Name: Siddharth Parshia

CWID: 20005837

CS 583 Assignment-1

Q1)

1) squared l-2 norm of x

$$\|x\|_2^2 = \sum_{i=1}^{n} x_i^2$$

$= 5^2 + (-3)^2 + (-1)^2 + (2)^2$

$= 25 + 9 + 1 + 4$

$= 39$

$\boxed{Ans = 39}$

2) l_1-norm of x

$$\|x\|_1 = \sum_{i=1}^{n} |x_i|$$

$= |5| + |-3| + |-1| + |2|$

$= 5 + 3 + 1 + 2$

$= 11$

$\boxed{Ans = 11}$

3.) inner product of x and $a = [4, -2, 6, -1]^T$

$$a^T x = a^T \cdot x$$

$$= [4, -2, 6, -1] \cdot [5, -3, -1, 2]$$

$$= 4 \times 5 + (-2) \times (-3) + 6 \times (-1) + (-1) \times 2$$

$$= 20 + 6 + (-6) + (-2)$$

$$= 18$$

$$\boxed{\text{Ans} = 18}$$

Q 2)

1) matrix vector product:

$$Ab = \begin{bmatrix} 6 & 1 & -2 \\ -5 & 7 & 9 \end{bmatrix} \times \begin{bmatrix} -4 \\ 5 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -24 + 5 - 4 \\ 20 + 35 + 18 \end{bmatrix}$$

$$= \begin{bmatrix} -23 \\ 73 \end{bmatrix}$$

$$\boxed{\text{Ans} = \begin{bmatrix} -23 \\ 73 \end{bmatrix}}$$

2) matrix - matrix product

$$AA^T = \begin{bmatrix} 6 & 1 & -2 \\ -5 & 7 & 9 \end{bmatrix} \times \begin{bmatrix} 6 & -5 \\ 1 & 7 \\ -2 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 36+1+4 & -30+7-18 \\ -30+7-18 & 25+49+81 \end{bmatrix}$$

$$= \begin{bmatrix} 41 & -41 \\ -41 & 155 \end{bmatrix}$$

$$\boxed{\text{Ans} = \begin{bmatrix} 41 & -41 \\ -41 & 155 \end{bmatrix}}$$

Q3) $x = [x_1, x_2, x_3]$ $\qquad y = \frac{x_1^2}{2} + \ln(x_2) - \frac{x_1}{x_3}$

$\frac{\partial y}{\partial x}$ at $x = [9, 1, 1/2]$

$$\frac{\partial y}{\partial x} = \left[\underset{(\text{at } x_1 = 9)}{\frac{\partial y}{\partial x_1}}, \underset{(\text{at } x_2 = 1)}{\frac{\partial y}{\partial x_2}}, \underset{(\text{at } x_3 = 1/2)}{\frac{\partial y}{\partial x_3}} \right]$$

$$\frac{\partial y}{\partial x_1} = x_1 - \frac{1}{x_3} = 9 - 2 = 7$$

$$\frac{\partial y}{\partial x_2} = \frac{1}{x_2} = 1 \qquad \frac{\partial y}{\partial x_3} = +\frac{x_1}{x_3^2} = 36$$

$$\frac{\partial y}{\partial x} = [7, 1, 36]$$

$$\boxed{Ans = \frac{\partial y}{\partial x} = [7, 1, 36]}$$

Q4) $f(w) = \|Xw - y\|_2^2 + \lambda \|w\|_2^2$

$$\frac{\partial(f(w))}{\partial w} = \frac{\partial}{\partial w}\left(\|Xw - y\|_2^2 + \lambda\|w\|_2^2\right)$$

Now, we know that the squared l_2 norm of an vector $y = \sum_{i=1}^{n} (y_i)^2$

Thus if we consider the derivative of the squared l_2 norm w.r.t the vector, we can say that

$$\frac{\partial(\|y\|_2^2)}{\partial(y_i)} = \frac{\partial}{\partial y_i}\sum_{k=1}^{n} y_k^2 = \sum_{k=1}^{n}\frac{\partial(y_k^2)}{\partial y_j}$$

(now if $y = k$, $2k$
else if $y != k$, 0)

$$\therefore \frac{\partial(\|y\|^2)}{\partial y_i} = \sum_{k=1}^{n}\frac{\partial(y_k^2)}{\partial y_j} = 2y_j$$

$$\therefore \frac{\partial(\|y\|^2)}{\partial y} = 2y$$

Hence $\frac{\partial}{\partial w}(\|w\|^2) = 2w$

considering the first term now,

$$\frac{\partial}{\partial w_i}\left(\|Xw - y\|_2^2\right) = \frac{\partial}{\partial w_i}\left(\sum_{j=1}^{n}(x_j w_i - y_i)^2\right)$$

$$= \frac{\partial}{\partial w_i}\sum_{j=1}^{n}(x_j w_i - y_i)^2$$

$$= \sum_{j=1}^{n}\frac{\partial}{\partial w_i}(x_j w_i - y_i)^2$$

$$= \sum_{j=1}^{n} 2(x_j w_i - y_i) \cdot x_j$$

$$\therefore \frac{\partial}{\partial w_i}\left(\|Xw - y\|_2^2\right) = \sum_{j=1}^{n}\left(2x_j(x_j w_i - y_i)\right)$$

$$\therefore \frac{\partial}{\partial w}\left(\|Xw - y\|_2^2\right) = \begin{bmatrix} \sum_{j=1}^{n}\left(2x_j(x_j w_1 - y_i)\right) \\ \sum_{j=1}^{n}\left(2x_j(x_j w_2 - y_i)\right) \\ \vdots \\ \sum_{j=1}^{n}\left(2x_j(x_j w_d - y_i)\right) \end{bmatrix}$$

Hence first and second term both will be $d \times 1$ type of vectors,

$$\therefore \frac{\partial f(w)}{\partial w} = \begin{bmatrix} \sum_{i=1}^{n} (2x_i(x_i w_1 - y_i)) \\ \sum_{i=1}^{n} (2x_i(x_i w_2 - y_i)) \\ \vdots \\ \sum_{i=1}^{n} (2x_i(x_i w_d - y_i)) \end{bmatrix} + \begin{bmatrix} 2w_1 \\ 2w_2 \\ \vdots \\ 2w_d \end{bmatrix}$$

$$\therefore \boxed{\text{Ans} = \frac{\partial (f(w))}{\partial w} = \begin{bmatrix} \sum_{i=1}^{n} (2x_i(x_i w_1 - y_i)) \\ \sum_{i=1}^{n} (2x_i(x_i w_2 - y_i)) \\ \vdots \\ \sum_{i=1}^{n} (2x_i(x_i w_d - y_i)) \end{bmatrix} + \begin{bmatrix} 2w_1 \\ 2w_2 \\ \vdots \\ 2w_d \end{bmatrix}}$$