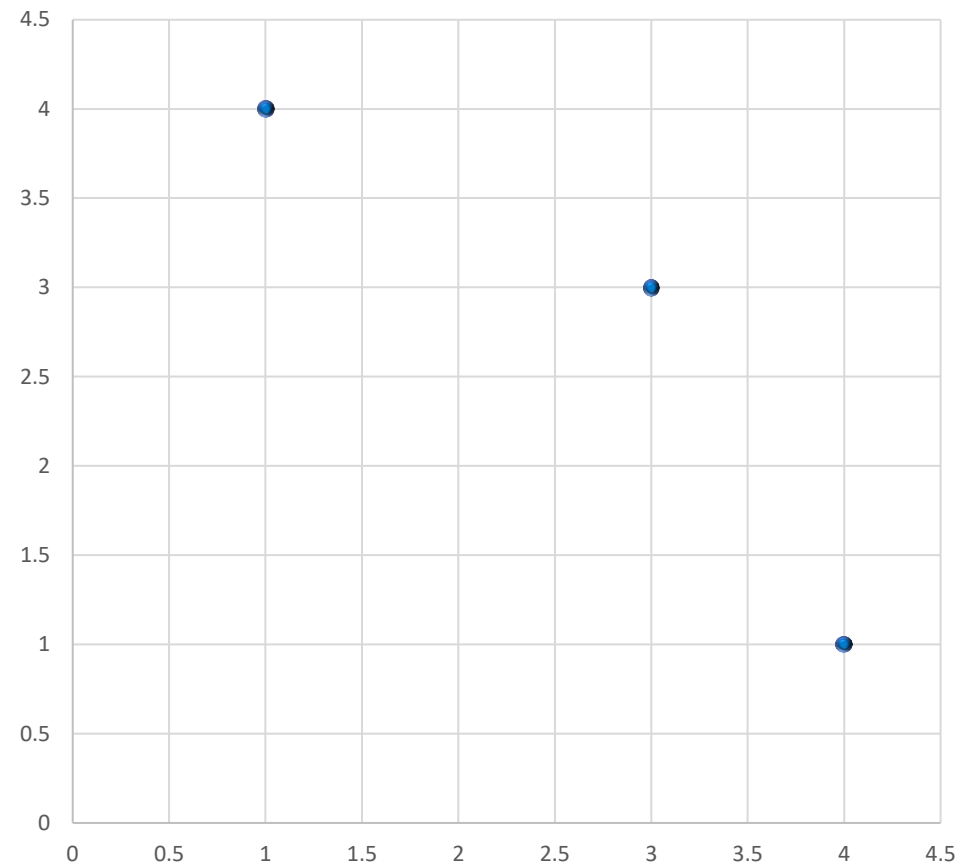
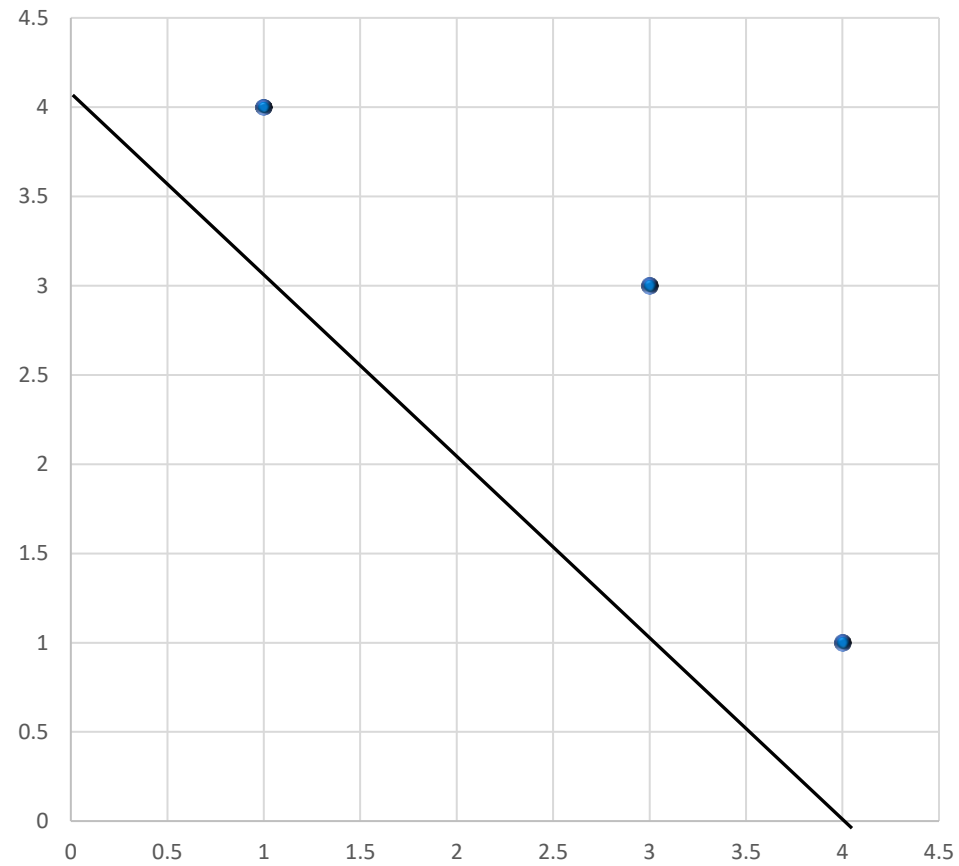


