Volatility-Managed Portfolio: Does It Really Work?*

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Abstract

Moreira and Muir (JF, 2017) propose a volatility-managed portfolio and show that it beats the market. In this paper, we identify a *look-ahead bias* in their procedure. After correcting the bias, we find that the strategy becomes very difficult to implement in practice as its maximum drawdown is 68–93% in almost all cases. Moreover, the strategy outperforms the market only during the financial crisis period. We also consider three alternative volatility-timing strategies and find that they do not outperform the market either. Our results show that one cannot easily beat the market by a simple volatility-timing strategy.

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1 Introduction

While the efficient-market hypothesis claims that it is impossible to beat the market consistently, there are countless studies that propose strategies to just attempt to do so. Along this line, Moreira and Muir (2017) provide an interesting volatility-timing strategy that exploits the well-known property of volatility persistence. In contrast to early studies of Fleming, Kirby, and Ostdiek (2001,2003) who consider daily asset allocation across stocks, the volatility-managed portfolio of Moreira and Muir (2017) is a leverage of the market, with greater weights assigned to the market when recent volatility is low, and with lower weights when recent volatility is high. They show that their strategy beats the market, yielding a positive alpha, a greater Sharpe ratio, and sizable utility gains for mean-variance investors who follow their volatility-timing strategy vs. the buy-and-hold. They show also that their strategy works for factor portfolios that are sorted on value, momentum, profitability, return on equity investment, etc., as well as for currency carry trade.

In this paper, we identify a *look-ahead bias* of their strategy. In their empirical results, they calibrate the weight parameter based on the unconditional volatility over the entire sample period. It appears first that this is innocuous, but a careful examination reveals that the results are highly sensitive to the calibrated parameter. To avoid the look-ahead bias and estimate the weight parameter at each time with only the data available as at that time, we have two popular choices of estimation windows, a fixed-window approach and a rolling-window approach. With either approach and with the window size ranging from 5 to 20 years, we find that the maximum drawdown of the volatility-managed portfolio is 68–93% in most cases, making it likely infeasible in practice. In contrast, without correcting the look-ahead bias, the maximum drawdown is only 56%, comparable to 50% of the market.

Assuming the strategy is feasible in spite of the large drawdowns, its performance is, however, disappointing. In terms of the Sharpe ratio, the volatility-managed portfolio do not outperform the market over the sample period from August 1936 to December 2017. For all estimation cases, the Sharpe ratio is only marginal higher (lower in one case), and the difference is never statistically significant.[1]

Breaking down the entire sample period into roughly 20-year subperiods, we find that the volatility-managed portfolio underperforms the market almost half of the time. However, it does outperform the market in the last period, driven largely by the extremely high volatility during the recent financial crisis. Since financial crises are rare and difficult to predict, the superior performance of the volatility-managed portfolio in this period alone does not support that it as a superior investment strategy overall. There are two reasons. First, an investment strategy that delivers no superior performance over long periods is unlikely to be adopted in practice. Second, the large drawdowns are of concern. The strategy can suffer a forced liquidation before a financial crisis (whose arrival time is difficult to anticipate) happens.

Moreira and Muir (2017) focus on using the alpha as the key performance evaluation measure. Although the conclusions are similar based on either alpha or the Sharpe ratio, we argue that the Sharpe ratio is a more informative measure. A strategy that has a positive alpha will not necessarily add to the investment value for an investor. It increases the investment value only if the strategy yields a greater Sharpe ratio or higher investor utility when it is combined with the market. Since the volatility-managed portfolio is already a portfolio of the market just with time-varying weights, the fact that it does not generally yield a higher Sharpe ratio than the market suggests that the value of the volatility-managed portfolio is limited.

Inspired by Moreira and Muir (2017), an important question is whether there are similar volatility-timing strategies that can perform better. To this end, we consider three alternative volatility-timing strategies in the literature, all of which do not seem to suffer a look-ahead bias. The first is a volatility-targeting strategy proposed by Barroso and Santa-Clara (2015), the second is an allocation strategy under estimation risk proposed by Kan and Zhou (2007), and the third is an optimal portfolio strategy utilizing conditional information proposed by Ferson and Siegel (2001). Applying all three strategies to real data, we find that they fail to outperform the market over the sample period from August 1936 to December 2017.

Overall, our results suggest that it is extremely difficult to beat the market. The

[1] An Online Appendix provides similar results for factor portfolios such as size and value.

volatility-managed strategy of Moreira and Muir (2017) fails to do so, and so do the three alternative strategies. Especially, even the unconditional optimal portfolio with conditional information of Ferson and Siegel (2001) fails to outperform the market, although it is theoretically designed to be the optimal strategy. This is due to the estimation errors that make the strategy less effective in practice.

This paper is organize as follows. The next section outlines the volatility-timing strategies. Section 3 reports empirical results, and Section 4 concludes.

2 Volatility-Timing Strategies

This section reviews four volatility-timing strategies in the literature. The first is the volatility-managed portfolio of Moreira and Muir (2017), the second is the volatility-targeting strategy of Barroso and Santa-Clara (2015), the third is the mean-variance portfolio allocation strategy under estimation risk of Kan and Zhou (2007), and the fourth is the unconditional optimal portfolio with conditional information of Ferson and Siegel (2001).

2.1 Volatility-Managed Portfolio

Moreira and Muir (2017) propose a volatility-managed portfolio constructed by scaling the excess return of the market or a factor portfolio by the inverse of the previous month's realized return variance. This strategy is motivated by observing that changes in volatility over time are not offset by proportional changes in expected returns. Moreira and Muir (2017) find that this strategy improves investment performance relative to the original portfolio by reducing risk exposure when volatility is high.

Theoretically, this volatility-managed strategy can be understood from the perspective of an optimal capital allocation problem between a risky portfolio and a risk-free asset. Suppose there exists a risky portfolio, whose return in excess of the risk-free rate, r_f, is a random variable $\tilde{r}_t$ over month t. The expected excess return is μ_t, and the return volatility is σ_t. Consider a mean-variance investor with risk aversion A, who chooses a weight w_t invested in the risky portfolio and $1 - w_t$ invested in the risk-free asset. Note that w_t can be viewed as the leverage level of this strategy. The investor faces the following mean-variance utility maximization problem:

$$\max_{w_t} U(w_t) = r_f + w_t \mu_t - \frac{A}{2} w_t^2 \sigma_t^2. \tag{1}$$

Solving (1) yields the optimal leverage

$$w_t = \frac{1}{A} \frac{\mu_t}{\sigma_t^2}.$$

Therefore, if μ_t and σ_t were known, the optimal leverage should be proportional to μ_t/σ_t^2.

