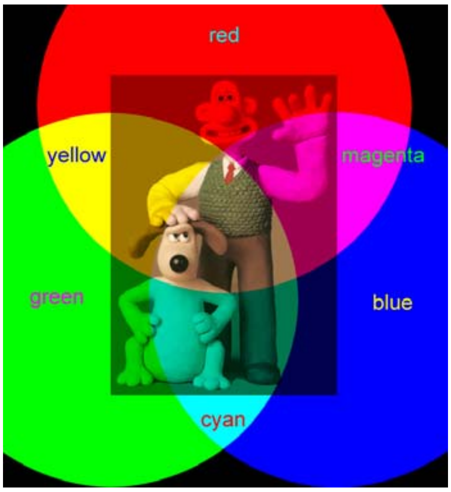
Image Processing

Lecture Notes: Color Correction

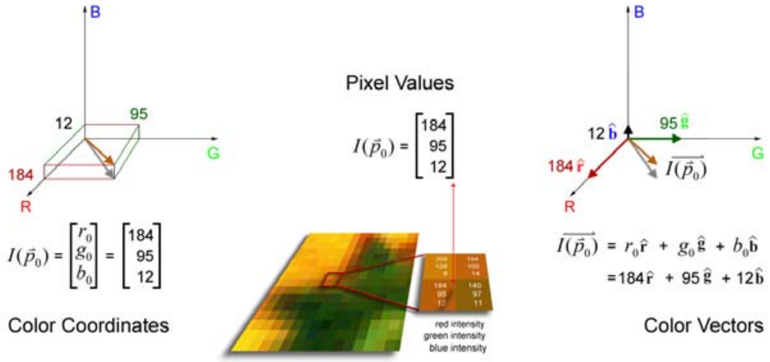
Kai-Lung Hua

Color Images

- Are constructed from three overlaid intensity maps.
- Each map represents the intensity of a different "primary" color.
- The actual hues of the primaries do not matter as long as they are distinct.
- The primaries are 3 vectors (or axes) that form a "basis" of the color space.



Vector-Valued Pixels

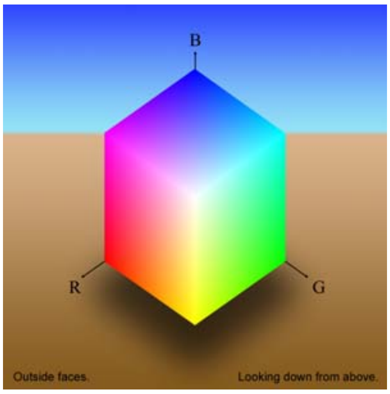


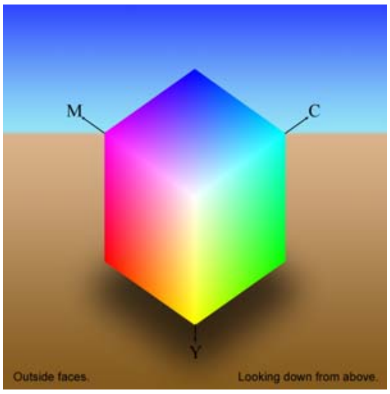
Each color corresponds to a point in a 3D vector space

Color Space for standard digital images

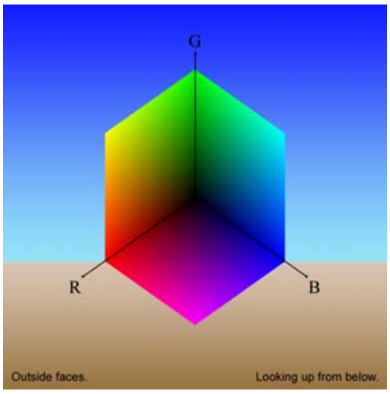
- primary image colors red, green, and blue
  - correspond to R,G, and B axes in color space.
- 8-bits of intensity resolution per color
  - correspond to integers 0 through 255 on axes.
- no negative values
  - color “space” is a cube in the first octant of 3-space.
- color space is discrete
  - 256^3 possible colors = 16,777,216 elements in cube.

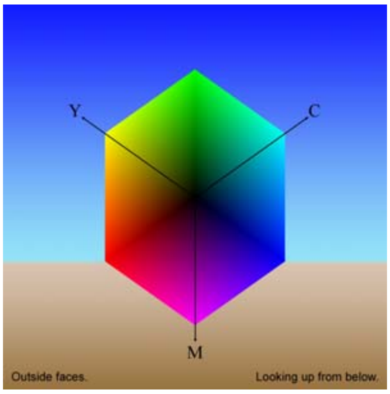
Color Cube: Faces (outer)



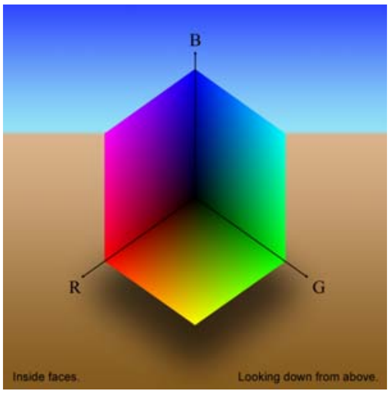


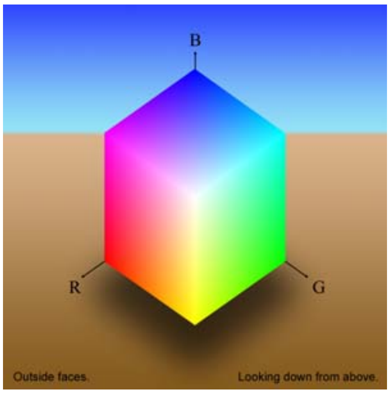
Color Cube: Faces (inner)



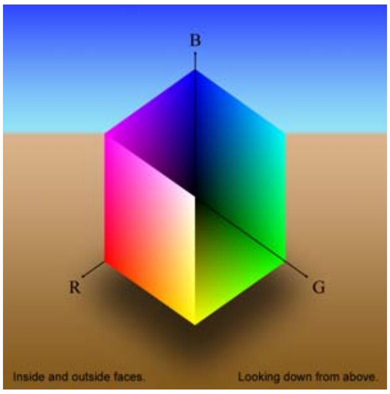


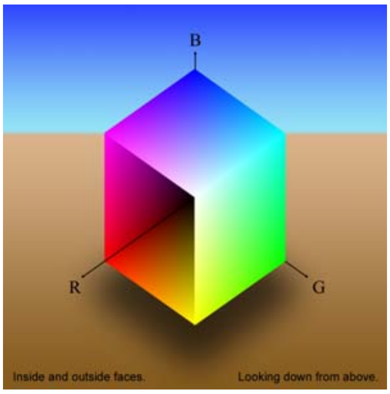
Color Cube: Faces (inner and outer)





Color Cube: Faces (inner and outer)





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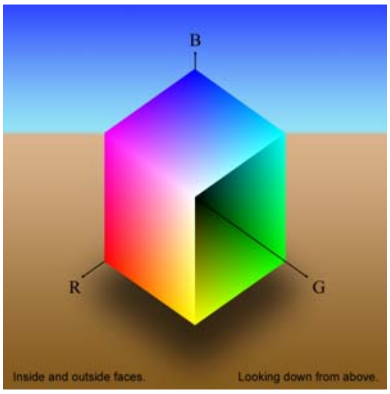

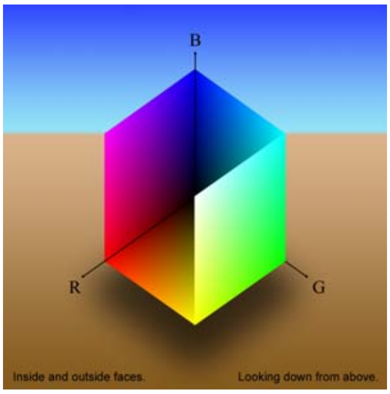

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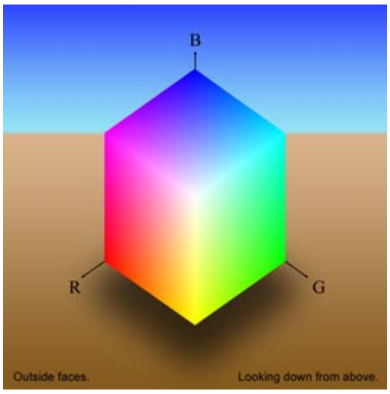

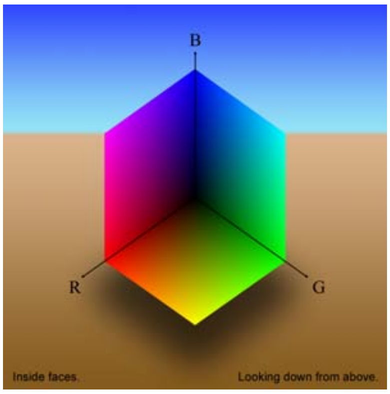

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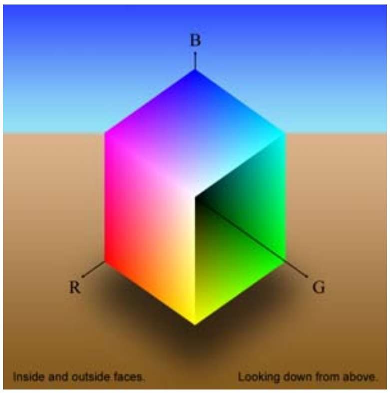

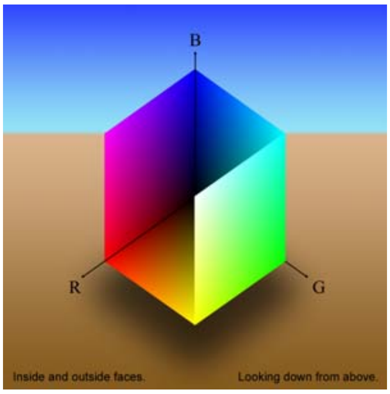

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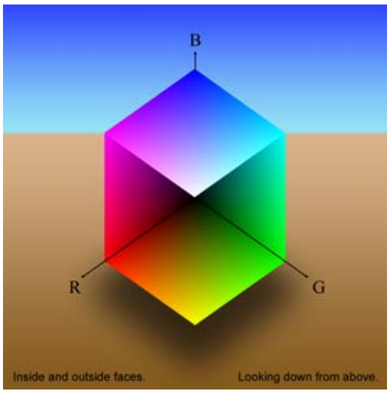

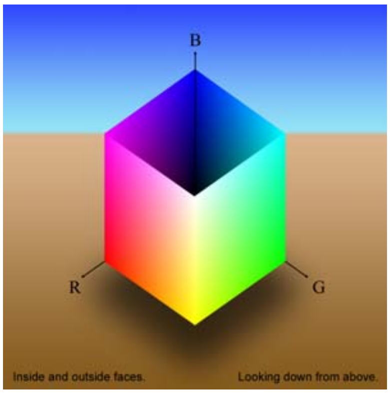

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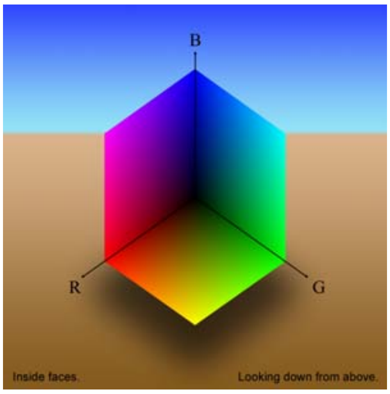
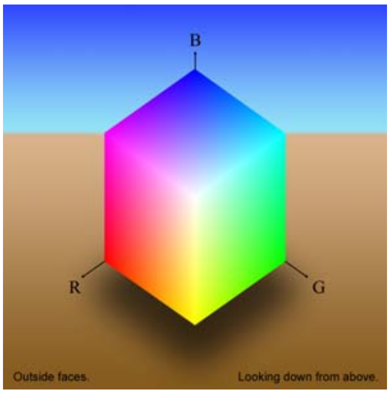
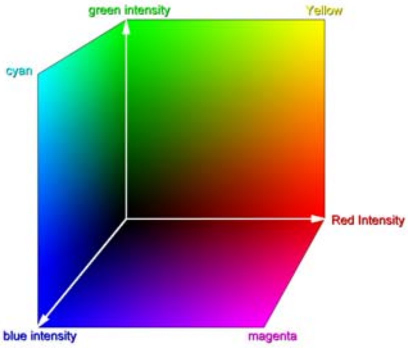
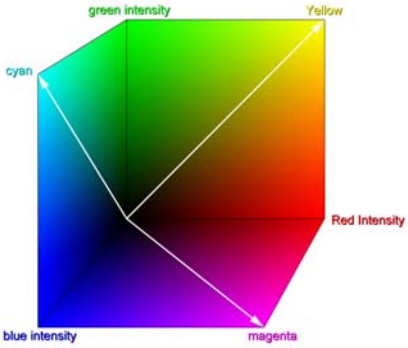




Different Axis Sets in Color Space

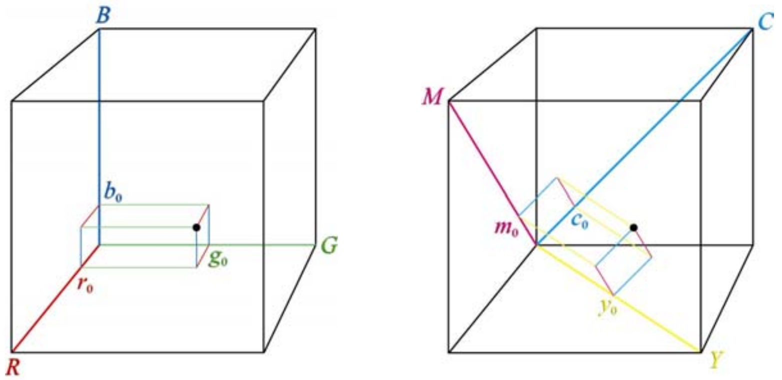


RGB axes



CMY axes

Color With Respect To Different Axes



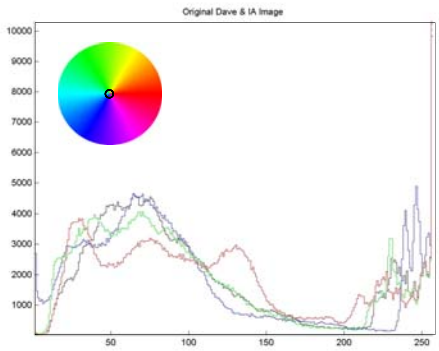
The same color has different RGB and CMY coordinates.

