AS5545 - Dynamics and Control of Spacecraft

Momentum Exchange Devices
Active Methods

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Reaction Thrusters Vs Momentum Exchange Devices

- **Attitude Deviations about each axis can be controlled using active devices such as (inertial angular momentum change (first 3) vs momentum exchange devices)**
  - **Reaction Thrusters**
  - **Magnetic Torque Coils**
  - **Natural Torques such as aerodynamic, solar radiation pressure etc**
  - **Momentum exchange devices (Momentum Wheels, Reaction Wheels, Control Moment Gyros etc)**
- **Reaction Thrusters – not linear – abrupt impulsive motions - once starts constant thrust unbounded– control requires sophisticated laws – PWPF Modulated etc**
- **Natural torques require flaps – more cumbersome – also torque level is much small and averaged torque level will be achieved after longer accumulation time**

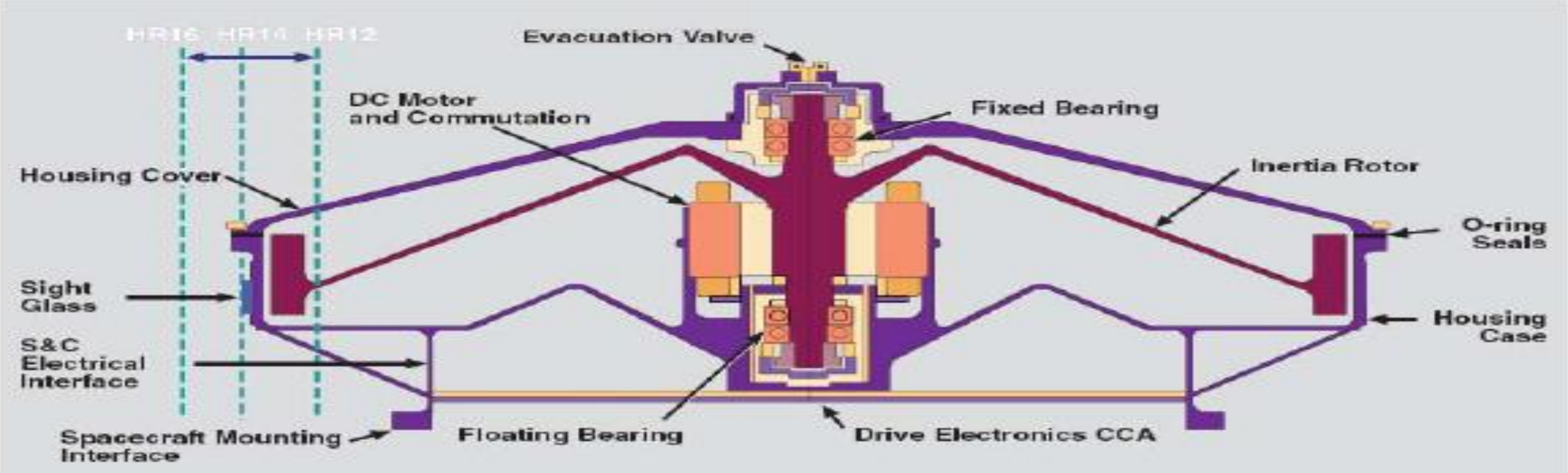
Reaction Thrusters Vs Momentum Exchange Devices

- **Natural torques through magnetic torque coils - require coils – current to be passed on to coils suitably to appropriately achieve torque directions – torque level is small and averaged torque level will be achieved after longer accumulation time – frequent change of directions required to effectively utilize this source – only earth based missions can utilize this**
- **In spite of difficulties, magnetic control is almost and always a very useful means and part of any earth based s/c control system**
- **Momentum exchange devices are angular momentum conserving devices of s/c; they are smooth and can be employed for accurate attitude control, moderately large angle attitude maneuvers etc**

Reaction Wheels

Spinning flywheels mounted on a central bearing whose rate of rotation can be adjusted by an electric motor.

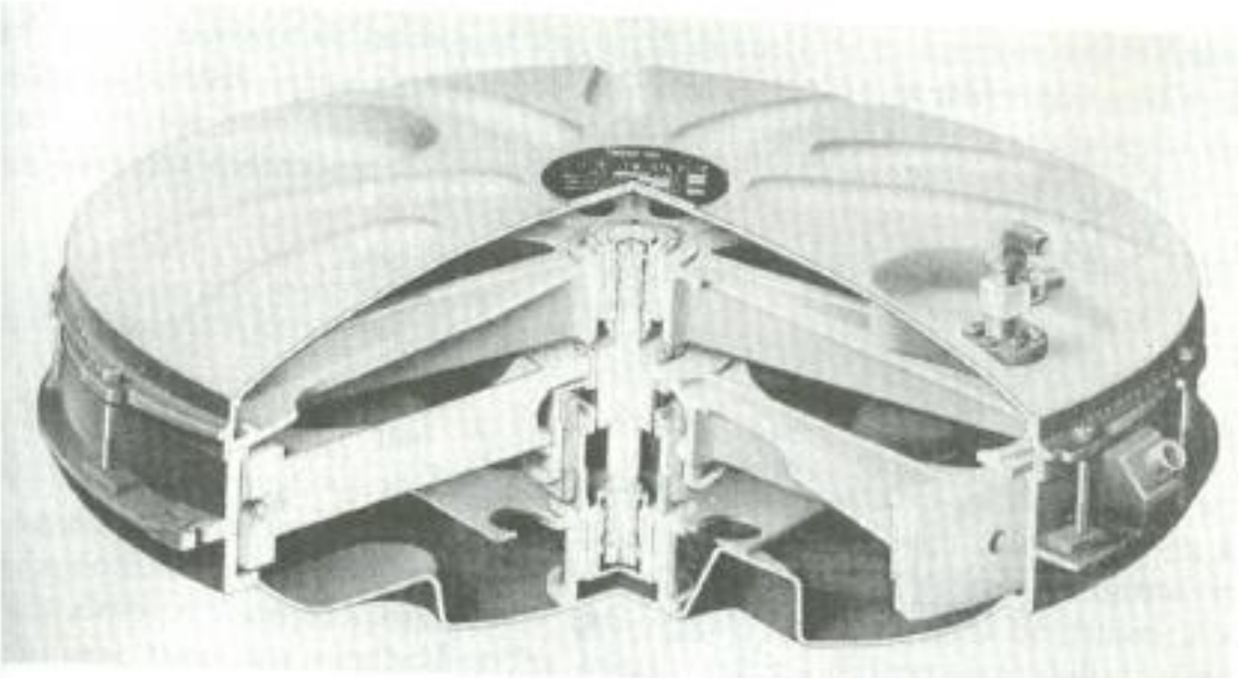
Exchange momentum with the spacecraft by changing wheel speed but **no influence on the total angular momentum**.



Mechanisms for Control

Reaction wheels

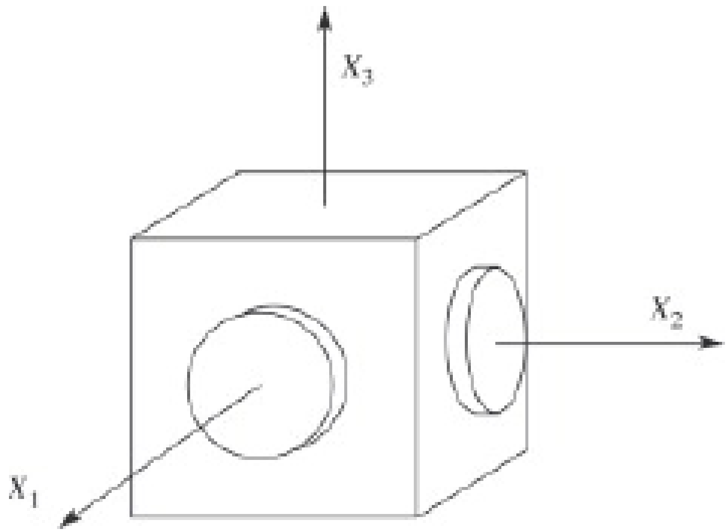
Because thrusters are not extremely accurate, they are rarely used to control the finer motions of attitude. Maintenance maneuvers are usually left to other mechanisms.



Reaction wheels: A momentum wheel rotates about a given axis aligned with the body-axis of the spacecraft.

Mechanisms for Control

Reaction wheels



Dynamics: Consider rotation about the x-axis.

- Let J_x be the moment of inertia of the Spacecraft.
- Let I_x be the moment of inertia of the flywheel.
- By conservation of angular momentum:

$$I_x(\omega_f + \omega_s) + J_x\omega_s = 0$$

  - ω_s is the angular velocity of the the craft in inertial space.
  - ω_f is the angular velocity of flywheel w/r to the craft.

Reaction Wheels: Usefulness

1. **Resist disturbing torques**: external torques give rise to unwanted angular momentum. The control system applies control torques to the reaction wheels to leave the spacecraft angular momentum unchanged.

   Example: when a clockwise disturbance torque is imposed on the spacecraft, the attitude control system holds attitude constant by rotating a reaction wheel counterclockwise.

Reaction Wheels: Usefulness

2. **Slewing maneuvers**: rotate the spacecraft by very small amounts (e.g., keep a telescope pointed at a star) by slowing down or accelerating the wheel.

   Example: from cruise attitude to target and return to cruise attitude: momentum is borrowed from the wheels and then returned to the wheel. There is no net change of momentum of the wheel in this case.

Momentum Dumping

When disturbing torques do not average out over one orbit, constant wheel speed increase is necessary to hold the spacecraft.

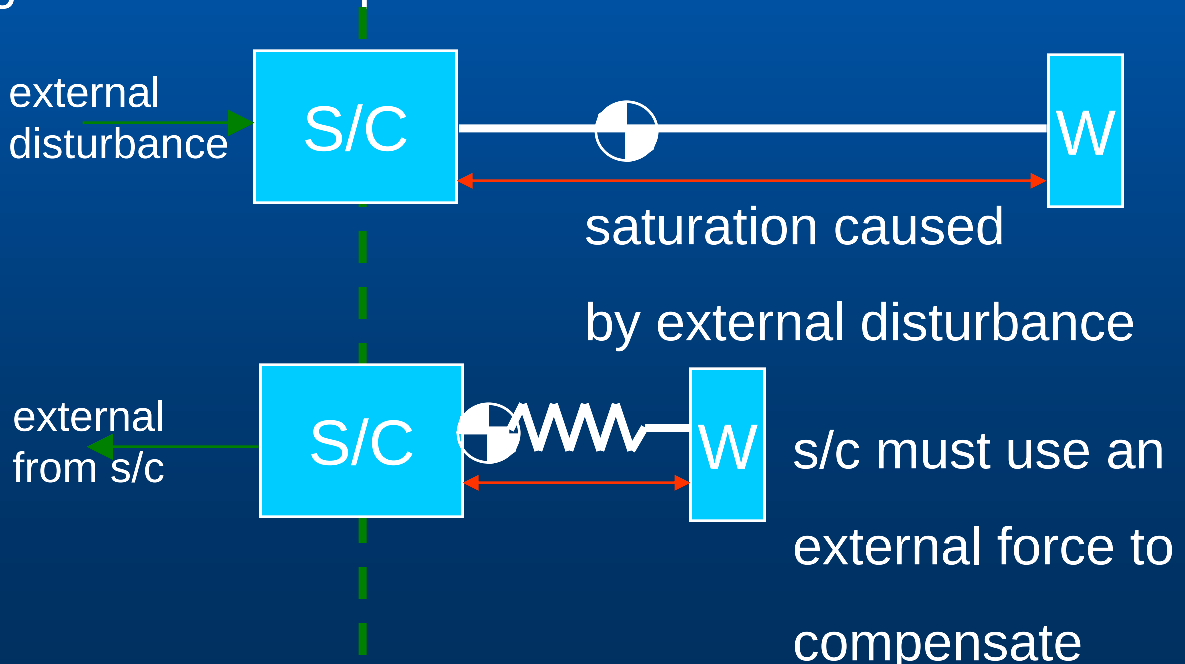
There is a risk to “saturate” the wheel; the wheel is spinning too fast and cannot counterbalance further disturbing torques.

The stored momentum needs to be cancelled; this process is called momentum dumping.

Thrusters or magneto-torquers are used to hold the spacecraft stationary while reducing wheel speed.

Momentum Dumping

- All external disturbances *change* the total momentum, which causes the wheels to spin up to saturation
- Therefore, all wheels must dump this extra momentum periodically, usually using an inertial torque
  - –torque coils or rods
  - –thrusters



Reaction Wheels: Some Remarks

Wheels are not operated near 0 rpm, because of nonlinear wheel friction.

The rotational axis of a wheel is usually aligned with a vehicle control axis; the vehicle must carry one wheel per axis for full attitude control.

Redundancy is usually desired, requiring four or more wheels, in a position oblique to all axes.