Observing that volatility is highly persistent over time and that it is difficult to predict returns, Moreira and Muir (2017) construct a volatility-managed portfolio by choosing

$$w_t^{MM} = \frac{L}{\hat{\sigma}_{t-1}^2}, \tag{2}$$

where L is a constant, and $\hat{\sigma}_{t-1}^2$ is the realized return variance in month $t-1$ computed from daily returns over the month (used as a proxy of σ_t^2 based on information available as of month $t-1$). Specifically, $\hat{\sigma}_{t-1}^2$ can be computed as

$$\hat{\sigma}_{t-1}^2 = \frac{22}{D_{t-1}} \sum_{d=1/D_{t-1}}^{1} \left(r_{t-1,d} - \frac{1}{D_{t-1}} \sum_{d=1/D_{t-1}}^{1} r_{t-1,d} \right)^2,$$

where D_{t-1} is the number of trading days in month $t-1$, $r_{t-1,d}$ is the excess return of the risky portfolio on date d of month $t-1$, and a multiplier 22 is included to convert daily variances into monthly values. The excess return of this strategy is thus given by

$$\tilde{r}_t^{MM} = \frac{L}{\hat{\sigma}_{t-1}^2} \tilde{r}_t. \tag{3}$$

The volatility-managed strategy is not implementable unless L is given a priori. Moreira and Muir (2017) choose L so that their strategy has the same unconditional volatility as the original portfolio over the full sample. The choice of L appears innocuous as it does not affect the Sharpe ratio. However, this introduces more complex issues in practice. First, choosing L based on the unconditional volatility over the entire period is an in-sample approach and is thus subject to a look-ahead bias. In practice, one must choose an out-of-sample method (i.e., using only information available at each time point) to estimate L in order to implement the strategy in real time. Different estimation methods lead to different outcomes, and it is unknown ex ante which estimation method will perform the best in the future. Second, the L estimate can potentially expose the strategy to too much tail risk in terms of drawdowns. In such cases, the strategy may be impossible to carry out to the end of the period, as investors will almost surely choose to liquidate when the drawdown reaches a level that is too high.

In short, choosing L based on the unconditional volatility over the full sample is subject to a look-ahead bias. To avoid the bias, one must estimate L using an out-of-sample approach, which may render the strategy very difficult to implement in practice due to large drawdowns. In addition, even ignoring the drawdowns we find that the volatility-managed portfolio cannot outperform the market in general, as will be demonstrated empirically in Section 3.

2.2 Volatility-Targeting Strategy

Barroso and Santa-Clara (2015) propose a volatility-timing strategy by setting a volatility target for the portfolio under consideration, and they show that it is effective in avoiding momentum crashes. Their strategy is based on the same idea of scaling an excess return by its conditional volatility. The key difference is that, instead of choosing the leverage size by requiring the unconditional volatility of the strategy to be equal to that of the original portfolio, they set some specific ex-ante target volatility σ_{target}, i.e.,

$$\tilde{r}_t^{BS} = \frac{\sigma_{target}}{\hat{\sigma}_{t-1}} \tilde{r}_t. \tag{4}$$

The leverage level of this strategy is thus

$$w_t^{BS} = \frac{\sigma_{target}}{\hat{\sigma}_{t-1}}. \tag{5}$$

Clearly, the choice of the target volatility is not subject to a look-ahead bias.

Similar to Moreira and Muir (2017), Barroso and Santa-Clara (2015) observe that the choice of σ_{target} does not affect the Sharpe ratio of the strategy and is thus unimportant. For implementation, they choose a target level corresponding to an annualized volatility of 12%. However, as in the case of Moreira and Muir (2017), the choice of σ_{target} has strong implications for the drawdowns.

2.3 Mean-Variance Portfolio Allocation Under Estimation Risk

The above strategies do not exploit any information on expected returns. We now examine whether such information may be incorporated to improve the performance of volatility timing. Since the expected return is unknown and has to be estimated from data, using the estimated expected return (i.e., sample mean) introduces an estimation risk. The greater the volatility, the more difficult it is to estimate the expected return, and so the greater the estimation risk. Intuitively, faced with estimation risk, mean-variance investors should lower their investment in the risky portfolio. This is qualitatively similar to the volatility-timing strategies of both Moreira and Muir (2017) and Barroso and Santa-Clara (2015).

Consider an estimated allocation strategy,

$$w_t^{KZ} = \frac{c}{A}\frac{\hat{\mu}_{t-1}}{\hat{\sigma}_{t-1}^2}, \tag{6}$$

where c is a pre-specified constant, and $\hat{\mu}_{t-1}$ and $\hat{\sigma}_{t-1}$ are the estimated expected excess return and volatility of the risky portfolio at $t-1$. When $c = 1$, this is the standard optimal portfolio rule. Intuitions suggest that c should be less than 1 when $\hat{\sigma}_{t-1} > 0$ due to risk involved in estimating $\hat{\mu}_{t-1}$.

Assume that the excess return of the risky portfolio follows a normal distribution, i.e.,

$$\tilde{r}_t \sim N(\mu_t, \sigma_t^2).$$

Kan and Zhou (2007) solve the optimal allocation problem under estimation risk. Here, utilizing the fact that volatility is highly persistent and much more predictable than the returns, we assume further that the volatility is known and is equal to the past realized volatility for simplicity. Then the optimal c that minimizes utility loss associated with estimation risk is

$$c = \frac{\hat{\theta}_{t-1}^2}{\hat{\theta}_{t-1}^2 + 1/T},$$

where

$$\hat{\theta}_{t-1}^2 = \frac{\hat{\mu}_{t-1}^2}{\hat{\sigma}_{t-1}^2},$$

and T is the sample size used to estimate $\hat{\mu}_{t-1}$. The analytical formula makes intuitive sense. Everything else equal, greater volatility yields a lower c, which in turn reduces exposure to the risky portfolio. As the sample size approaches infinity, the estimation risk disappears and the strategy converges to the standard optimal allocation rule.

