MIE 1622 Assignment 1 Report

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1 Investment Strategies

The buy and hold strategy was already implemented. The equally weighted strategy was done by setting the weight (proportion of the value of our portfolio invested in each asset) to be equal for all 20 assets. The minimum variance strategy was implemented by simply solving the quadratic optimization problem using the Q matrix in cplex, and then setting the weights equal to what the cplex solver gives. The maximum sharpe ratio was also computed using cplex, but adjusting the Q matrix, the A matrix (with the constraints that $k >= 0$, $\sum_i y_i = k$, and $\sum_i(\mu_i - r_f)y_i = 1$), and upper and lower bounds to solve the modified quadratic optimization problem. The risk-free rate r_f was determined to be the ten-year US Treasury yield rate of 0.00184.

For each strategy, the transaction costs were calculated and guaranteed to make our portfolio feasible (have positive cash value) given the total value of the portfolio using the following formula for the transaction cost constraint:

$$\sum_{i=0}^{n-1} n_i p_i + 0.005 \sum_{i=0}^{n-1} |n_i p_i - l_i p_i| = \alpha \tag{1}$$

where n_i represents the unknown new number of asset i, n is 20 in our case, p_i represents the current price of asset i, 0.005 is the transaction cost rate, l_i is the old number of asset i (before the current rebalancing), and α is the current total portfolio value. We note that:

$$n_i p_i = \frac{w_i}{w_0} n_0 p_0 \tag{2}$$

For any nonzero weighted asset 0, since we want the combined value of the number of stocks we buy for each asset to follow the weight w_i. The equation was then solved in matlab by iterating through the n+1 possible intervals that the unknown variable n_0 could be in, corresponding to the n absolute value terms, and finding the only solution that falls in the correct interval.

After determining the optimal values n_i, we first round it down to ensure that the rebalanced portfolio is feasible. Then, we consider all non-zero weighted assets, and try to round them up such that it does not exceed the total value of the portfolio. This is equivalent to the knapsack problem where the weight of each item is the same as its value, and the capacity is the cash we have after buying all n_i stocks rounded down and subtracting the transaction fee. The dynamic programming algorithm implemented in matlab has runtime O(nC), where n is the number of items (assets) and C is the cash we have left over.

2 Analysis

Here is the output for all the various strategies that were used to rebalance the porfolio after each 2 month period:

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Period 1: start date 01/03/05, end date 02/28/05
Strategy "Buy and Hold", value begin = $ 1008145.00, value end = $  994730.00
Strategy "Equally Weighted Portfolio", value begin = $ 1000706.70, value end = $  950415.22
Strategy "Minimum Variance Portfolio", value begin = $ 1001581.59, value end = $  974784.07
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Strategy "Maximum Sharpe Ratio Portfolio", value begin = $  999058.31, value end = $ 1416479.81
Strategy "Maximum Return Portfolio", value begin = $  999058.31, value end = $ 1416479.81
Strategy "Equally Weighted Buy and Hold", value begin = $ 1000706.70, value end = $  950415.22

Period 2: start date 03/01/05, end date 04/29/05
Strategy "Buy and Hold", value begin = $  998500.00, value end = $  834350.00
Strategy "Equally Weighted Portfolio", value begin = $  952630.81, value end = $  847875.56
Strategy "Minimum Variance Portfolio", value begin = $  977876.37, value end = $  910038.98
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1405112.81, value end = $ 1138619.81
Strategy "Maximum Return Portfolio", value begin = $ 1405112.81, value end = $ 1138619.81
Strategy "Equally Weighted Buy and Hold", value begin = $  953171.86, value end = $  848448.05

Period 3: start date 05/02/05, end date 06/30/05
Strategy "Buy and Hold", value begin = $  836625.00, value end = $  824920.00
Strategy "Equally Weighted Portfolio", value begin = $  855660.48, value end = $  879433.34
Strategy "Minimum Variance Portfolio", value begin = $  905556.65, value end = $  916373.30
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1138857.22, value end = $ 1506996.00
Strategy "Maximum Return Portfolio", value begin = $ 1138857.22, value end = $ 1506996.00
Strategy "Equally Weighted Buy and Hold", value begin = $  856404.29, value end = $  894463.80