Color Correction

Global changes in the coloration of an image to alter its tint, its hues or the saturation of its colors with minimal changes to its luminant features

Gamma Adjustment of Color Bands

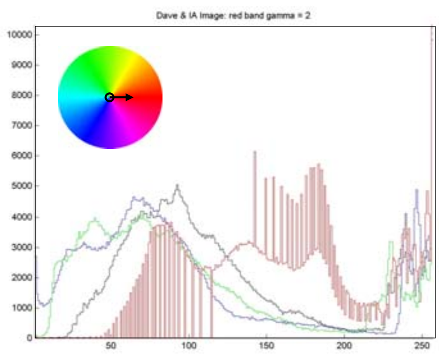
original



David Peters, producer, and representatives of the IA, The International Alliance of Theatrical Stage Employees, Moving Picture Technicians, Artists and Allied Crafts, on the set of *Frozen Impact* (PorchLight Entertainment, 2003).

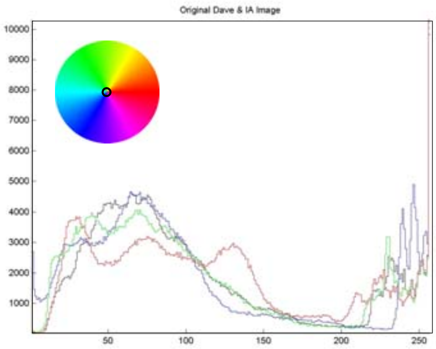
Gamma Adjustment of Color Bands

red γ=2



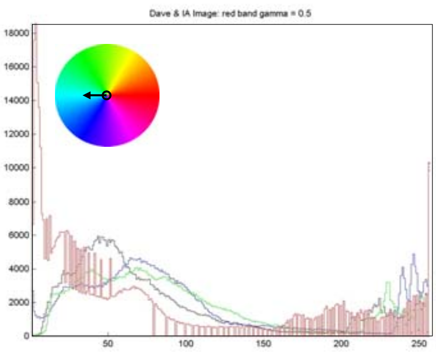
Gamma Adjustment of Color Bands

original



Gamma Adjustment of Color Bands

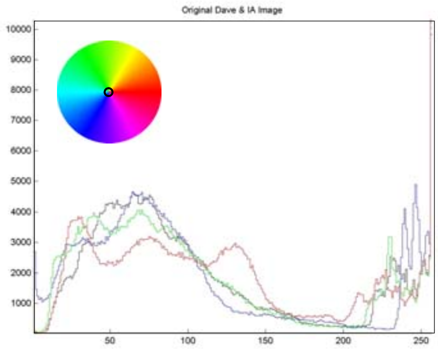
red γ=0.5



reduced red = increased cyan

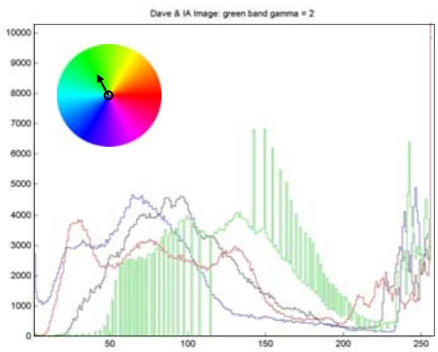
Gamma Adjustment of Color Bands

original



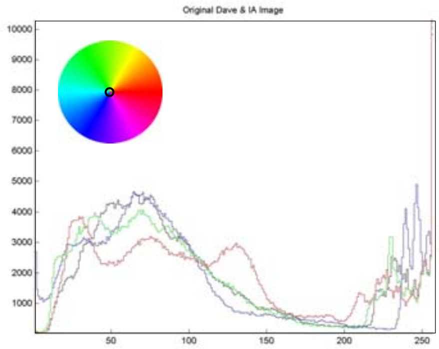
Gamma Adjustment of Color Bands

green γ=2



Gamma Adjustment of Color Bands

original



Gamma Adjustment of Color Bands

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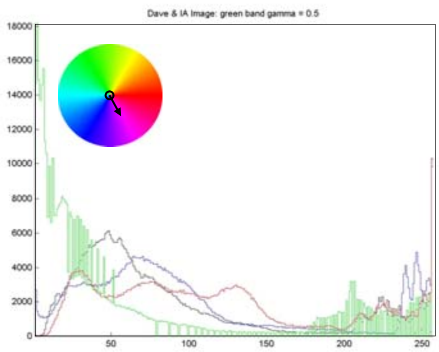
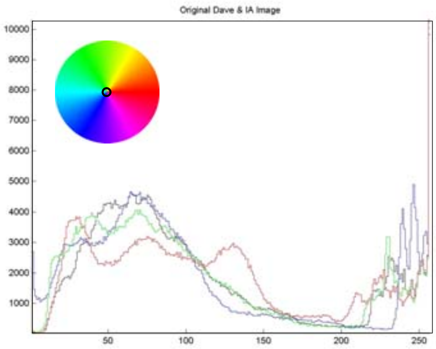


reduced green = incr. magenta

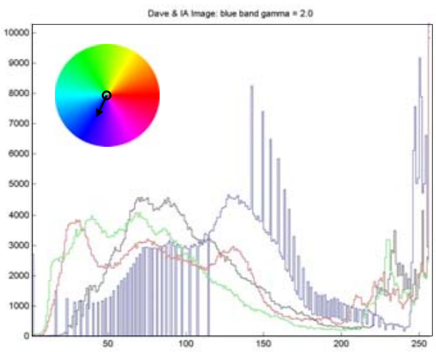
Gamma Adjustment of Color Bands

original



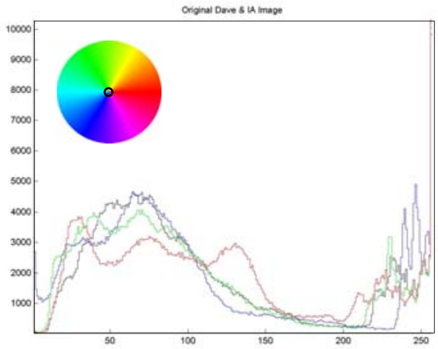
Gamma Adjustment of Color Bands

blue γ=2



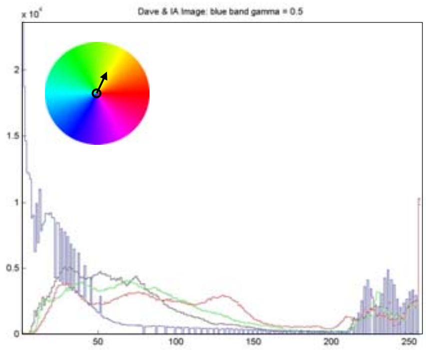
Gamma Adjustment of Color Bands

original



Gamma Adjustment of Color Bands

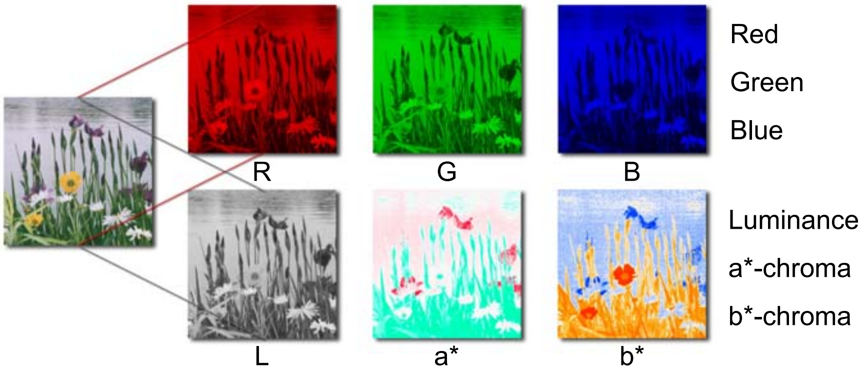
blue γ=0.5



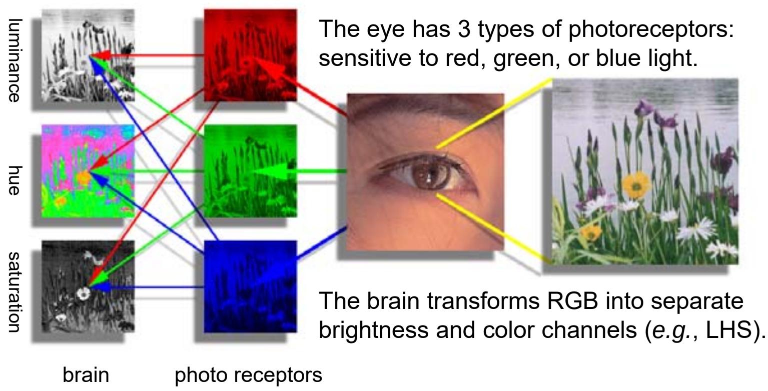
reduced blue = incr. yellow

Color Images

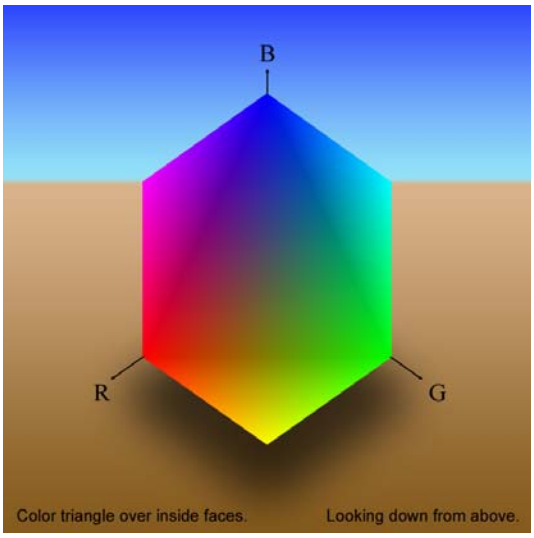
are represented by three bands (not uniquely) *e.g.*, R, G, & B or L, a*, & b*.



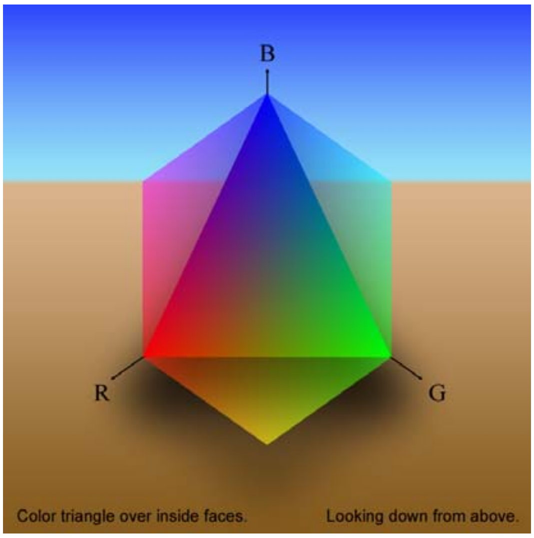
RGB to LHS: A Perceptual Transformation



Color Cube: Equi-value Triangle



Color Cube: Equi-value Triangle



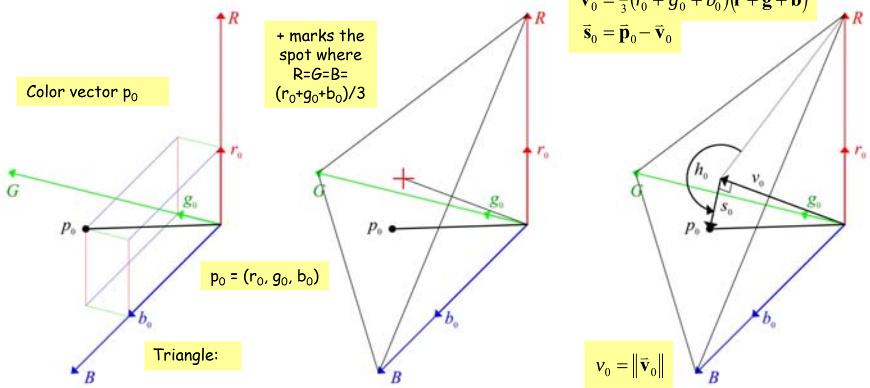
HSV Color Representation

$\vec{v}_0 = \frac{1}{3}(r_0 + g_0 + b_0)(\hat{\mathbf{r}} + \hat{\mathbf{g}} + \hat{\mathbf{b}})$

$\vec{s}_0 = \vec{p}_0 - \vec{v}_0$

Color vector p_0

+ marks the spot where R=G=B= $(r_0+g_0+b_0)/3$



$p_0 = (r_0, g_0, b_0)$

Triangle:

lies in plane $R+G+B=(r_0+g_0+b_0)$,

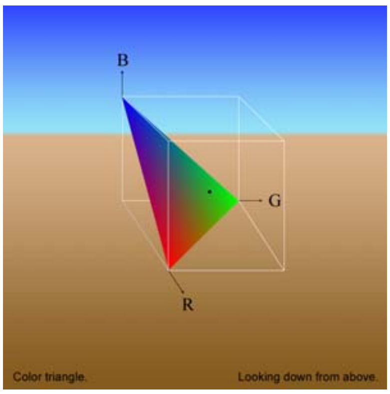
intersects R, G, & B at $r_0+g_0+b_0$, and contains p_0.