2.4 Unconditional Optimal Portfolio with Conditional Information

Ferson and Siegel (2001) show that conditional information can be utilized to improve the unconditional performance of portfolios. In particular, assume that the conditional expected excess return $\tilde{\mu}_{t-1}$ and the conditional volatility $\tilde{\sigma}_{t-1}^2$ follow stationary processes so that the expectation

$$\zeta = \mathrm{E}\left(\frac{\tilde{\mu}_{t-1}^2}{\tilde{\mu}_{t-1}^2 + \tilde{\sigma}_{t-1}^2}\right)$$

is well defined. Then, for any given unconditional target expected excess return μ_{target}, the optimal leverage yielding minimum unconditional variance is

$$w_t^{FS} = \frac{\mu_{target}}{\zeta}\left(\frac{\hat{\mu}_{t-1}}{\hat{\mu}_{t-1}^2 + \hat{\sigma}_{t-1}^2}\right),$$

where $\hat{\mu}_{t-1}$ and $\hat{\sigma}_{t-1}$ are the sample conditional expected excess return and conditional volatility estimated as before.

For implementation, we estimate ζ as the historical mean of $\hat{\mu}_{t-1}^2/\left(\hat{\mu}_{t-1}^2 + \hat{\sigma}_{t-1}^2\right)$. Clearly, it is of interest to develop better econometric models for estimating $\hat{\mu}_{t-1}$, $\hat{\sigma}_{t-1}$ and ζ. However, since the focus of this paper is to point out the look-ahead bias in Moreira and Muir (2017), and to demonstrate the empirical fact that all four simple volatility-timing strategies in the literature fail to beat the market, we leave the search for more sophisticated models of conditional information as future work.

3 Empirical Results

In this section, we examine the empirical performance of the volatility-timing strategies and show how their performance depends critically on the level of leverage. We focus on applying the strategies to the market excess return, while leaving discussions for other factor portfolios to the Online Appendix.

We start by comparing the performance of the volatility-managed portfolio of Moreira and Muir (2017) with L estimated in-sample, i.e., based on the unconditional volatility over the entire period, against that of the market excess return. We obtain the market excess returns from CRSP for the sample period from August 1926 to December 2017. The estimated value of L is around 0.0010. Since the volatility-managed strategy involves leverage, which may be subject to constraints in practice, following Moreira and Muir (2017), we consider two cases, one with unlimited leverage and one with limited leverage. In the limited leverage case, we impose a constraint that the leverage does not exceed 2, i.e., $w_t^{MM} \leq 2$. If w_t^{MM} estimated from (2) is higher than 2, we then truncate it at 2. Similar constraints are also imposed for other volatility-timing strategies.

Table 1 provides summary statistics of the market excess return and the corresponding volatility-managed portfolio for the full sample as well as for subsamples.[2] Over the full sample, the annualized Sharpe ratios (SR) of the volatility-managed portfolio with unlimited and limited leverage are 0.5143 and 0.5282, respectively. Neither is significantly different from the SR of the market, 0.4996, based on HAC standard errors (Ledoit and Wolf (2008)). We also examine the maximum drawdown (MDD) of the portfolios. Under unlimited and limited leverage, the MDD values of

[2]The full sample results would be the same as in Moreira and Muir (2017) if the data start in August 1926. We use the period from August 1936 to December 2017 to be consistent with our out-of-sample results later, where the first ten years' data are used to estimate L.

the volatility-managed strategy are 0.5642 and 0.5459, respectively. Although they are larger than that of the market index, 0.5039, they do not seem too extreme. We also estimate the risk-adjusted alphas of the strategy with respect to the market index, and the alphas are weakly significant based on Newey-West standard errors.

Breaking down the full sample into roughly 20-year subperiods shows that the only period during which the volatility-managed portfolio clearly dominates the market index is from year 2001 to 2017, driven largely by the financial crisis. For the other subperiods, there is no clear evidence that volatility timing improves performance even when L is estimated in-sample.

We then turn to estimating L out-of-sample and see how this would affect the performance of the volatility-managed strategy. We use two out-of-sample estimation approaches. The first is a fixed-window approach, which uses the first ten years between August 1926 and July 1936 as a training window. We choose L such that the volatility-managed portfolio has the same unconditional volatility as the market index over the training window, and we use this same value of L to determine leverage for all remaining months. The second approach is to estimate L using 10-year rolling windows, allowing time variations in the value of L. For example, we estimate L based on returns from August 1926 to July 1936 and use this L to determine the leverage size for August 1936, and so on.

Panel A of Table 2 reports results based on the 10-year fixed-window approach. The estimated L in this case is 0.0023, higher than the in-sample value of L. This leads to proportionally higher leverage in every single month, which does not affect the SR of the strategy without leverage constraint. However, this does have a substantial effect on the MDD of the strategy. Over the full sample, the MDD value is 0.9293 with unlimited leverage and 0.7596 with limited leverage, much higher than that of the market. In particular, an MDD of 0.9293 means that the strategy loses about 93% of its value during the worst decline, which would be unacceptable to investors in practice. To see how the performance of volatility timing is affected by the choice of the training window, Panels B and C report results using 5-year and 20-year windows for the estimation of L. The MDD values are 0.6781 and 0.6785, respectively. Both are substantially larger than the MDD of the market.

We then discuss results using rolling-window estimation of L. Figure 1 shows the time variation of L under the 10-year rolling-window approach. It ranges between 0.0004 and 0.0037, with a mean of 0.0011 and a median of 0.0007. It peaks around year 1940, and then it quickly drops below mean and stays low until year 2000 before picking up again. With time-varying values of L, the SR now differs from the in-sample level. As reported in Table 3, over the full sample, the SR of the volatility-managed portfolio with 10-year rolling windows is only 0.3866, much lower than that of the market. The worst monthly return is -115%, and it happened in May 1940, indicating that an investor using the volatility-managed strategy would go bankrupt in that month. Imposing the leverage constraint slightly improves performance. The SR becomes 0.4632, and the MDD is 0.7640, both of which are still worse than the market. Breaking down the full sample into subsamples shows that the poor performance mainly comes from the early years. Especially, from August 1936 to December 1960, the SRs of the volatility-managed strategy under both unlimited and limited leverage are significantly lower than that of the market at the 5% level. This is driven by the higher leverage during that period due to higher estimates of L. Using 5-year and 20-year rolling windows yields similar results.

Table 4 reports the performance of the volatility-targeting strategy of Barroso and Santa-Clara (2015) with various target volatility levels ranging between 12% and 20% annually. For all target levels, the SR of the strategy without leverage constraint is 0.5527, which is not significantly different from that of the market index. On the other hand, the MDD is very sensitive to the volatility target. This is because the choice of the target directly affects the overall level of leverage (see (5)). When a target of 12% is used, the MDD of the strategy is 0.5230, which is comparable to that of the market. When we increase the annualized volatility target to 16%, the MDD becomes 0.6519. When we further increase the target to 20%, the MDD now becomes 0.7567, about 50% higher than that of the market. Imposing the leverage constraint barely changes the result.