Momentum Biased S/C (MBS)

- Momentum biased s/c is 3-axis stabilized with a wheel providing gyroscopic stiffness about orbit normal
- MBS is a dual spin system but do not have two parts; the wheel is kept within the s/c and produces the momentum
- The torque capabilities of the wheel is used to control the attitude about orbit normal direction
- Most communication satellites are MBS

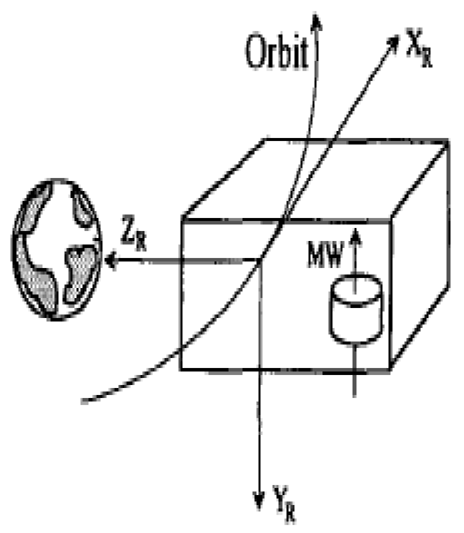


Figure 8.2.1 Definition of the reference frame and the direction of the MW axis

Quarter Orbit Roll-Yaw Coupling

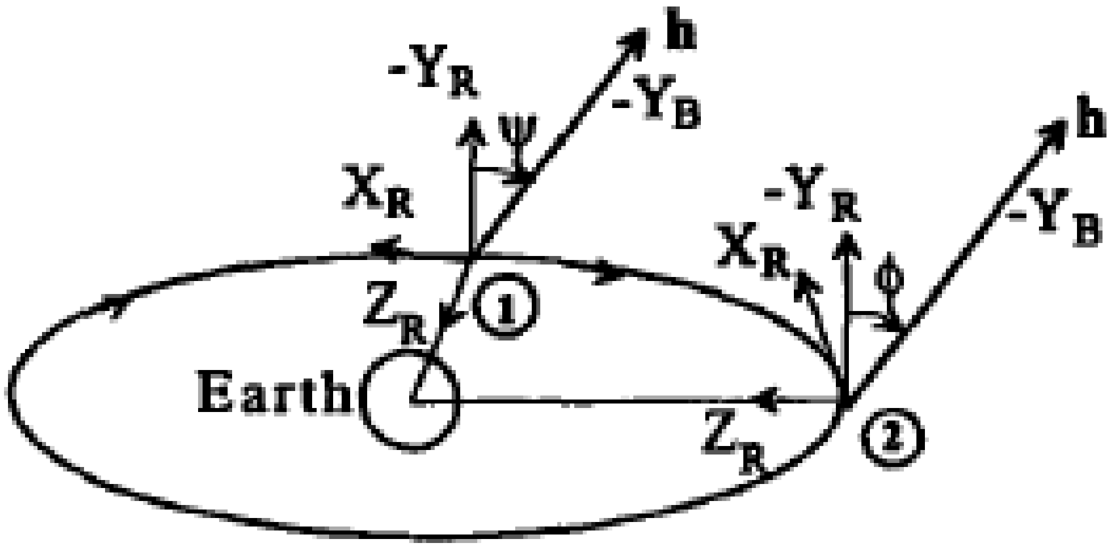


Figure 8.3.1 Change of the yaw error to roll error in a momentum-biased spacecraft.

Quarter Orbit Roll-Yaw Coupling due to Momentum Bias (No need of yaw sensor measurement)

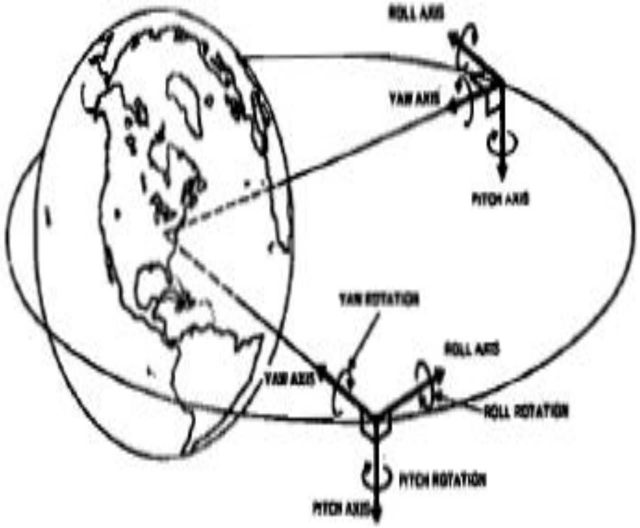


Fig. 2-4. Roll, Pitch, and Yaw (RPY) Coordinates

3 Nonspacecraft-Centered Coordinate Systems

Interchange of Yaw and Roll Attitude Components for a Momentum Wheel With Angular Momentum, h, Fixed in Inertial Space. The yaw error when the spacecraft is at A becomes a roll error when the spacecraft moves to B. (Compare with Fig. 2-4.)

Due to inertial angular momentum about pitch, the yaw error at one instant will be seen as roll error after a quarter orbit motion of the spacecraft in orbit. This is called quarter orbit roll-yaw coupling

Equation of Motion of MBS - continued

- Consider a s/c with a wheel aligned about pitch axis
- Total angular momentum = $\{J_x\omega_x, J_y\omega_y + J_w\omega_w, J_z\omega_z\}$
- The Euler's equation is

$J_c\dot{\bar{\omega}} + \bar{\omega} \times J_c\bar{\omega} = \bar{T}_d + \bar{T}_c$ where $\bar{T}_d, \bar{T}_c$ are disturbance torques and active control (thruster, RW) torques

- For Earth pointing satellite

$$\bar{\omega} = \begin{Bmatrix} \dot{\varphi} - \omega_0\psi \\ \dot{\theta} - \omega_0 \\ \dot{\psi} + \omega_0\varphi \end{Bmatrix}$$

- Note: In MBS the Momentum Wheel has the capacity to control pitch attitude by excursion about a bias speed by pre-defined rpm; that is $J_w\omega_w$ is not constant; $\dot{\omega}_w$ provides control torque for pitch control

Equation of Motion of MBS

- Accounting for Gravity Gradient Torque for disturbance torque we can rewrite the equation of motion as $(with\ h_s = J_w \omega_w)$

$$\begin{Bmatrix} T_{dx} \\ T_{dy} \\ T_{dz} \end{Bmatrix} + 3{\omega_0}^2 \begin{Bmatrix} (J_{cz} - J_{cy})\varphi \\ (J_{cz} - J_{cx})\theta \\ 0 \end{Bmatrix} + \begin{Bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{Bmatrix} =$$

$$\begin{Bmatrix} \boldsymbol{J_{cx}\ddot{\varphi} - (J_{cx} - J_{cy} + J_{cz})\omega_0}\dot{\psi} + \boldsymbol{{\omega_0}^2(J_{cy} - J_{cz})\varphi} - h_s(\omega_0\varphi + \dot{\psi}) \\ \boldsymbol{J_{cy}\ddot{\theta}} + \dot{h}_s \\ \boldsymbol{J_{cz}}\ddot{\psi} + \boldsymbol{(J_{cx} - J_{cy} + J_{cz})\omega_0\dot{\varphi} + {\omega_0}^2(J_{cy} - J_{cx})\psi + h_s}(\dot{\varphi} - \omega_0\psi) \end{Bmatrix}$$

Equation of Motion of MBS

- Rearranging

$$\begin{Bmatrix} T_{dx} \\ T_{dy} \\ T_{dz} \end{Bmatrix} + \begin{Bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{Bmatrix} =$$

$$\begin{Bmatrix} \boldsymbol{J_{cx}\ddot{\varphi} - (J_{cx} - J_{cy} + J_{cz})\omega_0}\dot{\psi} + \boldsymbol{4\omega_0{}^2(J_{cy} - J_{cz})\varphi} - h_s(\omega_0\varphi + \dot{\psi}) \\ \boldsymbol{J_{cy}\ddot{\theta}} + \dot{h}_{\boldsymbol{s}} + \boldsymbol{3\omega_0{}^2(J_{cx} - J_{cz})\theta} \\ \boldsymbol{J_{cz}}\ddot{\psi} + \boldsymbol{(J_{cx} - J_{cy} + J_{cz})\omega_0\dot{\varphi} + \omega_0{}^2(J_{cy} - J_{cx})\Psi + h_s}(\dot{\varphi} - \omega_0\psi) \end{Bmatrix}$$

- Defining a=$\boldsymbol{4\omega_0{}^2(J_{cy} - J_{cz})}$, b=$-\boldsymbol{(J_{cx} - J_{cy} + J_{cz})\omega_0}$, c= $\boldsymbol{\omega_0{}^2(J_{cy} - J_{cx})}$ and d= $\boldsymbol{3\omega_0{}^2(J_{cx} - J_{cz})}$ the compact form is

Equation of Motion of MBS

- Rearranging

$$\begin{Bmatrix} T_{dx} \\ T_{dy} \\ T_{dz} \end{Bmatrix} + \begin{Bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{Bmatrix} =$$

$$\begin{Bmatrix} \boldsymbol{J_{cx}\ddot{\varphi} - (-b + h_s)}\dot{\psi} + \boldsymbol{(a - h_s\omega_0)}\varphi \\ \boldsymbol{J_{cy}\ddot{\theta}} + \dot{h}_{\boldsymbol{s}} + \boldsymbol{d\theta} \\ \boldsymbol{J_{cz}}\ddot{\psi} + \boldsymbol{(-b + h_s)\dot{\varphi} + (c - h_s\omega_0)}\psi \end{Bmatrix}$$

- Note: Pitch motion is decoupled and can be handled and controlled separately
- The combined roll-yaw motion stability is to be understood
- In comparison to ‘a’, ‘b’, ‘c’ and ‘d’, the h_s is large and therefore they are negligible

Stability of Roll-Yaw motion

- **The roll-pitch-yaw equations after this assumption are**

$$\begin{Bmatrix} T_{dx} \\ T_{dy} \\ T_{dz} \end{Bmatrix} + \begin{Bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{J_{cx}\ddot{\varphi}} - (\boldsymbol{h_s})\dot{\psi} + (-\boldsymbol{h_s\omega_0})\varphi \\ \boldsymbol{J_{cy}\ddot{\theta}} + \dot{h}_s + \boldsymbol{d\theta} \\ \boldsymbol{J_{cz}}\ddot{\psi} + (\boldsymbol{h_s})\boldsymbol{\dot{\varphi}} + (-\boldsymbol{h_s\omega_0})\psi \end{Bmatrix}$$

- **Considering only roll-yaw dynamics the characteristic equation is {writing in state space form and taking det (A-λI) = 0 }**

$(\lambda^2 + \boldsymbol{\omega_0}^2)(\lambda^2 + \frac{\boldsymbol{h_s}^2}{\boldsymbol{J_{cx}J_{cz}}})$ **= 0** {the roots are purely imaginary and therefore periodic motion is the solution}

- **However, the disturbance torque may also have same orbital frequency $\boldsymbol{\omega_0}$ and hence the resonance may result and unstable motion may occur; Active control is required**

Control of Momentum Biased System using Earth Sensor and Gyroscope

- **Earth sensor measures roll and pitch errors directly**
- **The accuracy of such measurements can be improved to 0.08 deg if we model ad remove the systematic errors due to oblate nature of earth shape and carbon di-oxide radiation non-uniformity of earth**
- **Still noise of the measurement persists; this leads to not very good accuracy in control of the spacecraft**
- **Usually the control accuracy of the s/c with this system is about 0.2 deg in pitch and 0.5 deg about yaw. Pitch will be 0.005 deg since momentum wheel itself is used to control the pitch motion**
- **In order to make the control more smooth and better, momentum wheels themselves are mounted in special configurations such as ‘V’, or ‘L’ or gimballed to provide control over pitch and yaw.**

MBS with Two MW and a Redundant wheel

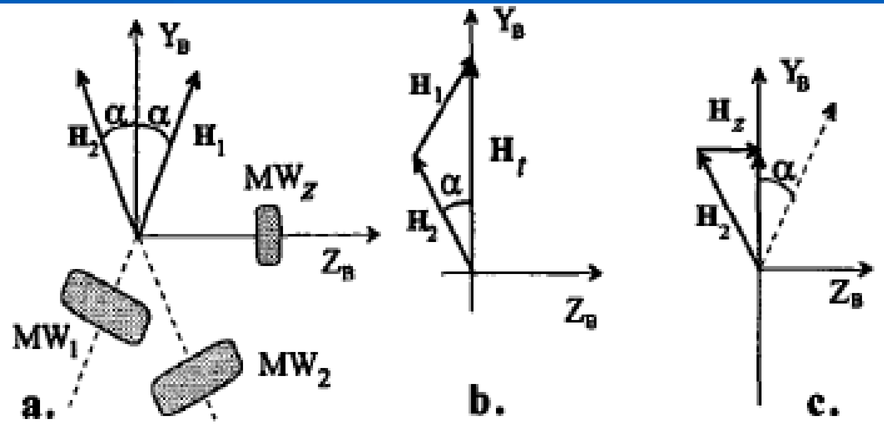


Figure 8.7.1 Two-wheel momentum bias arrangement, with a third wheel for backup.

- Total angular momentum about pitch axis
  $H_t = 2H_1 \cos\alpha$
- $h_y = (H_1 + H_2)\cos\alpha$
- $h_z = (H_1 - H_2)\sin\alpha$
- $T_{cy} = \left(\dot{H}_1 + \dot{H}_2\right)\cos\alpha$
- $T_{cz} = \left(\dot{H}_1 - \dot{H}_2\right) sin\,\alpha$

- The Two main wheels are in 'V' shaped configuration about pitch-yaw plane symmetric about pitch
- Allows both Pitch and Roll control using wheels
- In case of failure of one Main wheel, the small wheel about yaw can be used for control; until then it is held inactive.