$v_0 = \|\vec{v}_0\|$

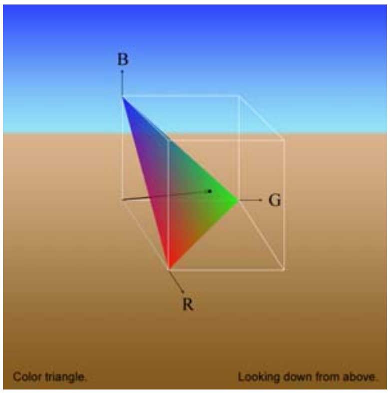
$s_0 = \|\vec{s}_0\|$

$h_0 = \angle\{[(r_0 + g_0 + b_0)\hat{\mathbf{r}} - \vec{v}_0], \vec{s}_0\}$

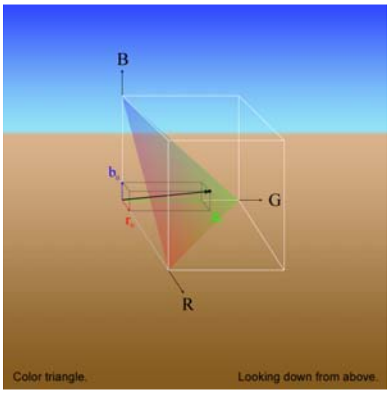
Color Point on Equivalue Triangle



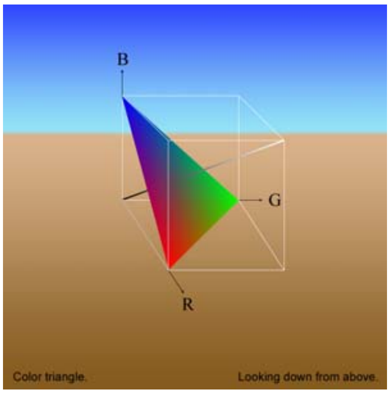
Color Vector Associated with Point



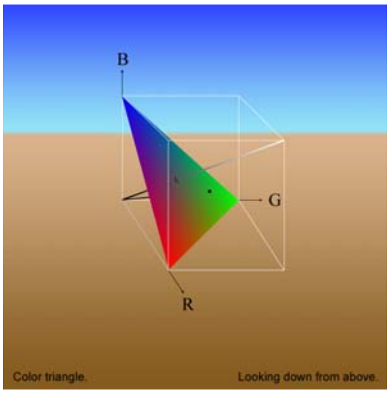
Color Coordinates and Component Vectors



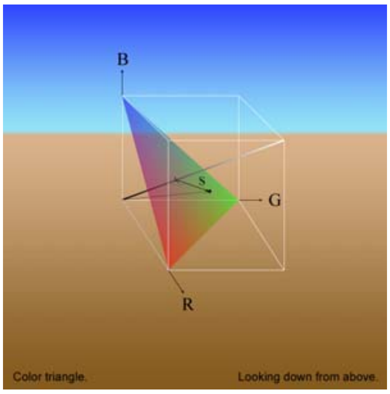
Color Cube, Equivalue Triangle, & Gray Line



Color Point and Gray Line



Saturation Component of Color Vector



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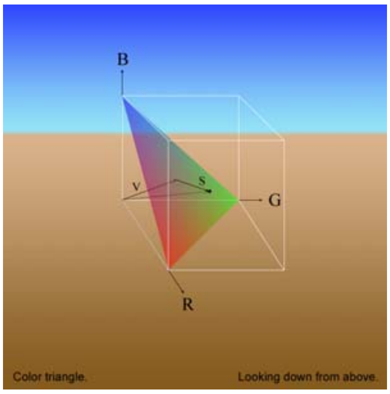
Saturation and Value Components of Color Vector



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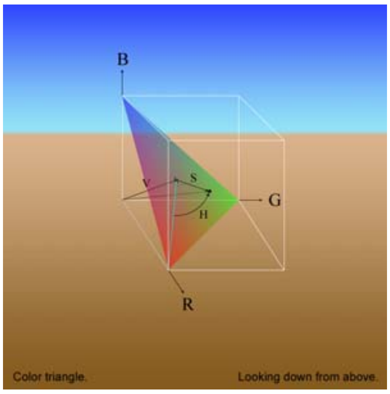
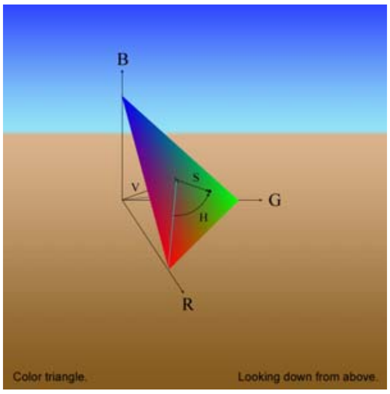
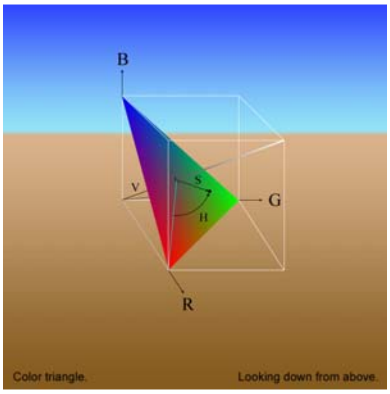
Hue, Saturation, and Value



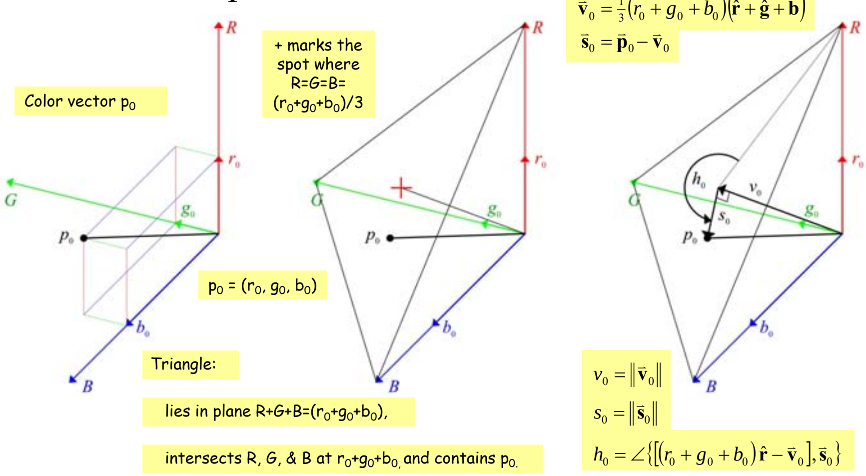
Hue and Saturation on Equivalue Plane



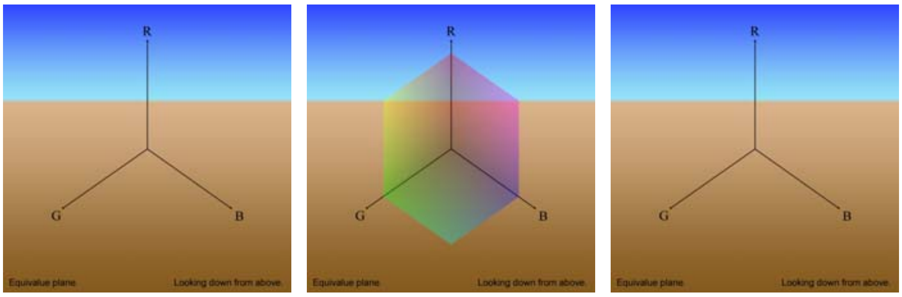
Hue, Saturation, and Value with Gray Line



HSV Color Representation



Equivalue Plane Intersecting Color Cube



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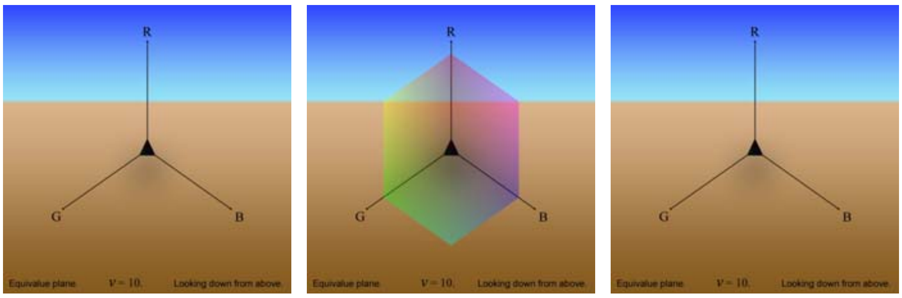

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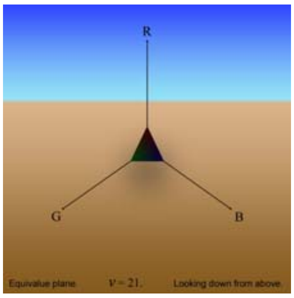

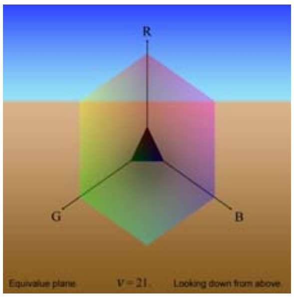

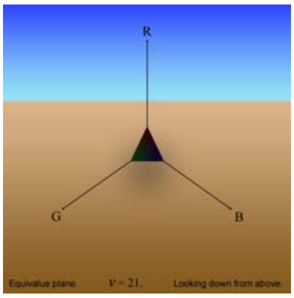

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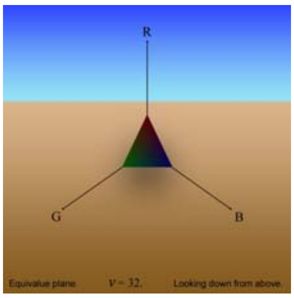
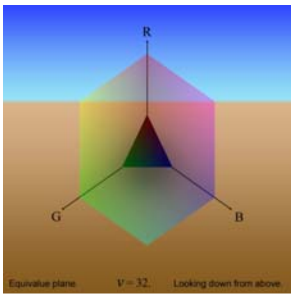
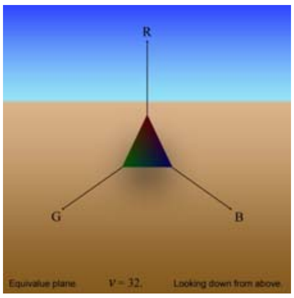
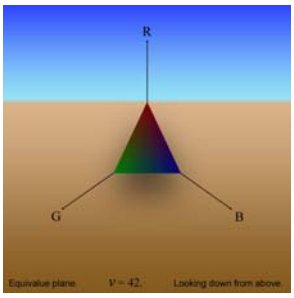
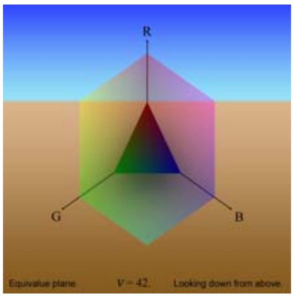


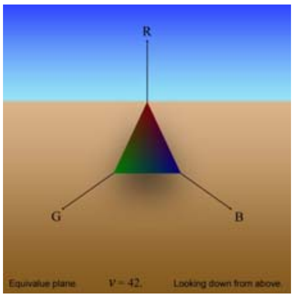




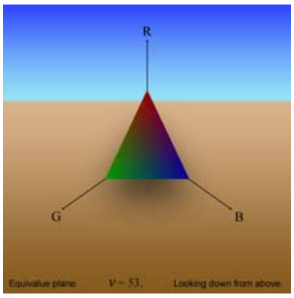
Equivalue Plane Intersecting Color Cube

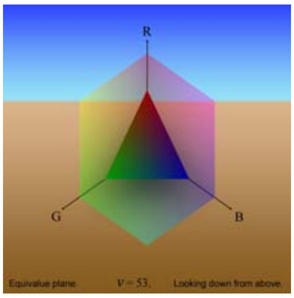


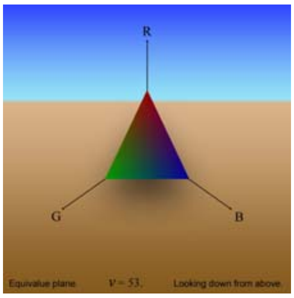




Equivalue Plane Intersecting Color Cube







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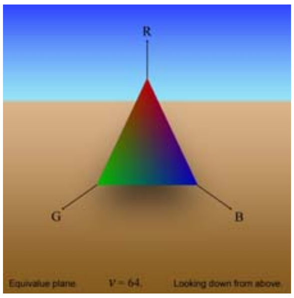
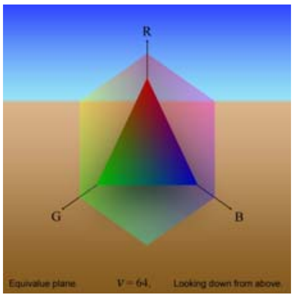
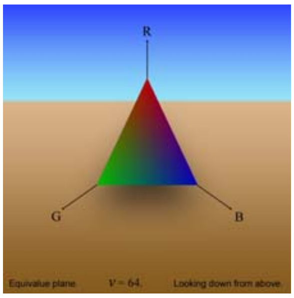
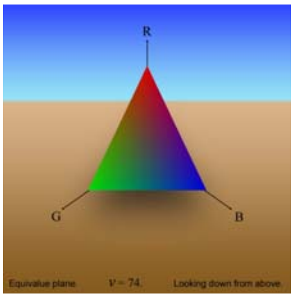
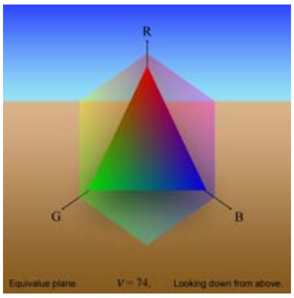


