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Fluctuations in level spectra—role of range

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Abstract. Matrix calculations are carried out for different ranges of the Yukawa interaction for eight particles distributed in the $N=2$ harmonic oscillator shell. The eigenvalue spectra, corresponding to an exact symmetry subspace of dimensionality 287, are analysed for fluctuation properties such as near-neighbour spacing distributions and short- and long-range correlation measures, Q and Δ_3 respectively. It is found that both short- and long-range ordering decrease as the range of the interaction is increased or decreased from the nuclear range.

1. Introduction

In an earlier paper (Venkataraman and Parikh 1980) we have discussed the effect of range on the level density and concluded that it departs more and more from a Gaussian with increasing range of interaction. This prompts one to inquire if the fluctuation properties also change with the range of the interaction. In a sequence of N levels, the fluctuation properties for a set of 'n' levels ($n<N$) are related to the local correlation function of these 'n' levels. For studying short-range correlations, one resorts to near-neighbour spacing distributions (Porter 1965, Mehta 1967) and the short-range statistic Q (Dyson and Mehta 1963). And for measuring the long-range correlation one evaluates the Δ_3 statistic (Dyson and Mehta 1963), which gives a measure of the average deviation of the staircase graph (cumulative level density) from a straight line. Any value of Δ_3 less than 1.0 would reflect what Dyson calls the 'crystalline structure of the eigenvalue lattice'. That is, given an energy span and the mean spacing one could predict the number of levels in that energy span exactly. Analysis of neutron resonance data (Liou *et al* 1973) in nuclei reveals that this crystalline structure is observed in some of the pure and complete sequences. There is an emphasis on the adjective 'pure' as these statistics, Δ_3 and Q, are found to be extremely sensitive to the presence of levels that belong to different symmetries. These statistics are also sensitive to the omission of levels of the same symmetry.

The shell model calculations (Wong and French 1972, Bohigas and Giannoni 1975) reveal that the fluctuation properties of two-body random ensembles (which correspond to two-body random interactions) are identical with the predictions of the random matrix theory. Hence it was concluded that fluctuation properties do not give us any physical information other than about hidden symmetries. We shall investigate the effect of the range of the interaction on fluctuation properties.

Section 2 deals with the short-range correlation measure Q and near-neighbour spacing distributions while § 3 is devoted to the study of the Δ_3 statistic and its variation with range. Concluding remarks are contained in § 4.

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2. Short-range correlations with range

With the three single-particle states in the $N = 2$ oscillator shell taken to be degenerate, the Hamiltonian consists of only the two-body interaction which has the Yukawa radial dependence

$$H = \sum_{i<j} - V_0 \exp(-r_{ij}/a)(r_{ij}/a)^{-1} \tag{1}$$

where $V_0 > 0$ is the strength of the interaction and $r_{ij} = |r_i - r_j|$. The range of the interaction 'a' is expressed in units of the oscillator size parameter. The matrices corresponding to eight nucleons and the total angular momentum $J = 0$ and the total isospin $T = 2$ are generated and diagonalised for the three different values of the ranges 0.04, 0.6 and 2.0. The level density for these three different ranges have been discussed in an earlier paper (Venkataraman and Parikh 1980).

For the purpose of analysis of fluctuations we choose 247 levels from the centre of these matrices (of dimensionality 287) leaving 20 on either side to eliminate end effects.

The first three near-neighbour spacing distributions for the three different ranges are given in figures 1–3. The spacing in each case is expressed in units of the nth-order local mean spacing ($n = 1$–3). The full curve in figure 1 is the Wigner (1967) surmise for the nearest-neighbour spacing distribution, given by

$$P(x) = \frac{\pi x}{2D^2} \exp(-\pi x^2/4D^2) \tag{2}$$

where D is the local mean spacing. Deviations from the Wigner surmise, even for the nuclear range (figure 1(b)), is attributed to inadequate statistics. The continuous curves in figure 2 are the theoretical distributions given by Kahn (1963) and those in figure 3 are the Gaussian approximation. With an increase in range, there is no significant change in the near-neighbour spacing distribution.