MBS with Two MW and a Redundant wheel (Equations of Motion)

he products of inertia are null. With these assumptions, Eqs. 4.8.14 become

$$T'_{cx}+T_{dx}=I_x\ddot{\phi}+4\omega_o^2(I_y-I_z)\phi+\omega_o(I_y-I_z-I_x)\dot{\psi}-\omega_o(H_1-H_2)\sin(\alpha), \qquad (8.7.12)$$

$$T'_{cz}+T_{dz}=I_z\ddot{\psi}+\omega_o(I_z+I_x-I_y)\dot{\phi}+\omega_o^2(I_y-I_x)\psi+(\dot{H}_1-\dot{H}_2)\sin(\alpha), \qquad (8.7.13)$$

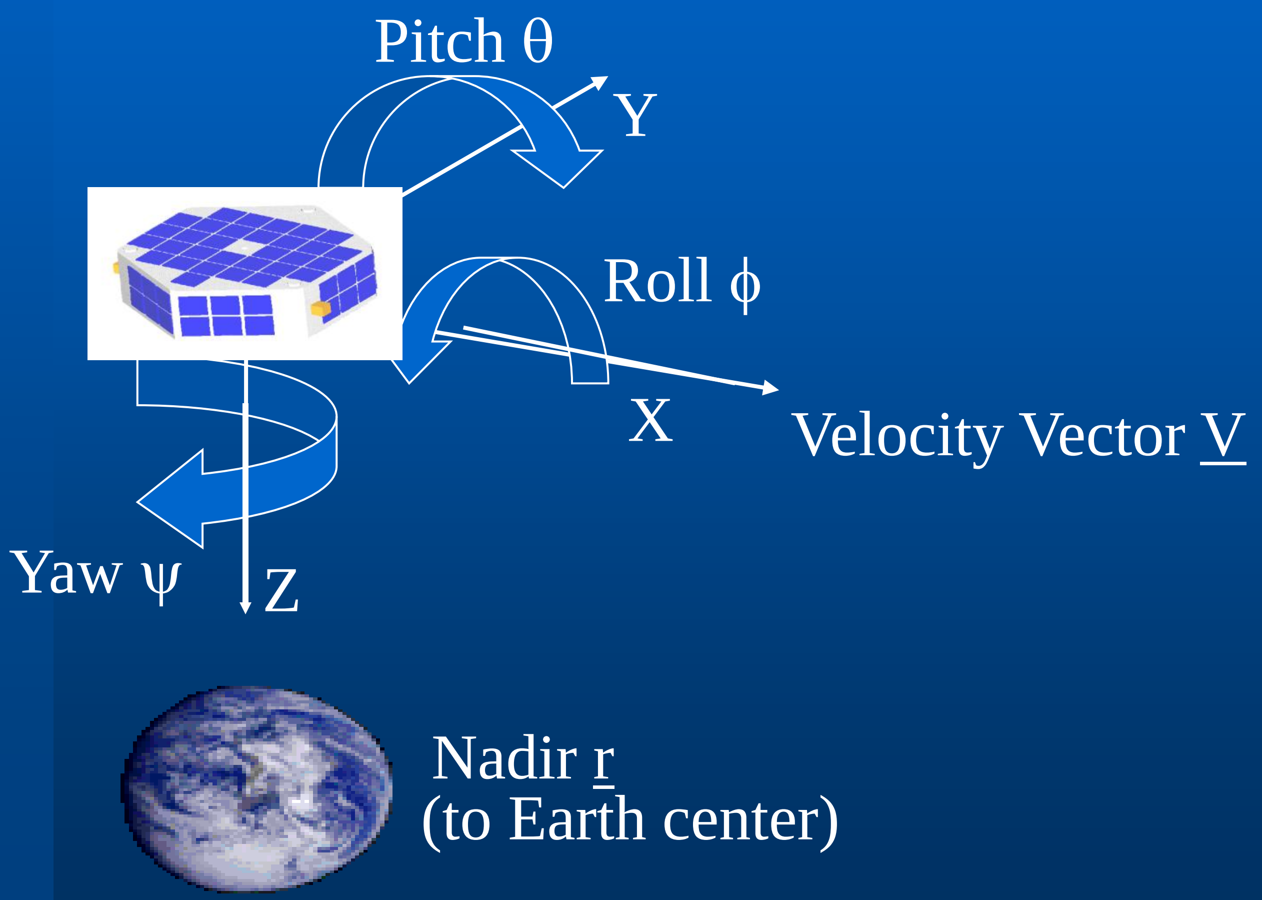
$$T'_{cy}+T_{dy}=I_y\ddot{\theta}+3\omega_o^2(I_x-I_z)\theta+(\dot{H}_1+\dot{H}_2)\cos(\alpha). \qquad (8.7.14)$$

- Total angular momentum about pitch axis
  $H_t = 2H_1 \cos\alpha$
- $h_y = (H_1 + H_2)\cos\alpha$
- $h_z = (H_1 - H_2)\sin\alpha$
- $T_{cy} = \left(\dot{H}_1 + \dot{H}_2\right)\cos\alpha$
- $T_{cz} = \left(\dot{H}_1 - \dot{H}_2\right) sin\,\alpha$

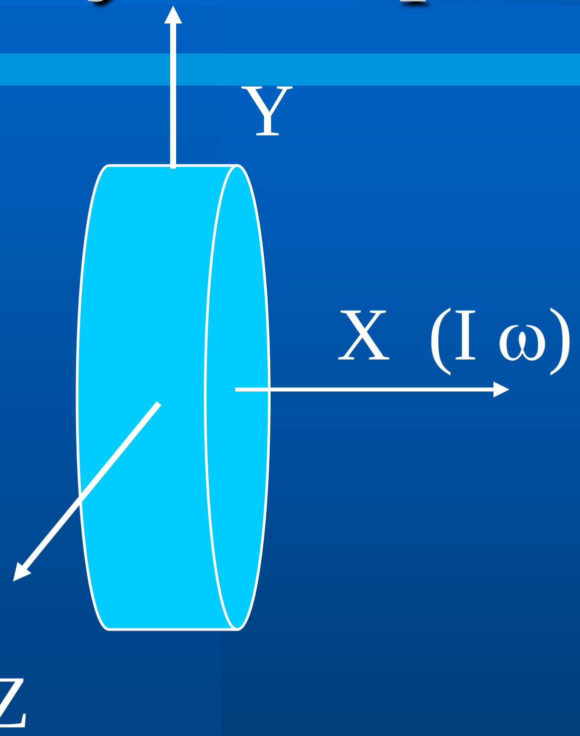
Three Axis AD using Earth Sensor and Gyros

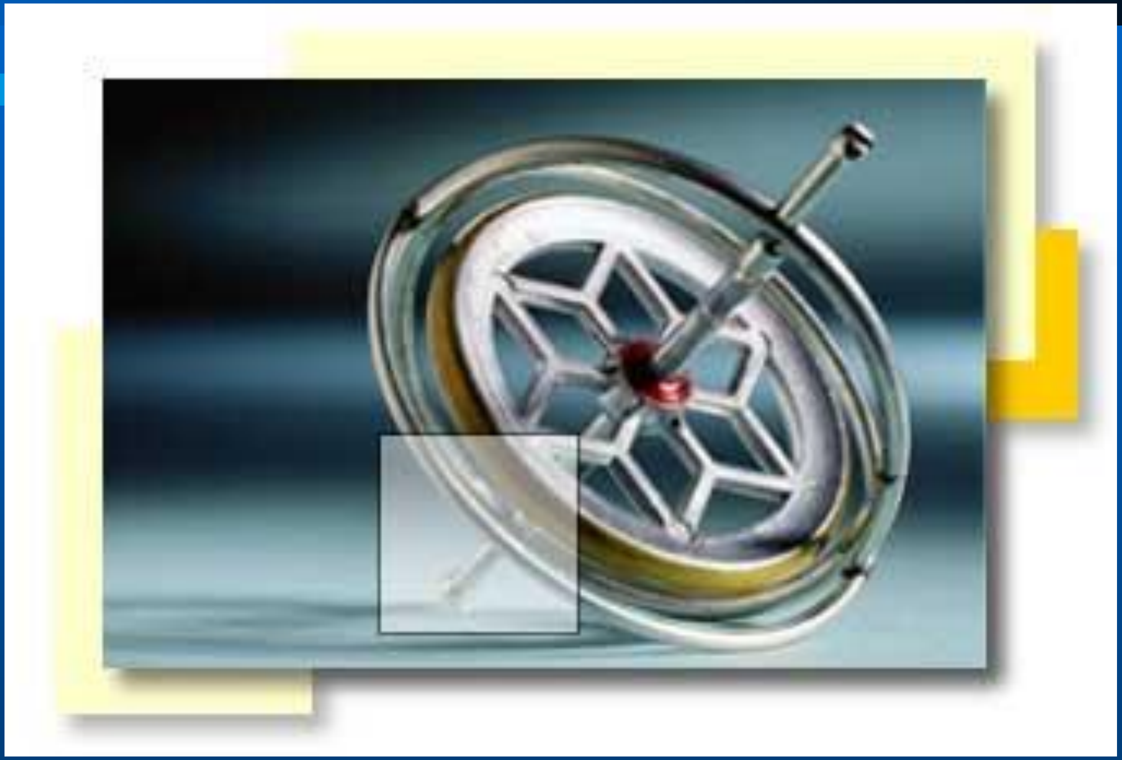
- **Earth Sensor Provides only Pitch and Roll Errors in body frame**
- **By suitable mounting of Sun Sensors one can obtain yaw errors at some instances of orbit**
- **Gyroscopes provide rates of the spacecraft continuously**
- **For Precision AD, Rates and Attitude Errors are fused by Kalman Filtering to get the best estimate**
- **In momentum biased system such as INSAT no need of attitude determination. Yaw – roll quarter orbit coupling of MB system used in controlling yaw indirectly.**

Basic Reference Frame



Gyroscopes

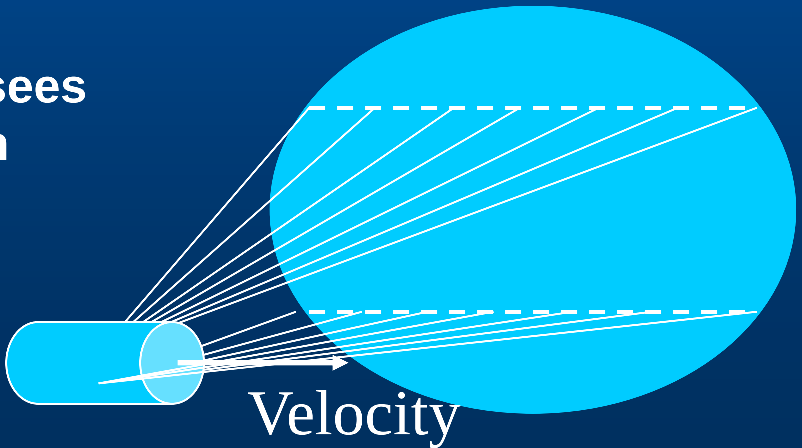




- **Angular Momentum Device**
- **Senses precessional motion**

Earth/Horizon sensor

- Distinguishes Earth's horizon, usually by its IR transition or horizon
- Can usually only provide two-axis knowledge
  - Very poor in yaw
- There are multiple types of horizon sensors.
- In a scanning sensor, two beams scan across the Earth, as shown below.
  - The difference in time, the absolute time, and the s/c relative angles at which the scan begins and ends can provide two-axis attitude knowledge.
- An Earth-sensing phototransistor sees the visual and/or infrared light from the Earth and outputs a binary trigger, tripping when the Earth is within the field of view.



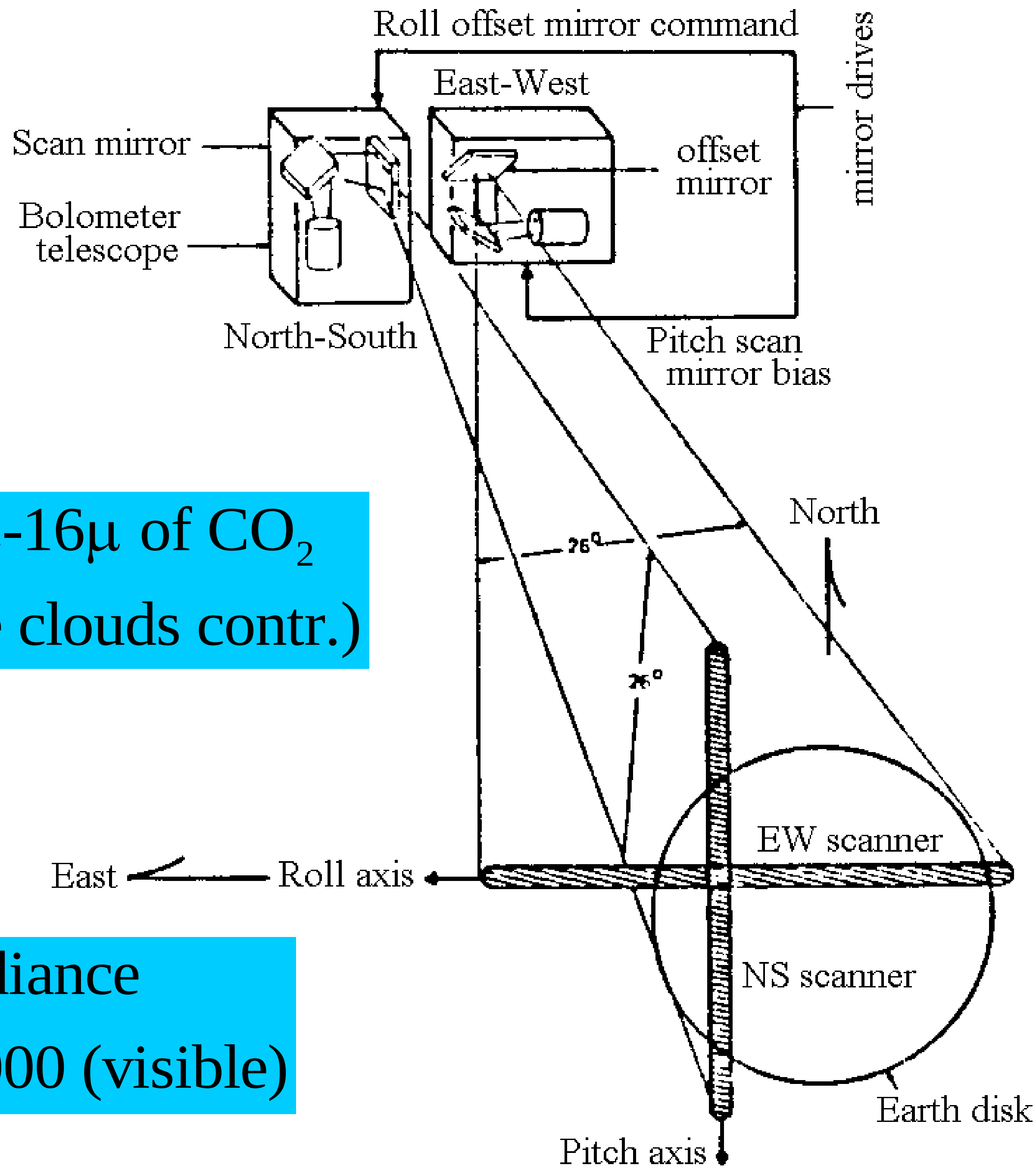
Earth horizon sensor

Best range: 14μ-16μ of CO_2

(less high altitude clouds contr.)