Table 5 reports the performance of the portfolio allocation strategy of Kan and Zhou (2007) accounting for estimation risk, with the expected returns $\hat{\mu}_{t-1}$ estimated as 10-year rolling sample means ($T = 120$). With risk version of $A = 3$ and $A = 5$, the strategy fails to outperform the market. While the SR of the strategy is higher than that of the market over the last period, the difference is not statistically significant. More importantly, the MDD makes the strategy even less attractive. With risk aversion of $A = 3$, the strategy goes bankrupt, rendering it infeasible in practice. With $A = 5$, the MDD improves to some extent, but remains substantially higher than that of the market.

Table 6 reports the performance of the unconditional optimal portfolio of Ferson and Siegel (2001) in the presence of conditional information. With annualized target expected excess returns of 6% and 10%, this strategy again fails to improve the SR, and the MDD is always higher than that of the market index. Our evidence indicates that while conditional information can theoretically be utilized to improve unconditional performance, empirically this fails due to estimation errors.

In summary, all four volatility-timing strategies do not deliver out-of-sample superior performance. The Sharpe ratio is never significantly improved except for during the financial crisis period. The full-sample Sharpe ratios are at most slightly better (in some cases even lower), and the differences are statistically insignificant. In terms of the drawdowns, they are generally much worse than the market, making the strategies infeasible in most cases in the real world.

4 Conclusion

In this paper, we provide a thorough re-examination of the volatility-managed portfolio proposed by Moreira and Muir (2017). We find that there is a look-ahead bias in their analysis, and after correcting it the strategy worsens substantially. In general, the strategy cannot be easily implemented due to large drawdowns. Even if we ignore the drawdowns and assume that the strategy were implementable, it cannot outperform the market except for during the recent financial crisis period. For deeper understanding, we further examine the performance of three alternative volatility-timing strategies, and find that they are also unable to beat the market. Overall, while volatility timing is an appealing idea, it remains a challenging task to show it can work.

References

[1] Barroso, Pedro, and Pedro Santa-Clara, 2015. Momentum has its moments. *Journal of Financial Economics* 116, 111–120.

[2] Ferson, Wayne E., and Andrew F. Siegel, 2001. The efficient use of conditioning information in portfolios. *Journal of Finance* 56, 967–982.

[3] Fleming, Jeff, Chris Kirby, and Barbara Ostdiek, 2001. The economic value of volatility timing. *Journal of Finance* 56, 329–352.

[4] Fleming, Jeff, Chris Kirby, and Barbara Ostdiek, 2003. The economic value of volatility timing using "realized" volatility. *Journal of Financial Economics* 67, 473–509.

[5] Kan, Raymond, and Guofu Zhou, 2007. Optimal portfolio choice with parameter uncertainty. *Journal of Financial and Quantitative Analysis* 42, 621–656.

[6] Ledoit, Oliver, and Michael Wolf, 2008. Robust performance hypothesis testing with the Sharpe ratio. *Journal of Empirical Finance* 15, 850–859.

[7] Moreira, Alan, and Tyler Muir, 2017. Volatility-managed portfolios. *Journal of Finance* 72, 1611–1644.

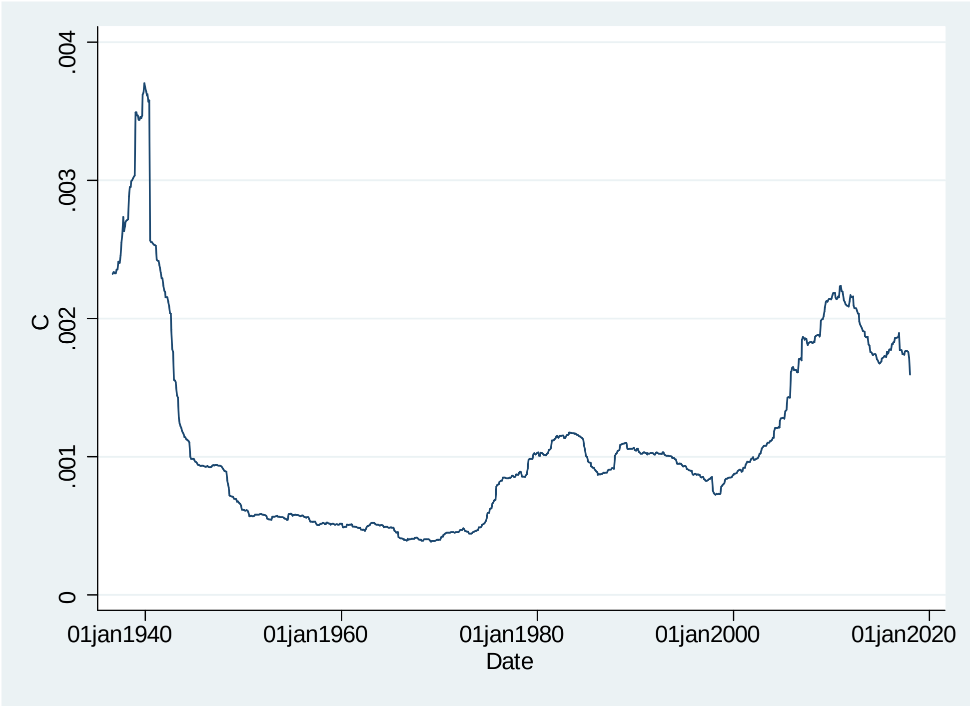

Figure 1: Rolling-Window Estimation of L for Volatility-Managed Portfolio

This figure plots the value of L for the volatility-managed portfolio (3) of Moreira and Muir (2017) applied to the market excess return with L estimated out-of-sample using 10-year rolling windows. The sample period is from August 1936 to December 2017.

Table 1: In-Sample Performance of Volatility-Managed Portfolio for Market

This table reports summary statistics of the market excess return (Panel A) and the volatility-managed portfolio of Moreira and Muir (2017) applied to the market excess return (Panel B) with L in (3) estimated in-sample. We report results for the full sample from August 1936 to December 2017 as well as for four different subsample periods. The summary statistics include the number of observations, mean, volatility, minimum, maximum, maximum drawdown (MDD), and the Sharpe ratio (SR). For the volatility-managed portfolio, we consider cases with unlimited leverage (UL) and limited leverage (LL) separately. We also report the risk-adjusted alpha of the volatility-managed portfolio with respect to the market index and the difference in the Sharpe ratio between the volatility-managed portfolio and the market index. Asterisks denote statistical significance based on Newey-West standard errors (for alpha) or HAC standard errors (for the Sharpe ratio test) at the 1% (***), 5% (**) and 10% (*) levels.