In table 1 we give the values of the short-range statistic Q for a uniform spectrum, the ensemble-averaged value for random sequences and for the shell model spectra corresponding to three different ranges of the interaction. The theoretical estimates of the mean and the RMS deviation of Q for 'n' levels calculated from

$$n(0.365 + \tfrac{1}{3}\pi^{-2}) \pm [n(-0.265 + \tfrac{1}{3}\pi^{-2})]^{1/2} \tag{3}$$

are also given in table 1. In all the cases, except the uniform one, the spectrum is unfolded i.e. each spacing is expressed in units of the local mean spacing which is evaluated by taking a couple of spacings on either side. For the uniform spectrum Q is much smaller than the theoretical estimate, whereas that of the random sequence is much higher. For the spectrum of the hydrogen atom the Q value is small again. In the case of shell model spectra, Q is smallest for the nuclear range and increases away from the nuclear range on either side. Bearing in mind the decoupling between level density and fluctuations for a small number 'n' ($n \ll N$, the matrix dimensionality) of levels (Dyson 1972) we would expect all short-range correlation properties to remain invariant with range. In view of this the variation in Q with range is not understood.

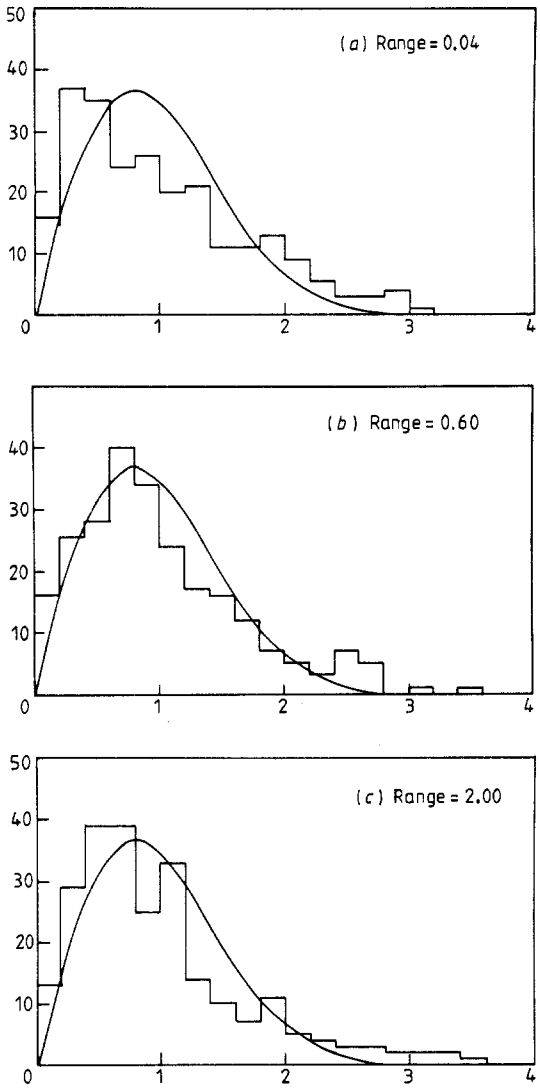

Figure 1. First near-neighbour spacing distributions for the range indicated. Full curves; Wigner surmise (1967).

3. Δ_3 statistic

The Δ_3 statistic gives a measure of the average deviation of the staircase graph (cumulative level density) from a straight line. For a set of 'n' levels running from $-L$ to L

$$\Delta_3 = \min_{A,B} \frac{1}{2L} \int_{-L}^{L} (N(E) - AE - B)^2 \, dE \tag{4}$$

where $N(E)$ is the cumulative level density. In terms of the eigenvalues E_i, Δ_3 is