Period 4: start date 07/01/05, end date 08/31/05
Strategy "Buy and Hold", value begin = $  826105.00, value end = $  901880.00
Strategy "Equally Weighted Portfolio", value begin = $  876966.79, value end = $  941970.63
Strategy "Minimum Variance Portfolio", value begin = $  902301.52, value end = $  966629.99
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1477292.48, value end = $ 1428574.90
Strategy "Maximum Return Portfolio", value begin = $ 1492139.30, value end = $ 1465243.55
Strategy "Equally Weighted Buy and Hold", value begin = $  891973.89, value end = $  959593.49

Period 5: start date 09/01/05, end date 10/31/05
Strategy "Buy and Hold", value begin = $  890300.00, value end = $  926200.00
Strategy "Equally Weighted Portfolio", value begin = $  937328.65, value end = $  962951.31
Strategy "Minimum Variance Portfolio", value begin = $  961463.66, value end = $  981472.79
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1380313.65, value end = $ 1304194.65
Strategy "Maximum Return Portfolio", value begin = $ 1451932.36, value end = $ 1661748.61
Strategy "Equally Weighted Buy and Hold", value begin = $  958085.54, value end = $ 1002088.98

Period 6: start date 11/01/05, end date 12/30/05
Strategy "Buy and Hold", value begin = $  919665.00, value end = $  971170.00
Strategy "Equally Weighted Portfolio", value begin = $  951021.46, value end = $ 1037966.37
Strategy "Minimum Variance Portfolio", value begin = $  969023.60, value end = $ 1027364.76
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1304280.40, value end = $ 1495424.32
Strategy "Maximum Return Portfolio", value begin = $ 1520576.81, value end = $ 1662780.65
Strategy "Equally Weighted Buy and Hold", value begin = $  990827.28, value end = $ 1086214.49

Period 7: start date 01/03/06, end date 02/28/06
Strategy "Buy and Hold", value begin = $  984040.00, value end = $  958750.00
Strategy "Equally Weighted Portfolio", value begin = $ 1061643.09, value end = $ 1080016.19
Strategy "Minimum Variance Portfolio", value begin = $ 1040249.93, value end = $ 1059306.79
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1461693.04, value end = $ 2203781.20
Strategy "Maximum Return Portfolio", value begin = $ 1727066.35, value end = $ 2061282.43
Strategy "Equally Weighted Buy and Hold", value begin = $ 1116178.86, value end = $ 1110063.34

Period 8: start date 03/01/06, end date 04/28/06
Strategy "Buy and Hold", value begin = $  961230.00, value end = $  959330.00
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Strategy "Equally Weighted Portfolio", value begin = $ 1092623.06, value end = $ 1070218.28
Strategy "Minimum Variance Portfolio", value begin = $ 1064626.88, value end = $ 1070302.39
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2260000.00, value end = $ 1877712.16
Strategy "Maximum Return Portfolio", value begin = $ 2114655.33, value end = $ 1756953.57
Strategy "Equally Weighted Buy and Hold", value begin = $ 1124992.71, value end = $ 1122734.74

Period 9: start date 05/01/06, end date 06/30/06
Strategy "Buy and Hold", value begin = $  956765.00, value end = $  877590.00
Strategy "Equally Weighted Portfolio", value begin = $ 1062727.15, value end = $ 1013322.10
Strategy "Minimum Variance Portfolio", value begin = $ 1058214.07, value end = $ 1003826.71
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1836765.86, value end = $ 1930825.58
Strategy "Maximum Return Portfolio", value begin = $ 1718639.63, value end = $ 1184122.55
Strategy "Equally Weighted Buy and Hold", value begin = $ 1117850.36, value end = $ 1017562.31

Period 10: start date 07/03/06, end date 08/31/06
Strategy "Buy and Hold", value begin = $  890780.00, value end = $  956745.00
Strategy "Equally Weighted Portfolio", value begin = $ 1025619.50, value end = $ 1095152.74
Strategy "Minimum Variance Portfolio", value begin = $ 1008098.71, value end = $ 1073631.72
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1929254.75, value end = $ 2342665.25
Strategy "Maximum Return Portfolio", value begin = $ 1189677.57, value end = $ 1444607.07
Strategy "Equally Weighted Buy and Hold", value begin = $ 1029287.17, value end = $ 1104745.66

