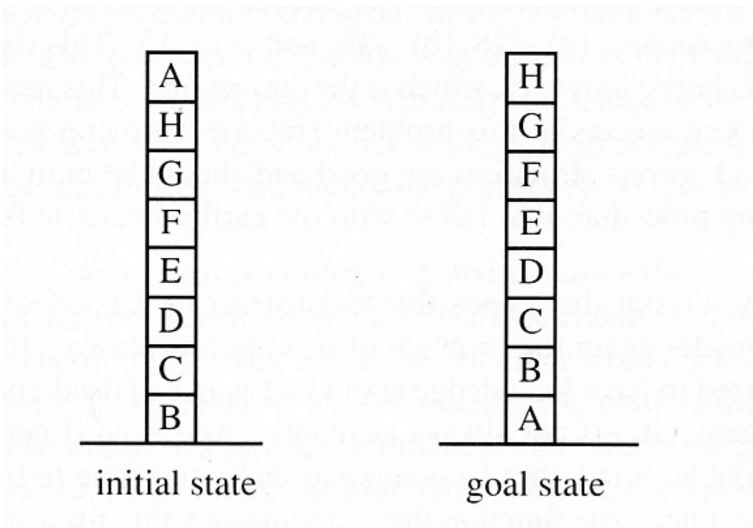
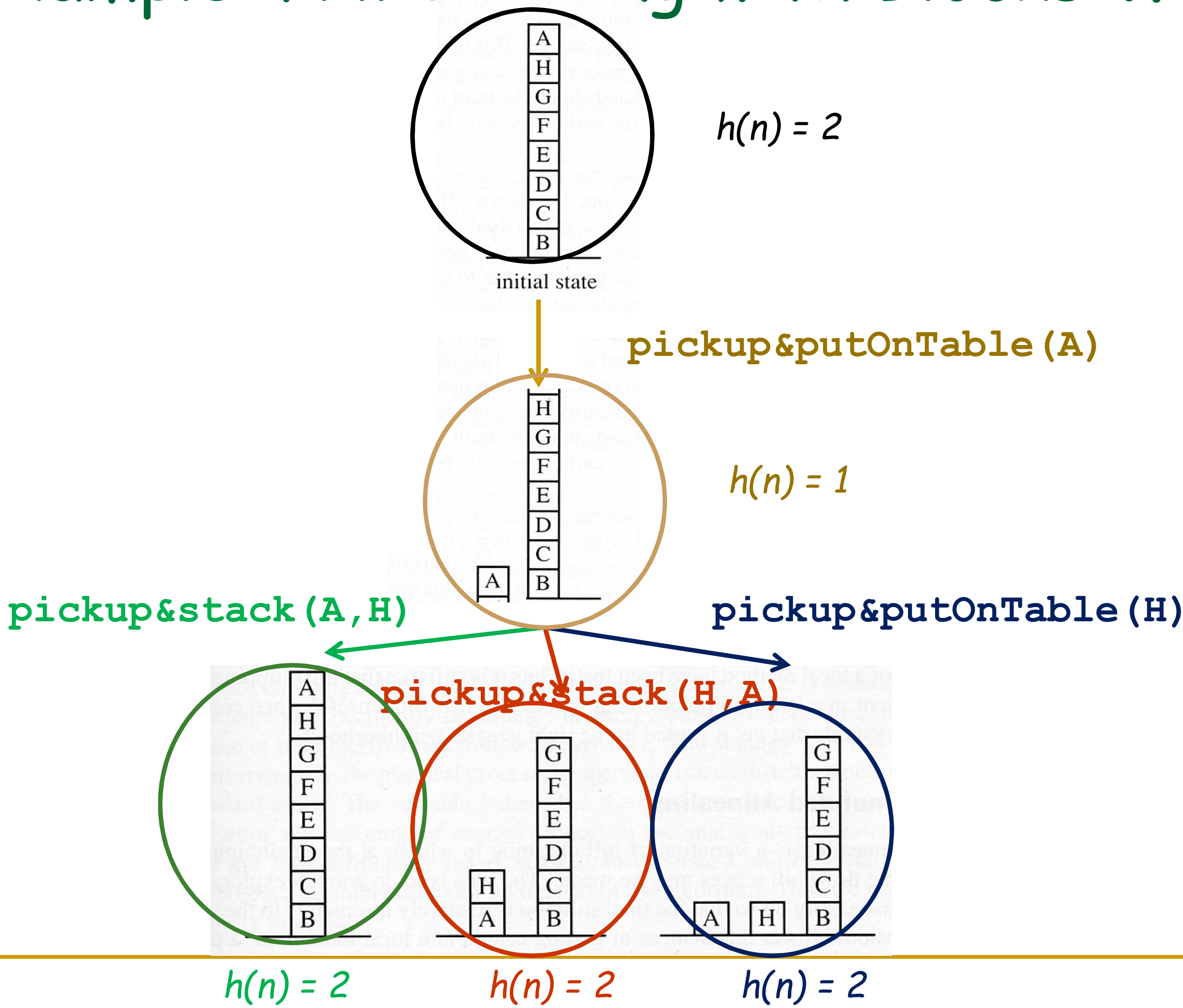
Example: Hill Climbing with Blocks World



- Operators:
  - **`pickup&putOnTable(Block)`**
  - **`pickup&stack(Block1,Block2)`**
- Heuristic:
  - 0pt if a block is sitting where it is supposed to sit
  - +1pt if a block is NOT sitting where it is supposed to sit
  - so lower h(n) is better
    - h(initial) = 2
    - h(goal) = 0

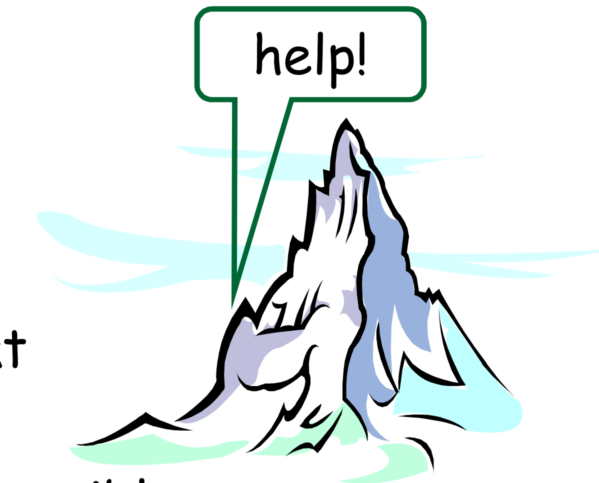
source: Rich & Knight, Artificial Intelligence, McGraw-Hill College 1991.