Panel A: Market Index

	Full Sample	Aug1936–Dec1960	Jan1961–Dec1980	Jan1981–Dec2000	Jan2001–Dec2017
#Obs	977	293	240	240	204
Mean	0.0065	0.0091	0.0034	0.0073	0.0054
Vol	0.0451	0.0478	0.0444	0.0445	0.0427
Min	-0.2382	-0.2382	-0.1290	-0.2324	-0.1723
Max	0.2387	0.2387	0.1610	0.1247	0.1135
MDD	0.5039	0.4935	0.4642	0.2991	0.5039
SR	0.4996	0.6610	0.2671	0.5710	0.4380

Panel B: Volatility-Managed Portfolio

	Full Sample		Aug1936–Dec1960		Jan1961–Dec1980		Jan1981–Dec2000		Jan2001–Dec2017	
	UL	LL	UL	LL	UL	LL	UL	LL	UL	LL
#Obs	977	977	293	293	240	240	240	240	204	204
Mean	0.0081	0.0066	0.0111	0.0096	0.0062	0.0036	0.0082	0.0069	0.0061	0.0057
Vol	0.0548	0.0435	0.0640	0.0520	0.0687	0.0483	0.0443	0.0398	0.0256	0.0239
Min	-0.3125	-0.3125	-0.3125	-0.3125	-0.2300	-0.1436	-0.1154	-0.1154	-0.0603	-0.0542
Max	0.3577	0.1876	0.2909	0.1876	0.3577	0.1576	0.1662	0.1370	0.1198	0.0972
MDD	0.5642	0.5459	0.5642	0.5459	0.4934	0.3933	0.1481	0.1410	0.1680	0.1680
Alpha	0.0027*	0.0017*	0.0023	0.0017	0.0026	0.0006	0.0025	0.0014	0.0040**	0.0037***
SR	0.5143	0.5282	0.6001	0.6370	0.3117	0.2583	0.6417	0.5971	0.8264	0.8296
ΔSR	0.0148	0.0286	-0.0608	-0.0239	0.0446	-0.0088	0.0708	0.0261	0.3884*	0.3916*

Table 2: Out-of-Sample Performance of Volatility-Managed Portfolio for Market: Fixed-Window

This table reports summary statistics of the volatility-managed portfolio of Moreira and Muir (2017) applied to the market excess return with L in (3) estimated out-of-sample using a fixed training window at the beginning of our sample period from August 1926 to December 2017. We report results for the full sample as well as for four different subsample periods. We use 10-year (Panel A), 5-year (Panel B), and 20-year (Panel C) training windows. We consider cases with unlimited leverage (UL) and limited leverage (LL) separately. The summary statistics include the number of observations, mean, volatility, minimum, maximum, maximum drawdown (MDD), and the Sharpe ratio (SR). We also report the risk-adjusted alpha of the volatility-managed portfolio with respect to the market index and the difference in the Sharpe ratio between the volatility-managed portfolio and the market index. Asterisks denote statistical significance based on Newey-West standard errors (for alpha) or HAC standard errors (for the Sharpe ratio test) at the 1% (***), 5% (**) and 10% (*) levels.

	Panel A: 10-Year Fixed Window									
	Full Sample		Aug1936–Dec1960		Jan1961–Dec1980		Jan1981–Dec2000		Jan2001–Dec2017	
	UL	LL	UL	LL	UL	LL	UL	LL	UL	LL
#Obs	977	977	293	293	240	240	240	240	204	204
Mean	0.0194	0.0096	0.0265	0.0143	0.0148	0.0044	0.0196	0.0099	0.0146	0.0088
Vol	0.1308	0.0654	0.1528	0.0721	0.1640	0.0704	0.1057	0.0681	0.0612	0.0418
Min	-0.7461	-0.4390	-0.7461	-0.4390	-0.5491	-0.2382	-0.2754	-0.2754	-0.1439	-0.1194
Max	0.8539	0.2494	0.6944	0.1876	0.8539	0.1995	0.3968	0.2494	0.2861	0.1114
MDD	0.9293	0.7596	0.9293	0.7596	0.9222	0.5800	0.8545	0.3765	0.4414	0.4088
Alpha	0.0065*	0.0013	0.0055	0.0022	0.0062	-0.0006	0.0059	-0.0005	0.0096**	0.0047**
SR	0.5143	0.5103	0.6001	0.6891	0.3117	0.2149	0.6417	0.5028	0.8264	0.7297
ΔSR	0.0148	0.0107	-0.0608	0.0281	0.0446	-0.0522	0.0708	-0.0682	0.3884*	0.2916
	Panel B: 5-Year Fixed Window									
	Full Sample		Aug1931–Dec1960		Jan1961–Dec1980		Jan1981–Dec2000		Jan2001–Dec2017	
	UL	LL	UL	LL	UL	LL	UL	LL	UL	LL
#Obs	1037	1037	353	353	240	240	240	240	204	204
Mean	0.0102	0.0076	0.0128	0.0100	0.0080	0.0040	0.0106	0.0081	0.0079	0.0069
Vol	0.0694	0.0498	0.0767	0.0561	0.0892	0.0549	0.0575	0.0486	0.0333	0.0291
Min	-0.4057	-0.4057	-0.4057	-0.4057	-0.2986	-0.1436	-0.1498	-0.1498	-0.0782	-0.0649
Max	0.4643	0.1876	0.3776	0.1876	0.4643	0.1630	0.2158	0.1778	0.1556	0.0972
MDD	0.6781	0.6527	0.6781	0.6527	0.6095	0.4558	0.2140	0.2140	0.2317	0.2317
Alpha	0.0047**	0.0028**	0.0065*	0.0047*	0.0033	0.0004	0.0032	0.0012	0.0052**	0.0043***
SR	0.5113	0.5259	0.5785	0.6205	0.3117	0.2507	0.6417	0.5762	0.8264	0.8170
ΔSR	0.0543	0.0690	0.0587	0.1007	0.0446	-0.0164	0.0708	0.0052	0.3884*	0.3790*
	Panel C: 20-Year Fixed Window									
	Full Sample		Aug1946–Dec1960		Jan1961–Dec1980		Jan1981–Dec2000		Jan2001–Dec2017	
	UL	LL	UL	LL	UL	LL	UL	LL	UL	LL
#Obs	857	857	173	173	240	240	240	240	204	204
Mean	0.0119	0.0082	0.0180	0.0143	0.0093	0.0040	0.0124	0.0087	0.0092	0.0073
Vol	0.0805	0.0522	0.0962	0.0583	0.1038	0.0587	0.0669	0.0543	0.0387	0.0321
Min	-0.3886	-0.1860	-0.3886	-0.1860	-0.3475	-0.1658	-0.1743	-0.1743	-0.0910	-0.0755
Max	0.5403	0.2056	0.3300	0.1876	0.5403	0.1630	0.2511	0.2056	0.1810	0.0972
MDD	0.6785	0.4965	0.4543	0.3268	0.6785	0.4965	0.3745	0.2629	0.2746	0.2746
Alpha	0.0037*	0.0017	-0.0037	-0.0006	0.0039	0.0000	0.0038	0.0009	0.0061**	0.0044**
SR	0.5131	0.5430	0.6494	0.8493	0.3117	0.2359	0.6417	0.5552	0.8264	0.7881
ΔSR	0.0018	0.0317	-0.2932	-0.0933	0.0446	-0.0312	0.0708	-0.0158	0.3884*	0.3501*