Period 11: start date 09/01/06, end date 10/31/06
Strategy "Buy and Hold", value begin = $  961475.00, value end = $ 1085615.00
Strategy "Equally Weighted Portfolio", value begin = $ 1097477.39, value end = $ 1160747.76
Strategy "Minimum Variance Portfolio", value begin = $ 1069427.81, value end = $ 1123606.07
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2364832.66, value end = $ 2627489.32
Strategy "Maximum Return Portfolio", value begin = $ 1458276.65, value end = $ 1823625.31
Strategy "Equally Weighted Buy and Hold", value begin = $ 1104249.55, value end = $ 1199884.25

Period 12: start date 11/01/06, end date 12/29/06
Strategy "Buy and Hold", value begin = $ 1074110.00, value end = $ 1141435.00
Strategy "Equally Weighted Portfolio", value begin = $ 1143093.46, value end = $ 1189931.79
Strategy "Minimum Variance Portfolio", value begin = $ 1109981.52, value end = $ 1129376.76
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2636641.12, value end = $ 2732735.02
Strategy "Maximum Return Portfolio", value begin = $ 1682503.51, value end = $ 1935738.74
Strategy "Equally Weighted Buy and Hold", value begin = $ 1174887.95, value end = $ 1229837.74
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As we can see from the graph of portfolio value over time, the maximum Sharpe Ratio rebalancing strategy far outperforms the other ones. However, it is more risky than the rest, as evidenced by large ups and down in portfolio value. The equally weighted and minimum variance strategies both perform better than the buy and hold strategy, and although the equally weighted portfolio performs slightly better than minimum variance, the plot of minimum variance shows very little variation in portfolio value, indicating that it is the choice with least risk (as expected). From the asset proportion graph of minimum variance, we can see that the proportion of assets changes frequently and dynamically over time, with most assets being represented by some proportion. This is to be expected, as a portfolio that is spread across many different assets is more likely to be less risky. The asset proportion graph for max sharpe shows that this strategy often recommends choosing one stock only, which although had a greater yield, is far more risky.

