Comp. Methods in Mech. Eng.

MCG 4127

Assignment # 9

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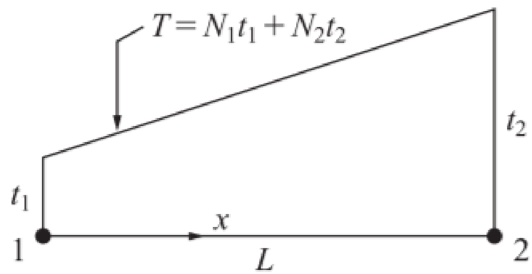
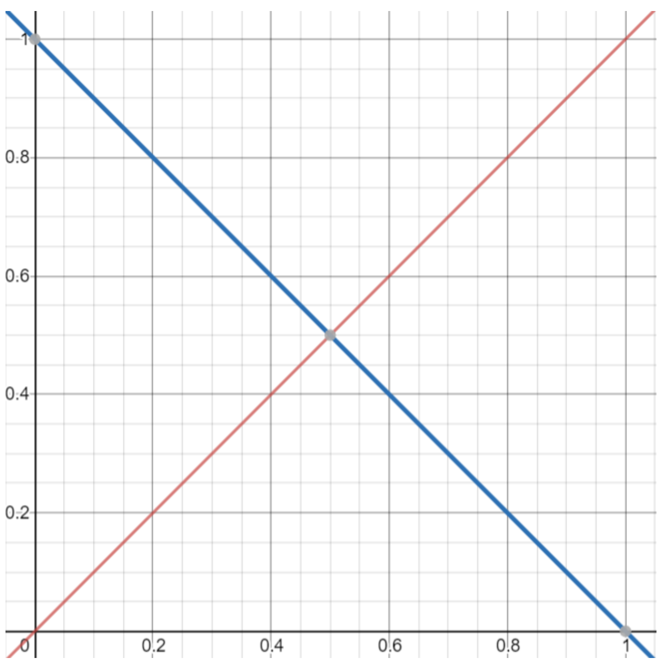
In this assignment we attempt to model 1-D transient FEM model of a rod subjected to varying temperatures and the temperature is studied as a function of x and t.

We assume that the shape function is linear therefore between each node we get a temperature variation like the following:



Where T is the temperature function and has the form $T(x) = N_1 t_1 + N_2 t_2$

If we assume a linear basis then there are two equations for N1 and N2, this can be graphically be seen in the image below.



Taking the Lagrange we of the two linear functions (in one the root is at $x = 0$ and the other is $x = 1$) we obtain the following two equations in terms of x obtain:

$$h_1(r) = \frac{x - x_1}{x_0 - x_1} = \frac{1}{2}(1 - r)$$

$$h_2(r) = \frac{x - x_2}{x_1 - x_0} = \frac{1}{2}(1 + r)$$

Therefore, the shape matrix takes the form of:

$[N] = [\frac{1}{2}(1 - r) \quad \frac{1}{2}(1 + r)]$

The derivative of this matric is given by

$[B] = [-\frac{1}{2} \quad \frac{1}{2}]$

The assemblage matrix is given by the following equation:

$$[A][T] = [B][S]$$

Where A is the stiffness matrix, T is the unknown temperature, B is the boundary condition and S is the forcing function (or the source). The equation for A and B are the following, respectively:

$$A_{ij}^k = -\frac{2}{L^k}\int_{-1}^{1} \frac{\partial h_i}{\partial r}\frac{\partial h_j}{\partial r} dr$$

$$B_{ij}^k = \frac{L^k}{2}\int_{-1}^{1} h_i h_j dr$$

Therefore

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\left(\frac{1}{\Delta x}\right)$$

And

$$B = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix} \Delta x$$

Using the above equation, it now can be implemented in the transient equation

$$\frac{\partial T}{\partial t} = \frac{T_i^{l+1} - T_i^l}{\Delta t}$$

The transient heat conduction is given by the following equation:

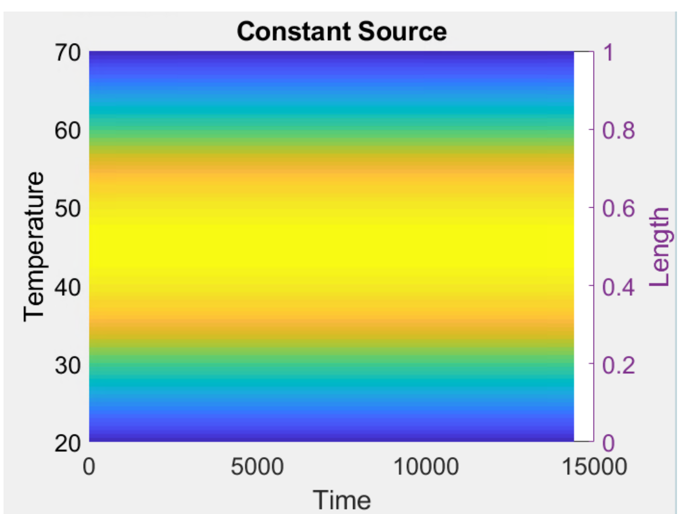
$$\frac{\rho C_p \partial T}{\partial t} = \frac{k \partial^2 T}{\partial x^2} + S$$

