Homework 1

Due: September 28, 2022

Note: No late homework will be accepted. You may discuss with your classmates but you may not plagiarize. You need to turn in your analysis and also your code written in Octave or Matlab.

In Parts A, B, C, D we will use Lagrange polynomial interpolation and cubic spline interpolation to interpolate $(N+1=11)$ data points (x_i, y_i), where $i=0,1,2,...,10$. The independent variable x is in the range of $[-1,1]$. There are two sets of $(N+1=11)$ data points hw1AB.dat and hw1CD.dat you may download.

In Part E we will use Lagrange polynomial interpolation and trigonometric functions to interpolate $(N+1=11)$ data points (x_i, y_i), where $i=0,1,2,...,10$. The independent variable x is in the range of $[0,2\pi]$. There is one set of $(N+1=11)$ data points hw1E.dat you may download.

Part A. (20%)

Please refer to the file hw1AB.dat for the 11 data points. Here x_i, where $i=0,1,2,...,10$, are uniformly distributed in $[-1,1]$.

A.1 Plot the Lagrange polynomial $L_j(x)$, where $j=0,1,2,...,10$. Note that $L_j(x_i)=0$ when $i\neq j$ and $L_j(x_i)=1$ when $i=j$.

A.2 Plot the interpolating polynomial that goes through the 11 data points

$$P(x)=\sum_{j=0}^{10} y_j L_j(x)$$

Part B. (20%)

Please refer to the file hw1AB.dat for the 11 data points. Here x_i, where $i=0,1,2,...,10$, are uniformly distributed in $[-1,1]$.

B.1 Here we use cubic spline interpolation and we should assume the second derivative at the points, $g''(x_i)$ where $i=0,1,2,...,10$, as unknowns. Let's use free run-out condition for $g''(x_0)=g''(x_{10})=0$. What are the values of $g''(x_i)$ where $i=1,2,...,9$?

B.2 Plot the cubic spline interpolation for the whole range of $x\in[-1,1]$.

Part C. (20%)

Please refer to the file hw1CD.dat for the 11 data points. Here x_i, where $i=0,1,2,...,10$, are non-uniformly distributed in $[-1,1]$.

C.1 Plot the Lagrange polynomial $L_j(x)$, where $j=0,1,2,...,10$. Note that $L_j(x_i)=0$ when $i\neq j$ and $L_j(x_i)=1$ when $i=j$.

C.2 Plot the interpolating polynomial that goes through the 11 data points

$$P(x) = \sum_{j=0}^{10} y_j L_j(x)$$

Part D. (20%)

Please refer to the file hw1CD.dat for the 11 data points. Here x_i, where $i = 0, 1, 2, ..., 10$, are non-uniformly distributed in $[-1, 1]$.

D.1 Here we use cubic spline interpolation and we should assume the second derivative at the points, $g''(x_i)$ where $i = 0, 1, 2, ..., 10$, as unknowns. Let's use free run-out condition for $g''(x_0) = g''(x_{10}) = 0$. What are the values of $g''(x_i)$ where $i = 1, 2, ..., 9$?

D.2 Plot the cubic spline interpolation for the whole range of $x \in [-1, 1]$.

Part E. (20%)

Please refer to the file hw1E.dat for the 11 data points. Here x_i, where $i = 0, 1, 2, ..., 10$, are uniformly distributed in $[0, 2\pi]$.

E.1 Plot the Lagrange polynomial $L_j(x)$, where $j = 0, 1, 2, ..., 10$. Note that $L_j(x_i) = 0$ when $i \neq j$ and $L_j(x_i) = 1$ when $i = j$.

E.2 Plot the interpolating polynomial that goes through the 11 data points

$$P(x) = \sum_{j=0}^{10} y_j L_j(x)$$

E.3 Let's suppose that the 11 data points are the representation of a periodic function $f(x)$, which can be expressed as $f(x) = a \times \cos(x) + 3.6 \times \sin(2x)$. Can you find out the value of a by trial and error? Plot the interpolating function $f(x)$ that goes through the 11 data points for the whole range of $x \in [0, 2\pi]$.