Table 3: Out-of-Sample Performance of Volatility-Managed Portfolio for Market: Rolling-Window

This table reports summary statistics of the volatility-managed portfolio of Moreira and Muir (2017) applied to the market excess return with L in (3) estimated out-of-sample using rolling windows. We report results for the full sample from August 1926 to December 2017 as well as for four different subsample periods. We use 10-year (Panel A), 5-year (Panel B), and 20-year (Panel C) rolling windows. We consider cases with unlimited leverage (UL) and limited leverage (LL) separately. The summary statistics include the number of observations, mean, volatility, minimum, maximum, maximum drawdown (MDD), and the Sharpe ratio (SR). We also report the risk-adjusted alpha of the volatility-managed portfolio with respect to the market index and the difference in the Sharpe ratio between the volatility-managed portfolio and the market index. Asterisks denote statistical significance based on Newey-West standard errors (for alpha) or HAC standard errors (for the Sharpe ratio test) at the 1% (***), 5% (**) and 10% (*) levels.

Panel A: 10-Year Rolling Window

	Full Sample		Aug1936–Dec1960		Jan1961–Dec1980		Jan1981–Dec2000		Jan2001–Dec2017	
	UL	LL	UL	LL	UL	LL	UL	LL	UL	LL
#Obs	977	977	293	293	240	240	240	240	204	204
Mean	0.0074	0.0060	0.0066	0.0064	0.0040	0.0030	0.0078	0.0066	0.0118	0.0083
Vol	0.0660	0.0450	0.1012	0.0602	0.0380	0.0345	0.0445	0.0402	0.0461	0.0348
Min	-1.1501	-0.4390	-1.1501	-0.4390	-0.1111	-0.1111	-0.1086	-0.1086	-0.1143	-0.0966
Max	0.4421	0.1843	0.4421	0.1843	0.1664	0.1465	0.1669	0.1591	0.2112	0.0972
MDD	Broke	0.7640	Broke	0.7640	Broke	0.2056	Broke	0.1813	Broke	0.2525
Alpha	0.0012	0.0009	-0.0070	-0.0034	0.0019	0.0010	0.0021	0.0011	0.0082***	0.0052***
SR	0.3866	0.4632	0.2259	0.3703	0.3663	0.2999	0.6103	0.5670	0.8886	0.8307
ΔSR	-0.1130	-0.0363	-0.4351**	-0.2906**	0.0993	0.0328	0.0393	-0.0040	0.4506**	0.3927*

Panel B: 5-Year Rolling Window

	Full Sample		Aug1931–Dec1960		Jan1961–Dec1980		Jan1981–Dec2000		Jan2001–Dec2017	
	UL	LL	UL	LL	UL	LL	UL	LL	UL	LL
#Obs	1037	1037	353	353	240	240	240	240	204	204
Mean	0.0087	0.0066	0.0113	0.0074	0.0037	0.0033	0.0071	0.0061	0.0120	0.0096
Vol	0.0614	0.0461	0.0843	0.0587	0.0445	0.0379	0.0418	0.0384	0.0495	0.0376
Min	-0.6935	-0.4390	-0.6935	-0.4390	-0.1577	-0.1312	-0.1035	-0.1035	-0.1649	-0.0842
Max	0.5133	0.1812	0.5133	0.1812	0.2212	0.1576	0.1645	0.1645	0.1862	0.1114
MDD	0.9110	0.7658	0.9110	0.7658	0.3905	0.2292	0.1902	0.1750	0.3111	0.3111
Alpha	0.0036*	0.0022*	0.0040	0.0014	0.0012	0.0010	0.0018	0.0009	0.0079***	0.0061***
SR	0.4923	0.4952	0.4659	0.4369	0.2882	0.3041	0.5917	0.5513	0.8365	0.8812
ΔSR	0.0354	0.0383	-0.0539	-0.0830	0.0211	0.0371	0.0207	-0.0197	0.3985*	0.4432**

Panel C: 20-Year Rolling Window

	Full Sample		Aug1946–Dec1960		Jan1961–Dec1980		Jan1981–Dec2000		Jan2001–Dec2017	
	UL	LL	UL	LL	UL	LL	UL	LL	UL	LL
#Obs	857	857	173	173	240	240	240	240	204	204
Mean	0.0078	0.0063	0.0113	0.0101	0.0036	0.0023	0.0079	0.0066	0.0095	0.0073
Vol	0.0468	0.0368	0.0713	0.0504	0.0360	0.0314	0.0413	0.0365	0.0370	0.0284
Min	-0.2973	-0.1860	-0.2973	-0.1860	-0.1231	-0.1231	-0.1055	-0.1055	-0.0861	-0.0767
Max	0.3457	0.1815	0.3457	0.1815	0.1863	0.0957	0.1565	0.1220	0.2171	0.0972
MDD	0.4610	0.3278	0.4610	0.3278	0.2225	0.2225	0.1446	0.1379	0.1728	0.1728
Alpha	0.0032**	0.0021**	-0.0042	-0.0022	0.0017	0.0005	0.0027	0.0016	0.0069***	0.0048***
SR	0.5760	0.5903	0.5485	0.6958	0.3478	0.2576	0.6652	0.6272	0.8937	0.8842
ΔSR	0.0647	0.0791	-0.3940**	-0.2468*	0.0808	-0.0094	0.0942	0.0562	0.4557**	0.4462**