Relative radiance

400 (IR) and 30,000 (visible)



Static Earth Sensor

Conical Earth Sensor

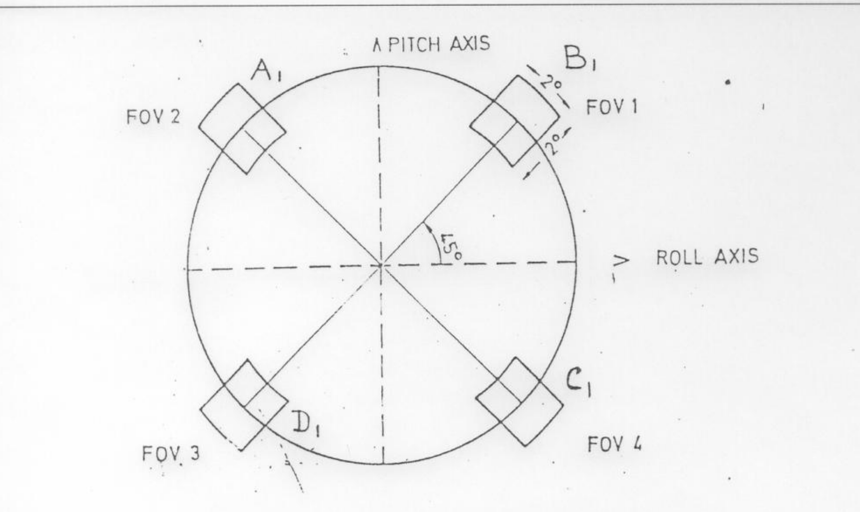


FIG 13.7 STATIC HORIZON SENSOR GEOMETRY

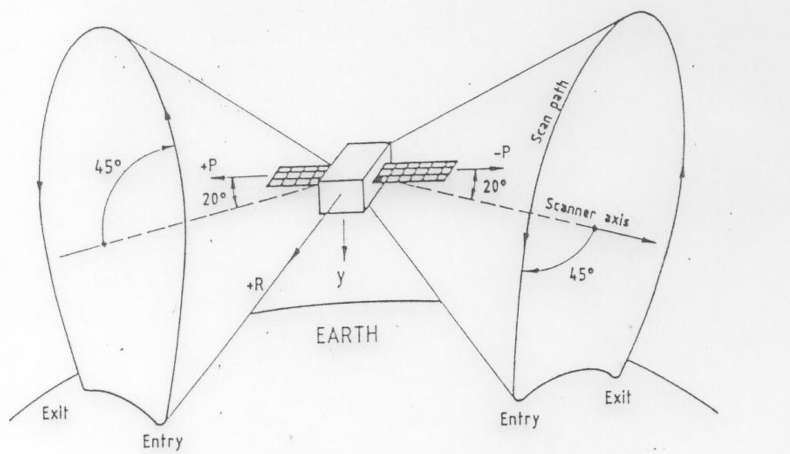
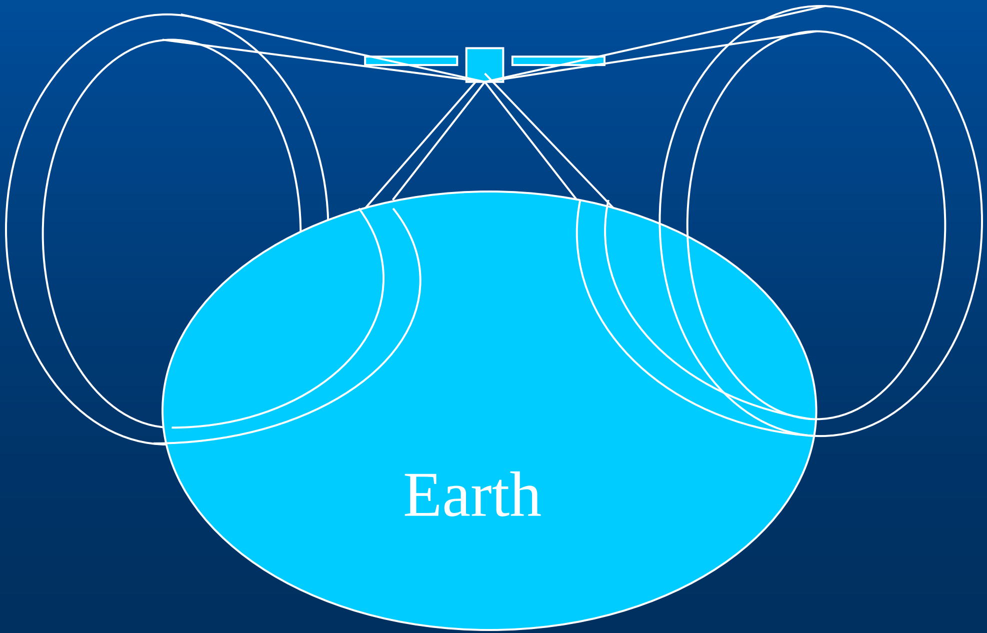


FIG 13.6 HORIZON SCANNER GEOMETRY



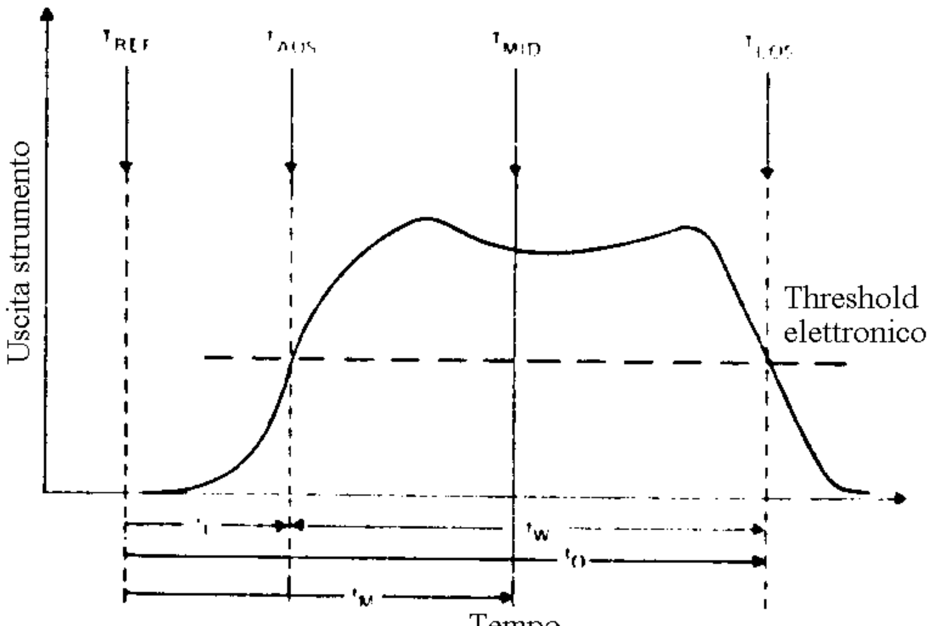
Earth Horizon sensor

Measure two crossing times t_1 and t_2

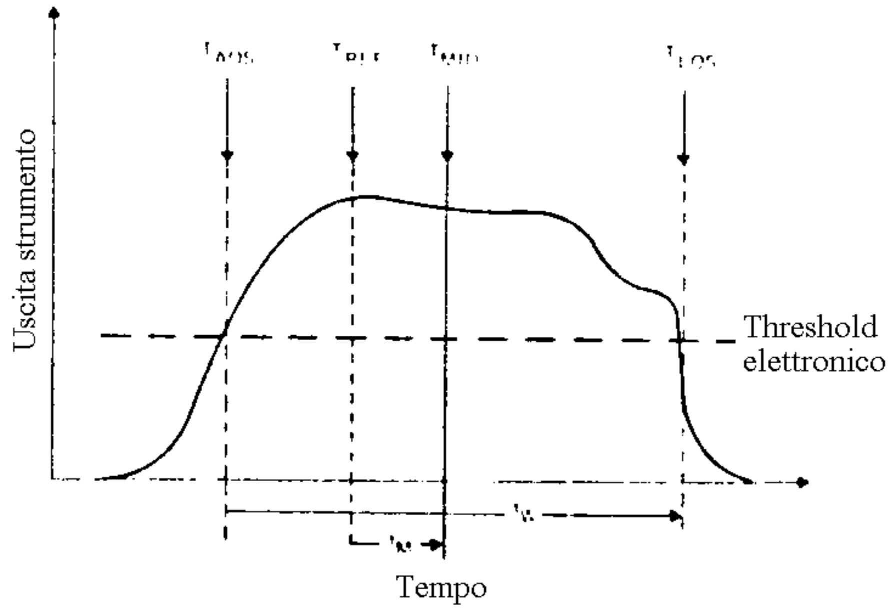
With angular speed ω, the measured angle is $\varphi = \omega(t_2 - t_1)$

Two measured angles, φ_1 and φ_2

allow to evaluate the Nadir direction.

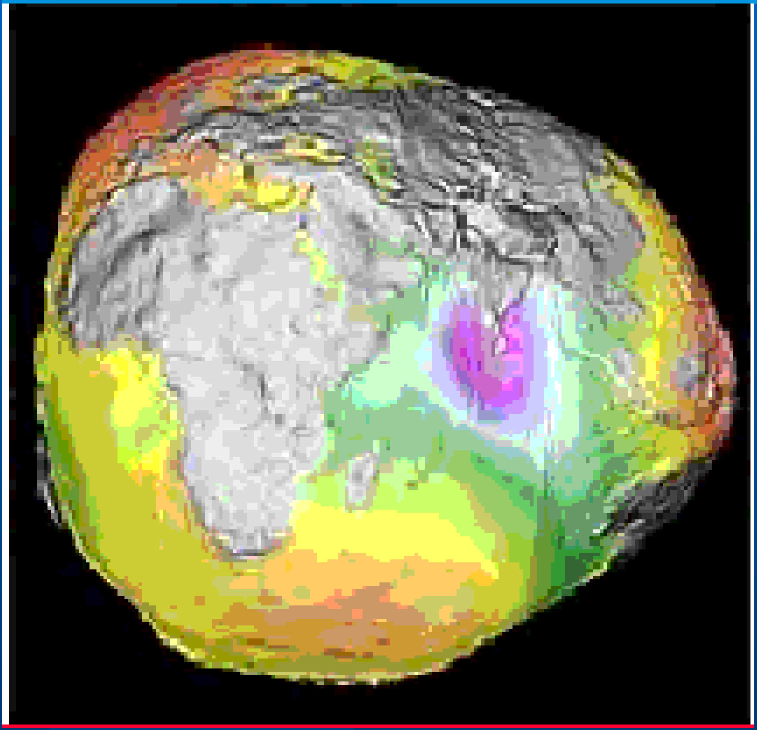


(a) Output from a body-mounted horizon sensor with a sun reference pulse



(b) Output from a wheel -mounted horizon sensor with a magnetic pickoff reference pulse