Example: Hill Climbing with Blocks World



Hill Climbing

- General hill climbing strategy:
  - as soon as you find a position that is better than the current one, select it.
  - Does not maintain a list of next nodes to visit (an *open* list)
  - Similar to climbing a mountain in the fog with amnesia … always go higher than where you are now, but never go back…



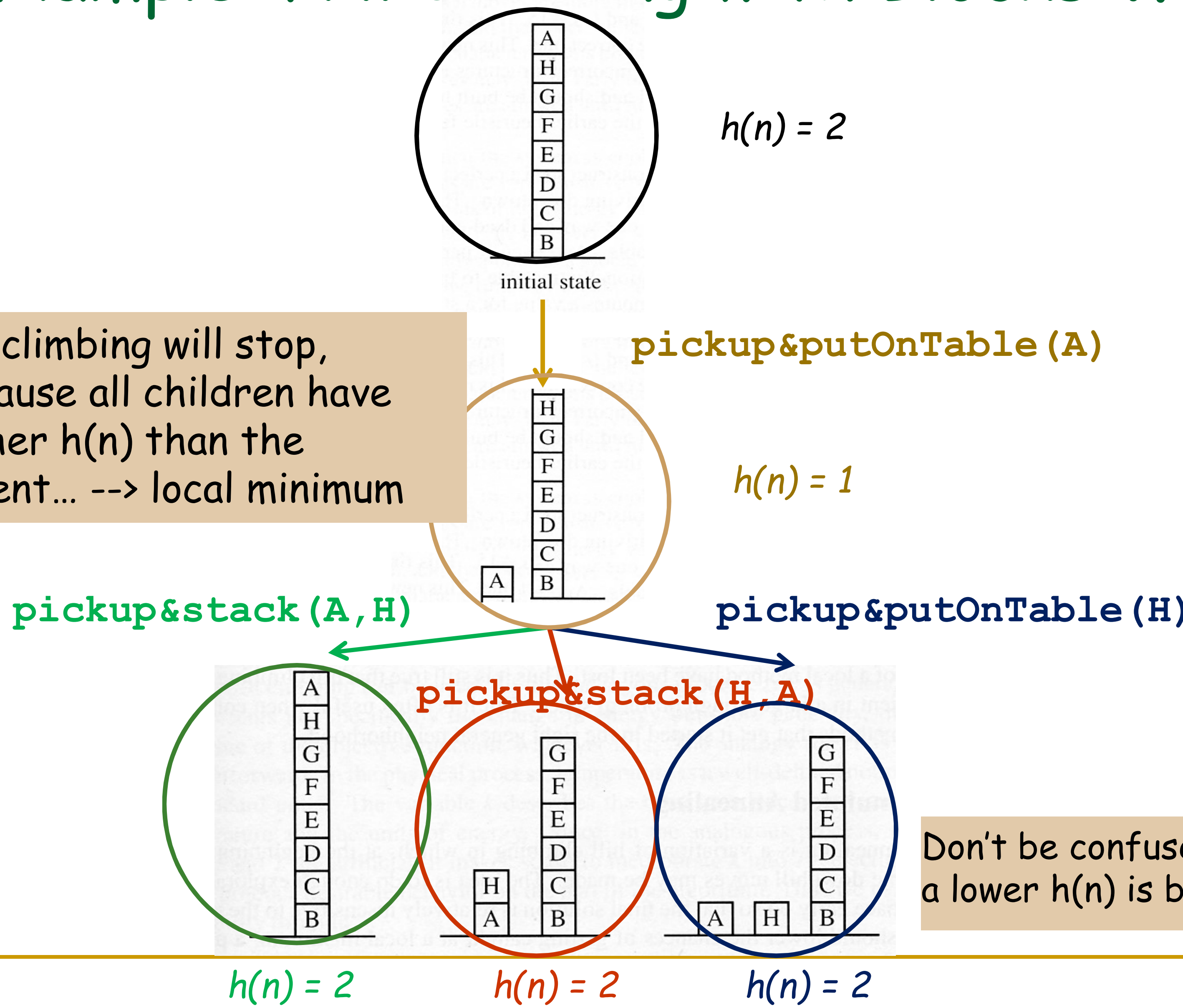
- Steepest ascent hill climbing:
  - instead of moving to the first position that is better than the current one
  - pick the best position out of all the next possible moves

Steepest Ascent Hill Climbing

```
currentNode = startNode;
    loop do
        L = CHILDREN(currentNode);
        nextEval = -INFINITY;
        nextNode = NULL;

        for all c in L
           if (HEURISTIC-VALUE(c) < nextEval) // lower h is better
                nextNode = c;
                nextEval = HEURISTIC-VALUE(c);

          if nextEval >= HEURISTIC-VALUE(currentNode)
             // Return current node since no better child state exist
             return currentNode;

          currentNode = nextNode;
```

Adapted from https://en.wikipedia.org/wiki/Hill_climbing

Example: Hill Climbing with Blocks World

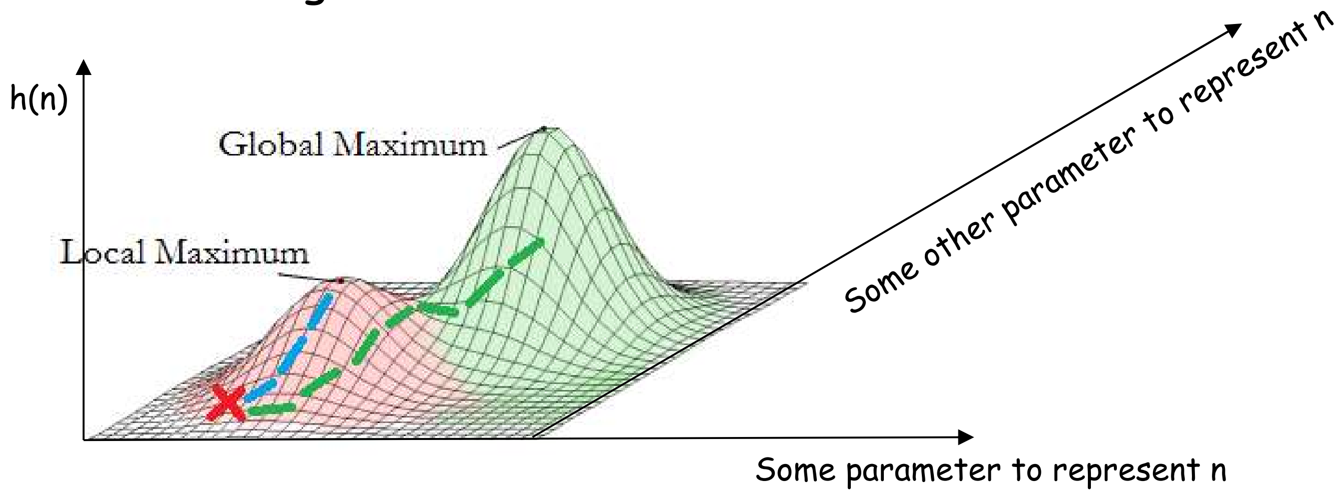


hill-climbing will stop, because all children have higher h(n) than the parent… --> local minimum

