Exercise 1 : Modelling

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ANSWER FOR SYSTEM A:

Let us consider the mass is M, coefficient of spring stiffness is k and coefficient of damper is b.

Task1:

Energy Variables

- $\frac{1}{2}M(\frac{dx_1}{dt})^2$
- $\frac{1}{2}kx_1^2$

Task 2:

List of state variables

- x_1
- x_2

Task 3:

State equations

$\frac{dx_1}{dt} = x_2$

$\Rightarrow \dot{x}_1 = x_2$

$M\frac{d^2x_1}{dt^2} + b\frac{dx_1}{dt} + kx_1 = 0$

$\Rightarrow \frac{d^2x_1}{dt^2} + \frac{b}{M}\frac{dx_1}{dt} + \frac{k}{M}x_1 = 0$

$\Rightarrow \frac{dx_2}{dt} + \frac{b}{M}\frac{dx_1}{dt} + \frac{k}{M}x_1 = 0$

$\Rightarrow \frac{dx_2}{dt} = -\frac{b}{M}\frac{dx_1}{dt} - \frac{k}{M}x_1$

$\Rightarrow \dot{x}_2 = -\frac{b}{M}\dot{x}_1 - \frac{k}{M}x_1$

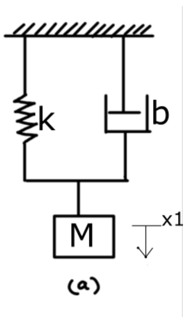

Figure 1: System A

ANSWER FOR SYSTEM B:

Task 1:

Energy Variables

- $\frac{1}{2}L_1 i_1^2$
- $\frac{1}{2}L_2 i_2^2$
- $\frac{1}{2}C_1 V_{C1}^2$
- $\frac{1}{2}C_2 V_{C2}^2$

Task 2:

List of state variables

1. i_1
2. i_2
3. V_{C1}
4. V_{C2}

Task 3:

State equation

Applying KVL in Loop i1:

$V_1 - i_1 R_1 - L_1 \frac{di_1}{dt} - V_{C_1} = 0$

$\Rightarrow \frac{di_1}{dt} = \frac{V_1}{L_1} - \frac{V_{C_1}}{L_1} - \frac{i_1 R_1}{L_1}$

$\Rightarrow \dot{i}_1 = \frac{V_1}{L_1} - \frac{V_{C_1}}{L_1} - \frac{i_1 R_1}{L_1}$

At node Vc1:

$V_{C_1} = \frac{1}{C_1} \int (i_1 - i_2) dt$

Differentiating both side with respect to t

$\Rightarrow \frac{dV_{C_1}}{dt} = \frac{i_1}{C_1} - \frac{i_2}{C_1}$

$\Rightarrow \dot{V}_{C_1} = \frac{i_1}{C_1} - \frac{i_2}{C_1}$

Applying KVL in Loop i2:

$V_{C_1} - i_2 R_2 - L_2 \frac{di_2}{dt} - V_{C_2} = 0$

$\Rightarrow \frac{di_2}{dt} = \frac{V_{C_1}}{L_2} - \frac{V_{C_2}}{L_2} - \frac{i_2 R_2}{L_2}$

$\Rightarrow \dot{i}_2 = \frac{V_{C_1}}{L_2} - \frac{V_{C_2}}{L_2} - \frac{i_2 R_2}{L_2}$

At node Vc2:

$V_{C_2} = \frac{1}{C_2} \int (i_2 - i_3) dt$

Differentiating both side with respect to t

$\Rightarrow \frac{dV_{C_2}}{dt} = \frac{i_2}{C_2} - \frac{i_3}{C_2}$

$\Rightarrow \frac{dV_{C_2}}{dt} = \frac{i_2}{C_2} - \frac{V_{C_2}}{R_3 C_2}$ as $i_3 = \frac{V_{C_2}}{R_3}$

$\Rightarrow \dot{V}_{C_2} = \frac{i_2}{C_2} - \frac{V_{C_2}}{R_3 C_2}$

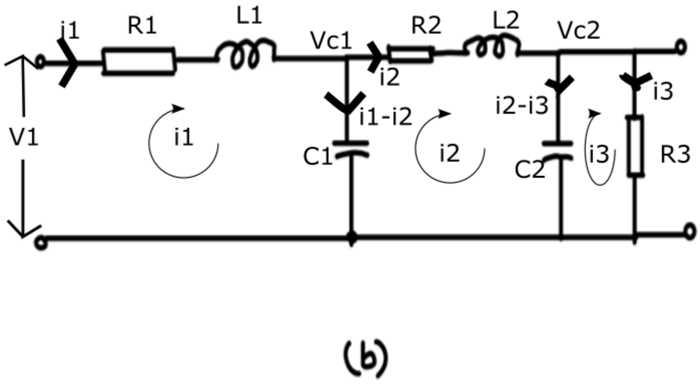

Figure 2: System B

ANSWER FOR SYSTEM C:

Task 1:

Energy Variables

- $\frac{1}{2}Li^2$
- $\frac{1}{2}Jw_1^2$

Task 2:

List of state variables

- i
- ω_2

Task 3:

Let us assume $E_b$is back EMF of the motor and J is the moment of inertia of the flywheel.

$E_b = k\omega_1$

State equation

Now considering the gear ratio is $N_1 : N_2$ so the speed would be ω_1:ω_2. Now, the torque developed by the motor is given by

$T_1 = k_m i$ [considering flux constant]

The torque transformed to flywheel side will be given by

$T_2 = \frac{T_1 N_1}{N_2}$

Or

$T_1\omega_1 = T_2\omega_2$

$\Rightarrow \omega_1 = \frac{T_2\omega_2}{T_1}$

which is also given by

$\mathrm{T}_2 = J\frac{d\omega_2}{dt}$

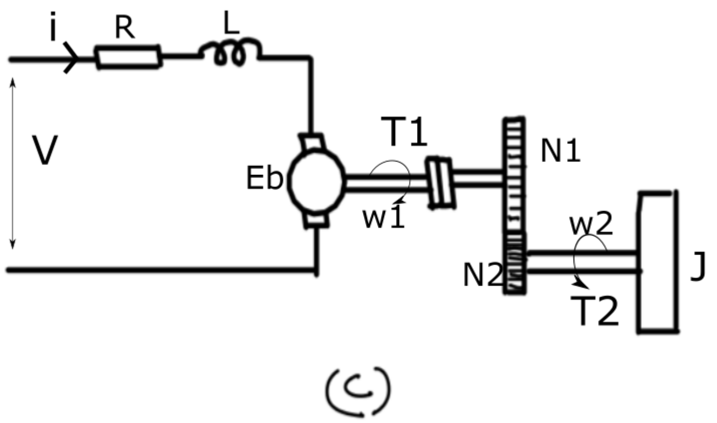

Figure 3: System C

So,

$\frac{d\omega_2}{dt} = \frac{T_1 N_1}{J N_2}$

$\Rightarrow \frac{d\omega_2}{dt} = \frac{k_m N_1 i}{J N_2}$

$\Rightarrow \dot{\omega}_2 = \frac{k_m N_1 i}{J N_2}$

Apllying KVL in the electrical motor circuit loop

$V = iR + L\frac{di}{dt} + E_b$

$\Rightarrow \frac{di}{dt} = \frac{V}{L} - \frac{iR}{L} - \frac{k\omega_1}{L}$

$\Rightarrow \dot{i} = \frac{V}{L} - \frac{iR}{L} - \frac{kT_2\omega_2}{T_1 L}$