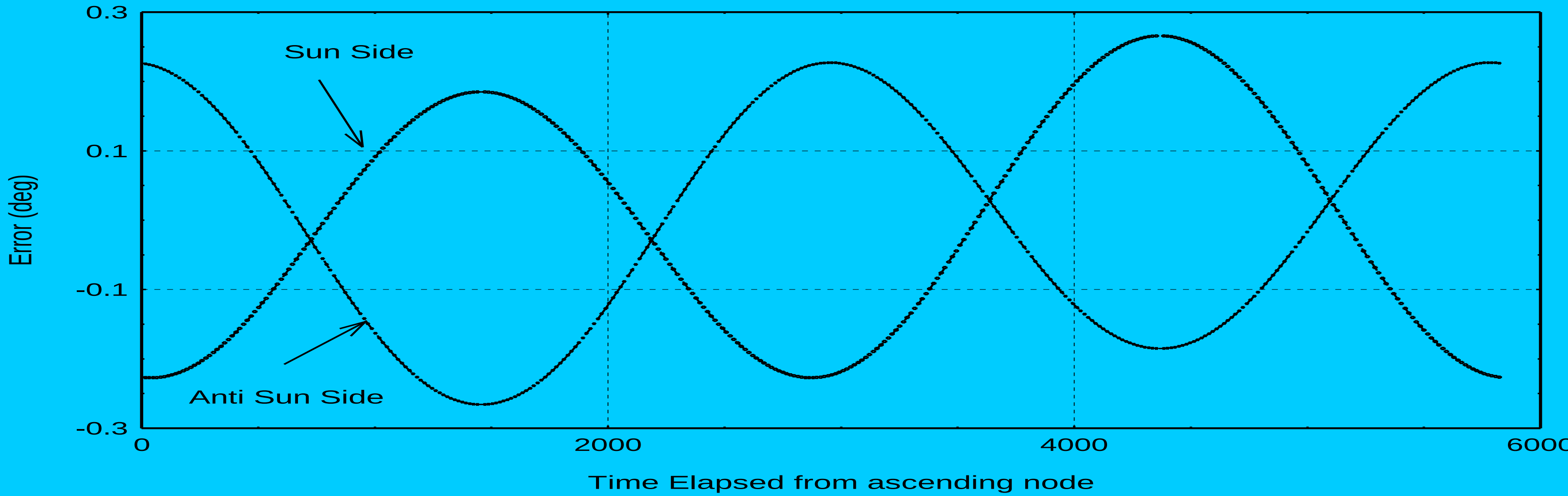
Earth Sensor Systematic Errors



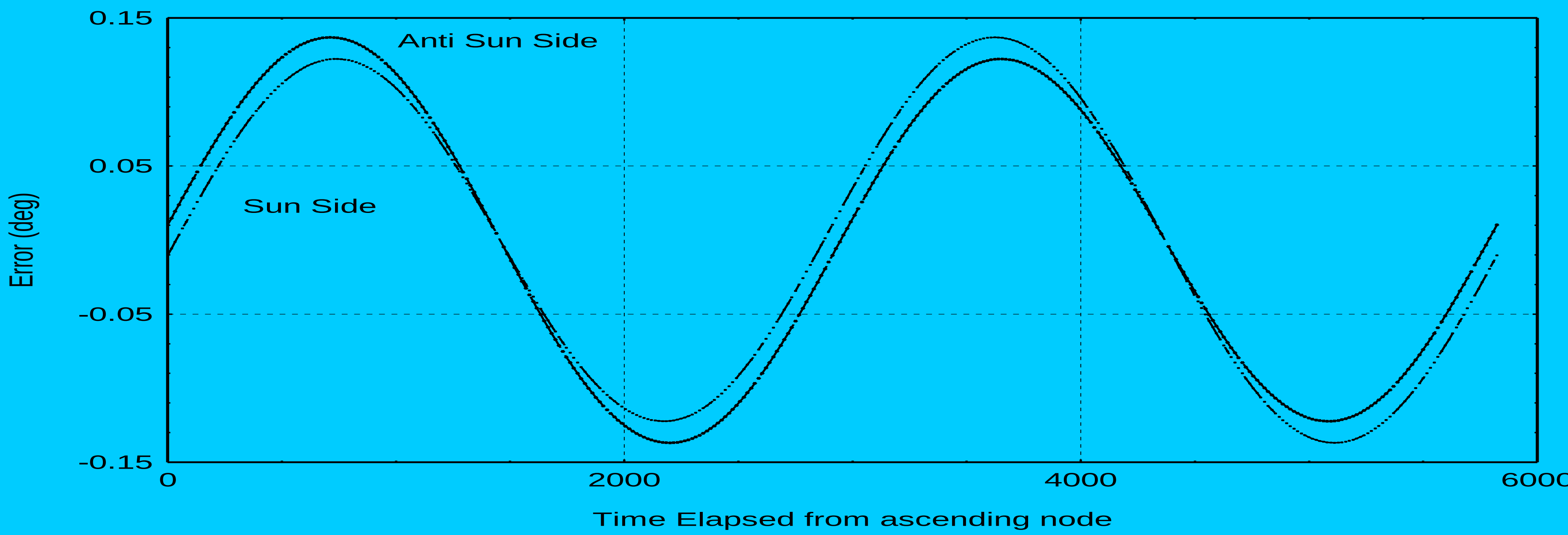
Earth's Asphericity: Computed Earth Potential model in 19 March 2002

- Earth is not a perfect sphere
- The actual shape is shown here
- The shape introduces systematic errors in the roll and pitch measurements
- Further, in IR radiation type earth sensors, another radiation based systematic error is also introduced.
- These systematic errors must be modelled and removed before this is used in control system
- These errors are latitude dependent and is shown in the next two slides

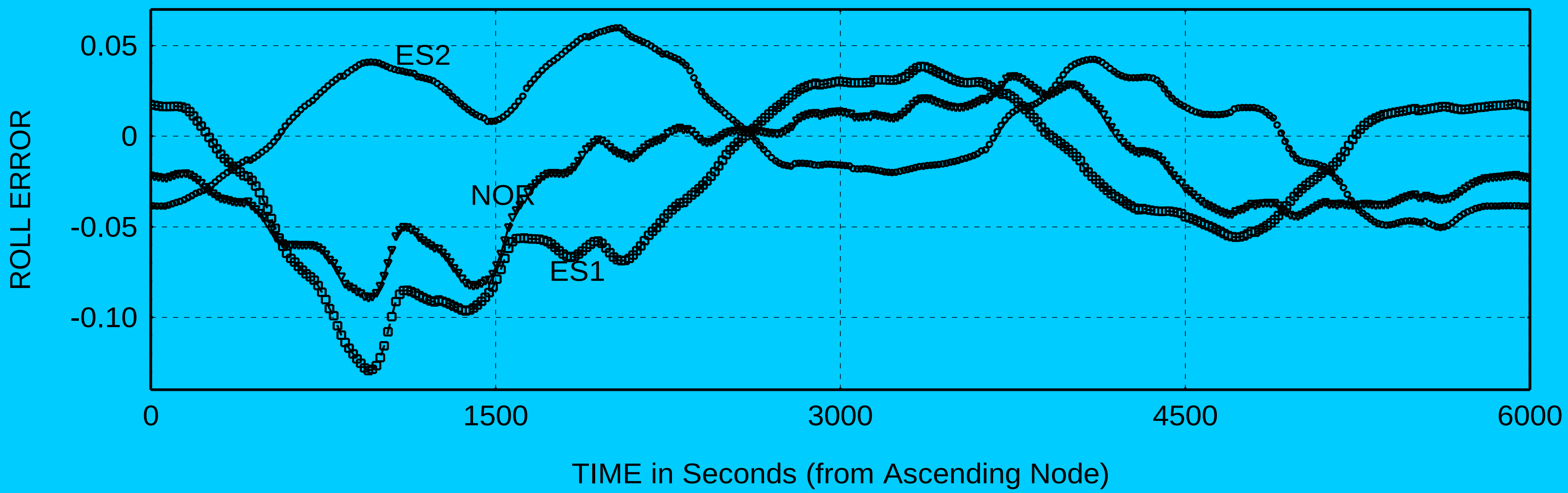
SYSTEMATIC ERROR IN ROLL DUE TO EARTH OBLATNESS FOR IRS-P5
(ONLY OBLATNESS)



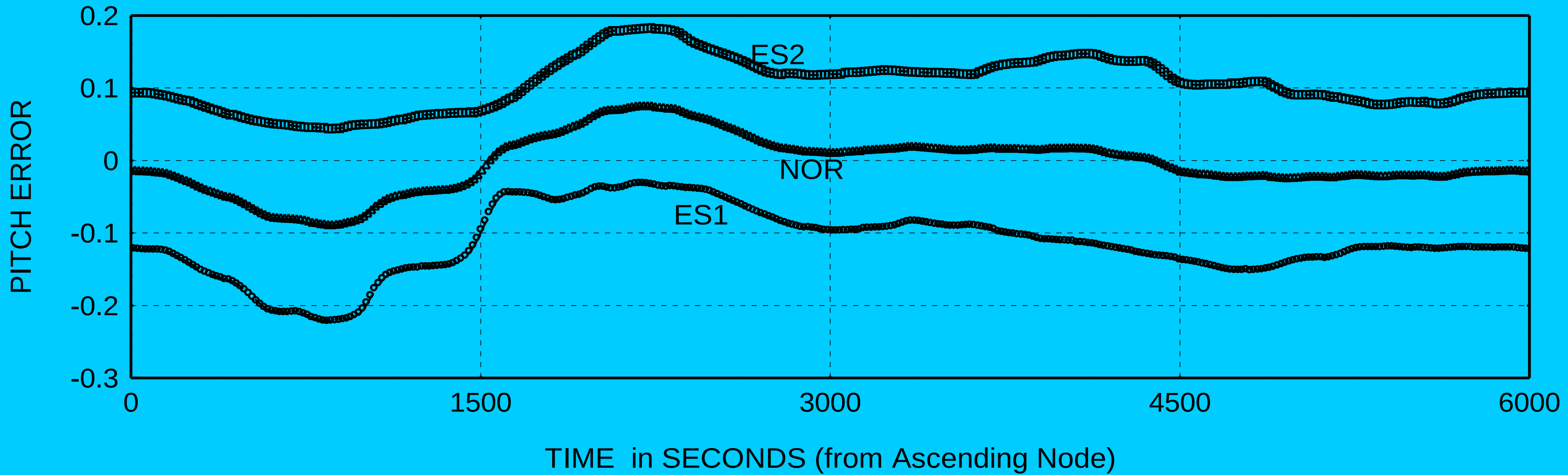
SYSTEMATIC ERROR IN PITCH DUE TO EARTH OBLATENESS FOR IRS-P5
(ONLY OBLATENESS)



SYSTEMATIC ERROR DUE TO EARTH RADIANCE IN ROLL MEASUREMENTS



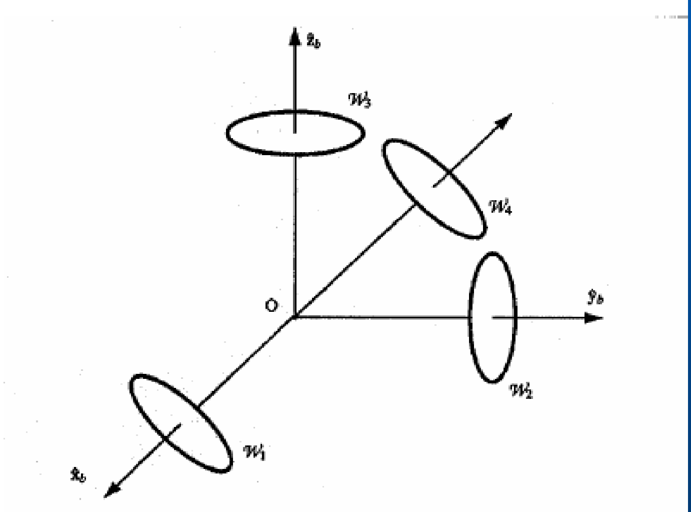
01-Dec PitchSYSTEMATIC ERROR DUE TO EARTH RADIANCE IN PITCH MEASUREMENTS



Three Axis Zero Momentum Biased Dynamics (Complete RW based control)

Total Angular Momentum of the 4 RW satellite is

$$\bar{H}_T = \bar{H}_S + \bar{H}_{RW}$$



Configuration of energy/momentum wheels.

where $\bar{H}_S = \bar{I}_S \ \bar{\omega}_S$

$$\bar{H}_{RW} = \begin{pmatrix} h_x + \frac{h_s}{\sqrt{3}} \\ h_y + \frac{h_s}{\sqrt{3}} \\ h_z + \frac{h_s}{\sqrt{3}} \end{pmatrix} \qquad \bar{\omega}_s = \begin{pmatrix} \dot{\sigma} + \omega_0 \varphi \\ \dot{\varphi} - \omega_0 \sigma \\ \dot{\theta} - \omega_0 \end{pmatrix}$$

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4 RW – Dynamical Equations

Using Euler's angular momentum conservation principle

$$\bar{\mathrm{T}} = \left.\frac{d\bar{\mathrm{H}}_T}{dt}\right]_b + \bar{\omega}_s \times \bar{\mathrm{H}}_T$$

Finally the equations look like

$$\mathrm{T}_x = \mathrm{I}_x\dot{\omega}_x + \dot{h}_x + \frac{\dot{h}_s}{\sqrt{3}} + (\mathrm{I_z} - \mathrm{I}_y)\omega_y\,\omega_z + \left(h_z + \frac{h_s}{\sqrt{3}}\right)\omega_y - \left(h_y + \frac{h_s}{\sqrt{3}}\right)\omega_z$$

$$\mathrm{T}_y = \mathrm{I}_y\dot{\omega}_y + \dot{h}_y + \frac{\dot{h}_s}{\sqrt{3}} + (\mathrm{I_x} - \mathrm{I}_z)\omega_x\,\omega_z - \left(h_z + \frac{h_s}{\sqrt{3}}\right)\omega_x + \left(h_x + \frac{h_s}{\sqrt{3}}\right)\omega_z$$

$$\mathrm{T}_z = \mathrm{I}_z\dot{\omega}_z + \dot{h}_z + \frac{\dot{h}_s}{\sqrt{3}} + (\mathrm{I_y} - \mathrm{I}_x)\omega_y\,\omega_z + \left(h_y + \frac{h_s}{\sqrt{3}}\right)\omega_x - \left(h_x + \frac{h_s}{\sqrt{3}}\right)\omega_y$$

Distribution Matrix

- **Torques produced about the three axes are**

$$\begin{Bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \alpha \\ 0 & 1 & 0 & \beta \\ 0 & 0 & 1 & \gamma \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} \text{ where } \alpha = \beta = \gamma = \frac{1}{\sqrt{3}}$$

- **Then the distribution matrix is given by**

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \frac{1}{1+\alpha^2+\beta^2+\gamma^2} \begin{bmatrix} 1+\beta^2+\gamma^2 & -\alpha\beta & -\alpha\gamma \\ -\alpha\beta & 1+\alpha^2+\gamma^2 & -\beta\gamma \\ -\alpha\gamma & -\beta\gamma & 1+\alpha^2+\beta^2 \\ \alpha & \beta & \gamma \end{bmatrix} \begin{Bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \\ 0 \end{Bmatrix}$$

Pyramid 4-RW Zero-Momentum Biased System Configuration

- **Let T_1, T_2, T_3 *and* T_4 *are* the torques produced by wheels**
- **Wheels mounted with an oblique angle β wrt 1, 2, 3, and 4 axes (see figure)**
- **This way it provides control about all three axes and redundancy provided in case of failure of one wheel**
- $h_z = (T_1 + T_2 + T_3 + T_4)/\sin\beta$
- $h_x = (T_1 - T_3)\cos\beta$
- $h_y = (T_2 - T_4)\cos\beta$

$$\begin{Bmatrix} T_{cx}/\cos\beta \\ T_{cy}/\cos\beta \\ T_{cz}/\sin\beta \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix}$$

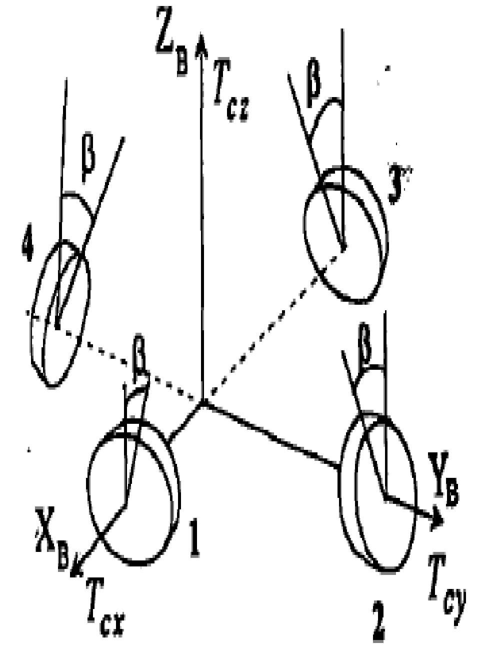


Figure 7.3.5 Attitude control system with four reaction wheels.

