# Econ 3001B - Winter 2023 Name: Nick Cooley. Student number: 101021174.

**Due:** 01 March 2023

**Question 1:**

(a)

$$\begin{bmatrix} 27 & 44 & 51 \\ 35 & 39 & 62 \\ 33 & 50 & 47 \end{bmatrix} + \begin{bmatrix} 25 & 42 & 48 \\ 33 & 40 & 66 \\ 35 & 48 & 50 \end{bmatrix}$$

$$\begin{bmatrix} 27+25 & 44+42 & 51+48 \\ 35+33 & 39+40 & 62+66 \\ 33+35 & 50+48 & 47+50 \end{bmatrix}$$

$$\begin{bmatrix} 52 & 86 & 99 \\ 68 & 79 & 128 \\ 68 & 98 & 97 \end{bmatrix}$$

(b) Solve AB and BA, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1*1+2*1 & 1*0+2*0 \\ 2*1+1*1 & 2*0+1*0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 0 \\ 3 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1*1+0*2 & 1*2+0*1 \\ 1*1+0*2 & 1*2+0*1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

In general, matrix multiplication is noncommutative as $BA \neq AB$.

(c) Compute $(A+B)^T$, for A and B below:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1+3 & 2+1 \\ 3+-1 & 0+1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$$

$$(A+B)^T - B^T = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$$

$$A^T + B^T - B^T = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A^T + B^T - B^T = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Additive inverse on $M(\mathbb{R})_{2x2}$ yields:

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$$

and,

$$B^T = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$$

Thus: $A^T + B^T = (A+B)^T$ which was to be shown.

**Question 2:**

(a) Compute the following limits:

(a) $\lim_{x \to -2}(x^2 + 5x)$
$\lim_{x \to -2^+}(x^2 + 5x)$

$$((2^+)^2 + 5(2^+))$$

$$(4 + 10)$$

$$\lim_{x \to -2^+}(x^2 + 5x) = 14$$

$\lim_{x\to -2^-}(x^2+5x)$

$$((2^-)^2+5(2^-))$$

$$(4+10)$$

$$\lim_{x\to -2^-}(x^2+5x)=14$$

$$\lim_{x\to -2^-}(x^2+5x)=14=\lim_{x\to -2^+}(x^2+5x)$$

$$\lim_{x\to -2}(x^2+5x)=14$$

(b) $\lim_{x\to 4}\frac{2x^{\frac{3}{2}}-x^{\frac{1}{2}}}{x^2-15}$

$$\lim_{x\to 4^+}\frac{2x^{\frac{3}{2}}-x^{\frac{1}{2}}}{x^2-15}$$

$$\frac{2(2^+))^{\frac{3}{2}}-(4^+)^{\frac{1}{2}}}{x^2-15}$$

$$\frac{2(2^+)(2^+)(2^+)-(2^+)}{(16^+)-15}$$

$$\frac{16-2}{1}$$

$$\lim_{x\to 4^+}\frac{2x^{\frac{3}{2}}-x^{\frac{1}{2}}}{x^2-15}=14$$

$$\lim_{x\to 4^-}\frac{2x^{\frac{3}{2}}-x^{\frac{1}{2}}}{x^2-15}$$

$$\frac{2(2^-)(2^-)(2^-)-(2^-)}{(16^-)-15}$$

$$\frac{16-2}{1}$$

$$\lim_{x\to 4^+}\frac{2x^{\frac{3}{2}}-x^{\frac{1}{2}}}{x^2-15}=14=\lim_{x\to 4^-}\frac{2x^{\frac{3}{2}}-x^{\frac{1}{2}}}{x^2-15}$$

$$\lim_{x\to 4}\frac{2x^{\frac{3}{2}}-x^{\frac{1}{2}}}{x^2-15}=14$$

(c) $\lim_{x\to a}(Ax^n)$

We need to be careful, as the following does not hold for $\forall[x,a,n] \in \mathbb{R}$, for example if $a = 0$ and $n < 0$ then we are dividing by zero which is undefined as the left sided and right sided limits diverge from each other.

Asumming $x, a \geq 0$.

$$\lim_{x\to a+}(Ax^n)$$

$$\lim_{x\to a+}(Ax^n) = A(a^+)^n$$

$$= (Ax^n)$$

Asumming $x, a > 0$.

$$\lim_{x\to a^-}(Ax^n)$$

$$\lim_{x\to a^-}(Ax^n) = A(a^-)^n$$

$$= (Ax^n)$$

$$\lim_{x\to a^-}(Ax^n) = Aa^n = \lim_{x\to a+}(Ax^n)$$

$$\lim_{x\to a}(Ax^n) = Aa^n$$

If and only if $x, a > 0$, if $n < 0$ otherwise if $n > 0$ then $x, a$ are free and the limits converge.