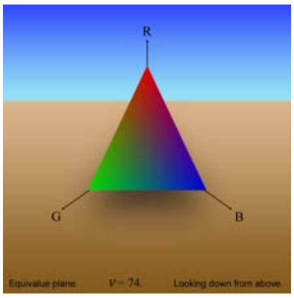




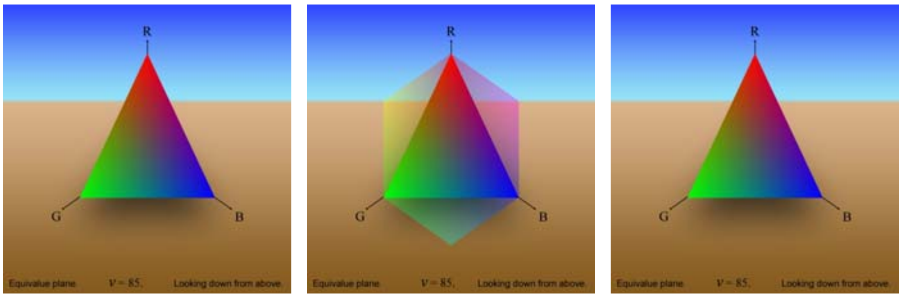
Equivalue Plane Intersecting Color Cube







Equivalue Plane Intersecting Color Cube



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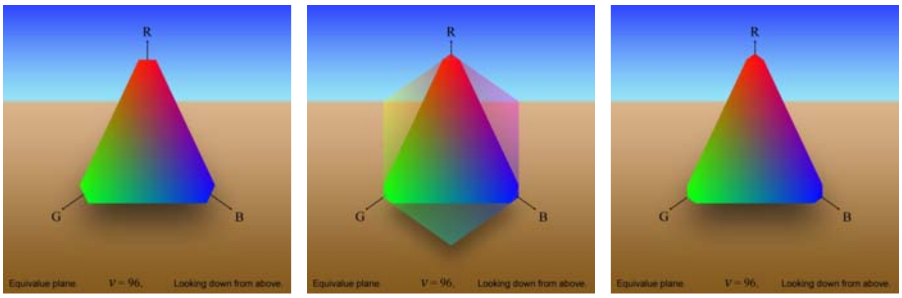
Equivalue Plane Intersecting Color Cube



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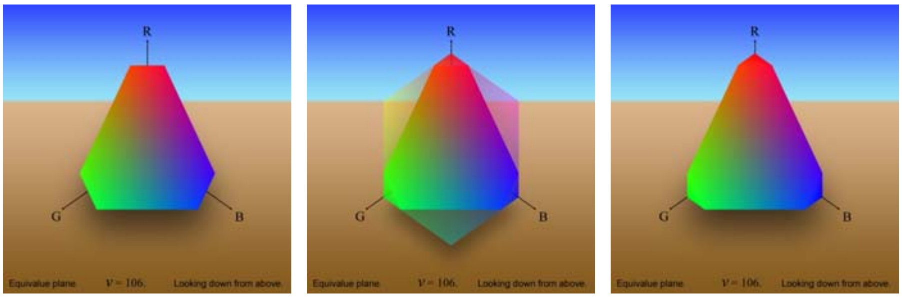
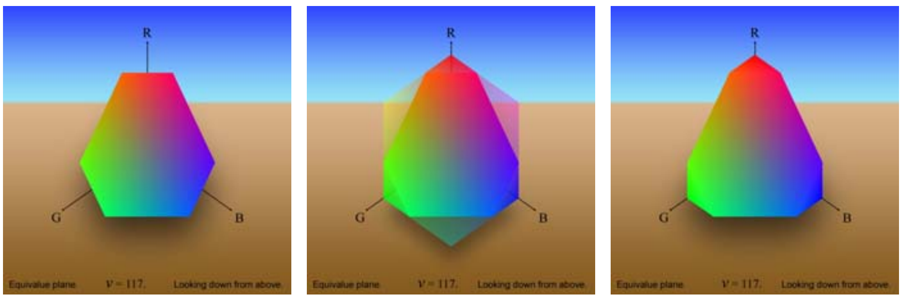
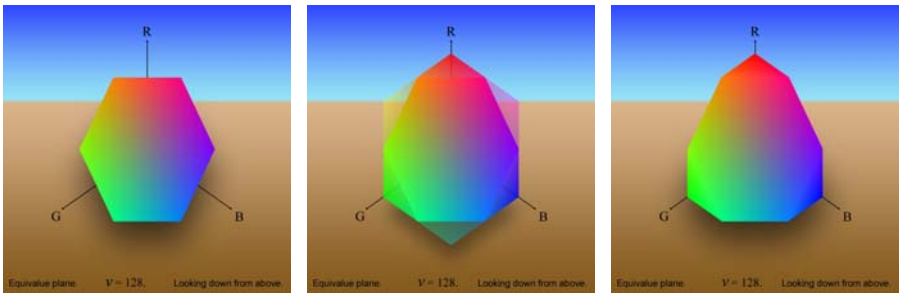
Equivalue Plane Intersecting Color Cube



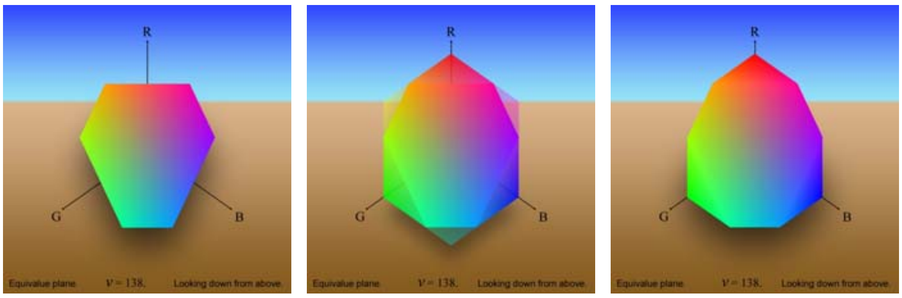
Equivalue Plane Intersecting Color Cube



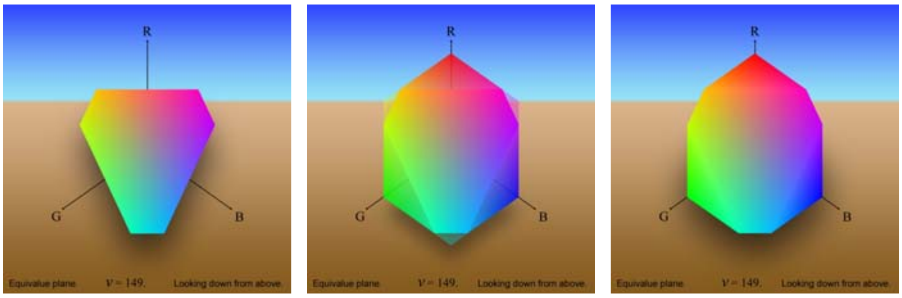
Equivalue Plane Intersecting Color Cube



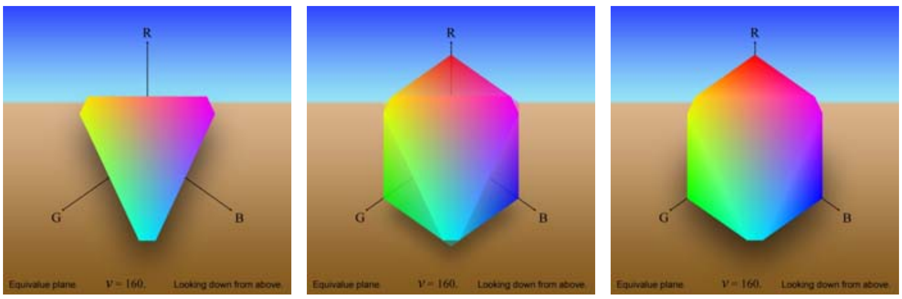
Equivalue Plane Intersecting Color Cube



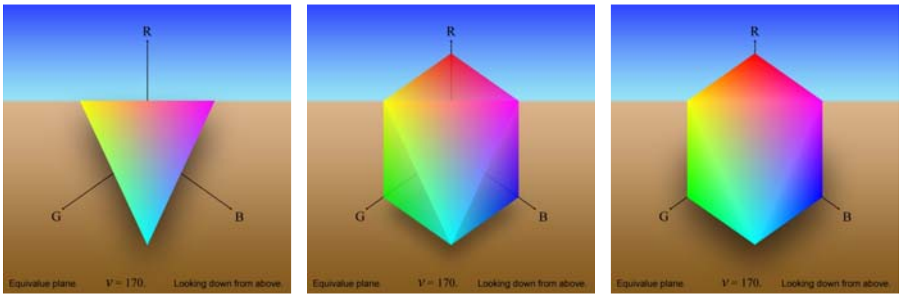
Equivalue Plane Intersecting Color Cube



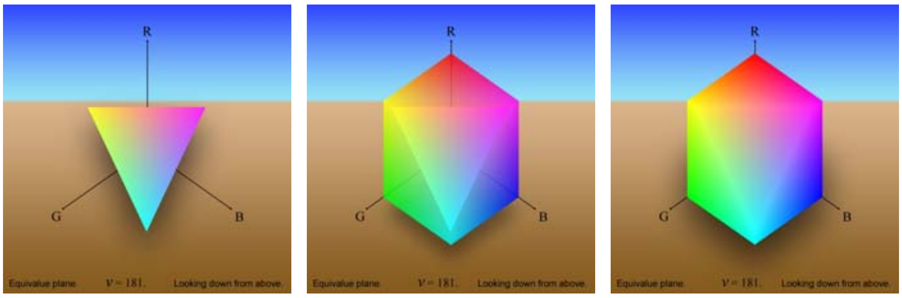
Equivalue Plane Intersecting Color Cube



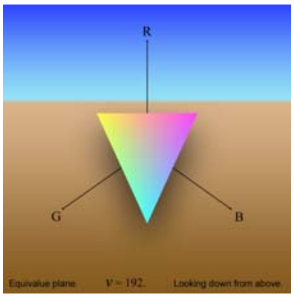
Equivalue Plane Intersecting Color Cube

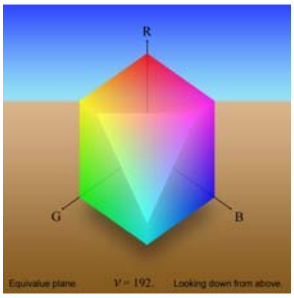


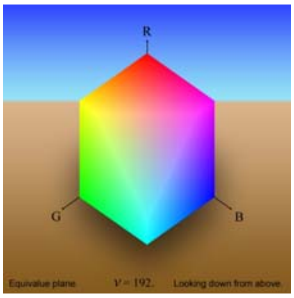
Equivalue Plane Intersecting Color Cube



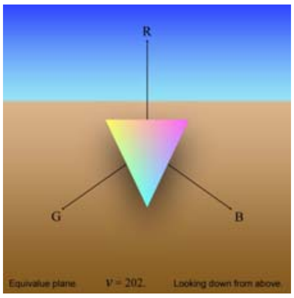
Equivalue Plane Intersecting Color Cube

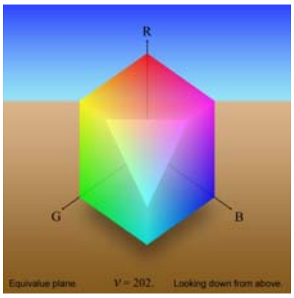


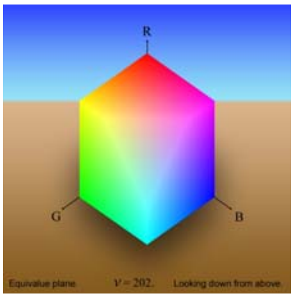




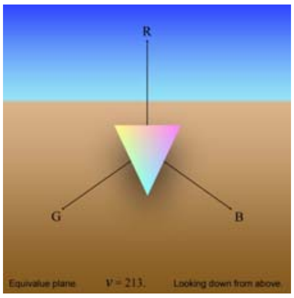
Equivalue Plane Intersecting Color Cube

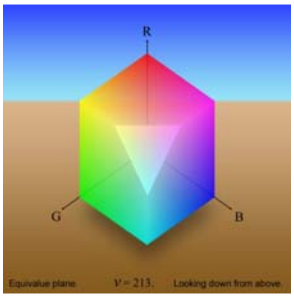


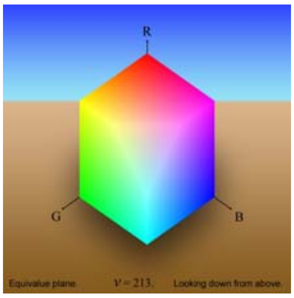




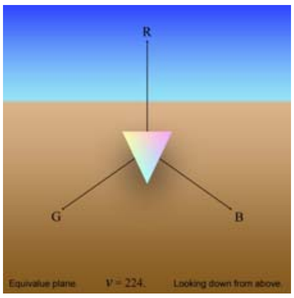
Equivalue Plane Intersecting Color Cube

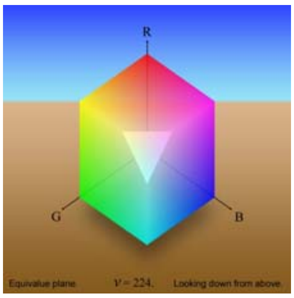


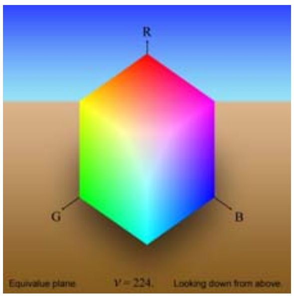




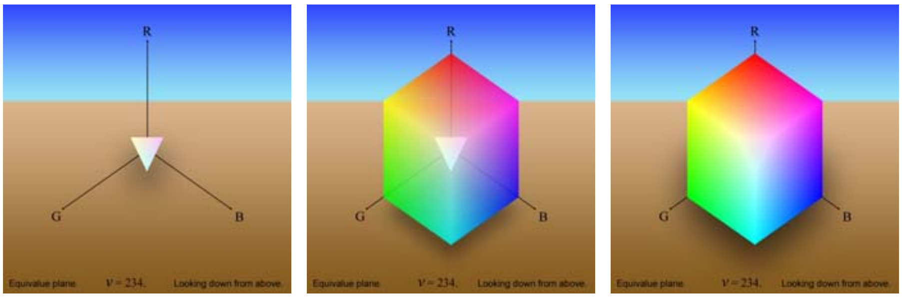
Equivalue Plane Intersecting Color Cube



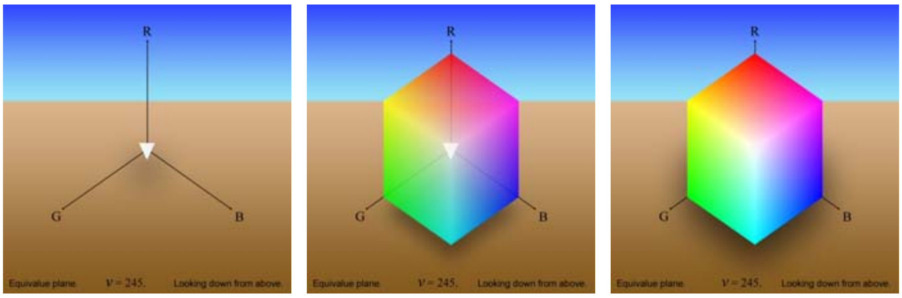




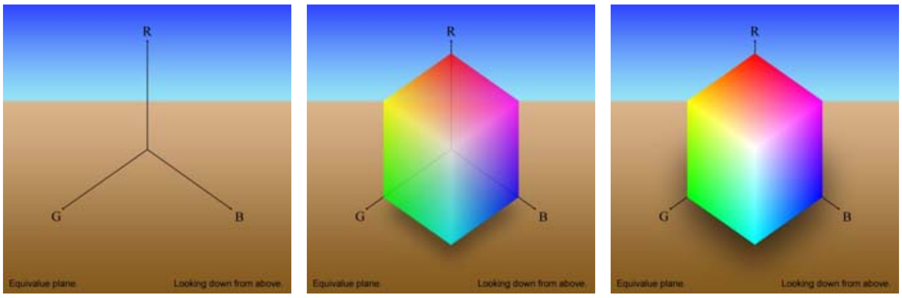
Equivalue Plane Intersecting Color Cube



Equivalue Plane Intersecting Color Cube



Equivalue Plane Intersecting Color Cube



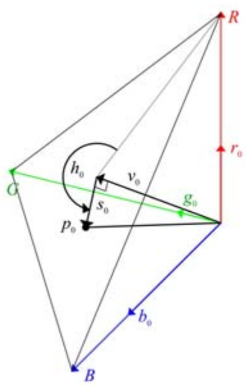
RGB to HSV Conversion

$$\mathbf{v}_0 = \tfrac{1}{3}\begin{bmatrix} c \\ c \\ c \end{bmatrix}, \text{ where } c = r_0 + g_0 + b_0.$$

