Linear Algebra Programming Project

2022-12-17

■ There are two iterative methods to estimate an eigenvalue. One is the power method for estimating a strictly dominant eigenvalue. The other is the inverse power method for estimating an eigenvalue λ with roughly estimated eigenvalues. The iterative process of these two methods are described as below.

THE POWER METHOD FOR ESTIMATING A STRICTLY DOMINANT EIGENVALUE

1. Select an initial vector $\mathbf{x}_0$ whose largest entry is 1.
2. For $k = 0, 1, \ldots,$
   a. Compute $A\mathbf{x}_k$.
   b. Let μ_k be an entry in $A\mathbf{x}_k$ whose absolute value is as large as possible.
   c. Compute $\mathbf{x}_{k+1} = (1/\mu_k)A\mathbf{x}_k$.
3. For almost all choices of $\mathbf{x}_0$, the sequence $\{\mu_k\}$ approaches the dominant eigenvalue, and the sequence $\{\mathbf{x}_k\}$ approaches a corresponding eigenvector.

THE INVERSE POWER METHOD FOR ESTIMATING AN EIGENVALUE λ OF A

1. Select an initial estimate α sufficiently close to λ.
2. Select an initial vector $\mathbf{x}_0$ whose largest entry is 1.
3. For $k = 0, 1, \ldots,$
   a. Solve $(A - \alpha I)\mathbf{y}_k = \mathbf{x}_k$ for $\mathbf{y}_k$.
   b. Let μ_k be an entry in $\mathbf{y}_k$ whose absolute value is as large as possible.
   c. Compute $\nu_k = \alpha + (1/\mu_k)$.
   d. Compute $\mathbf{x}_{k+1} = (1/\mu_k)\mathbf{y}_k$.
4. For almost all choices of $\mathbf{x}_0$, the sequence $\{\nu_k\}$ approaches the eigenvalue λ of A, and the sequence $\{\mathbf{x}_k\}$ approaches a corresponding eigenvector.

1. (15pt) Estimate a strictly dominant eigenvalue of a matrix A with initial vector $\mathbf{x}_0$ described as following.

$$A = \begin{bmatrix} 6 & 5 \\ 1 & 2 \end{bmatrix}, \mathbf{x}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Q1.1. What kind of method do you choose? Explain the reason of your selection.

The power method, 이 방식이 a strictly dominant eigenvalue 추정에 더 효율적이기 때문이다.

Q1.2. Write a code to estimate a strictly dominant eigenvalue of A with initial vector $\mathbf{x}_0$.

제출 파일 hw1.py 참고

Q1.3. Fill in the blanks in the table below. **소수점 6자리까지 표현(7짜리에서 반올림함)**

Iteration	1	2	3	4	5
$\mathbf{x}_k$	$\begin{bmatrix} 1 \\ 0.4 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0.225 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0.0.203509 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0.2005 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0.200071 \end{bmatrix}$
$A\mathbf{x}_k$	$\begin{bmatrix} 8 \\ 1.8 \end{bmatrix}$	$\begin{bmatrix} 7.125 \\ 1.45 \end{bmatrix}$	$\begin{bmatrix} 7.017544 \\ 1.407018 \end{bmatrix}$	$\begin{bmatrix} 7.0025 \\ 1.401 \end{bmatrix}$	$\begin{bmatrix} 7.000357 \\ 1.400143 \end{bmatrix}$
μ_k	8	7.125	7.017544	7.0025	7.000357

2. (15pt) Estimating an eigenvalue λ, which are the two smallest eigenvalues of A with initial vector $\mathbf{x}_0$ described as following.

$$A = \begin{bmatrix} 10 & -8 & -4 \\ -8 & 13 & 4 \\ -4 & 5 & 4 \end{bmatrix}, \mathbf{x}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ with roughly estimated eigenvalues 21, 3.3, and 1.9

Q2.1. What kind of method do you choose? Explain the reason of your selection.

The inverse power method, 이 방식이 임의의 eigenvalue 추정에 더 효율적이기 때문이다.

Q2.2. Write a code to estimate the two smallest eigenvalues of A with initial vector $\mathbf{x}_0$.

제출 파일 hw2.py참고

Q2.3. Draw two tables to estimate the two smallest eigenvalues of A with initial vector $\mathbf{x}_0$.

소수점 6자리까지 표현(7짜리에서 반올림함) α=1.9

Iteration	0	1	2	3	4
$\mathbf{x}_k$	$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0.57359 \\ 0.064649 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0.505364 \\ 0.004453 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0.500378 \\ 0.000313 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0.500027 \\ 0.000022 \\ 1 \end{bmatrix}$
$\mathbf{y}_k$	$\begin{bmatrix} 4.450374 \\ 0.501601 \\ 7.758805 \end{bmatrix}$	$\begin{bmatrix} 5.013058 \\ 0.044172 \\ 9.9197 \end{bmatrix}$	$\begin{bmatrix} 5.001247 \\ 0.003126 \\ 9.994931 \end{bmatrix}$	$\begin{bmatrix} 5.000089 \\ 0.00022 \\ 9.999646 \end{bmatrix}$	$\begin{bmatrix} 5.000006 \\ 0.000015 \\ 9.999975 \end{bmatrix}$
μ_k	7.758805	9.9197	9.994931	9.999646	9.999975
v_k	2.028886	2.00081	2.000051	2.000004	2

소수점 6자리까지 표현(7짜리에서 반올림함) α=3.3

Iteration	0	1	2	3	4
$\mathbf{x}_k$	$\begin{bmatrix}1\\1\\1\end{bmatrix}$	$\begin{bmatrix}1\\0.793369\\0.082172\end{bmatrix}$	$\begin{bmatrix}1\\0.786921\\0.095893\end{bmatrix}$	$\begin{bmatrix}1\\0.787014\\0.095667\end{bmatrix}$	$\begin{bmatrix}1\\0.787012\\0.095671\end{bmatrix}$
$\mathbf{y}_k$	$\begin{bmatrix}41.045365\\32.564103\\3.372781\end{bmatrix}$	$\begin{bmatrix}47.486662\\37.368252\\4.553654\end{bmatrix}$	$\begin{bmatrix}47.119503\\37.083691\\4.507785\end{bmatrix}$	$\begin{bmatrix}47.125168\\37.088078\\4.508501\end{bmatrix}$	$\begin{bmatrix}47.125075\\37.088006\\4.508489\end{bmatrix}$
μ_k	41.045365	47.486662	47.119503	47.125168	47.125075
v_k	3.324363	3.321059	3.321223	3.32122	3.32122