Inductive Definitions

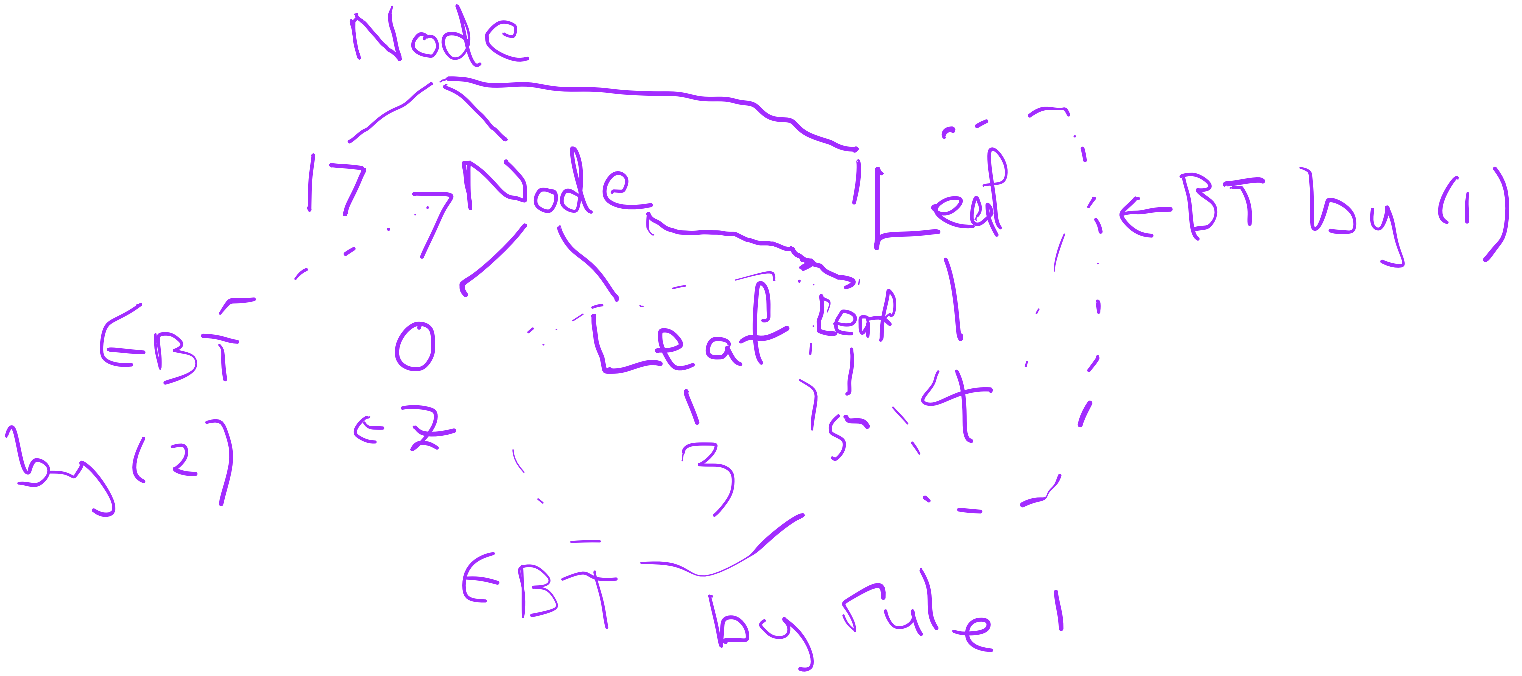
Haskell's version

Example: binary trees [botched by prof last lecture]

Inductively define the set `BT` of binary trees with integer "info" as follows.

```
     n ∈ Z             n ∈ Z    t1 ∈ BT    t2 ∈ BT
(1)-------------  (2)---------------------------------
  Leaf n ∈ BT                Node n t1 t2 ∈ BT
```

Example tree with derivation and tree picture:



Another view of incr

Last time, we had a pseudo-Python function for adding *1* to the numbers in a BT.

```
incr(t):
  if t is Leaf n:
    n+1
  else t is Node n t1 t2:
    Node (n+1) (incr t1) (incr t2)  # recursive calls on smaller trees
```

Rewriting,

```
if t is Leaf n then incr(t) = Leaf (n+1)
if t is Node n t1 t2 then incr(t) = Node (n+1) (incr t1) (incr t2)
```

Haskell

```
incr(Leaf n) = Leaf (n+1)
incr(Node n t1 t2) = Node (n+1) (incr t1) (incr t2)
```

"pattern"

"pattern-matching + recursion"

We've arrived at Haskell

Haskell directly supports inductive definitions, pattern-matching case analysis and recursion.

types of parts

```
-- Type of binary trees
data BT = Leaf Int | Node Int BT BT
```

constructors, must be capitalized

This means exactly the same thing as the corresponding inductive definition.

I.e. the Haskell type BT has exactly the values generated by the rules below.

```
                 n ∈ Int  ⇒  Leaf n ∈ BT
n ∈ Int, t1 ∈ BT, t2 ∈ BT  ⇒  Node n t1 t2 ∈ BT
```

rules

Evaluation details: example

actual prog

Leaf

```
incr (Leaf n) = n+1
incr (Node n t1 t2) = Node (n+1) (incr t1) (incr t2)
```

Steps Haskell takes to evaluate `incr v` for some tree value `v`:

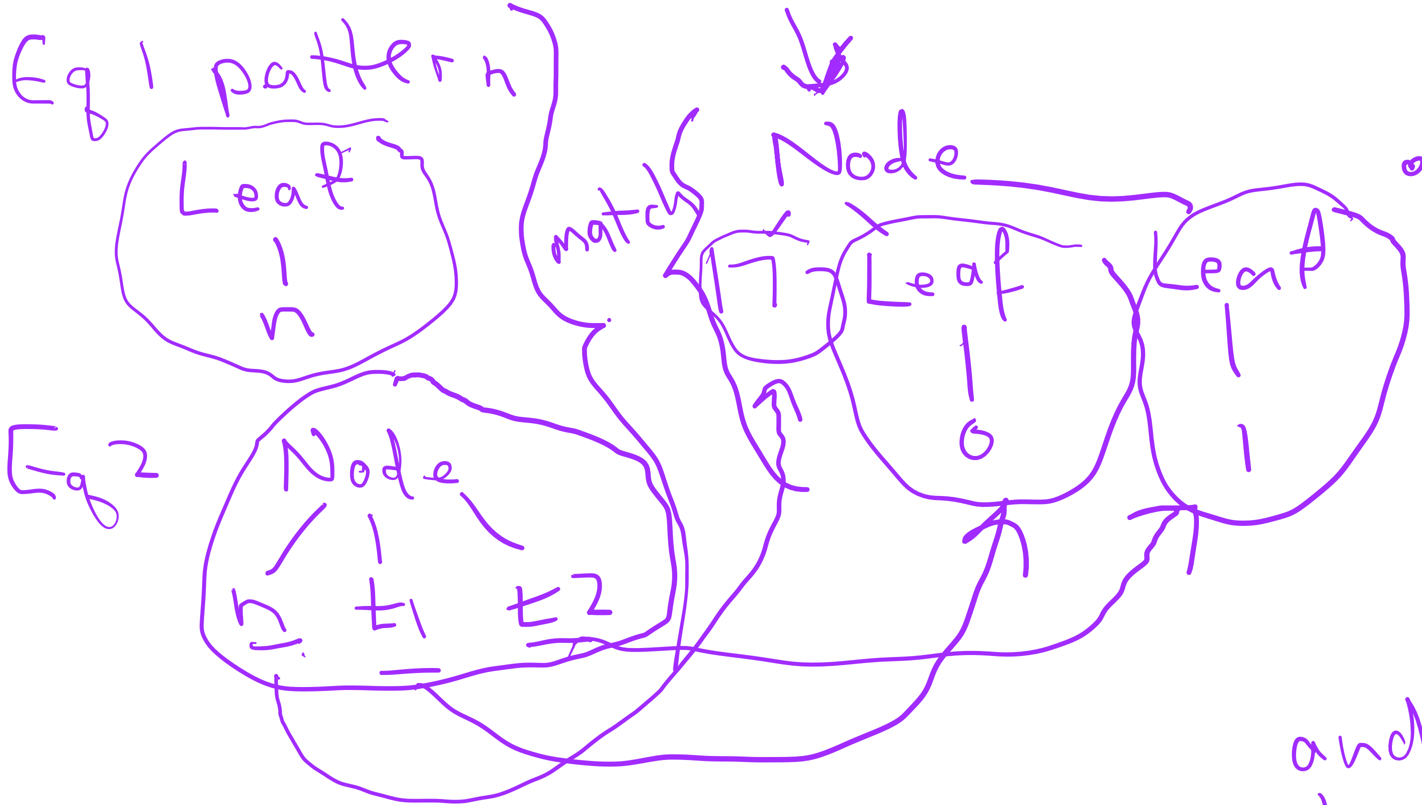
1. Find first equation whose lhs (left-hand-side) matches `v`.
2. Get the variable values from the match.
3. Plug the values in for the variables on the rhs and continue by evaluating the rhs.