Don't be confused… a lower h(n) is better…

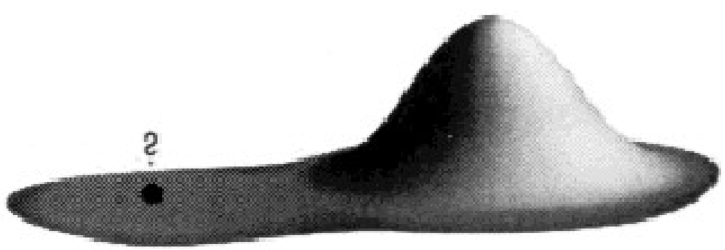
Problems with Hill Climbing

- Foothills (or local maxima)
  - reached a local maximum, not the global maximum
  - a state that is better than all its neighbors but is not better than some other states farther away.
  - at a local maximum, all moves appear to make things worse.
  - ex: 8-puzzle: we may need to move tiles temporarily out of goal position in order to place another tile in goal position
  - ex: TSP: "nearest neighbour" heuristic



Problems with Hill Climbing

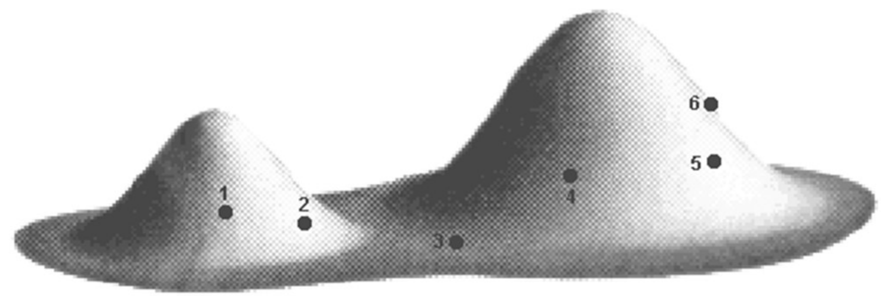
- Plateau
  - a flat area of the search space in which the next states have the same value.
  - it is not possible to determine the best direction in which to move by making local comparisons.



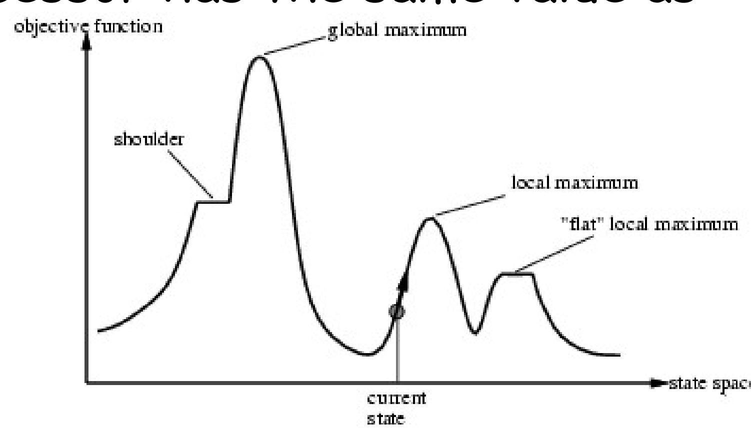
source: Rich & Knight, Artificial Intelligence, McGraw-Hill College 1991.

Some Solutions to Hill-Climbing

- Random-restart hill-climbing
  - random initial states are generated
  - run each until it halts or makes no significant progress.
  - the best result is then chosen.



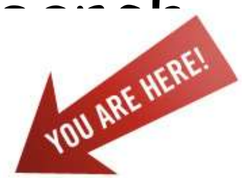
- keep going even if the best successor has the same value as current node
  - works well on a "shoulder"
  - but could lead to infinite loop on a plateau



source: Rich & Knight, Artificial Intelligence, McGraw-Hill College 1991. & Russel & Norvig (2003)

Today

- State Space Representation
- State Space Search
  - Uninformed search
    - Breadth-first and Depth-first
    - Depth-limited Search
    - Iterative Deepening
  - Informed search
    - Hill Climbing
    - Best-First
    - (Designing Heuristics)
    - A*