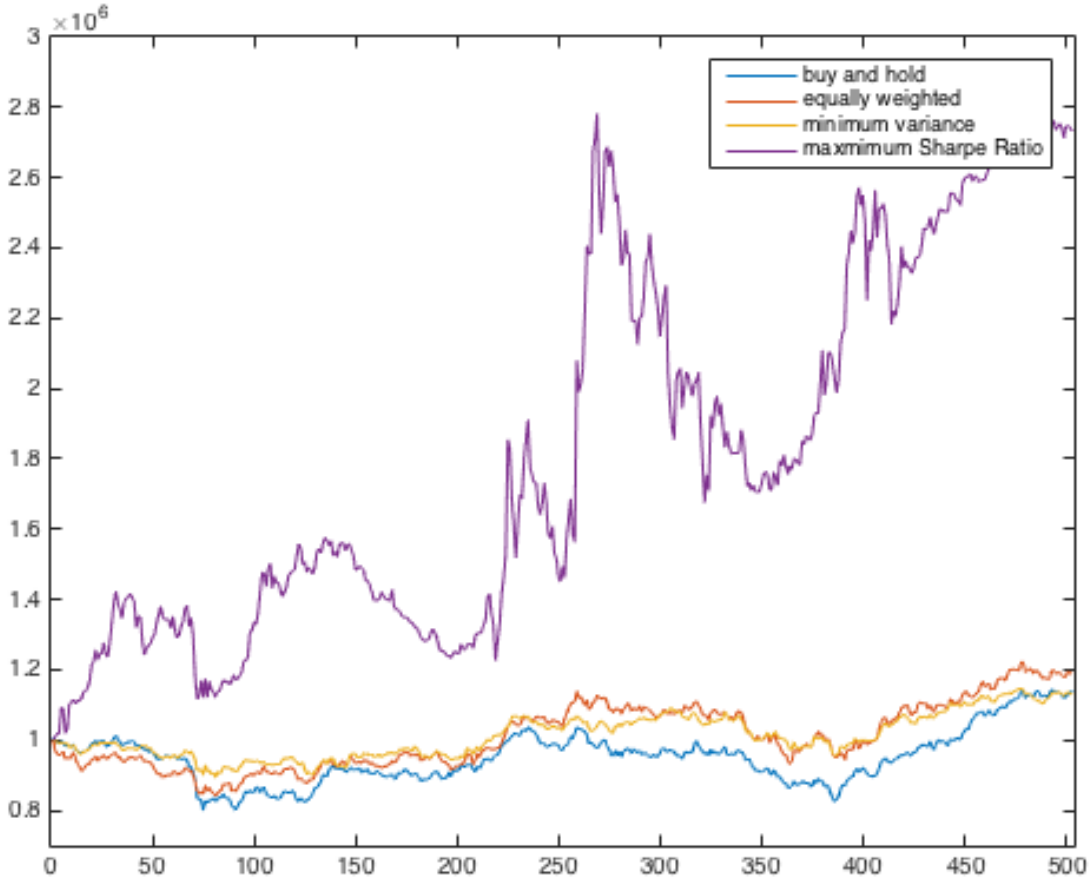

Figure 1: Graph showing the daily value of the porfolio over all 504 trading days for each of the 4 rebalancing strategies

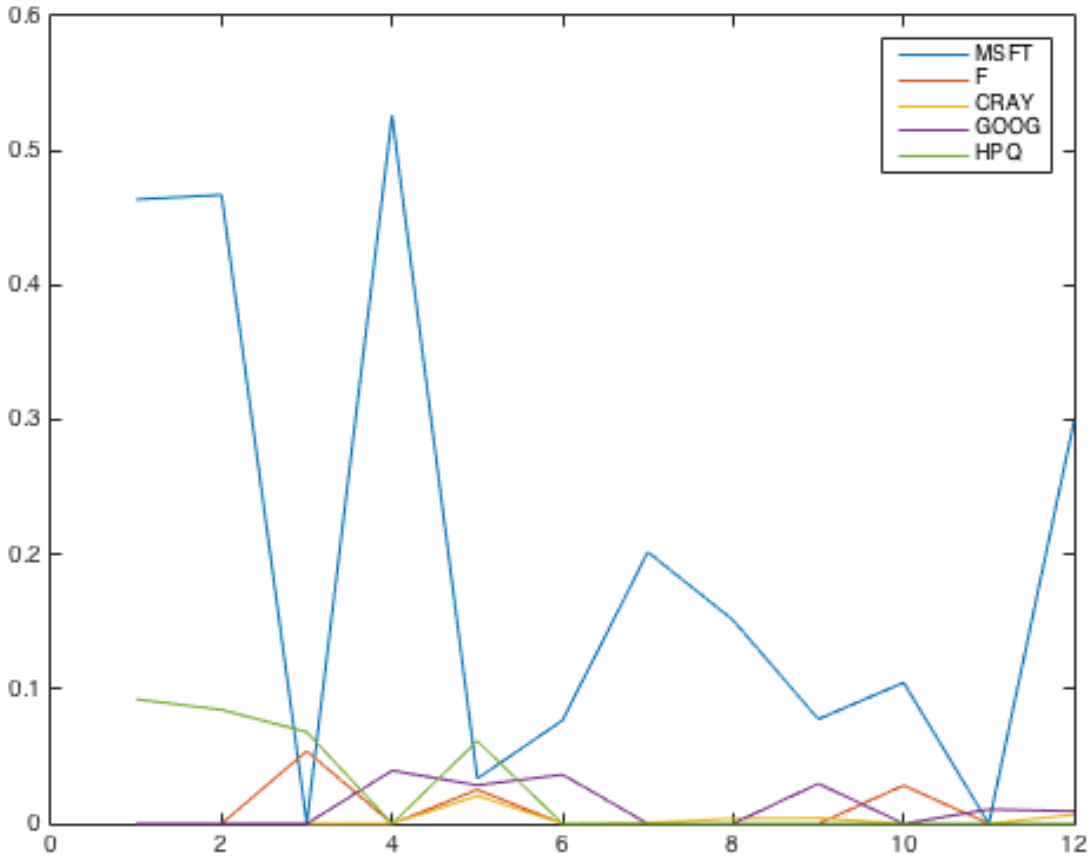

Figure 2: Graph showing the asset proportion over time with the minimum variance strategy (1-5)