Table 4: Performance of Volatility Targeting for Market

This table reports summary statistics of the volatility-targeting strategy of Barroso and Santa-Clara (2015) applied to the market excess return with annualized target volatility of 12% (Panel A), 16% (Panel B), and 20% (Panel C). We report results for the full sample from August 1936 to December 2017 as well as for four different subsample periods. We consider cases with unlimited leverage (UL) and limited leverage (LL) separately. The summary statistics include the number of observations, mean, volatility, minimum, maximum, maximum drawdown (MDD), and the Sharpe ratio (SR). We also report the risk-adjusted alpha of the volatility-targeting strategy with respect to the market index and the difference in the Sharpe ratio between the volatility-targeting strategy and the market index. Asterisks denote statistical significance based on Newey-West standard errors (for alpha) or HAC standard errors (for the Sharpe ratio test) at the 1% (***), 5% (**) and 10% (*) levels.

Panel A: Annualized Target Volatility 12%

	Full Sample		Aug1936–Dec1960		Jan1961–Dec1980		Jan1981–Dec2000		Jan2001–Dec2017	
	UL	LL	UL	LL	UL	LL	UL	LL	UL	LL
#Obs	977	977	293	293	240	240	240	240	204	204
Mean	0.0073	0.0071	0.0103	0.0102	0.0042	0.0037	0.0078	0.0076	0.0062	0.0062
Vol	0.0459	0.0452	0.0523	0.0518	0.0507	0.0492	0.0438	0.0436	0.0295	0.0294
Min	-0.2910	-0.2910	-0.2910	-0.2910	-0.1368	-0.1318	-0.1819	-0.1819	-0.0742	-0.0742
Max	0.1590	0.1590	0.1590	0.1590	0.1248	0.1248	0.1398	0.1398	0.0804	0.0804
MDD	0.5230	0.5225	0.5230	0.5225	0.4177	0.4037	0.1981	0.1981	0.3018	0.3018
Alpha	0.0014*	0.0012	0.0014	0.0013	0.0006	0.0002	0.0010	0.0009	0.0029**	0.0029**
SR	0.5527	0.5445	0.6822	0.6799	0.2855	0.2584	0.6146	0.6061	0.7275	0.7257
ΔSR	0.0531	0.0449	0.0213	0.0189	0.0184	-0.0086	0.0437	0.0351	0.2895**	0.2877**

Panel B: Annualized Target Volatility 16%

	Full Sample		Aug1936–Dec1960		Jan1961–Dec1980		Jan1981–Dec2000		Jan2001–Dec2017	
	UL	LL	UL	LL	UL	LL	UL	LL	UL	LL
#Obs	977	977	293	293	240	240	240	240	204	204
Mean	0.0098	0.0091	0.0137	0.0129	0.0056	0.0047	0.0104	0.0097	0.0083	0.0080
Vol	0.0612	0.0573	0.0698	0.0649	0.0675	0.0612	0.0584	0.0569	0.0393	0.0387
Min	-0.3880	-0.3880	-0.3880	-0.3880	-0.1824	-0.1694	-0.2426	-0.2426	-0.0989	-0.0989
Max	0.2121	0.1876	0.2121	0.1876	0.1664	0.1576	0.1864	0.1864	0.1072	0.0972
MDD	0.6519	0.6442	0.6519	0.6442	0.5387	0.4980	0.2692	0.2692	0.3999	0.3999
Alpha	0.0019*	0.0015*	0.0018	0.0016	0.0009	0.0003	0.0014	0.0008	0.0039**	0.0037**
SR	0.5527	0.5482	0.6822	0.6889	0.2855	0.2675	0.6146	0.5885	0.7275	0.7172
ΔSR	0.0531	0.0487	0.0213	0.0279	0.0184	0.0005	0.0437	0.0175	0.2895**	0.2792**

Panel C: Annualized Target Volatility 20%

	Full Sample		Aug1936–Dec1960		Jan1961–Dec1980		Jan1981–Dec2000		Jan2001–Dec2017	
	UL	LL	UL	LL	UL	LL	UL	LL	UL	LL
#Obs	977	977	293	293	240	240	240	240	204	204
Mean	0.0122	0.0103	0.0172	0.0147	0.0070	0.0051	0.0130	0.0110	0.0103	0.0092
Vol	0.0764	0.0664	0.0872	0.0735	0.0844	0.0699	0.0730	0.0682	0.0491	0.0466
Min	-0.4850	-0.4390	-0.4850	-0.4390	-0.2280	-0.2117	-0.3032	-0.3032	-0.1236	-0.1236
Max	0.2651	0.2329	0.2651	0.2164	0.2079	0.1867	0.2329	0.2329	0.1340	0.1016
MDD	0.7567	0.7173	0.7567	0.7173	0.6610	0.5821	0.3649	0.3612	0.4867	0.4867
Alpha	0.0024*	0.0013	0.0023	0.0016	0.0011	-0.0000	0.0017	0.0003	0.0049**	0.0039**
SR	0.5527	0.5365	0.6822	0.6927	0.2855	0.2535	0.6146	0.5606	0.7275	0.6819
ΔSR	0.0531	0.0369	0.0213	0.0317	0.0184	-0.0136	0.0437	-0.0104	0.2895**	0.2439*

Table 5: Performance of Portfolio Allocation Under Estimation Risk for Market

This table reports summary statistics of the mean-variance portfolio allocation strategy under estimation risk of Kan and Zhou (2007) applied to the market excess return with risk aversion levels of $A = 3$ (Panel A) and $A = 5$ (Panel B). We report results for the full sample from August 1936 to December 2017 as well as for four different subsample periods. We consider cases with unlimited leverage (UL) and limited leverage (LL) separately. The summary statistics include the number of observations, mean, volatility, minimum, maximum, maximum drawdown (MDD), and the Sharpe ratio (SR). We also report the risk-adjusted alpha of the portfolio allocation strategy with respect to the market index and the difference in the Sharpe ratio between the portfolio allocation strategy and the market index. Asterisks denote statistical significance based on Newey-West standard errors (for alpha) or HAC standard errors (for the Sharpe ratio test) at the 1% (***), 5% (**) and 10% (*) levels.