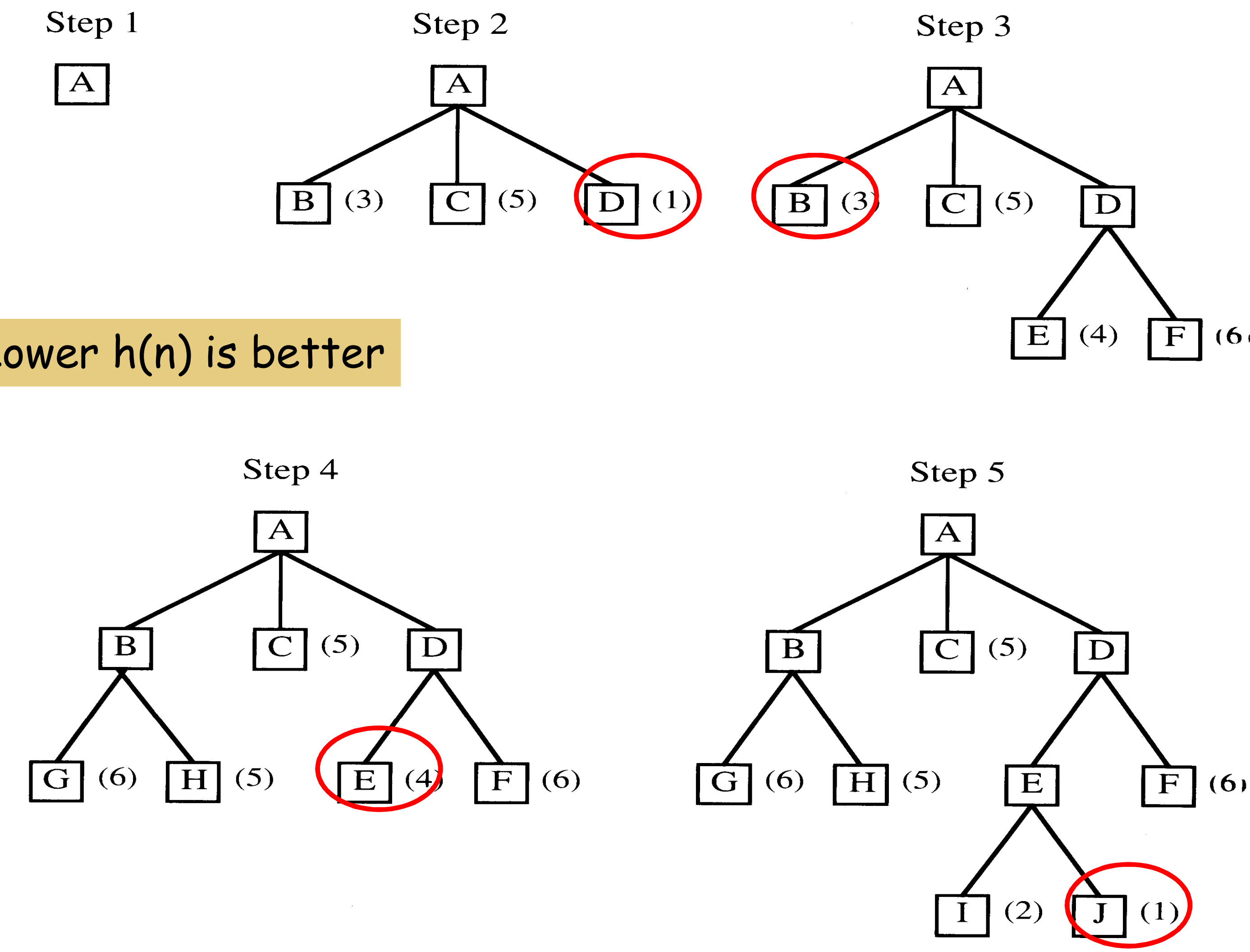
- Summary

Best-First Search

- problem with hill-climbing:
  - one move is selected and all others are forgotten.
- solution to hill-climbing:
  - use "open" as a priority queue
  - this is called best-first search

- Best-first search:
  - Insert nodes in *open* list so that the nodes are sorted in ascending **h(n)**
  - Always choose the next node to visit to be the one with the best h(n) -- regardless of where it is in the search space

Best-First: Example



Lower h(n) is better

source: Rich & Knight, Artificial Intelligence, McGraw-Hill College 1991.

Notes on Best-first

- If you have a good h(n), best-first can find the solution very quickly

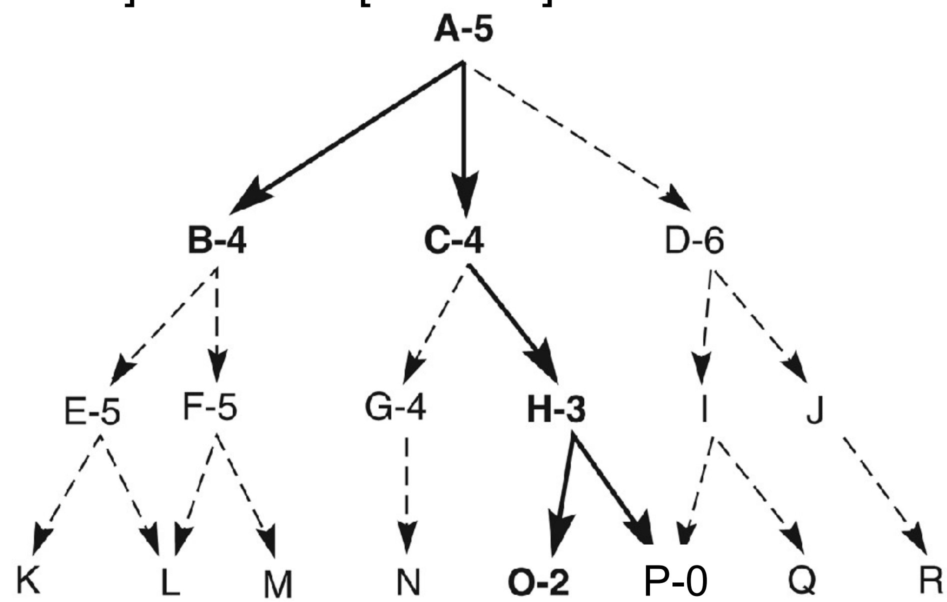
- The first solution may not be the best,
  but there is a good chance of finding it quickly

- It is an exhaustive search …
  - will eventually try all possible paths

Best-First Search: Example

1. open = [A-null-5] closed = []
2. open = [B-A-4 C-A-4 D-A-6] *(arbitrary choice)* closed [A]
3. open = [C-A-4 E-B-5 F-B-5 D-A-6] closed = [B A]
4. open = [H-C-3 G-C-4 E-B-5 F-B-5 D-A-6] closed = [C B A]
5. open = [P-H-0 0-H-2 G-C-4 E-B-5 F-B-5 D-A-6] closed = [H C B A]
6. goal P found

solution path: A C H P



Lower h(n) is better

source: adapted from G. Luger (2005)

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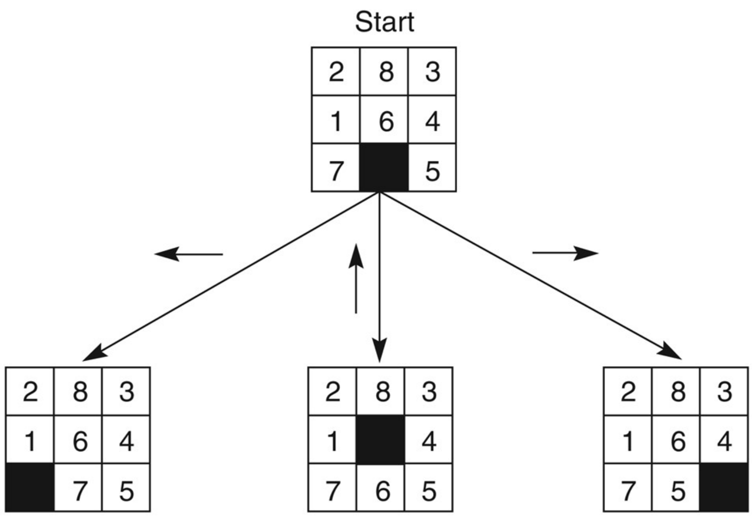
- Summary

Designing Heuristics

- Heuristic evaluation functions are highly dependent on the search domain
- In general: the more informed a heuristic is, the better the search performance
- Bad heuristics lead to frequent backtracking
- So how do we design a "good" heuristic?