$$v_0 = \tfrac{1}{3}c, \text{ or } v_0 = \|\mathbf{v}_0\| = \tfrac{\sqrt{3}}{3}c.$$

$$\mathbf{s}_0 = \mathbf{p}_0 - \mathbf{v}_0 = \begin{bmatrix} r_0 - \frac{1}{3}c \\ g_0 - \frac{1}{3}c \\ b_0 - \frac{1}{3}c \end{bmatrix}. \qquad \mathbf{p}_0 = \begin{bmatrix} r_0 \\ g_0 \\ b_0 \end{bmatrix}.$$

$$s_0 = \|\mathbf{s}_0\| = \sqrt{\left(r_0 - \tfrac{1}{3}c\right)^2 + \left(g_0 - \tfrac{1}{3}c\right)^2 + \left(b_0 - \tfrac{1}{3}c\right)^2}.$$



RGB to HSV Conversion

$\begin{bmatrix} c \end{bmatrix}$ …here $c = r_0 + g_0$ …

c/3 is the usual value-image intensity (the average of *r*, *g*, & *b*) …

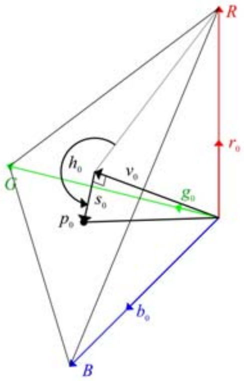
… either def. of v_0 can be used, but c/3 has the advantage of being in the range [0 , 255].

$$v_0 = \tfrac{1}{3}c, \text{ or } v_0 = \|\mathbf{v}_0\| = \tfrac{\sqrt{3}}{3}c.$$

… c √3/3 is the length of the value vector…

$$\mathbf{s}_0 = \mathbf{p}_0 - \mathbf{v}_0 = \begin{bmatrix} \ldots \\ g_0 - \tfrac{1}{3}c \\ b_0 - \tfrac{1}{3}c \end{bmatrix}. \qquad \mathbf{p}_0 = \begin{bmatrix} r_0 \\ g_0 \\ b_0 \end{bmatrix}.$$

$$s_0 = \|\mathbf{s}_0\| = \sqrt{\left(r_0 - \tfrac{1}{3}c\right)^2 + \left(g_0 - \tfrac{1}{3}c\right)^2 + \left(b_0 - \tfrac{1}{3}c\right)^2}.$$

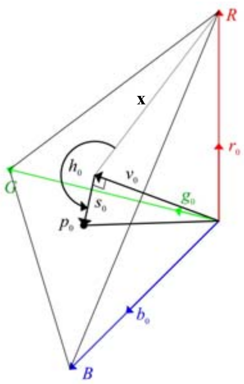


RGB to HSV Conversion

$$\mathbf{x} = \mathbf{R} - \mathbf{v}_0 = \begin{bmatrix} c \\ 0 \\ 0 \end{bmatrix} - \tfrac{1}{3}\begin{bmatrix} c \\ c \\ c \end{bmatrix} = \frac{c}{3}\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$$

$$\mathbf{s}_0 = \mathbf{p}_0 - \mathbf{v}_0 = \begin{bmatrix} r_0 - \tfrac{1}{3}c \\ g_0 - \tfrac{1}{3}c \\ b_0 - \tfrac{1}{3}c \end{bmatrix}.$$

$$h_0 = \angle(\mathbf{s}_0, \mathbf{x}) = \cos^{-1}\left(\frac{\mathbf{s}_0 \cdot \mathbf{x}}{\|\mathbf{s}_0\|\|\mathbf{x}\|}\right).$$



RGB to HSV Conversion

In summary,

$$v_0 = \tfrac{1}{3}c, \text{ or } v_0 = \|\mathbf{v}_0\| = \tfrac{\sqrt{3}}{3}c,$$

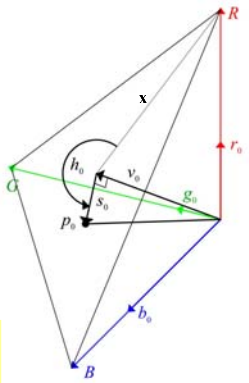
where $c = r_0 + g_0 + b_0$,

$$s_0 = \sqrt{\left(r_0 - \tfrac{1}{3}c\right)^2 + \left(g_0 - \tfrac{1}{3}c\right)^2 + \left(b_0 - \tfrac{1}{3}c\right)^2},$$

and

$$h_0 = \cos^{-1}\left(\frac{\mathbf{s}_0 \cdot \mathbf{x}}{\|\mathbf{s}_0\|\|\mathbf{x}\|}\right).$$

Usually, s_0 is normalized to lie in the interval (0,1) and h_0 is shifted to lie in $(0,2\pi)$.



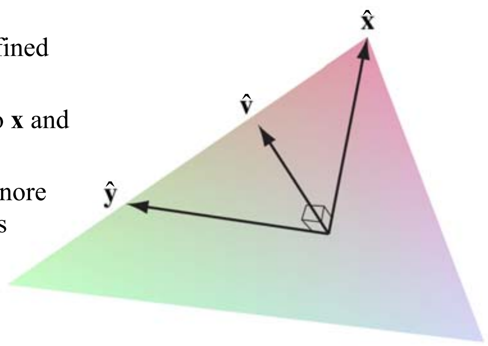
HSV to RGB Conversion

The equivalue plane is perpendicular to the value vector, **v**.

The plane contains vector **x** defined on slide 45.

Therefore, **v** is perpendicular to **x** and **y** = **v** × **x** is also in the plane.

If we keep the directions but ignore the magnitudes, the unit vectors



$$\hat{\mathbf{x}} = \begin{bmatrix}1\\0\\0\end{bmatrix}, \quad \hat{\mathbf{y}} = \begin{bmatrix}0\\1\\0\end{bmatrix}, \quad \text{and} \quad \hat{\mathbf{v}} = \begin{bmatrix}0\\0\\1\end{bmatrix},$$

form an orthonormal basis with respect to the equivalue plane.

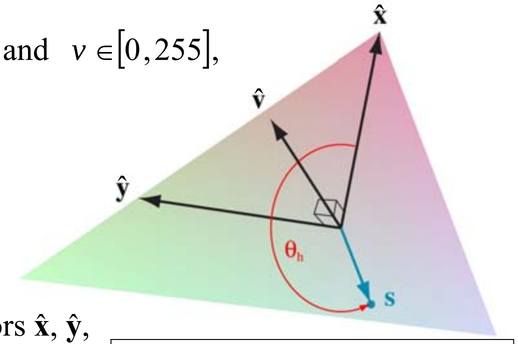
HSV to RGB Conversion

Given values *h*, *s*, and *v*, where

$$h \in [0, 2\pi), \quad s \in [0,1], \quad \text{and} \quad v \in [0, 255],$$

the saturation vector is

$$[\mathbf{s}]_{\mathbf{xyv}} = \begin{bmatrix} s\cos(h) \\ s\sin(h) \\ 0 \end{bmatrix}_{\mathbf{xyv}},$$



with respect to unit vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{v}}$, in the equivalue plane.

$$\boxed{\mathbf{s} = s\cos(h)\,\hat{\mathbf{x}} + s\sin(h)\,\hat{\mathbf{y}} + 0\,\hat{\mathbf{v}}.}$$

HSV to RGB Conversion

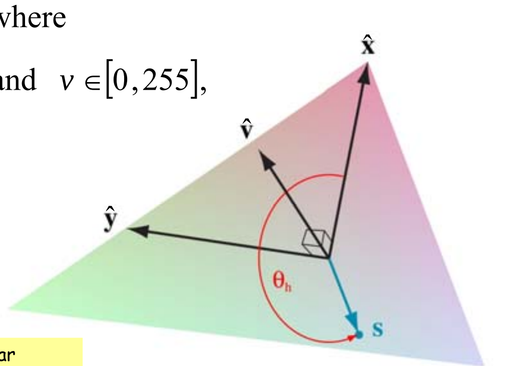
Given values *h*, *s*, and *v*, where

$$h \in [0, 2\pi), \quad s \in [0,1], \quad \text{and} \quad v \in [0, 255],$$

the

These are the coordinates of **s** with respect to $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, & $\hat{\mathbf{v}}$.

$$[\mathbf{s}]_{\mathbf{xyv}} = \begin{bmatrix} s\cos(h) \\ s\sin(h) \\ 0 \end{bmatrix}_{\mathbf{xyv}},$$



with res

This is **s** written as a linear combination of vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, & $\hat{\mathbf{v}}$.

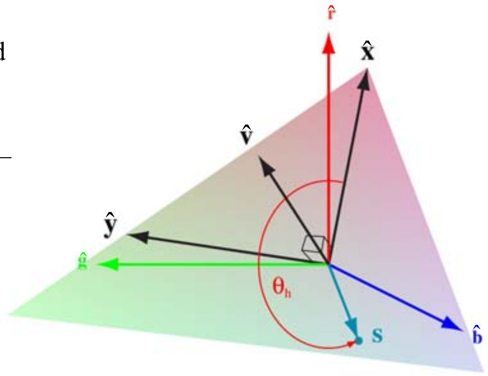
and $\hat{\mathbf{v}}$, in the equivalue plane.

$$\boxed{\mathbf{s} = s\cos(h)\,\hat{\mathbf{x}} + s\sin(h)\,\hat{\mathbf{y}} + 0\,\hat{\mathbf{v}}.}$$

HSV to RGB Conversion

$\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{v}}$, are not in the same directions as the red, green, and blue unit vectors, $\hat{\mathbf{r}}$, $\hat{\mathbf{g}}$, and $\hat{\mathbf{b}}$.

Therefore, $[\mathbf{s}]_{\mathbf{xyv}}$ — which we know — is not equal to $[\mathbf{s}]_{\mathbf{rgb}}$ — which we need in order to find the color, $\mathbf{p}_0$, with respect to $\hat{\mathbf{r}}$, $\hat{\mathbf{g}}$, and $\hat{\mathbf{b}}$.



$$[\mathbf{s}]_{\mathbf{rgb}} = [r_0 \quad g_0 \quad b_0]^{\mathrm{T}}$$

$$\mathbf{s} \leftrightarrow r_0\hat{\mathbf{r}} + g_0\hat{\mathbf{g}} + b_0\hat{\mathbf{b}},$$

$$\mathbf{s} \leftrightarrow s\cos(h)\hat{\mathbf{x}} + s\sin(h)\hat{\mathbf{y}} + 0\,\hat{\mathbf{v}}.$$

We need to find r_0, g_0, & b_0.

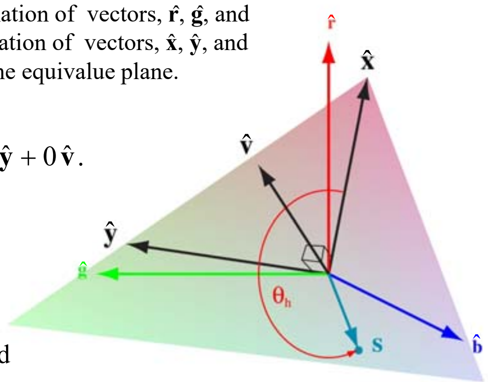
HSV to RGB Conversion

Vector $\mathbf{s}$ written as a linear combination of vectors, $\hat{\mathbf{r}}$, $\hat{\mathbf{g}}$, and $\hat{\mathbf{b}}$, and $\mathbf{s}$ written as a linear combination of vectors, $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{v}}$ both refer to the same point on the equivalue plane.

$$\mathbf{s} \leftrightarrow r_0\hat{\mathbf{r}} + g_0\hat{\mathbf{g}} + b_0\hat{\mathbf{b}},$$

$$\mathbf{s} \leftrightarrow s\cos(h)\hat{\mathbf{x}} + s\sin(h)\hat{\mathbf{y}} + 0\,\hat{\mathbf{v}}.$$

The specific numbers in $[\mathbf{s}]_{\mathbf{rgb}}$ and in $[\mathbf{s}]_{\mathbf{xyv}}$ (that represent the point w.r.t. the two coordinate systems) are, however, different.