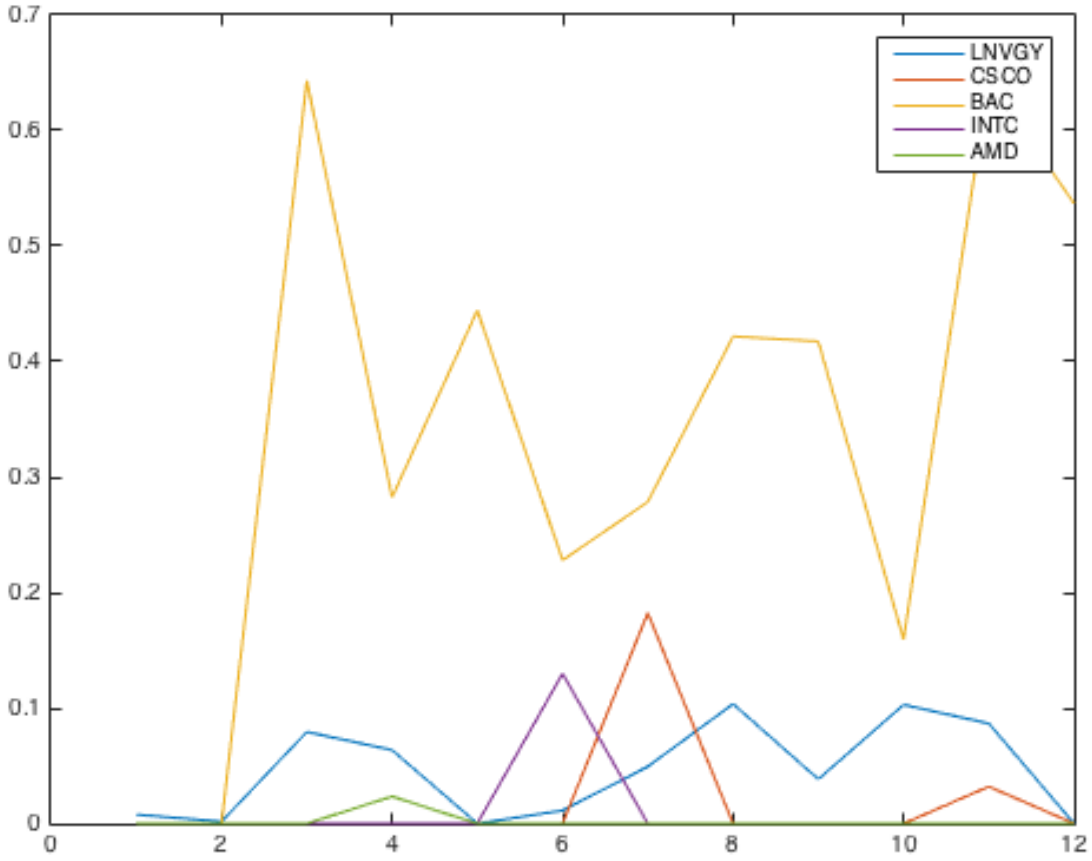

Figure 3: Graph showing the asset proportion over time with the minimum variance strategy (11-15)