Control Torque computation

- **The control torque realization is**

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0.5 & 0.5 \\ 0 & 1 & 0.5 & -0.5 \\ -1 & 0 & 0.5 & 0.5 \\ 0 & -1 & 0.5 & -0.5 \end{bmatrix} \begin{Bmatrix} \widehat{T}_{cx} \\ \widehat{T}_{cy} \\ \widehat{T}_{cz} \\ 0 \end{Bmatrix}$$

Zero-Momentum Biased System
Tetrahedron Wheel Configuration

- **This wheel assembly can provide twice as much of maximum torque on a single axis that a wheel can provide**

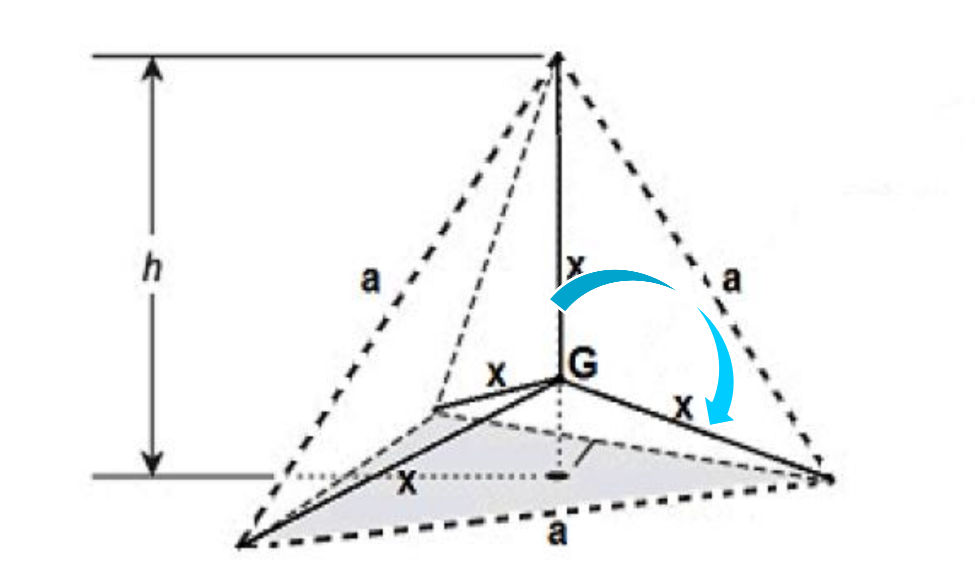
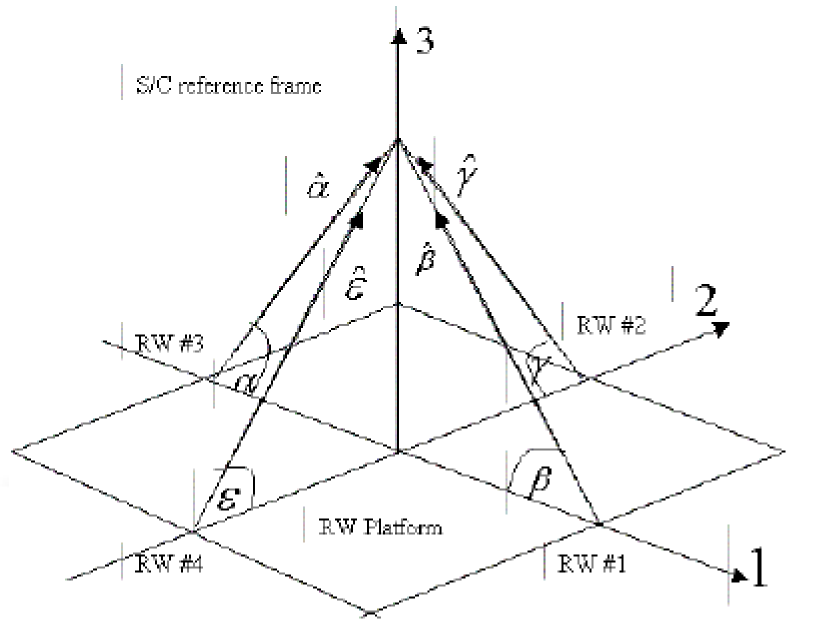


Figure B.2 – Tetrahedron Configuration



Angles: 109.47 deg
30.0 / 45.0 / 60.0 deg

Zero-Momentum Biased System Tetrahedron Wheel Configuration

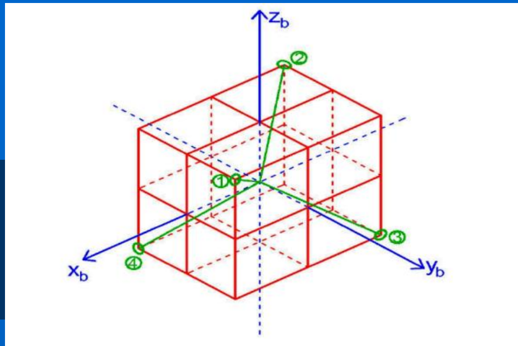
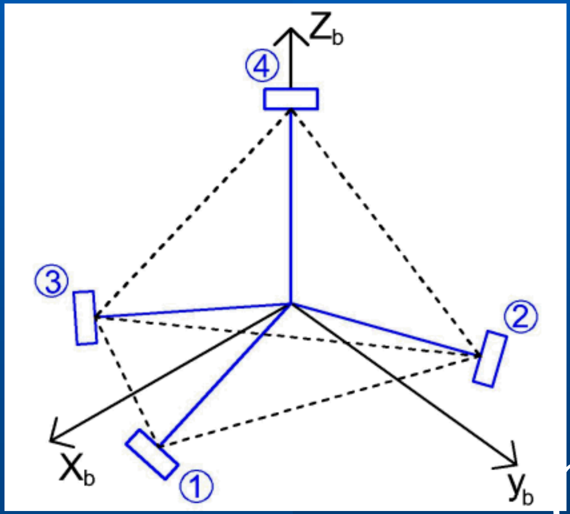


Figure B.5 – Tetrahedron configuration of reaction wheels

- **This wheel assembly can provide torque in the following distribution matrix form**



$$\begin{Bmatrix} T_{cx} \\ T_{cy} \\ T_{cz} \end{Bmatrix} = \begin{bmatrix} \sqrt{3}/3 & -\sqrt{3}/3 & -\sqrt{3}/3 & \sqrt{3}/3 \\ \sqrt{3}/3 & -\sqrt{3}/3 & \sqrt{3}/3 & -\sqrt{3}/3 \\ \sqrt{3}/3 & \sqrt{3}/3 & -\sqrt{3}/3 & -\sqrt{3}/3 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix}$$

Distribution matrix is

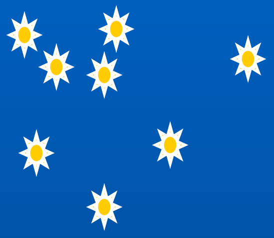
$$\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \frac{1}{4} \begin{bmatrix} \sqrt{3} & \sqrt{3} & \sqrt{3} & 1 \\ -\sqrt{3} & -\sqrt{3} & \sqrt{3} & 1 \\ -\sqrt{3} & \sqrt{3} & -\sqrt{3} & 1 \\ \sqrt{3} & -\sqrt{3} & -\sqrt{3} & 1 \end{bmatrix} \begin{Bmatrix} \hat{T}_{cx} \\ \hat{T}_{cy} \\ \hat{T}_{cz} \\ 0 \end{Bmatrix}$$

Tetrahedron Configuration Advantages

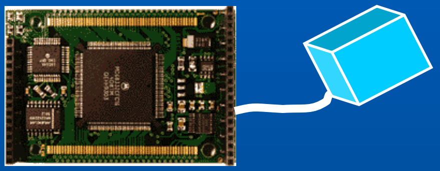
- Always one wheel redundancy
- In case of failure of a wheel, the percentage of increase in power consumption is much lower
- This 4-wheel configuration requires less power than 3-wheel configuration system
- Provides greater torque capacity and momentum exchange capability – closer to twice a single wheel capacity
- More wheels allow longer time intervals between momentum unloads for s/c

Star Sensor based Attitude Determination

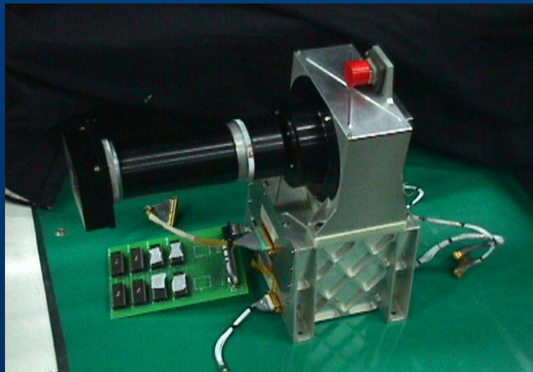
- Usually a digital or CCD type camera



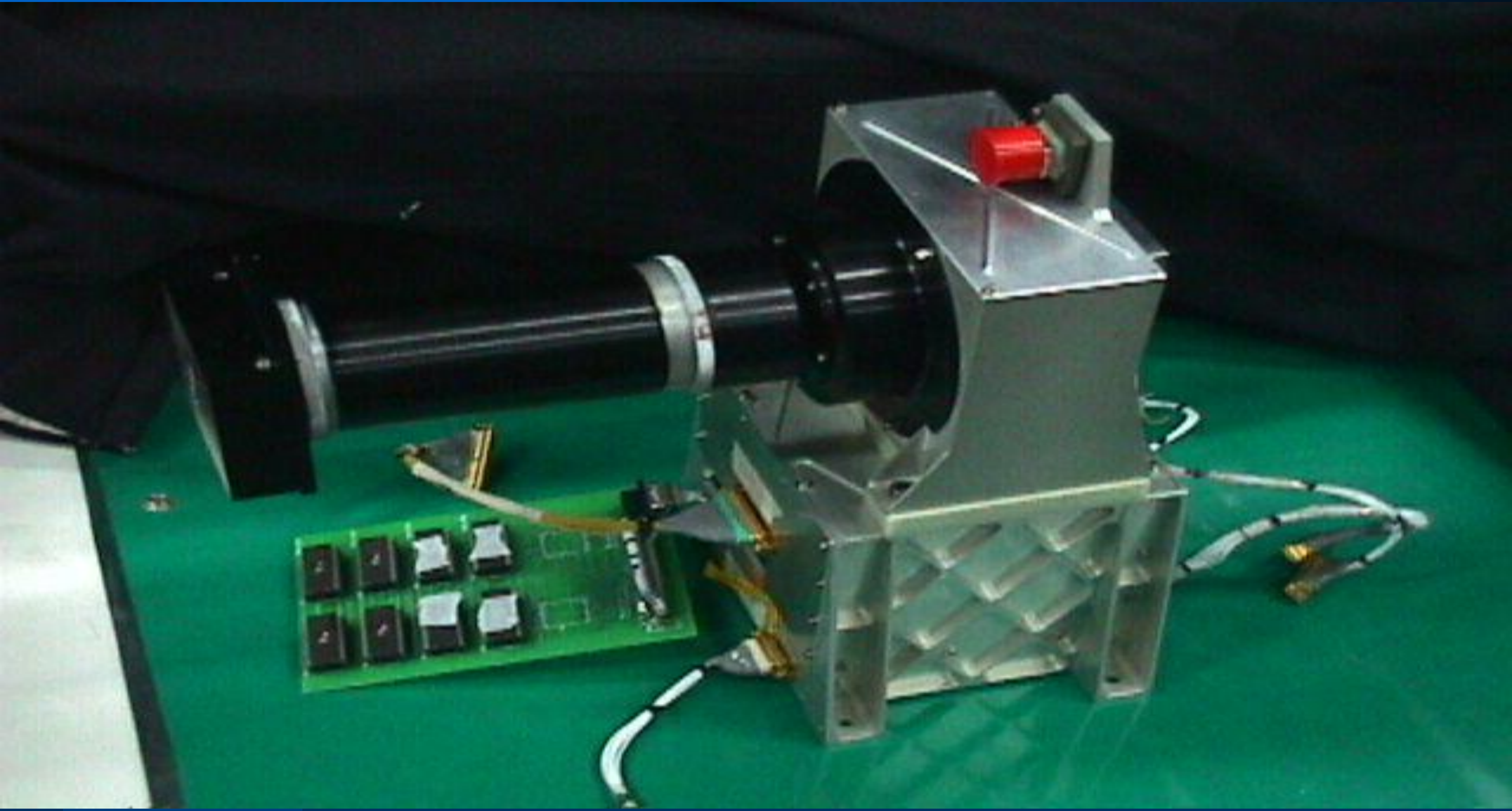
Star map in memory



- Locks on to bright stars.
  - Star map is held in computer memory
  - Requires computer time to process map algorithm, match picture with map
  - Provides amazingly accurate pointing knowledge.
- One star identified:
  - Provides two-axis knowledge
- Three or more stars identified:
  - Provide three-axis knowledge
- Sensitive to sun and moon



Star Sensor (ISRO)



3-AXIS ATTITUDE DETERMINATION

TWO TYPES:

- **DETERMINISTIC (POINT-by-POINT) APPROACH**
  - ✓ **Attitude Based on two or more vector observations from a single point of time**
- **FILTERS, RECURSIVE STOCHASTIC ESTIMATORS**
  - ✓ **Statistically combine measurements from several sensors with dynamic / kinematic models in order to achieve the best estimate of the attitude**

STAR AD TASKS:

- **GENERATION OF MISSION STAR CATALOG**
- **LOST IN SPACE STAR IDENTIFICATION TECHNIQUES**
- **QUEST ATTITUDE ESTIMATION ALGORITHM DETAILS**

Onboard Core Mission Catalogs - Necessity and Properties

- STORAGE OF ENTIRE SKYMAP 2000 CATALOG ONBOARD - NOT FEASIBLE AND ALSO NOT REQUIRED
- THINNING OF CATALOG IS A MUST
- SUCH A CATALOG MUST MEET
  - GIVEN TIME A MINIMUM OF STARS IN FOV
  - UNIFORM DISTRIBUTION OF STAR IMAGES
  - MINIMUM OR NULL STAR GAPS
  - ADEQUATE STAR IMAGES SEPARATION