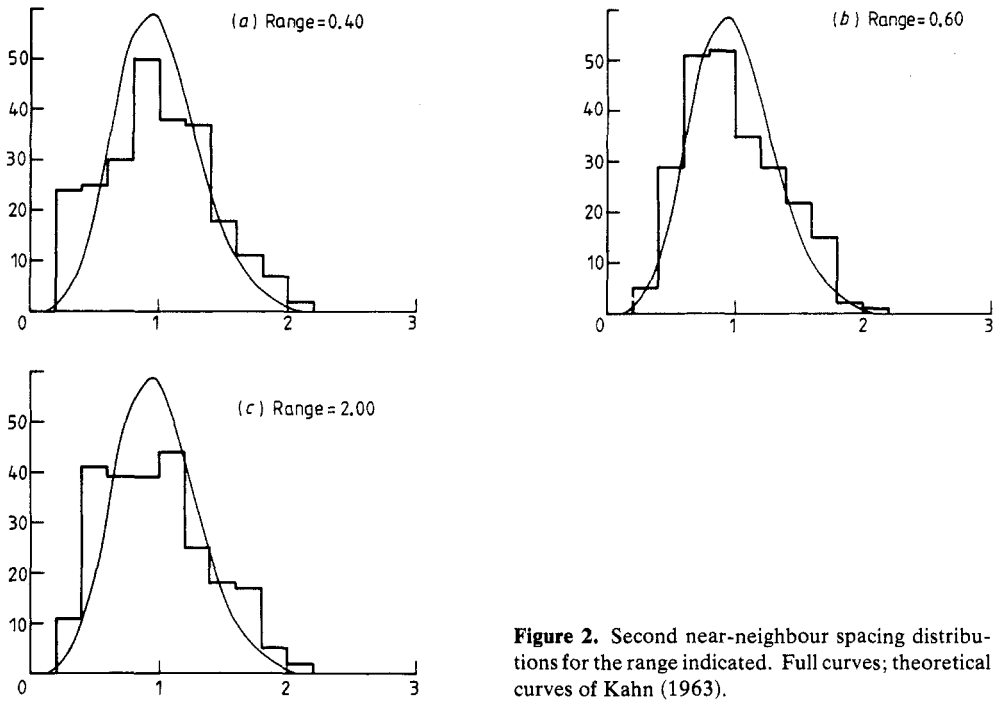

Figure 2. Second near-neighbour spacing distributions for the range indicated. Full curves; theoretical curves of Kahn (1963).

given by (Bohigas and Giannoni 1975)

$$\Delta_3 = \tfrac{1}{16}n^2 - \frac{1}{4L^2}\left(\sum_{i=1}^{n} E_i\right)^2 + \frac{3n}{8L^2}\sum_{i=1}^{n} E_i^2 - \frac{3}{16L^4}\left(\sum_{i=1}^{n} E_i^2\right)^2 + \frac{1}{2L}\sum_{i=1}^{n}(n-2i+1)E_i. \tag{5}$$

The theoretical estimates for the average value of Δ_3 and its RMS deviation are given by (Dyson and Mehta 1963)

$$\Delta_3 = \pi^{-2}(\ln n - 0.68) \pm 0.12 \tag{6}$$

which shows that for any value of 'n' below 20 000, Δ_3 is always less than 1.0. Thus, with the logarithmic dependence of Δ_3 on 'n', the deviation of the staircase graph from a straight line is on the average less than one spacing unit. Dyson calls this appropriately the crystalline structure of the eigenvalue lattice.

It should be borne in mind that the theoretical estimates of equation (6) are for a semicircular eigenvalue density, the central region of which does not have any secular variations. Hence, when one wants to include the levels at the end regions in the semicircular eigenvalue density, the secular variation in the eigenvalue density has to be removed. In general, when one wants to calculate the long-range fluctuation measure Δ_3 for any spectra, the secular variation in the eigenvalue density should be distinguished from fluctuations. For this purpose, the spectra are unfolded—each spacing is expressed in units of the local mean spacing.