Example: 8-Puzzle - Heuristic 1



- h_1: Simplest heuristic
  - Hamming distance : count number of tiles out of place when compared with goal

5		8
4	2	1
7	3	6

STATE n

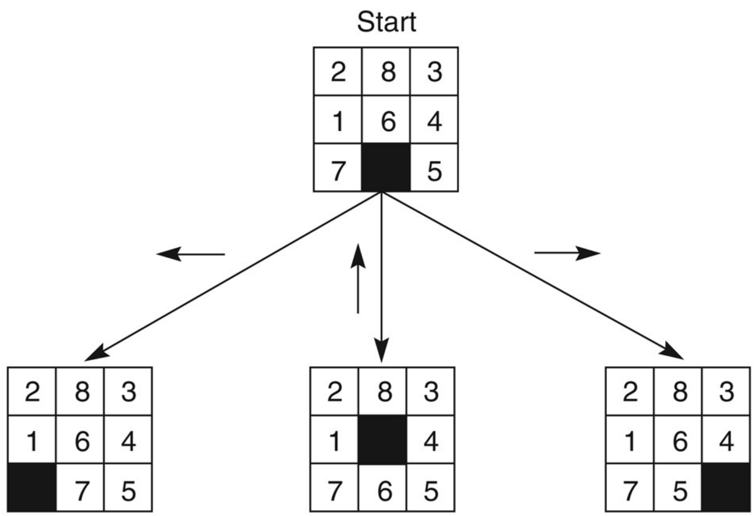
1	2	3
4	5	6
7	8	

Goal state

- $h_1(n) = 6$
  - does not consider the distance tiles have to be moved

source: *G. Luger (2005)*

Example: 8-Puzzle - Heuristic 2



- h_2: Better heuristic
  - Manhattan distance: sum up all the distances by which tiles are out of place

5		8
4	2	1
7	3	6

STATE n

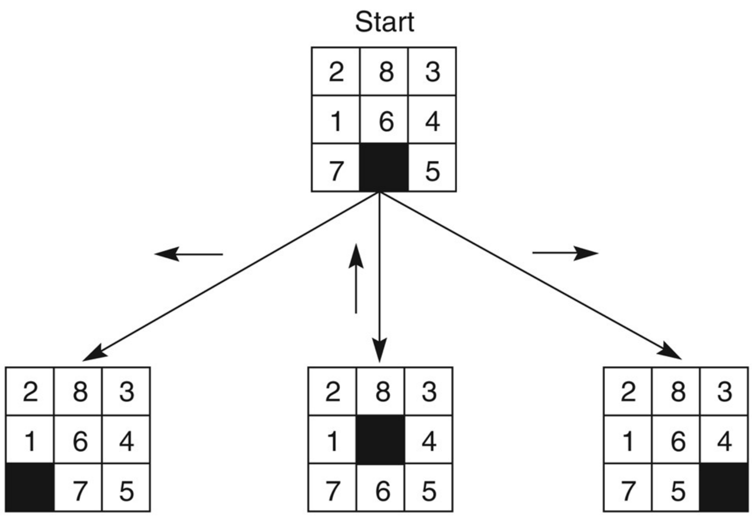
1	2	3
4	5	6
7	8	

Goal state

- $h_2(n) = 2+3+0+1+3+0+3+1$
  $= 13$

source: *G. Luger (2005)*

Example: 8-Puzzle - Heuristic 3



- h_3: Even Better
  - sum of permutation inversions
  - See next slide…

source: *G. Luger (2005)*

$h_3(N)$ = sum of permutation inversions

5		8
4	2	1
7	3	6

STATE n

→

5		8	4	2	1	7	3	6

- For each numbered tile, count how many tiles on its right should be on its left in the goal state.

- $$\begin{aligned} h_3(n) &= n_5 + n_8 + n_4 + n_2 + n_1 + n_7 + n_3 + n_6 \\ &= 4 + 6 + 3 + 1 + 0 + 2 + 0 + 0 \\ &= 16 \end{aligned}$$

1	2	3
4	5	6
7	8	

Goal state

Heuristics for the 8-Puzzle

5		8
4	2	1
7	3	6

STATE n

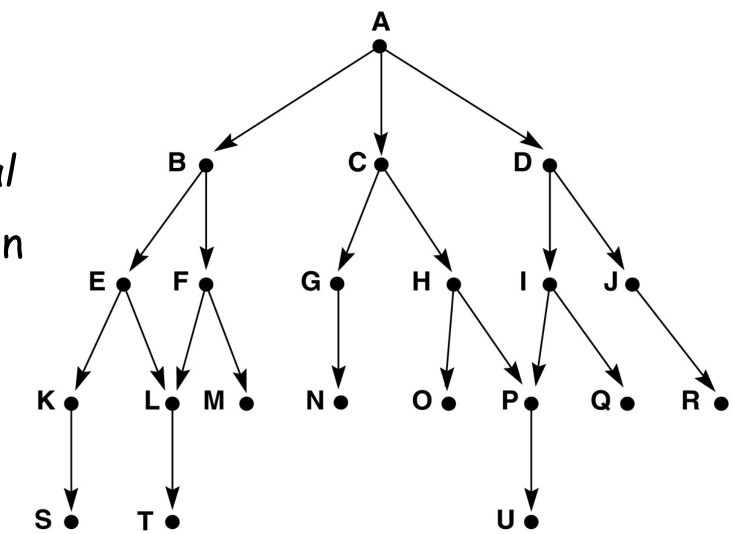
1	2	3
4	5	6
7	8	

Goal state

- $h_1(n)$ = misplaced numbered tiles
  = 6
- $h_2(n)$ = Manhattan distance
  = 2 + 3 + 0 + 1 + 3 + 0 + 3 + 1 = 13
- $h_3(n)$ = sum of permutation inversions
  $= n_5 + n_8 + n_4 + n_2 + n_1 + n_7 + n_3 + n_6$
  = 4 + 6 + 3 + 1 + 0 + 2 + 0 + 0 = 16

g(n), h(n) and f(n)