E.g. evaluate `incr (Node 17 (Leaf 0) (Leaf 1))`.

Tree view of pattern-matching

```
1) incr (Leaf n) = n+1
2) incr (Node n t1 t2) = Node (n+1) (incr t1) (incr t2)
```

```
incr (Node 17 (Leaf 0) (Leaf 1)).
```



∴ rhs of 2 becomes

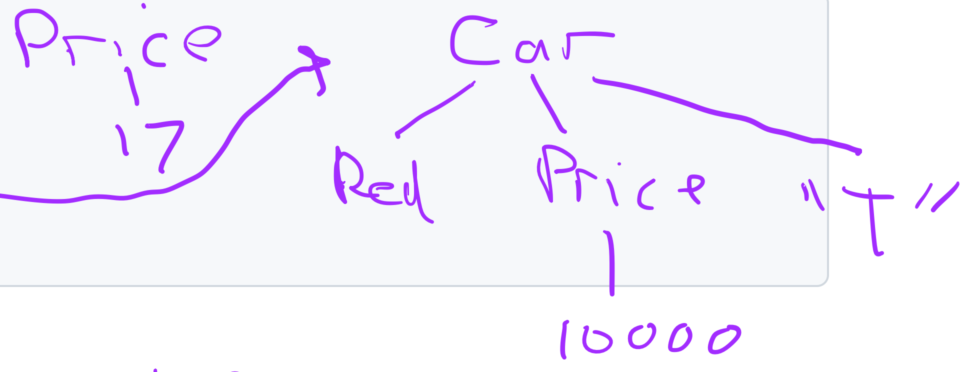
Node (17+1) (incr (Leaf 0)) (incr (Leaf 1))

and keep computing to get Node 18 (Leaf 1) (Leaf 2)

Example: cars!

single-node trees

```haskell
data Colour = Blue | Red | Yellow
data Price  = Price Int
data Car    = Car Colour Price String
data Fleet  = Empty | AddCar Car Fleet
```



Exercises: see the accompanying Haskell file.

Empty

Draw a tree for the fleet example

```
car0 = Car Red (Price 60000) "Lincoln Juggernaut"
car1 = Car Yellow (Price 120000) "BMW Highsnoot"
car2 = Car Blue (Price 10000) "Fiat Roadkill"
fleet = AddCar car0 (AddCar car1 (AddCar car2 Empty))
```