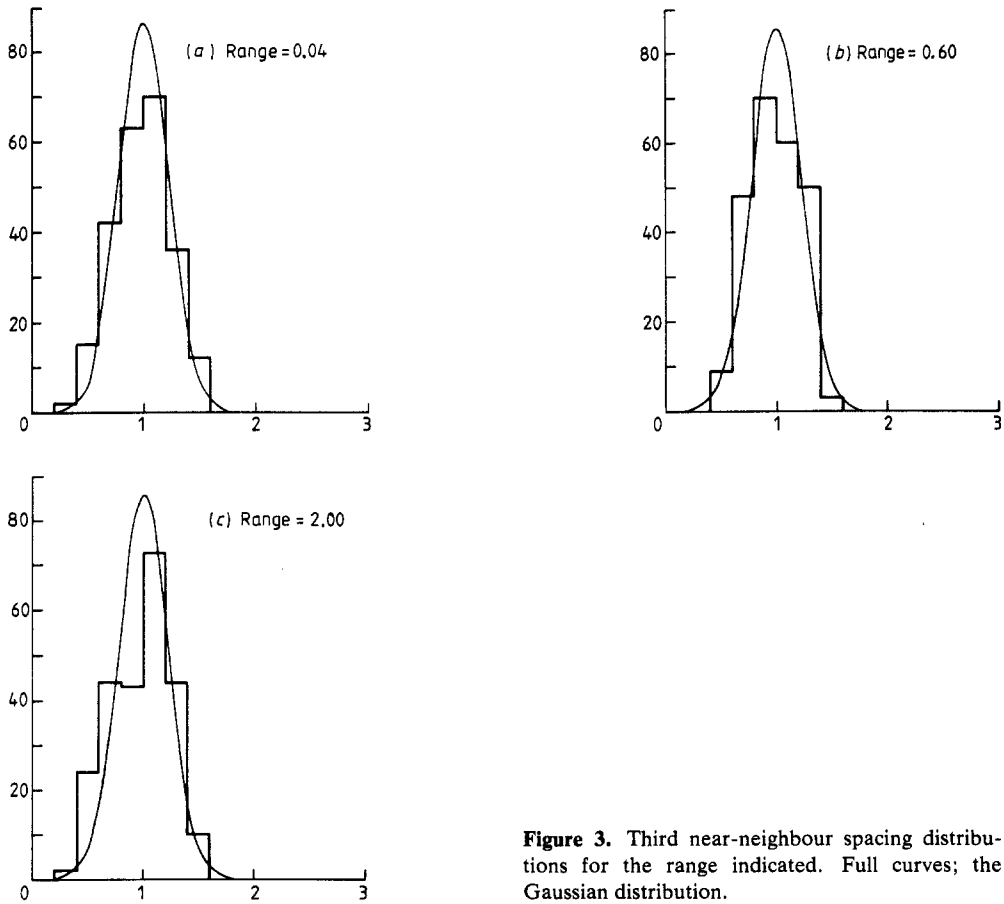

Figure 3. Third near-neighbour spacing distributions for the range indicated. Full curves; the Gaussian distribution.

Table 1. The values of Q for the cases indicated.

	Q
Shell model matrix $a = 0.04$	123.90
Shell model matrix $a = 0.60$	100.31
Shell model matrix $a = 2.00$	120.40
Hydrogen atom	1.54
Uniform spectrum	5.80
Random numbers	195.44 (20.00)
GOE (Theoretical)	82.50 (8.4)

If the number of levels is large, the local mean spacing D_i, for each spacing s_i, is found by taking an equal number of spacings, on either side. If D_i is the average of 'k' spacings, then

$$D_i = \frac{1}{k} \sum_{j=-(k-1)/2}^{(k-1)/2} s_{(i+j)} \tag{7}$$

$$x_i = s_i / D_i. \tag{8}$$

In this procedure, one has to eliminate $(k-1)/2$ spacings at either end while calculating the D_i for the end spacings. If on the other hand, the number of levels is small, one divides the spectrum into non-overlapping regions, equal or unequal, and calculates the mean spacing D for each of these regions. Every spacing s_i^l in the lth region is written as

$$x_i = s_i^l / D_l. \tag{9}$$

This is tantamount to fitting the staircase graph by more than one straight line and the calculated Δ_3 will then be the sum of the deviations of the different straight lines from the corresponding portions of the staircase. Evidently by fitting the staircase graph by many straight lines (or equivalently by taking a fine local average) one could always end up with a value of Δ_3 less than 1.0, which would imply long-range order. Hence, to retain the fluctuations in the process of unfolding, one should use an optimum number of spacings to include the largest possible number of levels. There is no unique prescription for such optimum unfolding. For every spectrum one has to find that out by trial and error methods.

Let the values of Δ_3 be calculated from equations (7), (8) and (5) for different values of k in equation (7). The optimum stage of unfolding is then found on the basis of two criteria. Firstly one looks for a minimum difference in the values of Δ_3 for two consecutive values of 'k'. Then any value of 'k' in between would be optimum provided that for that value of 'k', at least in random sequences, long-range order is violated for the same number of levels. At this juncture it must be mentioned that 'n' should be much greater than fifteen, as the Δ_3 value for the random sequence is given by (Dyson and Mehta 1963)

$$\langle \Delta_3 \rangle = n/15. \tag{10}$$

Table 2 gives the values of Δ_3 for NE levels when 'k' spacings are used to calculate the local mean spacing for each spacing. For $k = 5$, even random sequences give rise to long-range order. $k = 5$ is hence inadmissible. Using the two criteria mentioned above we have underlined the optimum values of Δ_3 for different cases. The spectrum of the hydrogen atom which exhibits short-range ordering does not exhibit long-range order. In the case of shell model spectra, even for the nuclear range, corresponding to 212 levels, long-range order is violated. Comparing the values of Δ_3 for a fixed 'k' (fixed NE) we find that long-range ordering decreases away from the nuclear range on either side.

Table 3 gives the values of Δ_3 for 50 levels from the centre of the shell model spectra. The values within brackets correspond to the pure spectra when no unfolding is done. For the nuclear range, even without unfolding, one finds long-range order. The values of Δ_3 for optimum 'k' are underlined for all the ranges. Comparison of Δ_3 for $k = 17$, again shows that long-range ordering decreases away from the nuclear range on either side.