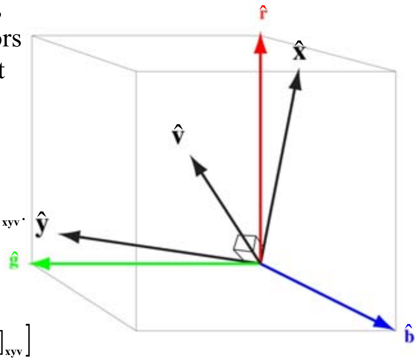
$$[\mathbf{s}]_{\mathbf{rgb}} = [r_0 \quad g_0 \quad b_0]^{\mathrm{T}} \quad \text{and}$$

$$[\mathbf{s}]_{\mathbf{xyz}} = [s\cos(h) \quad s\sin(h) \quad 0]^{\mathrm{T}} \text{ but } [\mathbf{s}]_{\mathbf{rgb}} \neq [\mathbf{s}]_{\mathbf{xyz}}$$

HSV to RGB Conversion

We can find r_0, g_0, and b_0, from h_0, s_0, and v_0, if we know how the unit vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{v}}$, are expressed with respect to $\hat{\mathbf{r}}$, $\hat{\mathbf{g}}$, and $\hat{\mathbf{b}}$. That relationship is in the form of a rotation matrix, A, such that,

$$[\hat{\mathbf{x}}]_{\mathbf{rgb}} = A[\hat{\mathbf{x}}]_{\mathbf{xyv}},\ [\hat{\mathbf{y}}]_{\mathbf{rgb}} = A[\hat{\mathbf{y}}]_{\mathbf{xyv}},\ [\hat{\mathbf{v}}]_{\mathbf{rgb}} = A[\hat{\mathbf{v}}]_{\mathbf{xyv}}.$$



Then,

$$\begin{aligned}[\mathbf{s}]_{\mathbf{rgb}} &= A[\mathbf{s}]_{\mathbf{xyv}} \\ &= A\left[s\cos(h)[\hat{\mathbf{x}}]_{\mathbf{xyv}} + s\sin(h)[\hat{\mathbf{y}}]_{\mathbf{xyv}} + 0[\hat{\mathbf{v}}]_{\mathbf{xyv}}\right] \\ &= s\cos(h)A[\hat{\mathbf{x}}]_{\mathbf{xyv}} + s\sin(h)A[\hat{\mathbf{y}}]_{\mathbf{xyv}} + 0\,A[\hat{\mathbf{v}}]_{\mathbf{xyv}} \\ &= s\cos(h)[\hat{\mathbf{x}}]_{\mathbf{rgb}} + s\sin(h)[\hat{\mathbf{y}}]_{\mathbf{rgb}} + 0[\hat{\mathbf{v}}]_{\mathbf{rgb}}.\end{aligned}$$

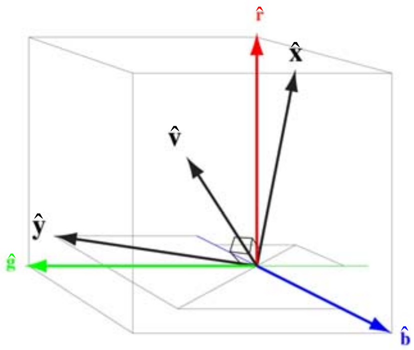
HSV to RGB Conversion

When written w.r.t the **xyz** coordinate system we have

$$\hat{\mathbf{x}} = \begin{bmatrix}1\\0\\0\end{bmatrix},\quad \hat{\mathbf{y}} = \begin{bmatrix}0\\1\\0\end{bmatrix},\quad \text{and}\quad \hat{\mathbf{v}} = \begin{bmatrix}0\\0\\1\end{bmatrix},$$

So that,

$$[\hat{\mathbf{x}}]_{\mathbf{rgb}} = A\begin{bmatrix}1\\0\\0\end{bmatrix},\ [\hat{\mathbf{y}}]_{\mathbf{rgb}} = A\begin{bmatrix}0\\1\\0\end{bmatrix},\ [\hat{\mathbf{v}}]_{\mathbf{rgb}} = A\begin{bmatrix}0\\0\\1\end{bmatrix}.$$



But that implies,

$$A = \left[[\hat{\mathbf{x}}]_{\mathbf{rgb}}\ \ [\hat{\mathbf{y}}]_{\mathbf{rgb}}\ \ [\hat{\mathbf{v}}]_{\mathbf{rgb}}\right].$$

HSV to RGB Conversion

$\hat{\mathbf{v}}$ is the unit vector in the direction $[1\ 1\ 1]^T$ when written w.r.t **rgb** coordinates.

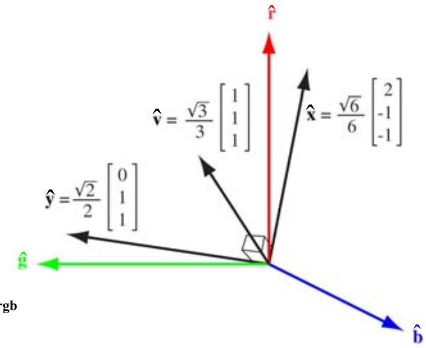
$$[\hat{\mathbf{v}}]_{\mathbf{rgb}} = \frac{\sqrt{3}}{3}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$\hat{\mathbf{x}}$ is perpendicular to $\hat{\mathbf{v}}$ and has equal $\hat{\mathbf{g}}$ and $\hat{\mathbf{b}}$ components.

$$[\hat{\mathbf{x}}]_{\mathbf{rgb}} = \frac{\sqrt{6}}{6}\begin{bmatrix}2\\-1\\-1\end{bmatrix}$$

$\hat{\mathbf{y}}$ is the cross product of $\hat{\mathbf{v}}$ with $\hat{\mathbf{x}}$.

$$[\hat{\mathbf{y}}]_{\mathbf{rgb}} = [\hat{\mathbf{v}}]_{\mathbf{rgb}} \times [\hat{\mathbf{x}}]_{\mathbf{rgb}} = \frac{\sqrt{2}}{2}\begin{bmatrix}0\\1\\-1\end{bmatrix}$$



HSV to RGB Conversion

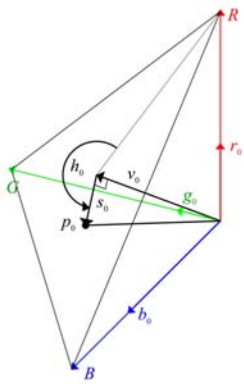
Therefore, the rotation matrix is

$$A = \frac{\sqrt{6}}{6}\begin{bmatrix}2 & 0 & \sqrt{2}\\ -1 & \sqrt{3} & \sqrt{2}\\ -1 & -\sqrt{3} & \sqrt{2}\end{bmatrix}$$

and

$$[\mathbf{s}]_{\mathbf{rgb}} = s\tfrac{\sqrt{6}}{6}\cos(h)\begin{bmatrix}2\\-1\\-1\end{bmatrix} + s\tfrac{\sqrt{2}}{2}\sin(h)\begin{bmatrix}0\\1\\-1\end{bmatrix} + 0\tfrac{\sqrt{3}}{3}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$= s\tfrac{\sqrt{6}}{6}\cos(h)\begin{bmatrix}2\\-1\\-1\end{bmatrix} + s\tfrac{\sqrt{2}}{2}\sin(h)\begin{bmatrix}0\\1\\-1\end{bmatrix}.$$

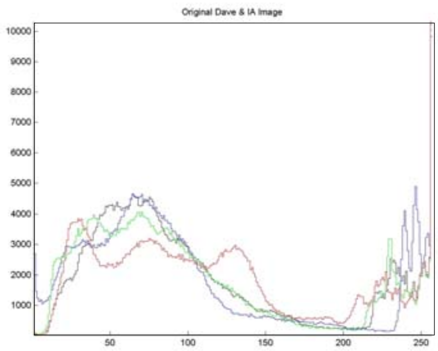


Finally, $[\mathbf{s}]_{\mathbf{rgb}}$ must be shifted to the value vector to obtain the **rgb** color of $\mathbf{p}_0$:

$\mathbf{p}_0 = [\mathbf{p}]_{\mathbf{rgb}} = [\mathbf{s}]_{\mathbf{rgb}} + [\mathbf{v}]_{\mathbf{rgb}}$, where $\mathbf{s}_0 = [\mathbf{s}]_{\mathbf{rgb}}$ and $[\mathbf{v}]_{\mathbf{rgb}} = \mathbf{v}_0$ as def'd. on slide 15.

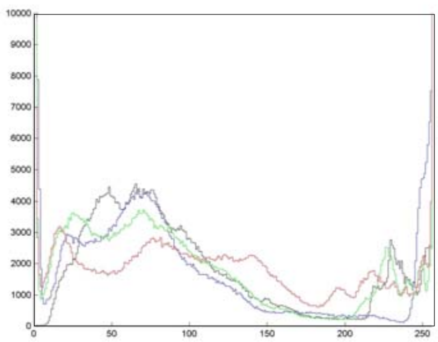
Saturation Adjustment

original



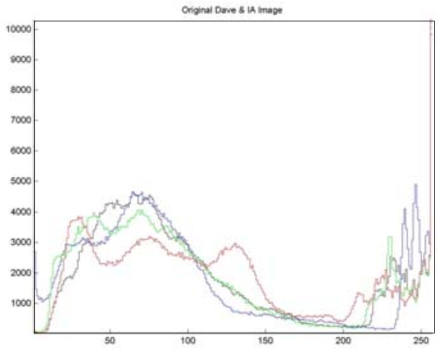
Saturation Adjustment

saturation + 50%



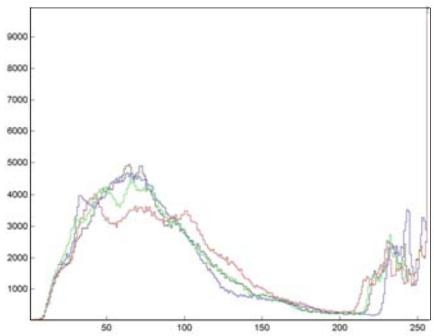
Saturation Adjustment

original



Saturation Adjustment

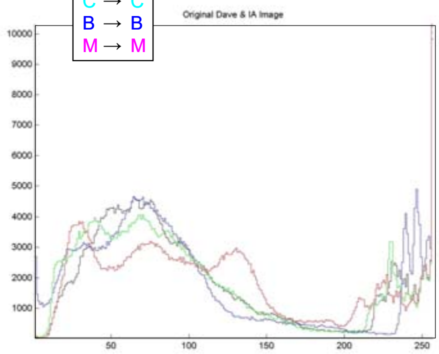
saturation - 50%



Hue Shifting

R → R
Y → Y
G → G
C → C
B → B
M → M

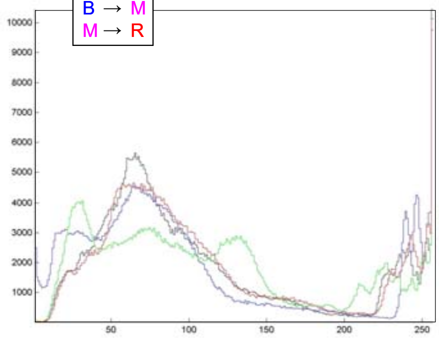
original



Hue Shifting

R → Y
Y → G
G → C
C → B
B → M
M → R

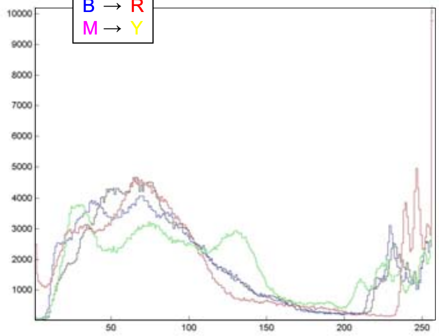
hue + 60°



Hue Shifting

R → G
Y → C
G → B
C → M
B → R
M → Y

hue + 120°



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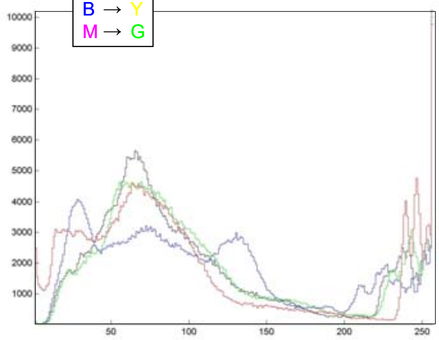
Hue Shifting

R → C
Y → B
G → M
C → R
B → Y
M → G

hue + 180°



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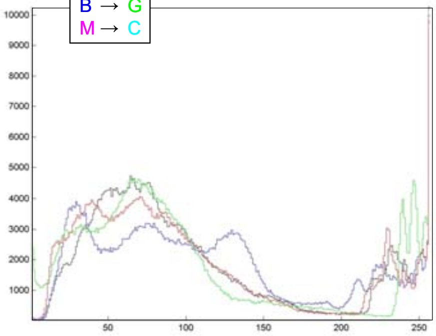
Hue Shifting