- Evaluation function **f(n) = g(n) + h(n)** for node **n**:
  - **g(n)** current cost from *start* to node *n*
  - **h(n)** estimate of the lowest cost from *n* to *goal*
  - **f(n)** estimate of the lowest cost of the solution path (from *start* to *goal* passing through *n*)
- Now consider **f*(n) = g*(n) + h*(n)**:
  - **g*(n)** cost of lowest cost path from *start* to node *n*
  - **h*(n)** actual lowest cost from *n* to *goal*
  - **f*(n)** actual cost of lowest cost of the solution path (from *start* to *goal* passing through *n*)



Evaluating Heuristics

1. Admissibility:
   - "optimistic"
   - never overestimates the actual cost of reaching the goal
   - guarantees to find the lowest cost solution path to the goal (if it exists)

2. Monotonicity:
   - "local admissibility"
   - guarantees to find the lowest cost path to each state n encountered in the search

3. Informedness:
   - measure for the "quality" of a heuristic
   - the more informed, the better

Admissibility

- A heuristic is admissible if it never overestimates the cost of reaching the goal
- i.e.:
  - $h(n) \leq h^*(n)$ for all n
- guarantees to find the lowest cost solution path to the goal (if it exists)
- Note: does not guarantee to find the lowest cost search path.
- e.g.: breadth-first is admissible -- it uses $f(n) = g(n) + 0$

Example: 8-Puzzle

5		8
4	2	1
7	3	6

n

1	2	3
4	5	6
7	8	

goal

- h1(n) = Hamming distance = number of misplaced tiles = 6
--> admissible

- h2(n) = Manhattan distance = 13
--> admissible

Monotonicity (aka consistent)

- An admissible heuristics may temporarily reach non-goal states along a suboptimal path
- A heuristic is monotonic if it always finds the optimal path to each state the 1st time it is encountered !
- guarantees to find the lowest cost path to each state n encountered in the <u>search</u>

- h is monotonic if for every node n and every successor n' of n:
  - $h(n) \leq c(n,n') + h(n')$
- i.e. f(n) is non-decreasing along any path

Informedness

- Intuition: number of misplaced tiles is less informed than Manhattan distance
- For two admissible heuristics h_1 and h_2
  - if $h_1(n) \leq h_2(n)$, for all states n
  - then h_2 is more informed than h_1
  - $h_1(n) \leq h_2(n) \leq h^*(n)$
- More informed heuristics search smaller space to find the solution path
- However, you need to consider the computational cost of evaluating the heuristic…
- The time spent computing heuristics must be recovered by a better search

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    - Be
    - (De euristics)
    - A*

- Summary

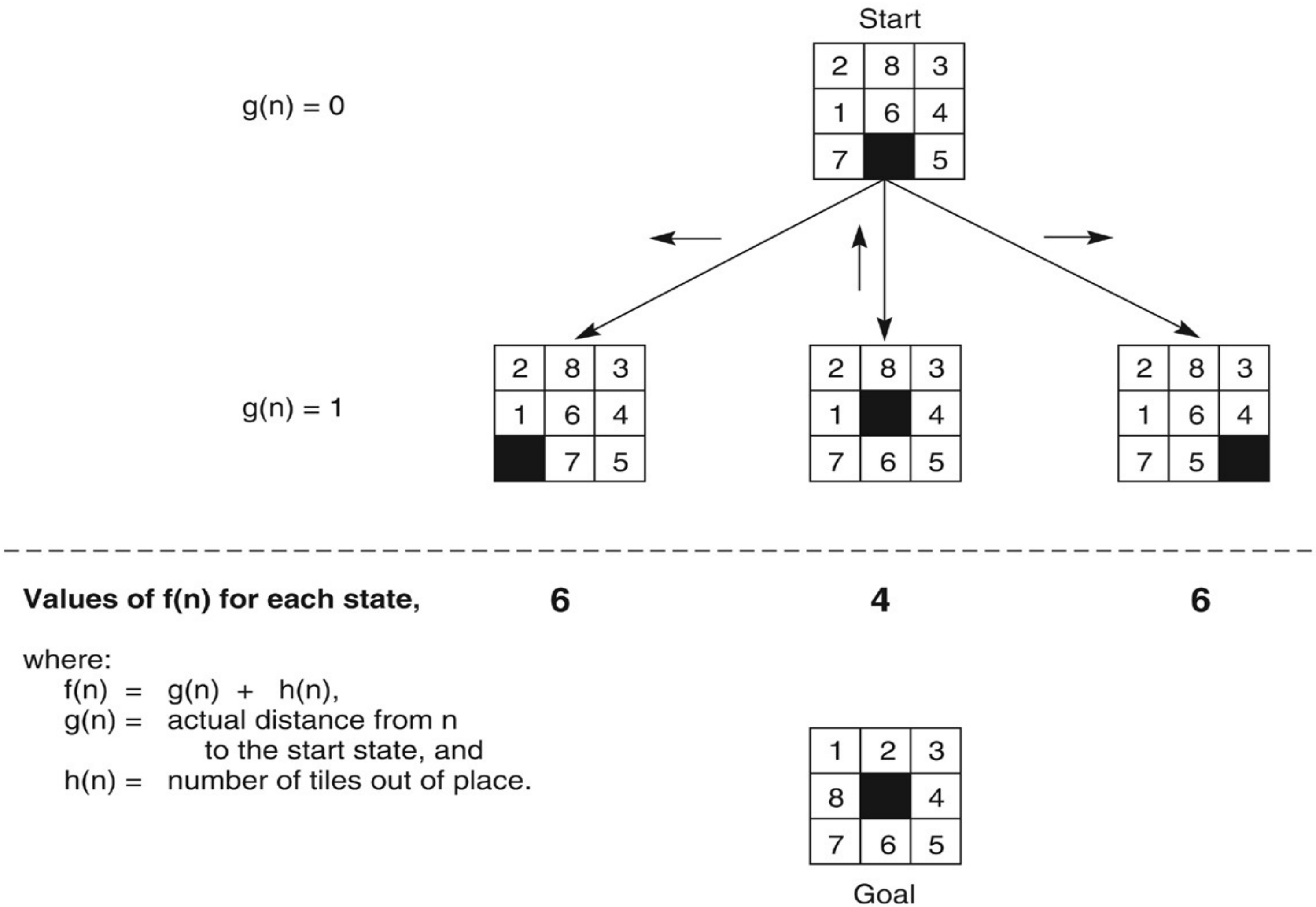
Algorithm A

- Heuristics might be wrong:
  - so search could continue down a wrong path
- Solution:
  - Maintain depth/cost count, i.e., give preference to shorter/least expensive paths
- Modified evaluation function **f**:

  f(n) = g(n) + h(n)