	Full Sample		Aug1936–Dec1960		Jan1961–Dec1980		Jan1981–Dec2000		Jan2001–Dec2017	
Panel A: Risk Aversion $A = 3$										
	UL	LL	UL	LL	UL	LL	UL	LL	UL	LL
#Obs	977	977	293	293	240	240	240	240	204	204
Mean	0.0222	0.0087	0.0400	0.0154	0.0175	0.0032	0.0160	0.0083	0.0092	0.0059
Vol	0.1547	0.0534	0.2161	0.0667	0.1735	0.0456	0.0907	0.0572	0.0447	0.0299
Min	-1.1402	-0.2714	-1.1402	-0.2714	-0.8301	-0.1730	-0.2487	-0.2487	-0.1247	-0.0937
Max	0.8775	0.1876	0.8669	0.1876	0.8775	0.1630	0.4359	0.1752	0.2777	0.0972
MDD	Broke	0.6905	Broke	0.6905	Broke	0.4313	Broke	0.2704	Broke	0.3630
Alpha	0.0112**	0.0032**	0.0158	0.0055**	0.0120	0.0011	0.0060	0.0004	0.0067**	0.0038*
SR	0.4967	0.5622	0.6419	0.7974	0.3496	0.2446	0.6127	0.5034	0.7139	0.6802
ΔSR	-0.0029	0.0627	-0.0191	0.1364	0.0825	-0.0225	0.0417	-0.0676	0.2759	0.2422
Panel B: Risk Aversion $A = 5$										
	Full Sample		Aug1936–Dec1960		Jan1961–Dec1980		Jan1981–Dec2000		Jan2001–Dec2017	
	UL	LL	UL	LL	UL	LL	UL	LL	UL	LL
#Obs	977	977	293	293	240	240	240	240	204	204
Mean	0.0133	0.0074	0.0240	0.0138	0.0105	0.0026	0.0096	0.0066	0.0055	0.0047
Vol	0.0928	0.0448	0.1296	0.0593	0.1041	0.0402	0.0544	0.0421	0.0268	0.0223
Min	-0.6841	-0.1860	-0.6841	-0.1860	-0.4980	-0.1730	-0.1492	-0.1492	-0.0748	-0.0562
Max	0.5265	0.1876	0.5202	0.1876	0.5265	0.1373	0.2615	0.1250	0.1666	0.0972
MDD	0.7676	0.5578	0.7324	0.5578	0.7676	0.3691	0.1897	0.1605	0.2825	0.2581
Alpha	0.0067**	0.0031**	0.0095	0.0056**	0.0072	0.0009	0.0036	0.0012	0.0040**	0.0033**
SR	0.4967	0.5724	0.6419	0.8081	0.3496	0.2273	0.6127	0.5443	0.7139	0.7282
ΔSR	-0.0029	0.0728	-0.0191	0.1472	0.0825	-0.0397	0.0417	-0.0267	0.2759	0.2902

Table 6: Performance of Unconditional Optimal Portfolio with Conditional Information for Market

This table reports summary statistics of the unconditional optimal portfolio with conditional information of Ferson and Siegel (2001) applied to the market excess return with annualized target expected returns of 6% (Panel A) and 10% (Panel B). We report results for the full sample from August 1936 to December 2017 as well as for four different subsample periods. We consider cases with unlimited leverage (UL) and limited leverage (LL) separately. The summary statistics include the number of observations, mean, volatility, minimum, maximum, maximum drawdown (MDD), and the Sharpe ratio (SR). We also report the risk-adjusted alpha of the unconditional optimal portfolio with respect to the market index and the difference in the Sharpe ratio between the unconditional optimal portfolio and the market index. Asterisks denote statistical significance based on Newey-West standard errors (for alpha) or HAC standard errors (for the Sharpe ratio test) at the 1% (***), 5% (**) and 10% (*) levels.

Panel A: Annualized Target Expected Return 6%

	Full Sample		Aug1936–Dec1960		Jan1961–Dec1980		Jan1981–Dec2000		Jan2001–Dec2017	
	UL	LL	UL	LL	UL	LL	UL	LL	UL	LL
#Obs	977	977	293	293	240	240	240	240	204	204
Mean	0.0030	0.0030	0.0055	0.0054	0.0012	0.0012	0.0026	0.0026	0.0019	0.0019
Vol	0.0272	0.0254	0.0453	0.0416	0.0143	0.0143	0.0148	0.0148	0.0092	0.0092
Min	-0.3295	-0.3295	-0.3295	-0.3295	-0.0670	-0.0670	-0.0477	-0.0477	-0.0233	-0.0233
Max	0.2120	0.1415	0.2120	0.1415	0.0556	0.0556	0.0558	0.0558	0.0482	0.0482
MDD	0.6679	0.6098	0.6679	0.6098	0.1255	0.1255	0.0443	0.0443	0.0516	0.0516
Alpha	0.0008	0.0008	-0.0004	-0.0003	0.0007	0.0007	0.0008	0.0008	0.0013**	0.0013**
SR	0.3808	0.4064	0.4167	0.4504	0.2991	0.2975	0.6173	0.6173	0.7253	0.7253
ΔSR	-0.1188	-0.0931	-0.2443	-0.2106	0.0320	0.0304	0.0463	0.0463	0.2873	0.2873

Panel B: Annualized Target Expected Return 10%

	Full Sample		Aug1936–Dec1960		Jan1961–Dec1980		Jan1981–Dec2000		Jan2001–Dec2017	
	UL	LL	UL	LL	UL	LL	UL	LL	UL	LL
#Obs	977	977	293	293	240	240	240	240	204	204
Mean	0.0050	0.0045	0.0091	0.0081	0.0021	0.0017	0.0044	0.0042	0.0032	0.0031
Vol	0.0453	0.0353	0.0756	0.0556	0.0238	0.0229	0.0246	0.0240	0.0154	0.0149
Min	-0.5491	-0.4390	-0.5491	-0.4390	-0.1116	-0.1116	-0.0795	-0.0795	-0.0388	-0.0388
Max	0.3534	0.1483	0.3534	0.1483	0.0926	0.0926	0.0929	0.0803	0.0803	0.0682
MDD	0.8745	0.7298	0.8745	0.7298	0.2153	0.2153	0.0805	0.0805	0.1280	0.1280
Alpha	0.0013	0.0013	-0.0007	-0.0000	0.0012	0.0008	0.0014	0.0012	0.0022**	0.0021**
SR	0.3808	0.4428	0.4167	0.5021	0.2991	0.2604	0.6173	0.6014	0.7253	0.7228
ΔSR	-0.1188	-0.0568	-0.2443	-0.1588	0.0320	-0.0066	0.0463	0.0304	0.2873	0.2848