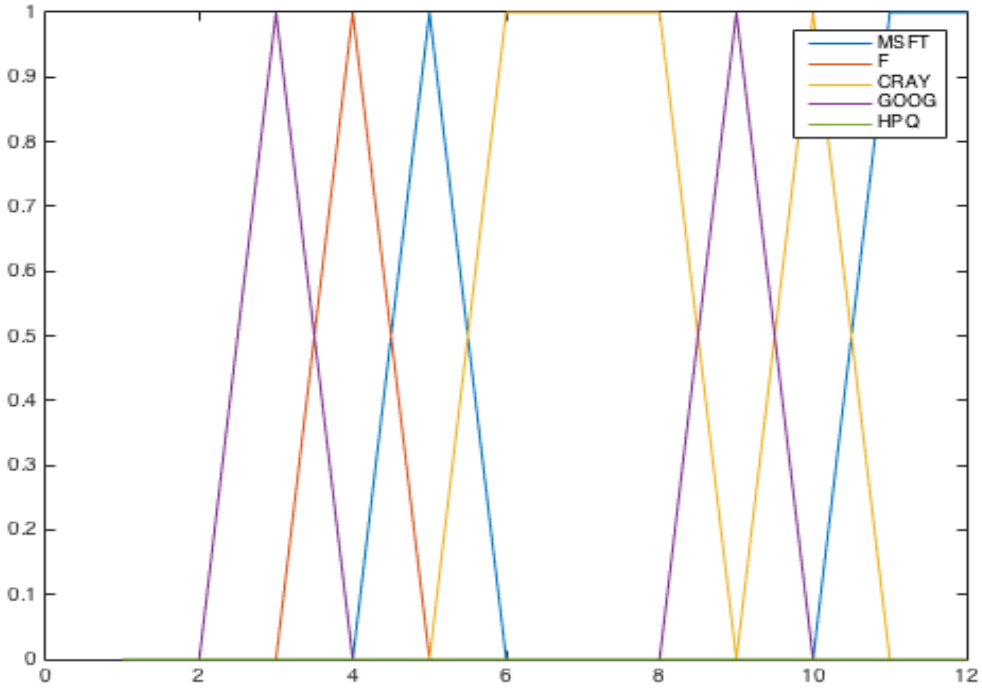

Figure 4: Graph showing the asset proportion over time with the max Sharpe strategy (selected assets)

3 Possible Improvements

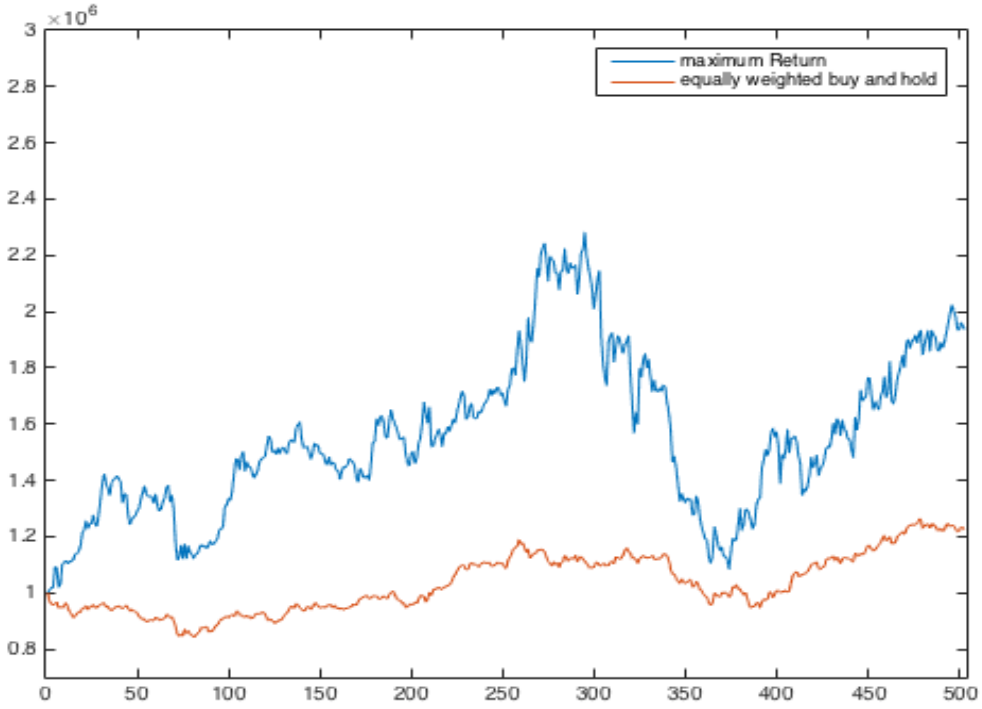

Figure 5: Graph showing the daily value of the porfolio over all 504 trading days for the max return and equally weighted buy and hold strategies

Two other rebalancing strategies were implemented. The maximum return strategy used cplex to maximize μ, the expected return of the portfolio. This strategy always selected the asset with the greatest expected return, and only suggested buying that asset. There are wild fluctuations in porfolio value, and although in the end the performance was better than minimum variance and equally weighted, it still performed worse than maximum Sharpe Ratio and was a lot riskier. The equally weighted buy and hold strategy bought equally weighted assets at the first rebalancing period, and simply held on to those stocks until the end of all the periods without further rebalancing. This strategy saved on transaction costs when compared to the equally weighted at every rebalancing strategy. The performance of all strategies, especially the minimum variance strategy, could be improved further by including the transaction cost in the cost function to minimize that along with the variance, or by introducing cardinality constraints.