$$\frac{T_i^{l+1} - T_i^l}{\Delta t} = [A][T] + [B][S]$$

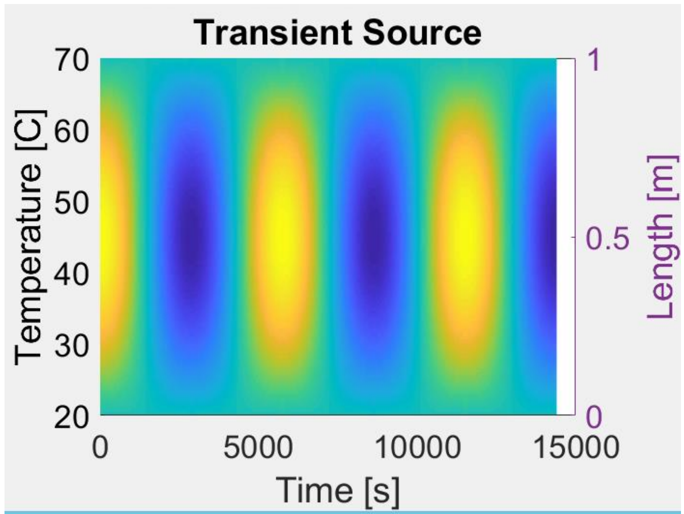
$$T_i^{l+1} = \Delta t([A][T] + [B][S]) + T_i^l$$

Programming the above equation on MATLAB yields the following graphs.

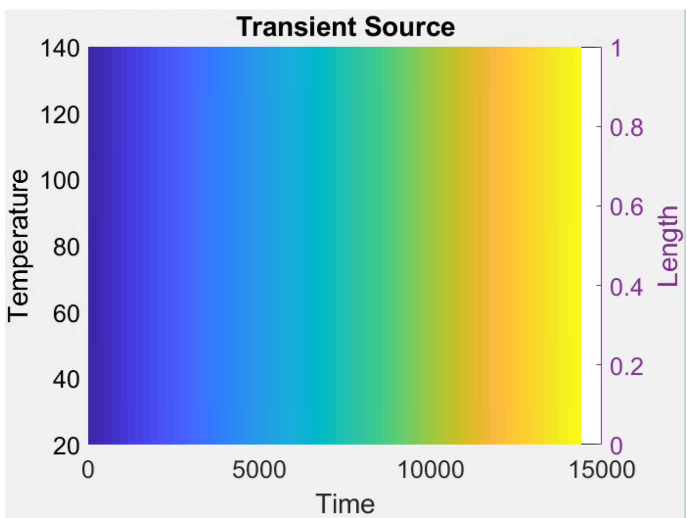
In the first graph a constant heat source is kept at $\sin(\pi x)$



In this second graph a time varying heat source is added to be $\sin(x)\sin(t)$



A linear transient heat source of t



MATLAB Code

```
% constants
L = 1; % length of rod
nelem = 100; % number of elements
nnode = nelem + 1; % number of nodes
del_x = L/nelem; %delta x
k = 10; % heat conductivity
rho = 1250; % density
Cp = 420;
T_o = 20; % initial temp
T_end = 70; % end temp

t = 7200*2; %number of seconds to do timesteps for
time_steps = 500; % number of time steps
del_t = t/(time_steps - 1); %delta t

% basis functions
syms h1(r);
h1(r) = 0.5*(1-r);
syms h2(r);
h2(r) = 0.5*(1+r);

% Setting up [A][t] = [B][S]
% A matrix for each element
a =[-2*int(diff(h1)*diff(h1),[-1,1]) -2*int(diff(h1)*diff(h2),[-1,1]);
    -2*int(diff(h2)*diff(h1),[-1,1]) -2*int(diff(h2)*diff(h2),[-1,1])]

% B matrix for each element
b = [int(h1*h1,[-1,1]) int(h1*h2,[-1,1]);
    int(h2*h1,[-1,1]) int(h2*h2,[-1,1])]

A = spalloc (nelem, nelem, nelem^2);
B = spalloc(nelem, nelem, nelem^2);

% Matrix assemblage
for i = 1:nelem-1
    A(i,i) = A(i,i) + a(1,1);
    A(i,i+1) = A(i,i+1) + a(1,2);
    A(i+1,i) = A(i+1,i) + a(2,1);
    A(i+1,i+1) = A(i+1,i+1) + a(2,2);

    B(i,i) + B(i,i) + b(1,1);
    B(i,i+1) = B(i,i+1) + b(1,2);
    B(i+1,i) = B(i+1,i) + b(2,1);
    B(i+1,i+1) = B(i+1,i+1) + b(2,2);
end

% Setting up source term
S = zeros(nelem,1);
for i = 1:nelem
    for j = 1:time_steps
```

```matlab
        S(i,j) = sin(pi*i/nelem);
    end
end

% Time marching
% Euler - forward method

T = linspace(T_o, T_end, nelem);
T = T';

for i = 1:time_steps
    if mod((i-1),(time_steps/10)) == 0
        plot(linspace(0,L,nelem),T)
    end
    hold on
    T_n = del_t*k/(Cp*rho*del_x)*(A*T+del_x*B*S(:,i))+T;
    T = T_n;

    % boundary conditions
    T(1) = T_o;
    T(end) = T_end;
end

% Graph properties
xlabel('Length')
ylabel('Temperature')
set(gca,'FontSize',15)
% Plot the source
hold on
yyaxis right
plot(linspace(0,L,nelem),S(:,1),'-.','LineWidth',5)
ylabel('Steady Heat Source [W/m^2]')

%% Plot the transient source surface
[X,Y] = meshgrid(linspace(0,t,time_steps),linspace(0,L,nelem));
s = surf(X,Y,S);
xlabel('Time')
ylabel('Length')
zlabel('Heat Flux')
title('Constant Source')
set(gca,'FontSize',15)
s.EdgeColor = 'none';
```