  - f(n) estimate of total cost along path through n
  - g(n) actual cost of path from start to node n
  - h(n) estimate of cost to reach goal from node n

Algorithm A on the 8-puzzle



source: *G. Luger (2005)*

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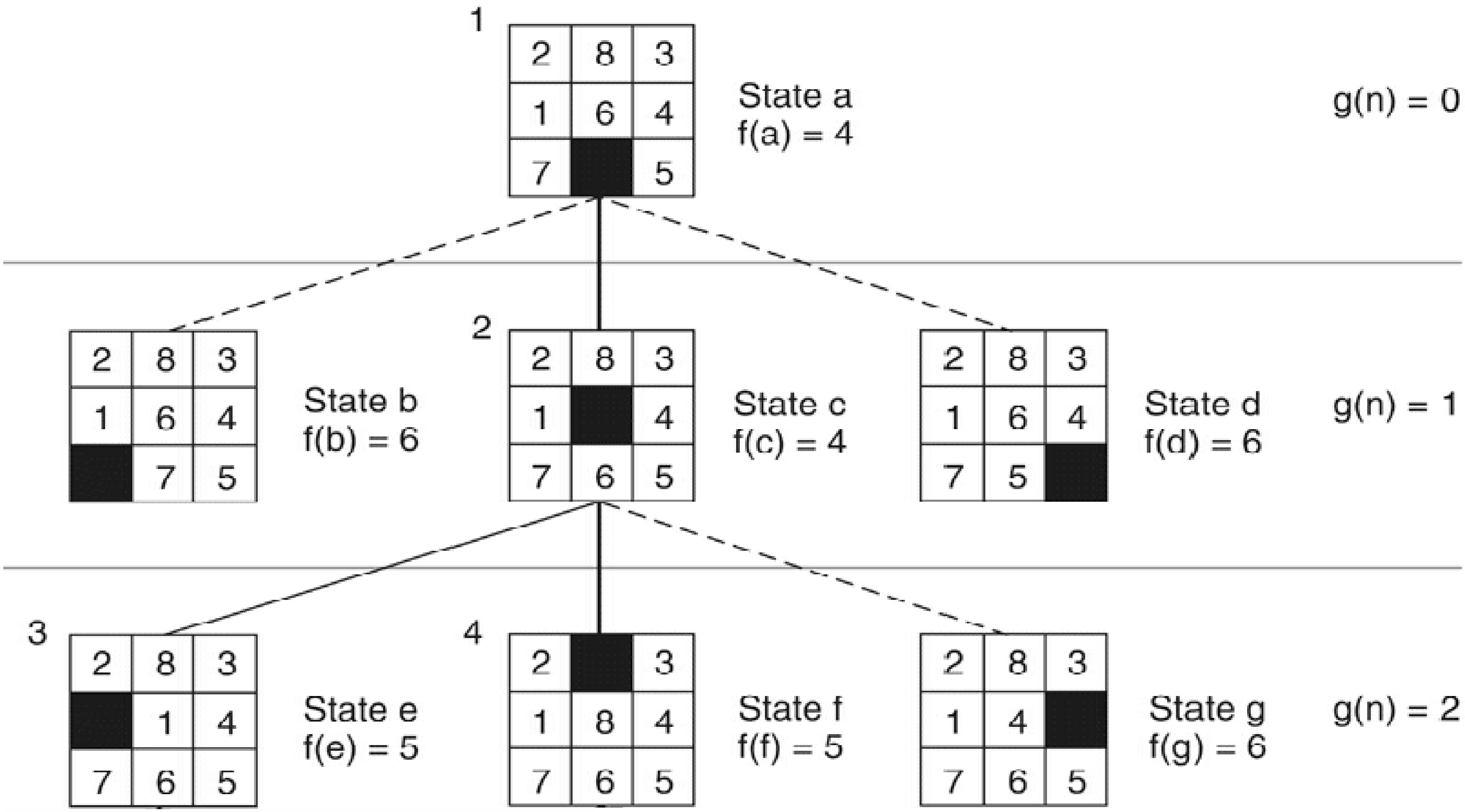