Since the value of Δ_3, for 212 levels corresponding to the nuclear range, is greater than 1.0, the extension of the long-range order or the crystalline structure to the ground-state region of nuclei is not valid. It should be added that such a feature arises in the case of the Gaussian eigenvalue density of a random matrix ensemble (Venkataraman 1981) as well. This result is in contradiction with the earlier investigations (Wong and French 1972). In their analysis, unfolding has been done using a four-moment function when the level density itself is well described by a four-moment

Table 2. The values of Δ_3 for *NE* levels of different spectra. The theoretical estimate for 242 levels is 0.55 ± 0.12.

k	*NE*	0.04	0.60	2.00	Hydrogen atom	Random sequence
5	242	0.76	0.37	0.43	0.14	0.94 (0.27)
11	236	1.10	0.62	0.74	0.43	1.33 (0.34)
17	230	1.60	0.94	1.32	1.12	1.76 (0.34)
23	224	2.30	1.27	1.78	2.04	2.20 (0.46)
29	218	3.01	1.64	2.57	3.15	2.70 (0.55)
35	212	4.02	1.82	3.53	4.42	3.17 (0.73)
41	206	5.11	2.01	4.20	5.77	3.70 (0.89)
47	200	5.95	2.33	4.89	7.15	4.17 (1.04)
53	194	6.68	2.92	5.31	8.51	4.62 (1.19)

Table 3. The values of Δ_3 for *NE* levels for the shell model spectra for the specified ranges of interaction. The theoretical estimate for 50 levels is 0.39 ± 0.12.

k	*NE*	$a = 0.04$	$a = 0.60$	$a = 2.00$
5	50	0.33 (2.11)	0.31 (0.93)	0.44 (2.92)
11	44	0.92	0.52	0.69
17	38	1.25	0.53	1.22
23	32	1.85	0.55	1.60

function (Chang and Zuker 1972). Thus even at the unfolding stage, the fluctuations are eliminated.

4. Conclusion

The analysis of fluctuations for different ranges of interaction reveals that both short- and long-range ordering decrease away from the nuclear range on either side. The variation of short-range ordering with range is not understood for reasons mentioned in § 2. It is not obvious as to why corresponding to the nuclear range both short- and long-range ordering are maximal. The effect of range for a general two-body random interaction (with fixed range) can be studied only through ensemble calculations. However, as we see for the Yukawa interaction for the nuclear range, long-range order is indeed observed in the centre of the spectrum. But, for a larger number 'n' ($n \sim N$, the matrix dimensionality) of levels, long-range order is violated. Thus,

we find that the extension of the long-range order to the ground-state region of nuclei is not valid. Just as level density depicts a variation with range, Δ_3 does as well. From this we would expect that the r-body random ensembles (RBRE) which give rise to different level densities for different values of 'r' would also give rise to different values of the long-range statistic Δ_3 for $n \sim N$, when the unfolding is done in an optimum way. We conclude therefore that in addition to information about hidden symmetries one should also be able to learn about the rank and range of the interaction from long-range fluctuation measures. Owing to the non-availability of complete and pure sequences, we have not been able to investigate the long-range ordering and hence the validity of the random matrix theory in atomic level spectra.

References

Bohigas O and Giannoni M J 1975 *Ann. Phys., NY* **89** 393–422
Chang F S and Zuker A 1972 *Nucl. Phys.* A **198** 417–29
Dyson F J 1972 *J. Math. Phys.* **13** 90–7
Dyson F J and Mehta M L 1963 *J. Math. Phys.* **4** 701–12
Kahn P B 1963 *Nucl. Phys.* **41** 159–66
Liou H I, Camarda H S, Wynchank S, Slagowitz M, Hacken G, Rahn F and Rainwater J 1973 *Phys. Rev.* C **5** 974–1001
Mehta M L 1967 *Random Matrices* (New York: Academic)
Porter C E 1965 *Statistical Theories of Spectra: Fluctuations* (New York: Academic)
Venkataraman R 1981 *PhD Thesis* University of Gujarat, India, submitted
Venkataraman R and Parikh J C 1980 *J. Phys. B: At. Mol. Phys.* **13** 1295–305
Wigner E P 1967 *SIAM Rev.* **9** 1–27
Wong S S M and French J B 1972 *Nucl. Phys.* A **198** 188–208