(b) Find an expression for $dz$ in terms of $dx$ and $dy$ in the following:

(a) $z = Ax^a + By^b$

$$z = f(x,y) = Ax^a + By^b$$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

$$dz = aAx^{a-1}dx + bBy^{b-1}dy$$

(b) $z = e^{xu}, where\ u = u(x,y).$

$$z = f(x,y) = e^{xu}, where\ u = u(x,y)$$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial u}\frac{\partial u}{\partial x}dx + \frac{\partial f}{\partial u}\frac{\partial u}{\partial y}dy$$

$$dz = \left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial u}\frac{\partial u}{\partial x}\right)dx + \frac{\partial f}{\partial u}\frac{\partial u}{\partial y}dy$$

$$dz = \left(ue^{xu} + xe^{xu}\frac{\partial u}{\partial x}\right)dx + xe^{xu}\frac{\partial u}{\partial y}dy$$

$u = u(x, y)$ is unknown so its derivative with respect to x and y are unknown.

(c) $z = ln(x^2 + y)$

$$z = f(x, y) = ln(x^2 + y)$$

let $u = x^2 + y$

$$dz = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x}dx + \frac{\partial f}{\partial u}\frac{\partial u}{\partial y}dy$$

$$dz = \frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy\right)$$

$$dz = \frac{1}{u}\left(2xdx + dy\right)$$

$$dz = \frac{2x + 1}{x^2 + y}$$

**Question 3:**

Find $A^{-1}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$det(A) = (1 * 1 * 1) + (2 * -1 * 1) + (3 * 0 * 2) - (2 * 0 * 1) - (1 * -1 * 2) - (3 * 1 * 1)$$

$$det(A) = 1 + -2 + 0 - 0 - -2 - 3 = -2$$

Matrix is invertible.

$$A^{-1} = \frac{1}{det(A)}cof(A)^T = \frac{1}{det(A)}adj(A)$$

$$minidets(A) = \begin{bmatrix} 3 & 1 & -1 \\ -4 & -2 & 0 \\ -5 & -1 & 1 \end{bmatrix}$$

$$cof(A) = \begin{bmatrix} 3 & -1 & -1 \\ 4 & -2 & 0 \\ -5 & 1 & 1 \end{bmatrix}$$

$$cof(A)^T = adjoint(A) = \begin{bmatrix} 3 & 4 & -5 \\ -1 & -2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} 3 & 4 & -5 \\ -1 & -2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{-3}{2} & -2 & \frac{5}{2} \\ \frac{1}{2} & 1 & \frac{-1}{2} \\ \frac{1}{2} & 0 & \frac{-1}{2} \end{bmatrix}$$

**Question 4:** Consider the National -Income model with 3 endogenous variables, Y (national income), C (consumption), and t (taxes).

$$Q_d = a - bp \quad (a, b > 0)$$
$$Q_s = -c + dp \quad (c, d > 0)$$

Endogenous variables are: $\{P, Q\}$ which are functions of the exogenous variables: $\{a, b, c, d\}$.

(a) Derive $P^*$ and $Q^*$ in equilibrium (when quantity supplied = to quantity demanded)

We want to find the intersection of the supply and demand curves.

$$D = \{(P, Q) | Q = a - bP\}$$

$$S = \{(P, Q) | Q - c + dP\}$$

$$D \cap S = (P^*, Q^*)$$

$$Q_d = a - bP = -c + dP = Q_s \quad (c, d > 0), (a, b > 0)$$

$$a + c = bP + dP$$

$$a + c = P(b + d)$$

$$\frac{a + c}{b + d} = P^*$$

$$Q^* = a - b\left(\frac{a + c}{b + d}\right)$$

$$Q^* = \frac{a(b+d) - b(a+c)}{b+d}$$

$$Q^* = \frac{ab + ad - ba - bc}{b+d}$$

$$Q^* = \frac{ab - ab + ad - bc}{b+d}$$

$$Q^* = \frac{ad - bc}{b+d} \quad (a, b, c, d > 0)$$

$$P^* = \frac{a+c}{b+d} \quad (a, b, c, d > 0)$$

(b) Examine the comparative-static properties of the equilibrium quantity and provide the economic meaning of it? (Note compute partial derivatives of $P^*$ with respect to parameters in the model. We discuss this in details in class during lecture)

This comparative-static model reflects the equilibrium point $(Q^*, P^*)$ with respect to a single commodity. The exogenous variables $a$ and $c$ reflect the Q axis intercept, while the remaining exogenous variables $b$ and $d$ reflect the change in quantity with respect to $P$.