Grid Generation - Quadrant Separation Method

- Entire celestial area divided into grids with area equal to area of FOV of sensor
- Each area of FOV divided into FOUR quadrants - retain the 2 bright stars for each quadrant - ensures wide separation
- Designed for a global mission - uniformity over entire celestial sphere requested
- assumption of maximum of 8 star images in FOV at any time though 5 is sufficient

G - Q Method - Procedure

- Stars in the unit sphere mapped onto rectangle area using

$$r_1 = z,$$

$$r_2 = \arctan\left(\frac{y}{x}\right) \qquad (1)$$

$$\text{where} = \left|(1 - z^2)^{\frac{1}{2}}\right|$$

- Rectangle is divided into orthogonal grid each with an area of size of FOV of sensor
- The brightest 2 stars in each quadrant retained. This ensures minimum 8 star images in FOV for initial acquisition

Random Method - Procedure

- Generate two uniformly generated random numbers - each for RA and declination
- Using the equation

$$z = r_1,$$
$$x = r\cos r_2,\ y = r\sin r_2, \qquad (2)$$
$$\text{where } r = \left|(1 - r_1^2)^{\frac{1}{2}}\right|$$

- Since each exposure requires 3 stars retain 3 brightest stars
- This process continued until no update in the total number of stars

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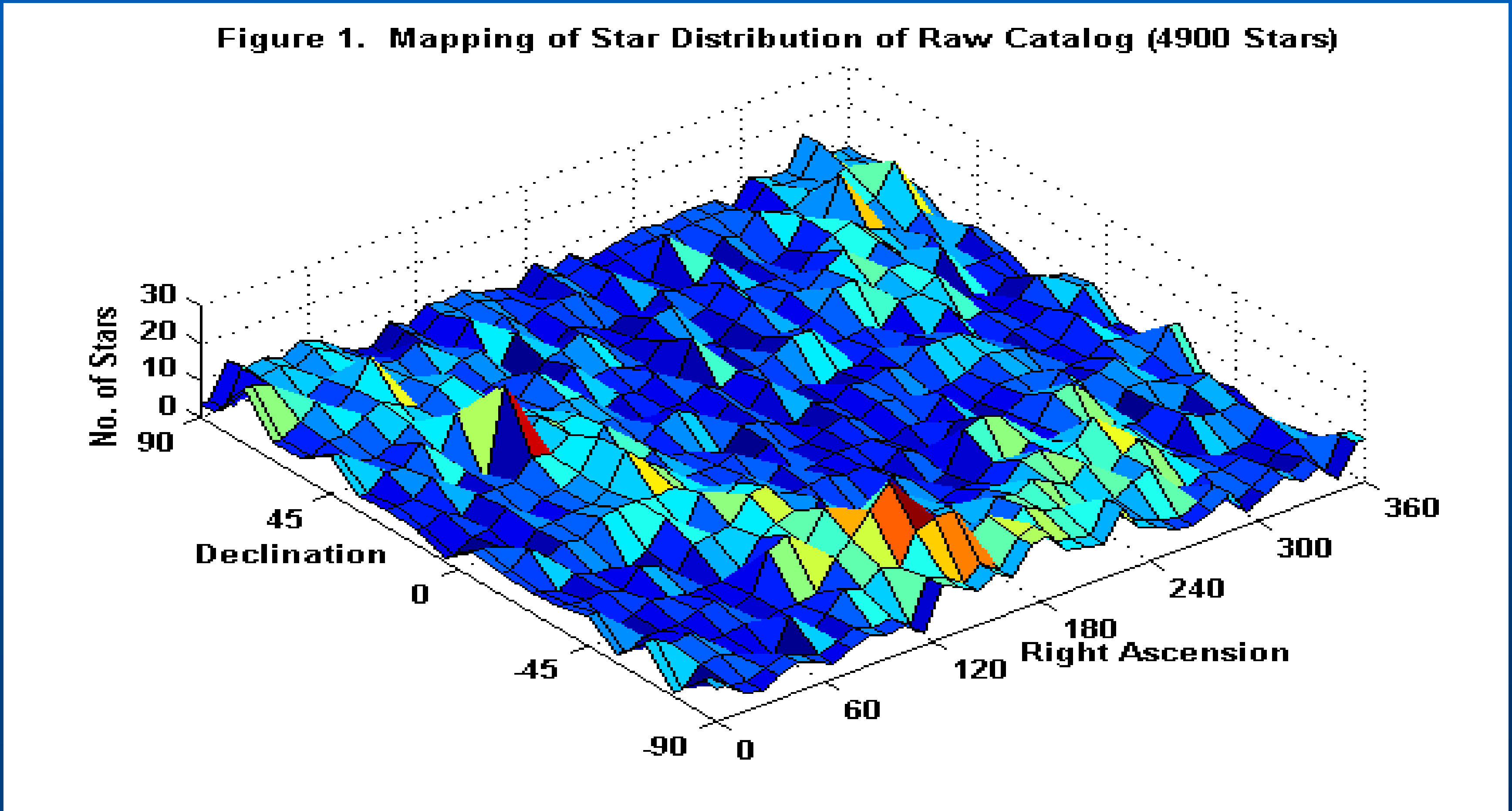
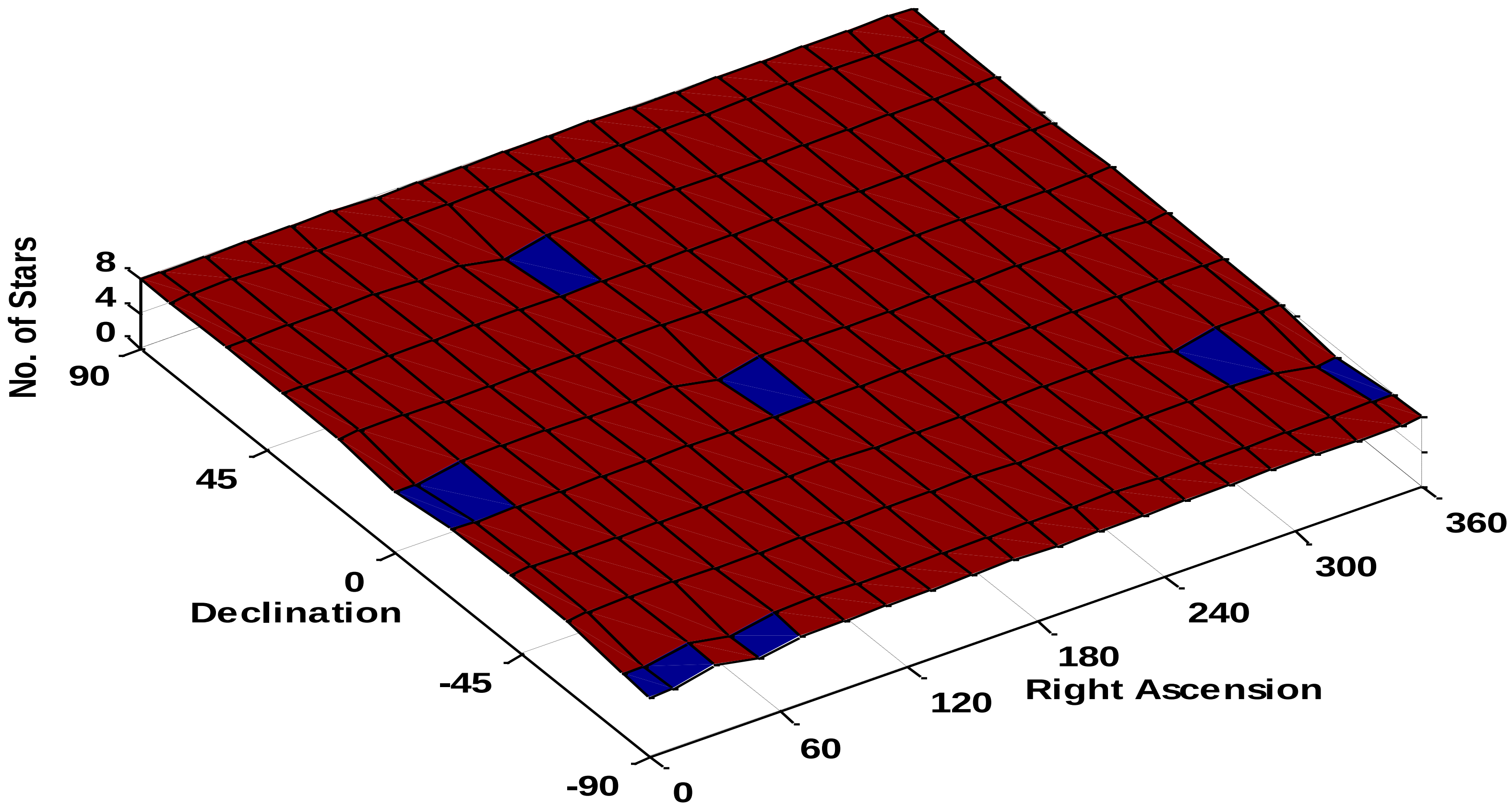
Surface Map of Raw Catalog



Thinned Core Catalog - G-Q Method

Figure 2. Mapping of Star Distribution of Thinned Catalog (GQ Method)



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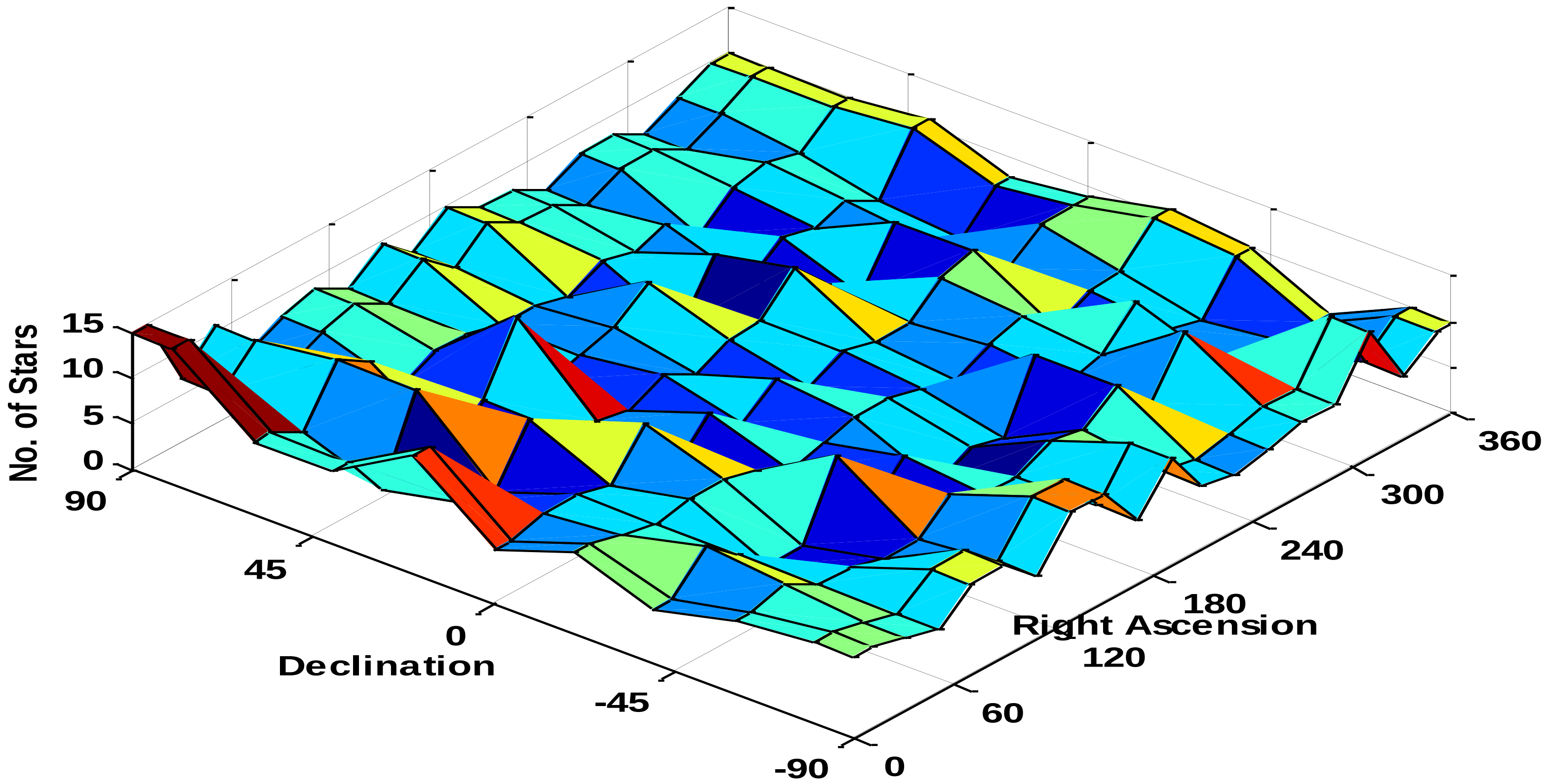
Thinned Core Catalog - Random Method

Figure 3. Mapping of Star Distribution of Thinned Catalog (Random Method)



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Computed Uniformity Measures of Thinned Catalogs

- **Global Uniformity Numbers**

Catalog Name	λ_1	λ_2	λ_3	Φ
Raw (4900 Stars)	0.3636	0.3785	0.2579	1.35e-2
G-Q Method (1289 Stars)	0.3358	0.3340	0.3302	2.47e-5
Random Method (1212 Stars)	0.3187	0.3211	0.3602	1.61e-3