# Support Vector Machine

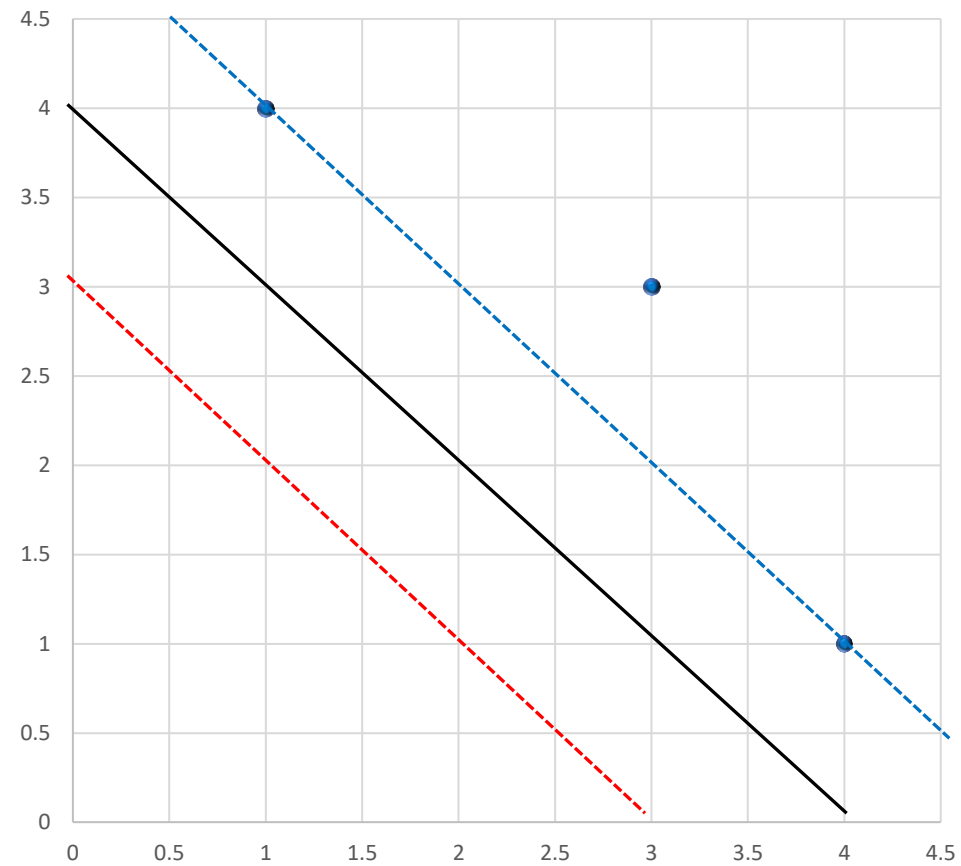
# Support Vector Machine



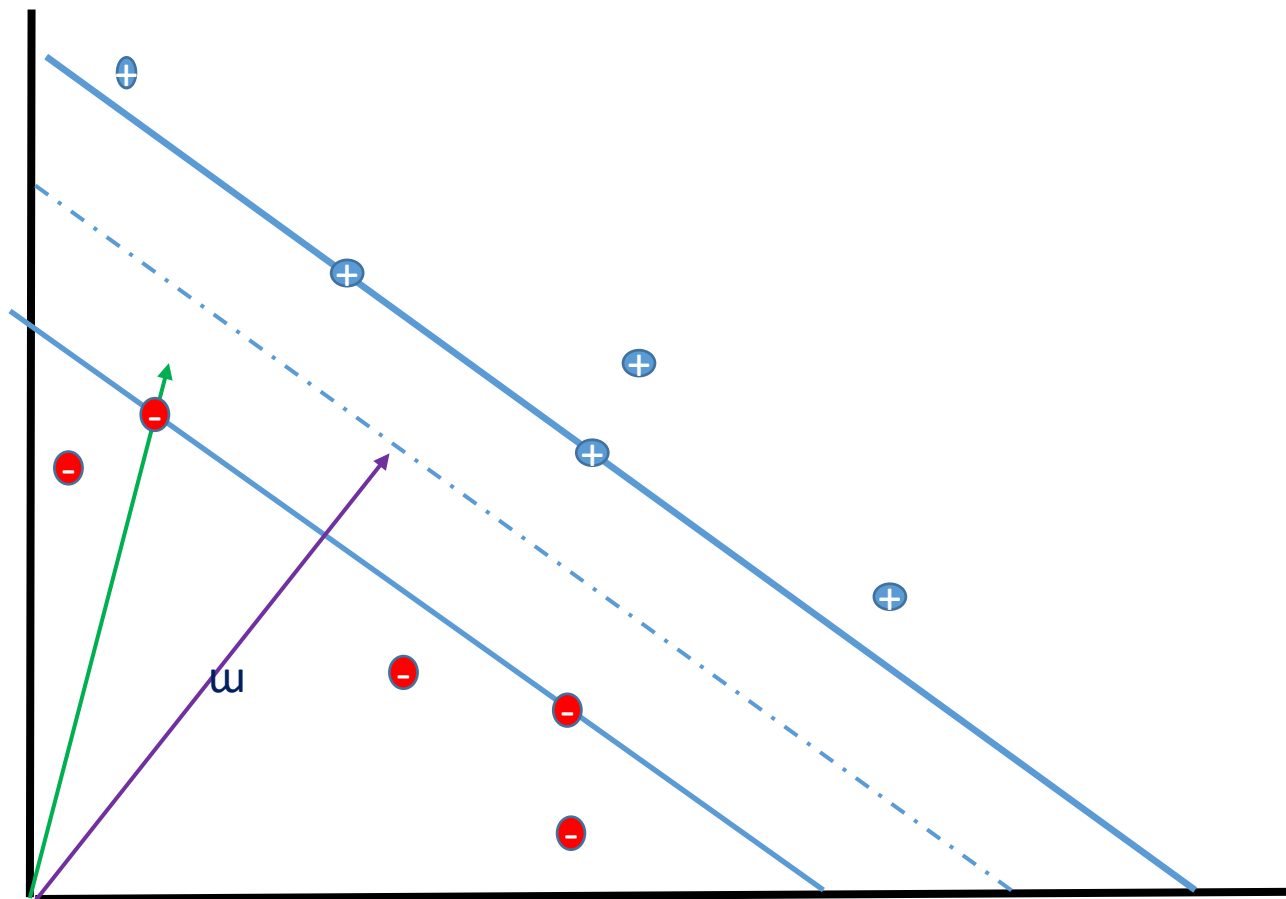
# Support Vector Machine



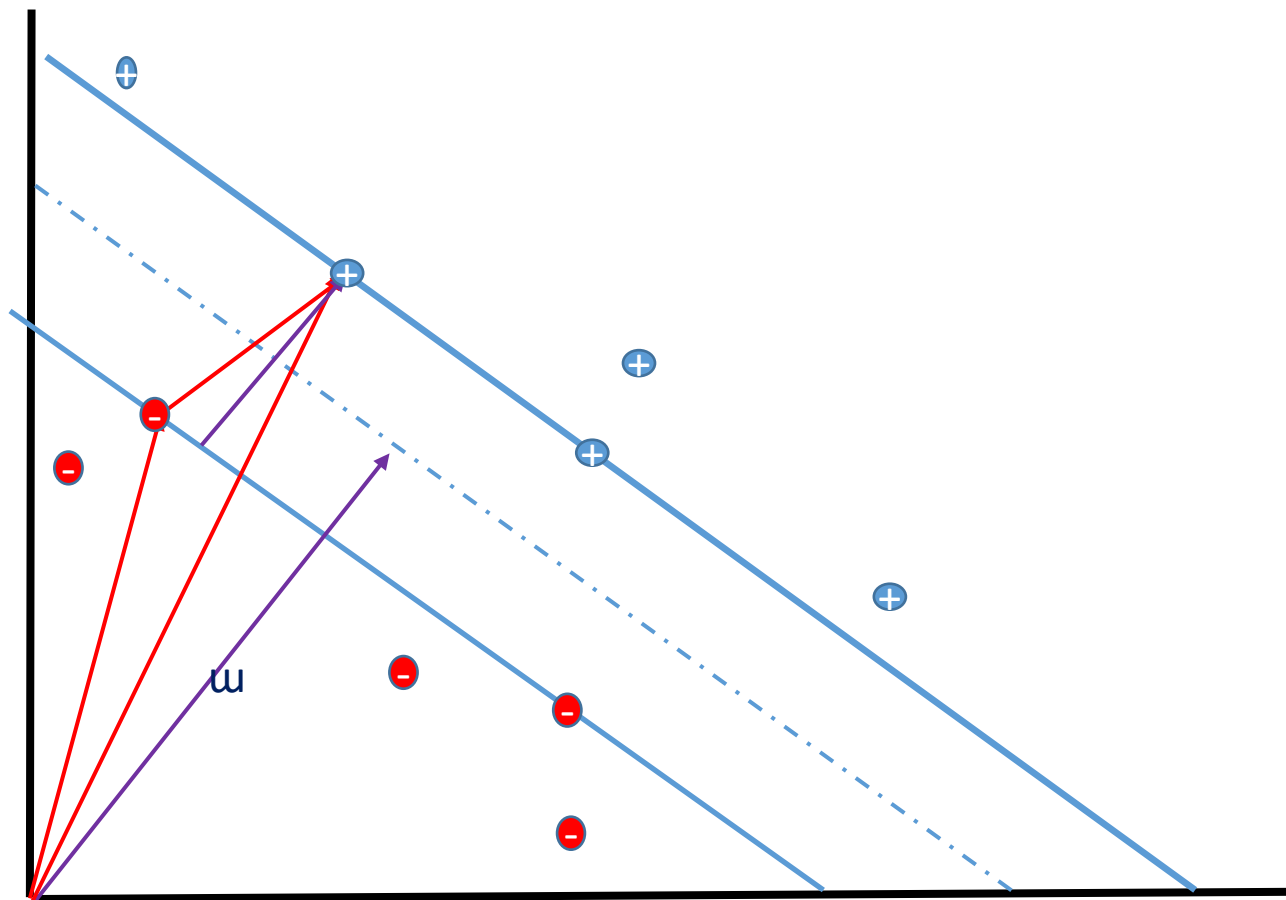
# Support Vector Machine



# Support Vector Machine



# Support Vector Machine



$$\bar{w} \cdot \bar{U} \geq c \quad b = -c$$

$$\bar{w} \cdot \bar{U} + b \geq 0$$

$$\bar{w} \cdot \bar{x}_i^+ + b \geq 1$$

$$\bar{w} \cdot \bar{x}_i^- + b \leq -1$$

Define  $y_i = 1$  for  $\bar{x}_i^+$   
 $y_i = -1$  for  $\bar{x}_i^-$

$$\begin{cases} y_i (\bar{w} \cdot \bar{x}_i^+ + b) \geq 1 \\ y_i (\bar{w} \cdot \bar{x}_i^- + b) \leq -1 \end{cases}$$

for  $x_i$  in the gutter

$$\begin{cases} y_i (\bar{w} \cdot \bar{x}_i^+ + b) - 1 = 0 \\ y_i (\bar{w} \cdot \bar{x}_i^- + b) - 1 = 0 \end{cases}$$

width of the street

$$(\bar{x}_i^+ - \bar{x}_i^-) \cdot \frac{\bar{w}}{\|\bar{w}\|} = \frac{\bar{w} \cdot \bar{x}_i^+ - \bar{w} \cdot \bar{x}_i^-}{\|\bar{w}\|}$$