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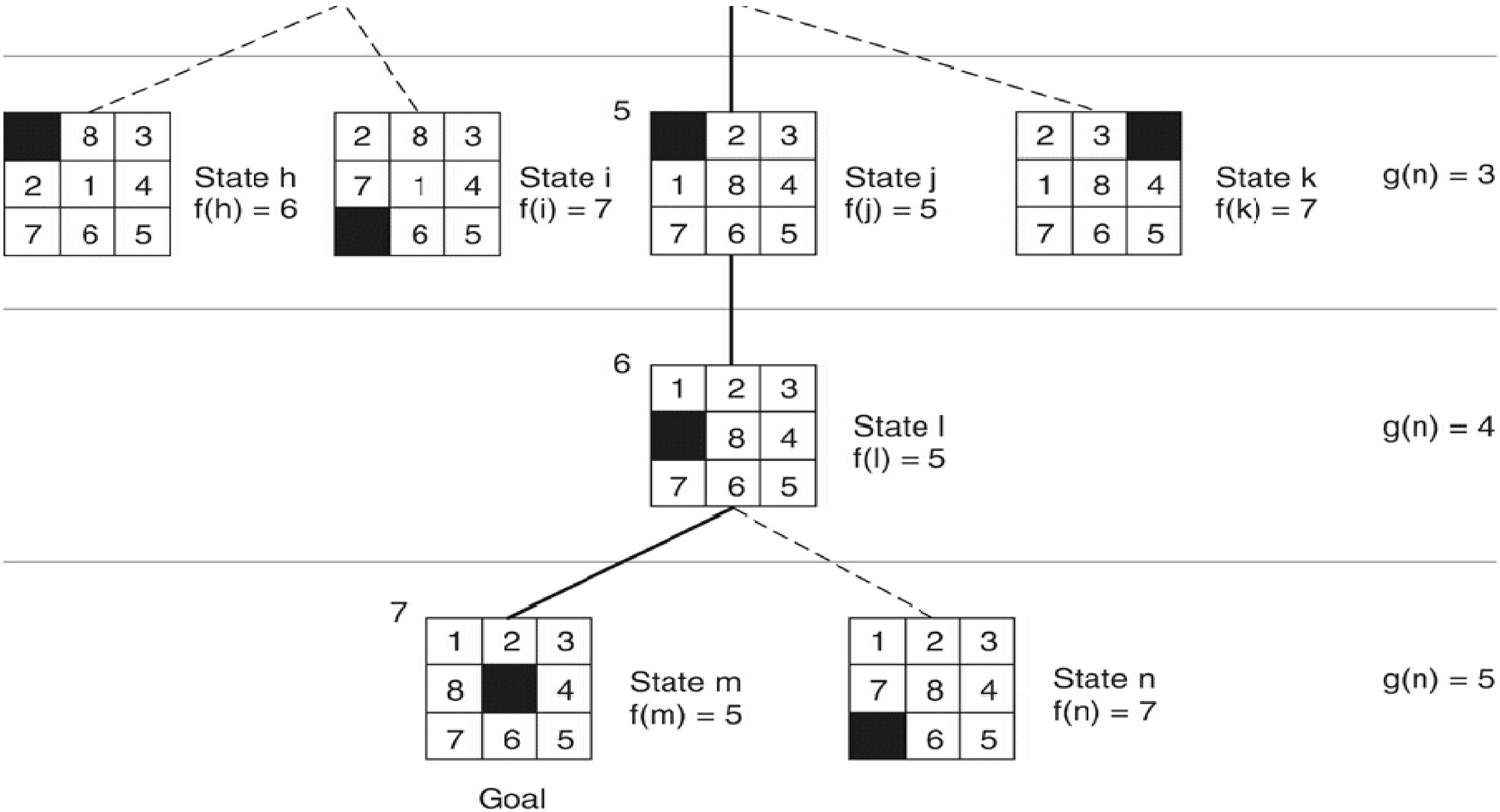
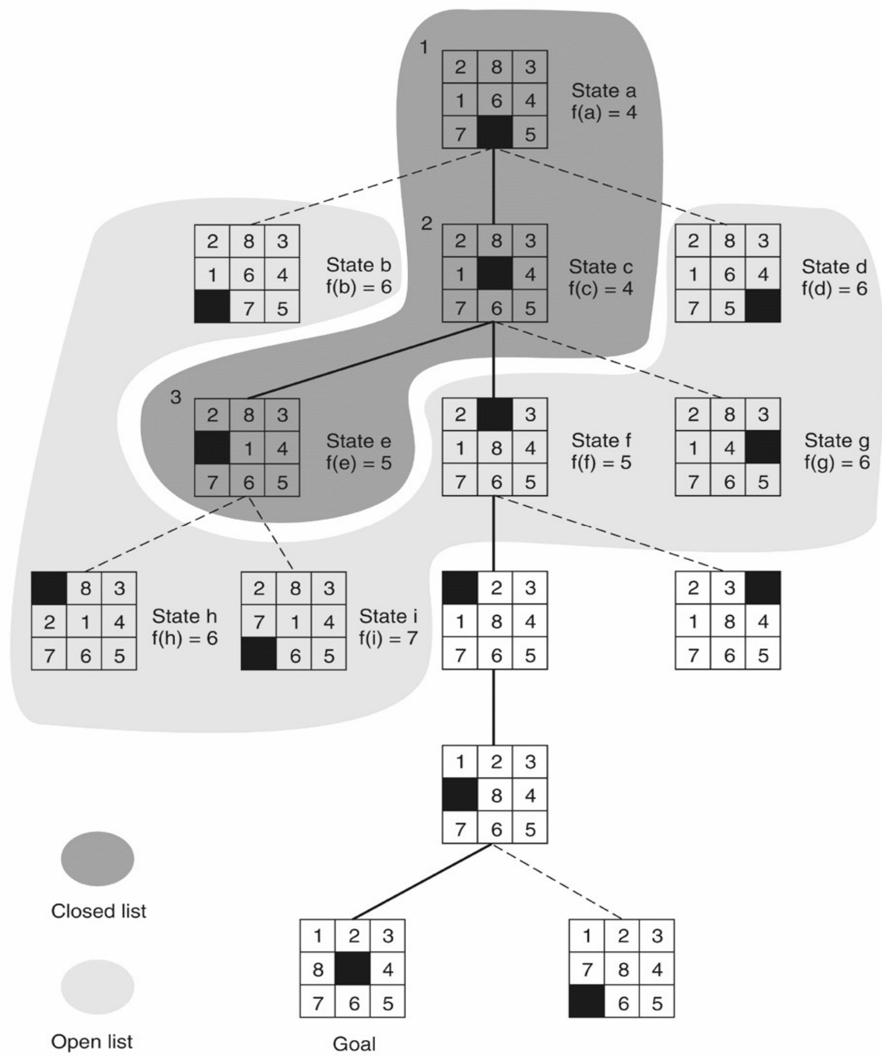
Algorithm A on the 8-puzzle



source: G. Luger (2005)

Algorithm A on the 8-puzzle



source: *G. Luger (2005)*

Algorithm A vs Algorithm A*

- if **$g(n) \geq g^*(n)$** for all n
  - best-first used with such a g(n) is called "algorithm A"
- if **$h(n) \leq h^*(n)$** for all n
  - i.e. h(n) never overestimates the true cost from n to a goal
  - algorithm **A** used with such an h(n) is called "algorithm A*"
  - → an A* algorithm is admissible
  - → i.e. it guarantees to find the lowest cost solution path from the initial state to the goal

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- Summary

Summary

Search	Uses h(n)?	Open is a…
Breadth-first	No	Queue
Depth-first	No	Stack
Depth-limited	No	Stack
Iterative Deepening	No	Stack
Uniform Cost	No	Priority queue sorted by g(n)
Hill Climbing	Yes	none
Best-First	Yes	Priority queue sorted by h(n)
Algorithm A - no constraints on h(n)	Yes	Priority queue sorted by f(n) f(n) = g(n) + h(n)
Algorithm A* - same as A, but h(n) must be admissible - guarantees to find the lowest cost solution path	Yes	Priority queue sorted by f(n) f(n) = g(n) + h(n)

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