R → B
Y → M
G → R
C → Y
B → G
M → C

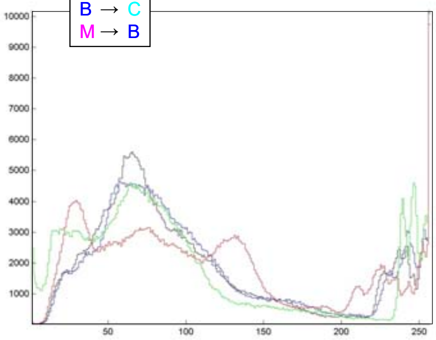
hue + 240°



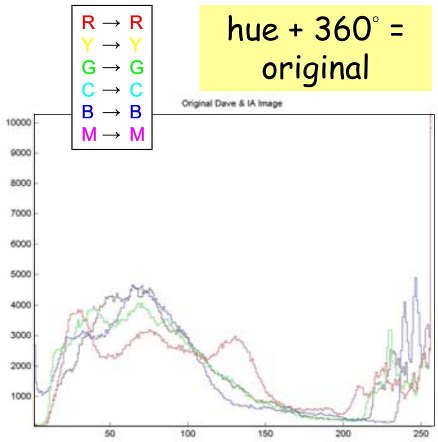
Hue Shifting

R → M
Y → R
G → Y
C → G
B → C
M → B

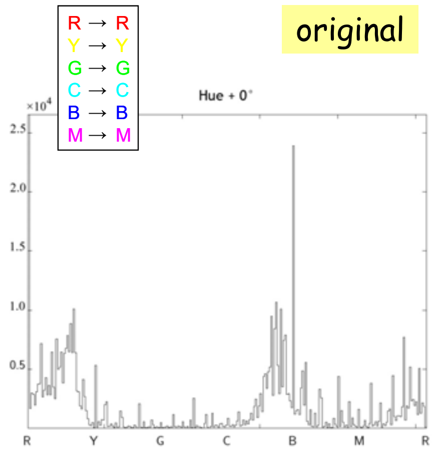
hue + 300°



Hue Shifting



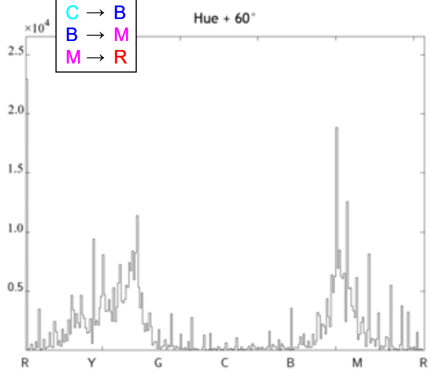
Hue Shifting



Hue Shifting

R → Y
Y → G
G → C
C → B
B → M
M → R

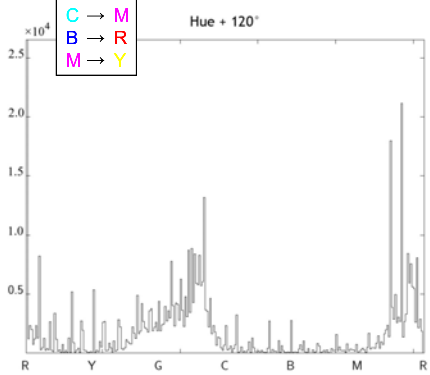
hue + 60°



Hue Shifting

R → G
Y → C
G → B
C → M
B → R
M → Y

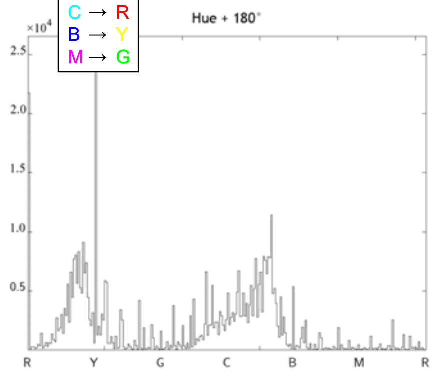
hue + 120°



Hue Shifting

R → C
Y → B
G → M
C → R
B → Y
M → G

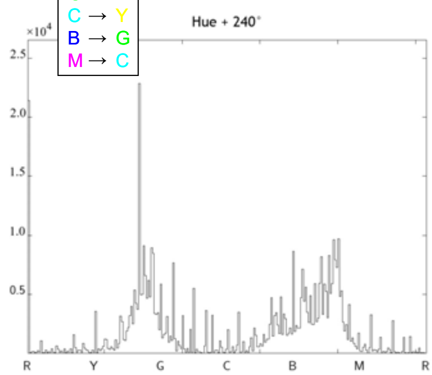
hue + 180°



Hue Shifting

R → B
Y → M
G → R
C → Y
B → G
M → C

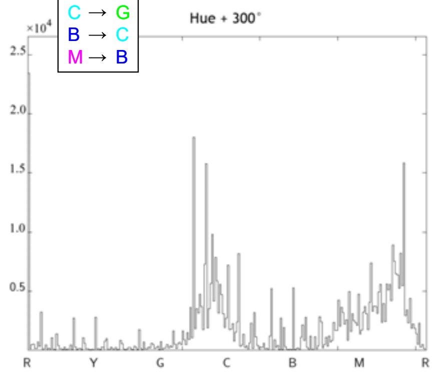
hue + 240°



Hue Shifting

R → M
Y → R
G → Y
C → G
B → C
M → B

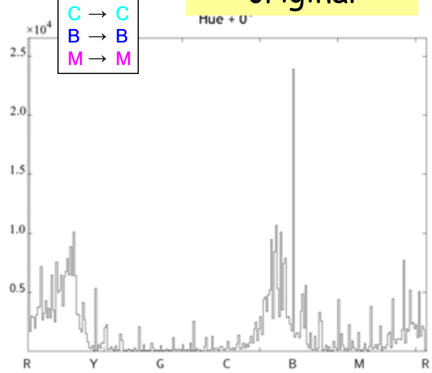
hue + 300°



Hue Shifting

R → R
Y → Y
G → G
C → C
B → B
M → M

hue + 360° = original



Color Correction Via Transformation

- is a point process; the transformation is applied to each pixel as a function of its color alone.

$$J(r,c) = \Phi[I(r,c)] \quad \forall (r,c) \in \mathrm{supp}(I)$$

- Each pixel is vector valued, therefore the transformation is a vector space operator.

$$I(r,c) = \begin{bmatrix} R_I(r,c) \\ G_I(r,c) \\ B_I(r,c) \end{bmatrix} \qquad J(r,c) = \begin{bmatrix} R_J(r,c) \\ G_J(r,c) \\ B_J(r,c) \end{bmatrix} = \Phi\{I(r,c)\} = \Phi\left\{ \begin{bmatrix} R_I(r,c) \\ G_I(r,c) \\ B_I(r,c) \end{bmatrix} \right\}$$

Linear Transformation of Color

Color Vector Space Operators

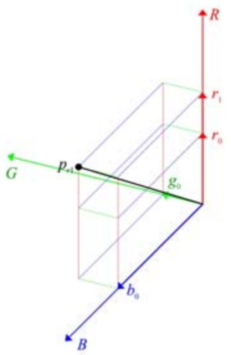
Linear operators are matrix multiplications

$$\begin{bmatrix} r_1 \\ g_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} r_0 \\ g_0 \\ b_0 \end{bmatrix}$$

$$\begin{bmatrix} r_1 \\ g_1 \\ b_1 \end{bmatrix} = 255 \cdot \begin{bmatrix} (r_0/255)^{1/\gamma_r} \\ (g_0/255)^{1/\gamma_g} \\ (b_0/255)^{1/\gamma_b} \end{bmatrix}$$

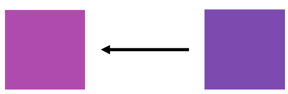
Example of a nonlinear operator: gamma correction

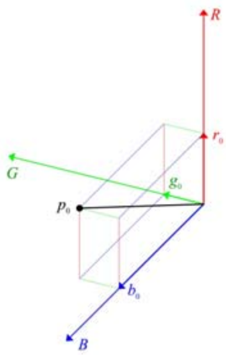
Linear Transformation of Color



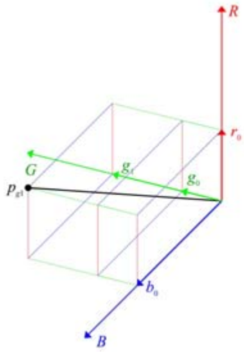
$$\begin{bmatrix} r_1 \\ g_0 \\ b_0 \end{bmatrix} = \begin{bmatrix} r_1/r_0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_0 \\ g_0 \\ b_0 \end{bmatrix}$$

$$\begin{bmatrix} 175 \\ 75 \\ 175 \end{bmatrix} \longleftarrow \begin{bmatrix} 125 \\ 75 \\ 175 \end{bmatrix}$$



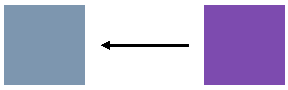


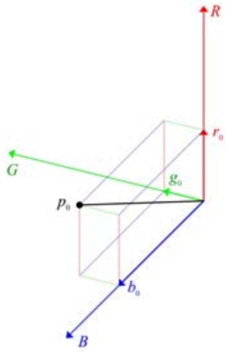
Linear Transformation of Color



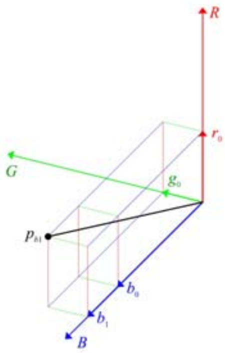
$$\begin{bmatrix} r_0 \\ g_1 \\ b_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & g_1/g_0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_0 \\ g_0 \\ b_0 \end{bmatrix}$$

$$\begin{bmatrix} 125 \\ 150 \\ 175 \end{bmatrix} \longleftarrow \begin{bmatrix} 125 \\ 75 \\ 175 \end{bmatrix}$$



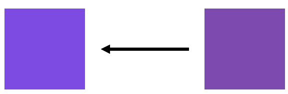


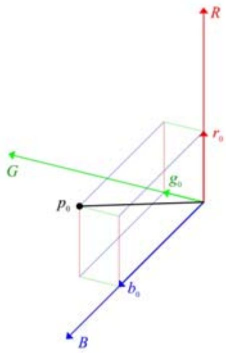
Linear Transformation of Color



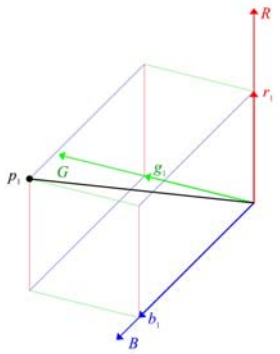
$$\begin{bmatrix} r_0 \\ g_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & b_1/b_0 \end{bmatrix} \begin{bmatrix} r_0 \\ g_0 \\ b_0 \end{bmatrix}$$

$$\begin{bmatrix} 125 \\ 75 \\ 225 \end{bmatrix} \longleftarrow \begin{bmatrix} 125 \\ 75 \\ 175 \end{bmatrix}$$



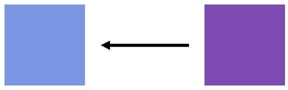


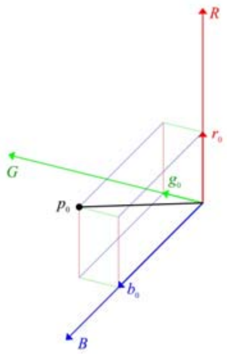
Linear Transformation of Color



$$\begin{bmatrix} r_1 \\ g_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} r_1/r_0 & 0 & 0 \\ 0 & g_1/g_0 & 0 \\ 0 & 0 & b_1/b_0 \end{bmatrix} \begin{bmatrix} r_0 \\ g_0 \\ b_0 \end{bmatrix}$$

$$\begin{bmatrix} 175 \\ 150 \\ 225 \end{bmatrix} \longleftarrow \begin{bmatrix} 125 \\ 75 \\ 175 \end{bmatrix}$$





Color Transformation

Assume J is a discolored version of image I such that $J = \Phi[\ I\ ]$. If Φ is linear then it is represented by a 3×3 matrix, A:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}.$$

Then $J = AI$ or, more accurately, $J(r,c) = AI(r,c)$ for all pixel locations (r,c) in image I.

Color Transformation

If at pixel location (r,c),

image $I(r,c)=\begin{bmatrix}\rho_I\\ \gamma_I\\ \beta_I\end{bmatrix}$ and

image $J(r,c)=\begin{bmatrix}\rho_J\\ \gamma_J\\ \beta_J\end{bmatrix}$,

then $J(r,c)=AI(r,c)$, or

$$\begin{bmatrix}\rho_J\\ \gamma_J\\ \beta_J\end{bmatrix}=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\ a_{21}&a_{22}&a_{23}\\ a_{31}&a_{32}&a_{33}\end{bmatrix}\begin{bmatrix}\rho_I\\ \gamma_I\\ \beta_I\end{bmatrix}$$

$$=\begin{bmatrix}a_{11}\rho_I&+&a_{12}\gamma_I&+&a_{13}\beta_I\\ a_{21}\rho_I&+&a_{22}\gamma_I&+&a_{23}\beta_I\\ a_{31}\rho_I&+&a_{32}\gamma_I&+&a_{33}\beta_I\end{bmatrix}.$$

Color Transformation

The inverse transform Φ^{-1} (if it exists) maps the discolored image, *J*, back into the correctly colored version, *I*, *i.e.*, $I = \Phi^{-1}[\, J\,]$. If Φ is linear then it is represented by the inverse of matrix *A*:

$$A^{-1}=\left(a_{11}a_{22}a_{33}-a_{11}a_{23}a_{32}-a_{12}a_{21}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}-a_{13}a_{22}a_{31}\right)^{-1}\cdot$$
$$\begin{bmatrix}a_{22}a_{33}-a_{23}a_{32}&a_{13}a_{32}-a_{12}a_{33}&a_{12}a_{23}-a_{13}a_{22}\\ a_{23}a_{31}-a_{21}a_{33}&a_{11}a_{33}-a_{13}a_{31}&a_{13}a_{21}-a_{11}a_{23}\\ a_{21}a_{32}-a_{22}a_{31}&a_{12}a_{31}-a_{11}a_{32}&a_{11}a_{22}-a_{12}a_{21}\end{bmatrix}.$$

Color Correction

Assume we know n colors in the discolored image, J, that correspond to another set of n colors (that we also know) in the original image, I.

$$\left\{ \begin{bmatrix} \rho_{J,k} \\ \gamma_{J,k} \\ \beta_{J,k} \end{bmatrix} \right\}_{k=1}^{n} \qquad \begin{bmatrix} \rho_{J,k} \\ \gamma_{J,k} \\ \beta_{J,k} \end{bmatrix} \leftrightarrow \begin{bmatrix} \rho_{I,k} \\ \gamma_{I,k} \\ \beta_{I,k} \end{bmatrix} \qquad \left\{ \begin{bmatrix} \rho_{I,k} \\ \gamma_{I,k} \\ \beta_{I,k} \end{bmatrix} \right\}_{k=1}^{n}$$

for $k = 1, \ldots, n$.

known wrong colors

known correspondence

known correct colors

Color Correction

To remap the discolored image so that the result matches the original image in a linearly optimal way, we need to find the matrix, A, that minimizes

$$\varepsilon^2 = \sum_{k=1}^{n} \left\| \begin{bmatrix} \rho_{I,k} \\ \gamma_{I,k} \\ \beta_{I,k} \end{bmatrix} - A^{-1} \begin{bmatrix} \rho_{J,k} \\ \gamma_{J,k} \\ \beta_{J,k} \end{bmatrix} \right\|^2$$

Color Correction

To find the solution of this problem, let

$$Y = \left[\begin{bmatrix} \rho_{I,1} \\ \gamma_{I,1} \\ \beta_{I,1} \end{bmatrix} \cdots \begin{bmatrix} \rho_{I,n} \\ \gamma_{I,n} \\ \beta_{I,n} \end{bmatrix} \right] \quad \text{and} \quad X = \left[\begin{bmatrix} \rho_{J,1} \\ \gamma_{J,1} \\ \beta_{J,1} \end{bmatrix} \cdots \begin{bmatrix} \rho_{J,n} \\ \gamma_{J,n} \\ \beta_{J,n} \end{bmatrix} \right].$$

Then X and Y are known $3\times n$ matrices such that

$$Y \approx A^{-1} X,$$

where A is the 3×3 matrix that we want to find.

