MATH 3800 D — Winter 2024

Assignment 2 (due 11:59pm Feb 6, 2024 in Brightspace)

Enter **your** your 9-digit student ID number and click "Generate":

101021174 Generate

Not using your student ID number to generate the questions could result in a grade of zero for the entire submission.

Note that you may not use any toolbox unless explicitly allowed by the question.

You must follow all the submission requirements when you submit your solutions for marking. **Submissions that do not follow all the requirements will receive a deduction of 10%.**

Name your MATLAB script file as `A2of101021174.m`. Failure to do so could result in loss of points for all questions requiring MATLAB code.

1. Let $\mathbf{A} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -3 & 5 \\ 1 & -3 & 4 \end{bmatrix}$. [MATH3800W24A2-101021174]

   a. (2 points) Use the MATLAB function `lu` to obtain a permutation matrix $\mathbf{P}$ so that $\mathbf{PA}$ has an LU-decomposition. Include in your write-up the matrix $\mathbf{P}$ *and* the fully-evaluated matrix product $\mathbf{PA}$.

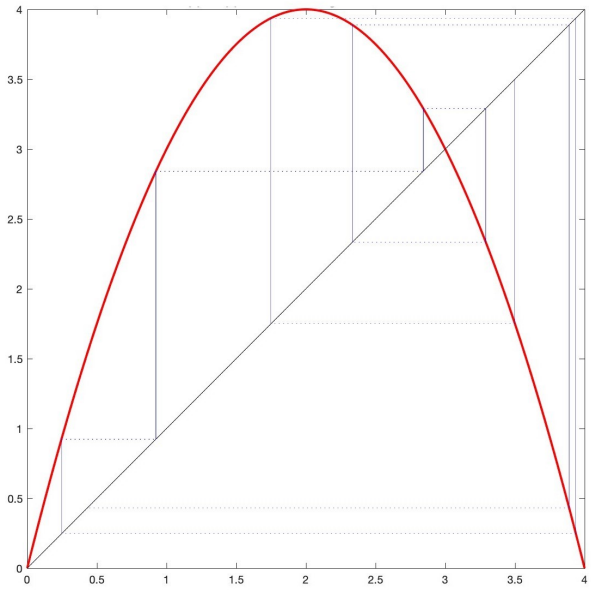
   b. (4 points) Obtain the **reduced row-echelon form** of $\mathbf{PA}$ by hand without performing any exchanges of rows.

   Marking Scheme.

   a. 1 point for executable MATLAB code that uses `lu`.
   0.5 point for the matrix $\mathbf{P}$ extracted from the MATLAB output.
   0.5 point for correctly forming $\mathbf{PA}$ in the write-up.
   b. 1 point for the correct reduced row-echelon form.
   1 point for not using any exchange of rows.
   2 points for completely correct row-reduction work; otherwise, 1 point for reasonable work.

2. (2 points) For this question, you will need the function `cobwebmodded` in the MATLAB script file `cobwebmodded.m` found in the module for Unit 4 in Brightspace.

   Consider the function $f(x) = -x^{5/3} + 3x$. Obtain a cobweb by applying `cobwebmodded` to f with domain $[0, 6]$ and starting value at 5.52 showing the first 8 iterates. The following is an example of what a cobweb looks like.



Include your MATLAB code in the MATLAB script file and save the image as a JPEG file and include it in your submission.. [MATH3800W24A2-101021174]

Marking Scheme.

0.5 point for a JPEG file showing a cobweb with proper domain.
0.5 point for having the correct number of iterates.
0.5 point for coding f correctly.
0.5 point for calling `cobwebmodded` correctly.

3. Recall that the iterative formula for Steffensen's method for root-finding is

$$x_{k+1} = x_k - \frac{f(x_k)^2}{f(x_k + f(x_k)) - f(x_k)}.$$

Let $f(x) = x^3 - 3x^2 - 13x$. [MATH3800W24A2-101021174]

a. (2 points) Use MATLAB to plot f. Adjust the axes so that all x-intercepts and critical points are visible.

b. (2 points) Use Wolfram Alpha to obtain the largest root of f **rounded to 8** decimal places. Include a screenshot of the results and explain your work.

c. (2 points) Let $x_1, x_2 \ldots$ be the sequence generated by Steffensen's method with $x_0 = 6$. Use MATLAB to obtain x_{2024}. How well does x_{2024} approximate the root in part (a)?

Marking Scheme.

a. 2 points for correct plot with all x-intercepts and critical points visible.; otherwise, 1.5 point for a reasonable attempt.
b. 2 points for having the correct value with a proper explanation of the work; otherwise; 1.5 point for a reasonable attempt.
c. 1.5 points for correct code; otherwise; 1 point for a reasonable attempt at obtaining

x_{2024}.
0.5 point for answering how well x_{2024} approximates the root in part (a).

4. Let $\mathbf{f}(x,y) = \begin{bmatrix} x^3 + y^2 - 7 \\ \frac{3}{1+x-2y} - 4 \end{bmatrix}$. [MATH3800W24A2-101021174]

a. (1 point) Use Wolfram Alpha to solve $\mathbf{f}(x,y) = \mathbf{0}$. Provide a screenshot in your submission.

b. (2 points) Obtain the Jacobian matrix of $\mathbf{f}$. No MATLAB code required for this part.

c. (3 points) Recall the high-dimensional Newton's iteration for root-finding:

$$\mathbf{v}_{k+1} = \mathbf{v}_k - \mathbf{J}_{\mathbf{f}}(\mathbf{v}_k)^{-1}\mathbf{f}(\mathbf{v}_k).$$

Choose **integers** a and b so that with $\mathbf{v}_0 = (a,b)$, $\mathbf{v}_3$ is a better approximation to the solution in part (a) than $\mathbf{v}_0$ is. Provide computational evidence in MATLAB. You might find the following code helpful.

```
%% Question 4
f = @(x,y) [ x^3 + y^2 - 7; 3/(1 + x - 2*y) - 4 ];
Jf = @(x,y) [ % Fill in the entries here.
            ];

a = % The value a
b = % The value b
v0 = [a; b];
v1 = % Code for obtaining v1
v2 = % Code for obtaining v2
v3 = % Code for obtaining v3

display(v3);
```

Marking Scheme.

a. 0.5 for providing a screenshot showing the prompt and the solutions.
0.5 point for correctly using Wolfram Alpha to solve the system.
b. If the matrix is 2-by-2, subtract 0.1 point for each incorrect entry.
If the dimensions of the matrix is incorrect, 1 point if most entries are correct partial derivatives and 0.5 point for a reasonable attempt.
c. 1.5 points for code that implement the high-dimensional Newton's iteration correctly; otherwise, 1 point for a reasonable attempt.
1 point for computational evidence that the chosen integers a and b work; otherwise, 0.5 for a reasonable attempt in obtaining such evidence.
0.5 point for discussion of computational evidence.