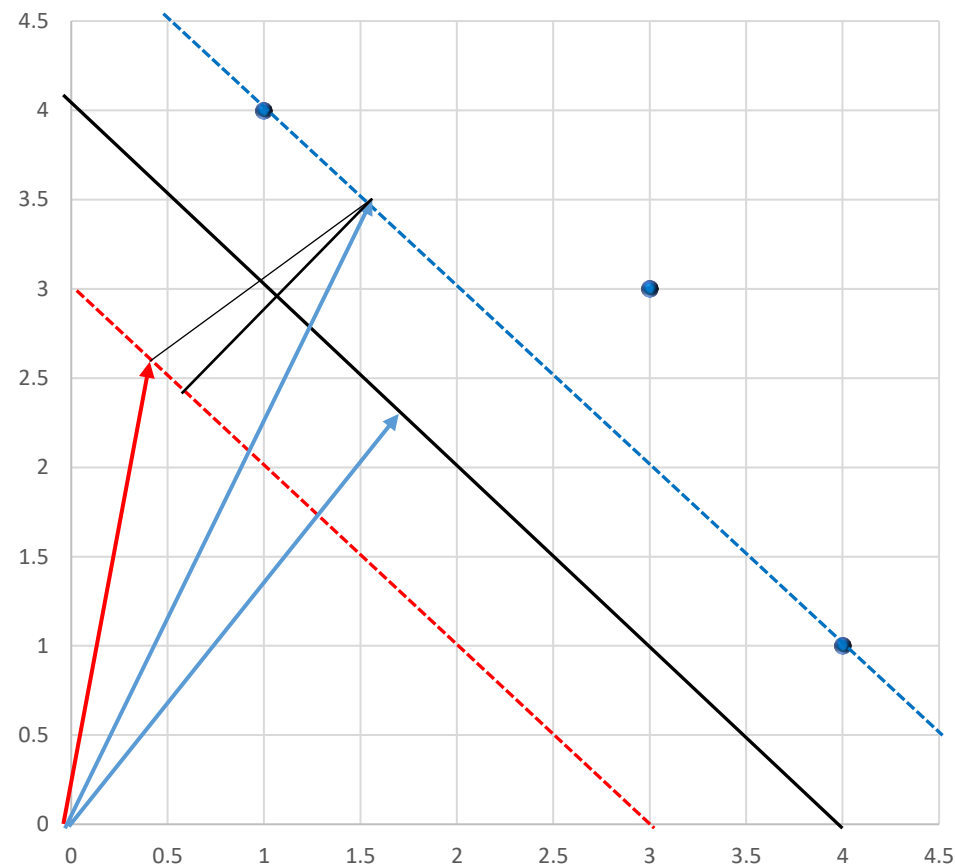
for  $x^+$   $y_i = 1$  then  $\bar{w} \cdot \bar{x}^+ = 1 - b$

for  $x^-$   $y_i = -1$  then  $\bar{w} \cdot \bar{x}^- = -1 - b$

$$\frac{\bar{w} \cdot \bar{x}^+ - \bar{w} \cdot \bar{x}^-}{\|\bar{w}\|} = \frac{(1-b) - (-1-b)}{\|\bar{w}\|} = \frac{2}{\|\bar{w}\|} = \text{width of the street}$$

$$\text{Max } \frac{2}{\|\bar{w}\|} \quad \text{s.t.} \quad y_i (\bar{w} \cdot \bar{x}_i + b) - 1 = 0$$

$$\Rightarrow \text{Min } \|\bar{w}\| \quad \text{s.t.} \quad y_i (\bar{w} \cdot \bar{x}_i + b) - 1 = 0$$



$$\Rightarrow \min \frac{1}{2} \|w\|^2$$

$$\text{s.t.} = y_i (\bar{w} x_i + b) - 1 = 0 \text{ for all } i$$

$$\Rightarrow \mathcal{L} = \min \frac{1}{2} \|w\|^2 - \sum_i \alpha_i [y_i (\bar{w} x_i + b) - 1]$$

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial w} = w - \sum_i \alpha_i y_i x_i = 0 \Rightarrow \boxed{w = \sum_i \alpha_i y_i x_i} \\ \frac{\partial \mathcal{L}}{\partial b} = \boxed{\sum_i \alpha_i y_i = 0} \end{cases}$$

$$\begin{aligned} \min \mathcal{L} = & \frac{1}{2} \sum_i \alpha_i y_i x_i \sum_j \alpha_j y_j x_j - \sum_i \alpha_i y_i x_i \sum_j \alpha_j y_j x_j \\ & - \sum_i \alpha_i y_i \vec{b} + \sum_i \alpha_i \end{aligned}$$

$$\max L = \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j x_i x_j$$

$$\text{Define } H_{ij} = y_i x_i \cdot y_j x_j$$

$$\Rightarrow L = \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i H_{ij} \alpha_j$$

$$\max_{\alpha} L = \sum \alpha_i - \frac{1}{2} \alpha^T H \alpha$$