Color Correction

The linearly optimal solution is the least mean squared solution that is given by

$$B = A^{-1} = YX^{T}\left(XX^{T}\right)^{-1}$$

where X^T represents the transpose of matrix X.

Notes:
1. n, the number of color pairs, must be ≥ 3,
2. XX^T must be invertible, *i.e.*, $\text{rank}(XX^T) = 3$,
3. If n=3, then $X^T(XX^T)^{-1} = X^{-1}$.

Color Correction

input colors (to be changed):

$$\left[\begin{bmatrix}\rho_{J,1}\\ \gamma_{J,1}\\ \beta_{J,1}\end{bmatrix} \cdots \begin{bmatrix}\rho_{J,n}\\ \gamma_{J,n}\\ \beta_{J,n}\end{bmatrix}\right] \cdot \begin{bmatrix}[\rho_{J,1} & \gamma_{J,1} & \beta_{J,1}]\\ & \vdots & \\ [\rho_{J,n} & \gamma_{J,n} & \beta_{J,n}]\end{bmatrix}$$

The linearly optimal soluti ... ared solution that is given by

$$B = A^{-1} = YX^T\left(XX^T\right)^{-1}$$

output colors (wanted):

$$\left[\begin{bmatrix}\rho_{I,1}\\ \gamma_{I,1}\\ \beta_{I,1}\end{bmatrix} \cdots \begin{bmatrix}\rho_{I,n}\\ \gamma_{I,n}\\ \beta_{I,n}\end{bmatrix}\right]$$

where X^T represe ... of matrix X.

Notes:
1. n, ... or pairs, must be ≥ 3,
2. XX^T must be invertible, *i.e.*, rank(XX^T) = 3,
3. If n=3, then $X^T(XX^T)^{-1} = X^{-1}$.

Color Correction

input colors (to be changed):

$$\left[\begin{bmatrix}\rho_{J,1}\\ \gamma_{J,1}\\ \beta_{J,1}\end{bmatrix} \cdots \begin{bmatrix}\rho_{J,n}\\ \gamma_{J,n}\\ \beta_{J,n}\end{bmatrix}\right] \cdot \begin{bmatrix}[\rho_{J,1} & \gamma_{J,1} & \beta_{J,1}]\\ & \vdots & \\ [\rho_{J,n} & \gamma_{J,n} & \beta_{J,n}]\end{bmatrix}$$

The linearly optimal soluti ... ared solution that is given by

$$B = A^{-1} = YX^T\left(XX^T\right)^{-1}$$

output colors (wanted):

$$\left[\begin{bmatrix}\rho_{I,1}\\ \gamma_{I,1}\\ \beta_{I,1}\end{bmatrix} \cdots \begin{bmatrix}\rho_{I,n}\\ \gamma_{I,n}\\ \beta_{I,n}\end{bmatrix}\right]$$

where X^T represe ... of matrix X.

Notes:
1. n, ... or pairs, must be ≥ 3,
2. XX^T must be invertible, *i.e.*, rank(XX^T) = 3,
3. If n=3, then $X^T(XX^T)^{-1} = X^{-1}$.

Color Correction

Then the image is color corrected by performing

$$I(r,c) = B\,J(r,c),\ \text{for all}\ (r,c) \in \text{supp}(J).$$

In `Matlab` this is easily performed by

```
I = reshape(((B*(reshape(J,R*C,3))')'),R,C,3);
```

where $B=A^{-1}$ is computed directly through the LMS formula on the previous page, and R & C are the number of rows and columns in the image.

Linear Color Correction

NASA Summer Faculty Fellows at Ellington Air Force Base, Houston, TX, July 2002. Airplane is a T-38.

Original Image

"Aged" Image

Color Mapping 1

Original Image

"Aged" Image

Color Mapping 2

Original Image

"Aged" Image

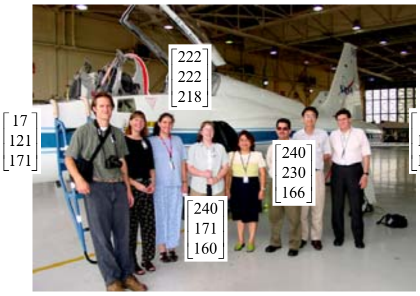
Color Mapping 3

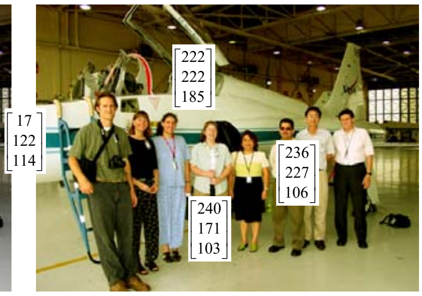
Original Image "Aged" Image

Color Mapping 4

Original Image "Aged" Image

Color Transformations

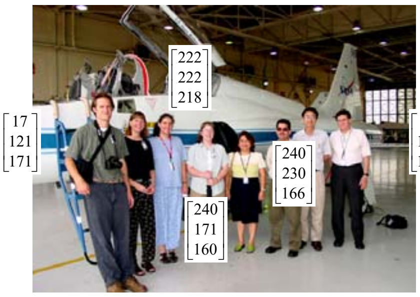


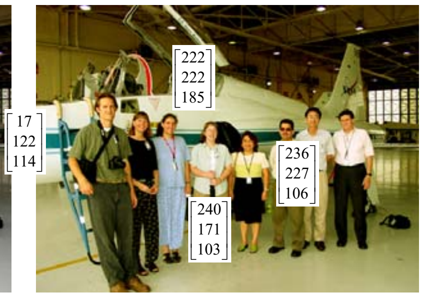


The aging process was a transformation, Φ, that mapped:

$$\begin{bmatrix}17\\122\\114\end{bmatrix}=\Phi\left\{\begin{bmatrix}17\\121\\171\end{bmatrix}\right\}\quad \begin{bmatrix}222\\222\\185\end{bmatrix}=\Phi\left\{\begin{bmatrix}222\\222\\218\end{bmatrix}\right\}\quad \begin{bmatrix}240\\171\\103\end{bmatrix}=\Phi\left\{\begin{bmatrix}240\\171\\160\end{bmatrix}\right\}\quad \begin{bmatrix}236\\227\\106\end{bmatrix}=\Phi\left\{\begin{bmatrix}240\\230\\166\end{bmatrix}\right\}$$

Color Transformations





To undo the process we need to find, Φ^{-1}, that maps:

$$\begin{bmatrix}17\\121\\171\end{bmatrix}=\Phi^{-1}\left\{\begin{bmatrix}17\\122\\114\end{bmatrix}\right\}\quad \begin{bmatrix}222\\222\\218\end{bmatrix}=\Phi^{-1}\left\{\begin{bmatrix}222\\222\\185\end{bmatrix}\right\}\quad \begin{bmatrix}240\\171\\160\end{bmatrix}=\Phi^{-1}\left\{\begin{bmatrix}240\\171\\103\end{bmatrix}\right\}\quad \begin{bmatrix}240\\230\\166\end{bmatrix}=\Phi^{-1}\left\{\begin{bmatrix}236\\227\\106\end{bmatrix}\right\}$$

Correction Using 3 Mappings

$$B = A^{-1} = YX^{-1}$$

$$X = \begin{bmatrix} 222 & 17 & 240 \\ 222 & 122 & 171 \\ 185 & 114 & 103 \end{bmatrix} \qquad Y = \begin{bmatrix} 222 & 17 & 240 \\ 222 & 121 & 171 \\ 218 & 171 & 160 \end{bmatrix}$$

Correction Using 3 Mappings

$$B = A^{-1} = YX^{-1}$$

original

corrected

$$X = \begin{bmatrix} 222 & 17 & 240 \\ 222 & 122 & 171 \\ 185 & 114 & 103 \end{bmatrix} \qquad Y = \begin{bmatrix} 222 & 17 & 240 \\ 222 & 121 & 171 \\ 218 & 171 & 160 \end{bmatrix}$$

Another Correction Using 3 Mappings

$$B = A^{-1} = YX^{-1}$$

$$X = \begin{bmatrix} 222 & 17 & 236 \\ 222 & 122 & 227 \\ 185 & 114 & 106 \end{bmatrix} \quad Y = \begin{bmatrix} 222 & 17 & 240 \\ 222 & 121 & 230 \\ 218 & 171 & 166 \end{bmatrix}$$

Another Correction Using 3 Mappings

$$B = A^{-1} = YX^{-1}$$

original

corrected

$$X = \begin{bmatrix} 222 & 17 & 236 \\ 222 & 122 & 227 \\ 185 & 114 & 106 \end{bmatrix} \quad Y = \begin{bmatrix} 222 & 17 & 240 \\ 222 & 121 & 230 \\ 218 & 171 & 166 \end{bmatrix}$$

Correction Using All 4 Mappings

$$B = A^{-1} = YX^T\left(XX^T\right)^{-1}$$

$$X = \begin{bmatrix} 222 & 17 & 236 & 240 \\ 222 & 122 & 227 & 171 \\ 185 & 114 & 106 & 103 \end{bmatrix} \quad Y = \begin{bmatrix} 222 & 17 & 240 & 240 \\ 222 & 121 & 230 & 171 \\ 218 & 171 & 166 & 160 \end{bmatrix}$$

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Correction Using All 4 Mappings

$$B = A^{-1} = YX^T\left(XX^T\right)^{-1}$$

original

corrected

$$X = \begin{bmatrix} 222 & 17 & 236 & 240 \\ 222 & 122 & 227 & 171 \\ 185 & 114 & 106 & 103 \end{bmatrix} \quad Y = \begin{bmatrix} 222 & 17 & 240 & 240 \\ 222 & 121 & 230 & 171 \\ 218 & 171 & 166 & 160 \end{bmatrix}$$

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Random Sampling of Color Values

```
>> rr = round(R*rand([1 n]));
>> rc = round(C*rand([1 n]));
>> idx = [rr;rc];
>> Y(:,1) = diag(I(rr,rc,1));
>> Y(:,2) = diag(I(rr,rc,2));
>> Y(:,3) = diag(I(rr,rc,3));
>> X(:,1) = diag(J(rr,rc,1));
>> X(:,2) = diag(J(rr,rc,2));
>> X(:,3) = diag(J(rr,rc,3));
```

R = number of rows in image
C = number of columns in image
n = number of pixels to select

rand([1 n]) : 1 × n matrix of random numbers between 0 and 1.

diag(I(rr,rc,1)): vector from main diagonal of matrix I(rr,rc,1).

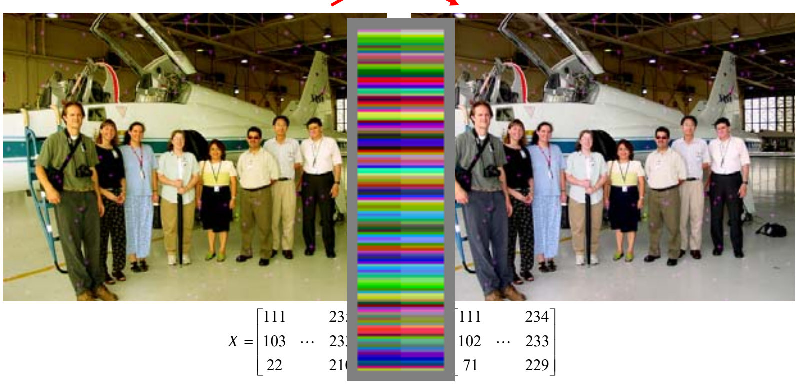
Correction Using 128 Mappings

$$B = A^{-1} = YX^T\left(XX^T\right)^{-1}$$

$$X = \begin{bmatrix} 111 & & 235 \\ 103 & \cdots & 233 \\ 22 & & 210 \end{bmatrix} \qquad Y = \begin{bmatrix} 111 & & 234 \\ 102 & \cdots & 233 \\ 71 & & 229 \end{bmatrix}$$

Correction Using 128 Mappings

$$B = A^{-1} = YX^{T}\left(XX^{T}\right)^{-1}$$



$$X = \begin{bmatrix} 111 & & 23 \\ 103 & \cdots & 23 \\ 22 & & 21 \end{bmatrix} \qquad \begin{bmatrix} 111 & & 234 \\ 102 & \cdots & 233 \\ 71 & & 229 \end{bmatrix}$$

Correction Using 128 Mappings

$$B = A^{-1} = YX^{T}\left(XX^{T}\right)^{-1}$$

original

corrected

$$X = \begin{bmatrix} 111 & & 235 \\ 103 & \cdots & 233 \\ 22 & & 210 \end{bmatrix} \qquad Y = \begin{bmatrix} 111 & & 234 \\ 102 & \cdots & 233 \\ 71 & & 229 \end{bmatrix}$$

Correction Using 4 Mappings

$$B = A^{-1} = YX^T\left(XX^T\right)^{-1}$$

original

corrected

$$X = \begin{bmatrix} 222 & 17 & 236 & 240 \\ 222 & 122 & 227 & 171 \\ 185 & 114 & 106 & 103 \end{bmatrix} \quad Y = \begin{bmatrix} 222 & 17 & 240 & 240 \\ 222 & 121 & 230 & 171 \\ 218 & 171 & 166 & 160 \end{bmatrix}$$

Linear Color Transformation Program

```
function J = LinTrans(I,A)
[R C B] = size(I);
I = double(I);
J = reshape(((A*(reshape(I,R*C,3))')'),R,C,3);
J = uint8(J);
return;
```