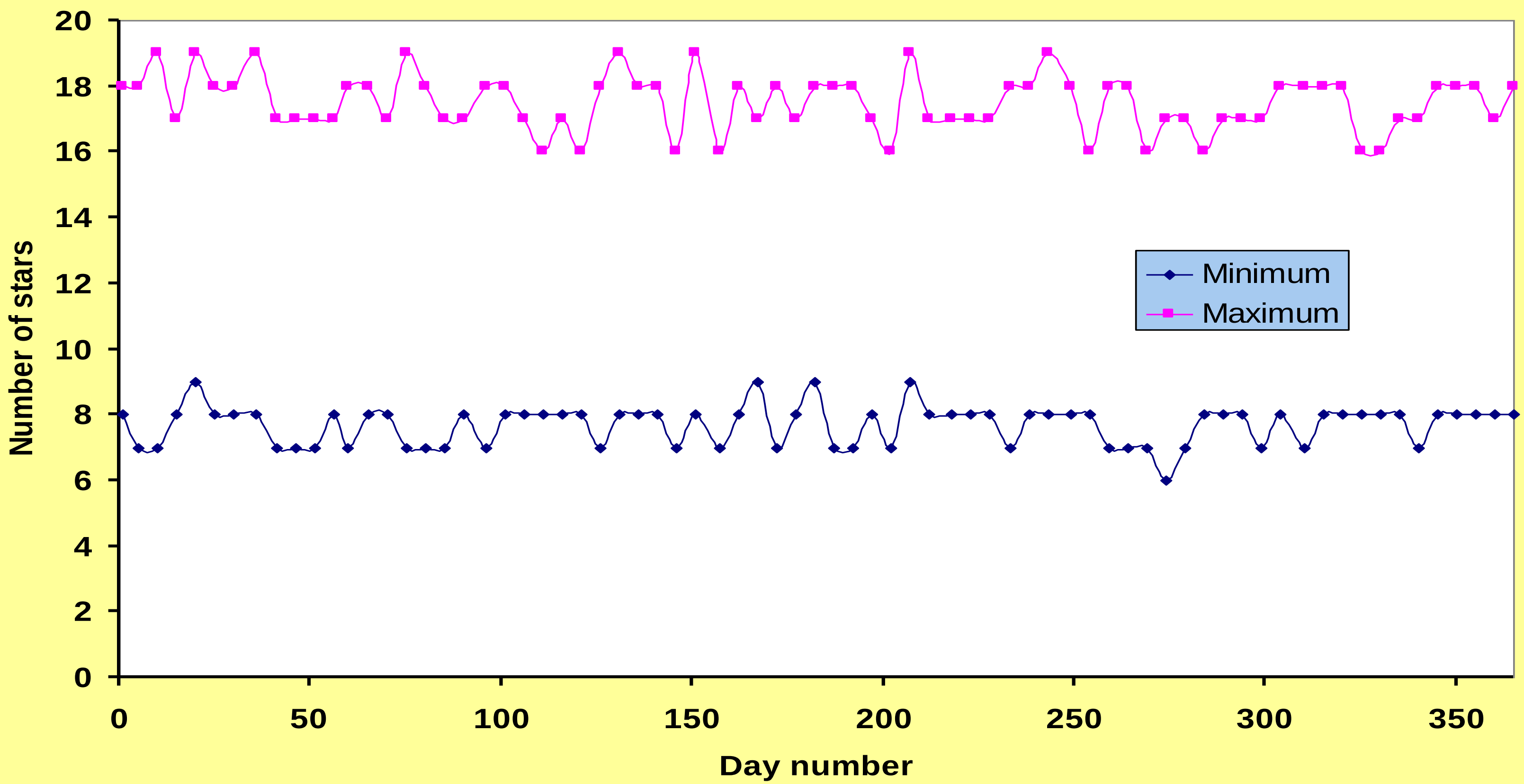
- **Local Uniformity Numbers**
  - **G-Q method (Ψ range 0.88 - 1.0)**
  - **Random method (Ψ range 0.4 - 2.0)**

Star Gap Assessment - Simulation Results for G-Q Method

- **A yearlong simulation carried out with actual orbit characteristics**
- **Actual Mounting of the Star Sensor for EOS is used**
- **G-Q generated thinned catalog is utilized**
- **For stars of magnitude 6.0 or brighter there was not an instant of star less period**
- **The lowest minimum number of stars in a FOV = 6**
- **The highest number of stars in FOV = 19**

Star Gap Assessment - Simulation Results for G-Q Method - contd...

Figure 14. Minimum and maximum number of stars sighted



Computed Uniformity Measures of Thinned Catalogs - Stars of VM 5.0

- Global Uniformity Numbers

Catalog Name	λ_1	λ_2	λ_3	Φ
Raw (1560 Stars)	0.3655	0.3792	0.2553	1.45e-2
G-Q Method (990 Stars)	0.3518	0.3456	0.3027	2.18e-3
Random Method (898 Stars)	0.3440	0.3299	0.3261	2.64e-4

- Local Uniformity Numbers
  - G-Q method (Ψ range 0.33 - 1.31)
  - Random method (Ψ range 0.36 - 2.18)

Star Identification Algorithms

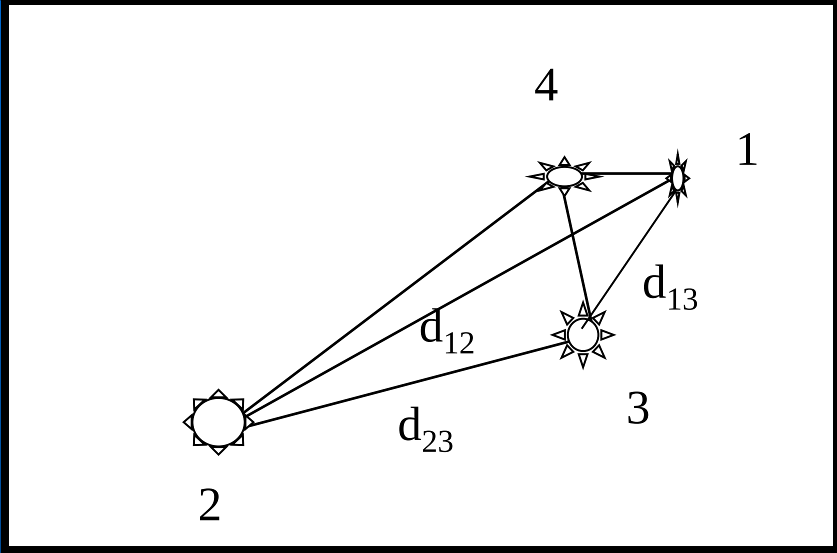
- **THREE IDENTIFICATION ALGORITHMS**
- TRIPLET IDENTIFICATION DUE TO LEOS
- QU**ATERNION** A**TTITUDE IN** L**OST** I**N SPACE (**QUALIS**) USING ANGULAR SEPARATION METHOD**
- **STAR** TR**IPLET** ID**ENTIFICATION FOR LOST IN SPACE (**STRIDE**) - TRIPLET PATTERN RECOGNITION METHOD**
- **ABOVE ALGORITHMS DO NOT REQUIRE A PRIORI ATTITUDE KNOWLEDGE FOR STAR IDENTIFICATION**

LEOS Triplet Identification Algorithm

- Cosine of Angular Separations of core stars computed, sorted and stored in ascending order
- 3 bright star images selected and angular distances computed with suitable tolerances
- 3 regions correspond to three angular distances identified in angular distance table
- First, a pair (s1,s2) in region 1 is selected and from region 2 look for one of the pair (s2) along with s3. In region 3, a pair (s1,s3) or (s2,s3) can be identified. Then the triad (s1,s2,s3) can be identified. Then the 4th image can be used in similar way and a polygon can be identified.
- When no success, another pair in region 1 is selected and the procedure continued.

Methodology of LEOS approach

d_{ij}	cosθ	Star1 Inertial position	Star2 Inertial position
0.96	0.951	s1	s2
-	0.963	p1	p2
-	0.972	q1	q2
-	0.973	r1	r2



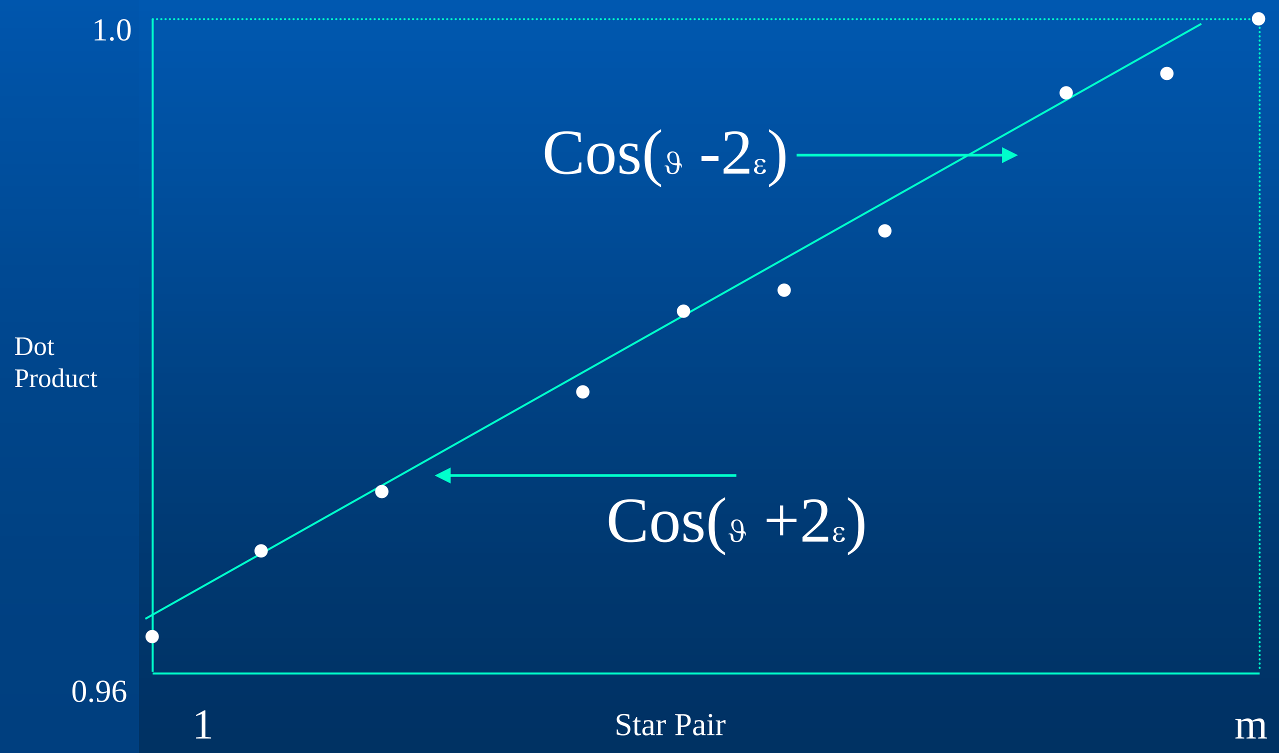
d_{ij}	cosθ	Star1 Inertial position	Star2 Inertial position
0.98	0.971	a1	a2
-	0.973	b1	b2
-	0.982	c1	c2
-	0.983	d1	d2

d_{ij}	cosθ	Star1 Inertial position	Star2 Inertial position
0.94	0.931	m1	m2
-	0.943	n1	n2
-	0.952	u1	u2
-	0.953	v1	v2

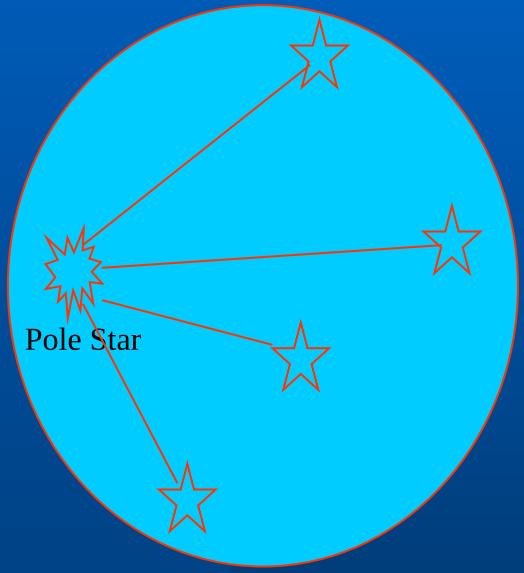
Features Of QUALIS

- HIGHLY SUCCESSFUL WHEN MORE THAN 7 STARS APPEAR IN FOV
- SEARCHLESS ALGORITHM - FAST COMPUTATION
- STAR PAIR CATALOG GENERATION - STORE IN ASCENDING ORDER
- STAR PAIR IDENTIFICATION TECHNIQUE (SPIT)
- STAR MATCHING IDENTIFICATION TECHNIQUE (SMIT)

SPIT Procedure



SMIT PROCEDURE



BIN1	BIN2	BIN3	BIN4
(U,V)	(X,Y)	(P,Q)	(L,M)
(U1,V1)	(X1,Y1)	(P1,Q1)	(L1,M1)
(U2,V2)	(X2,Y2)	(P2,Q2)	(L2,M2)
-	-	-	-
-	-	-	-
-	-	-	-

SMIT PROCEDURE --- CONTINUED

- STAR WHICH OCCURS ONCE IN EACH BIN IS THE POLE STAR AND CONSIDERED TO BE IDENTIFIED
- IF MORE THAN ONE STAR APPEARS IN EVERY BIN THERE IS AMBIQUITY
- TO OVERCOME THIS, MORE STARS IN THE FOV REQUIRED
- TO AVOID WRONG IDENTIFICATION, AGAIN MORE STARS IN THE FOV IS EXPECTED
- THE PROCEDURE IS REPEATED UNTIL ALL STARS IN FOV ARE CONSIDERED AND IDENTIFIED

QUALIS - GSAT 1 EXPERIENCES

- TO PHASE -GSAT 1 IS INERTIALLY POINTING
- SAME SET OF STARS FOR ENTIRE DURATION
- 5 STARS - 1 BINARY STAR - 4 OTHER STARS
- BINARY STAR MAG - 0.9 AND 1.4 (ANTARES)
- OTHER STARS - MAG ARE 2.3, 2.6, 2.7 AND 3.2
- SKY 2000 IS ADOPTED TO ACCOUNT BINARY STARS
- PRUNING OF CATALOG - KEEP THE BRIGHTEST OF THE BINARY STARS - DELETE THE OTHER STAR

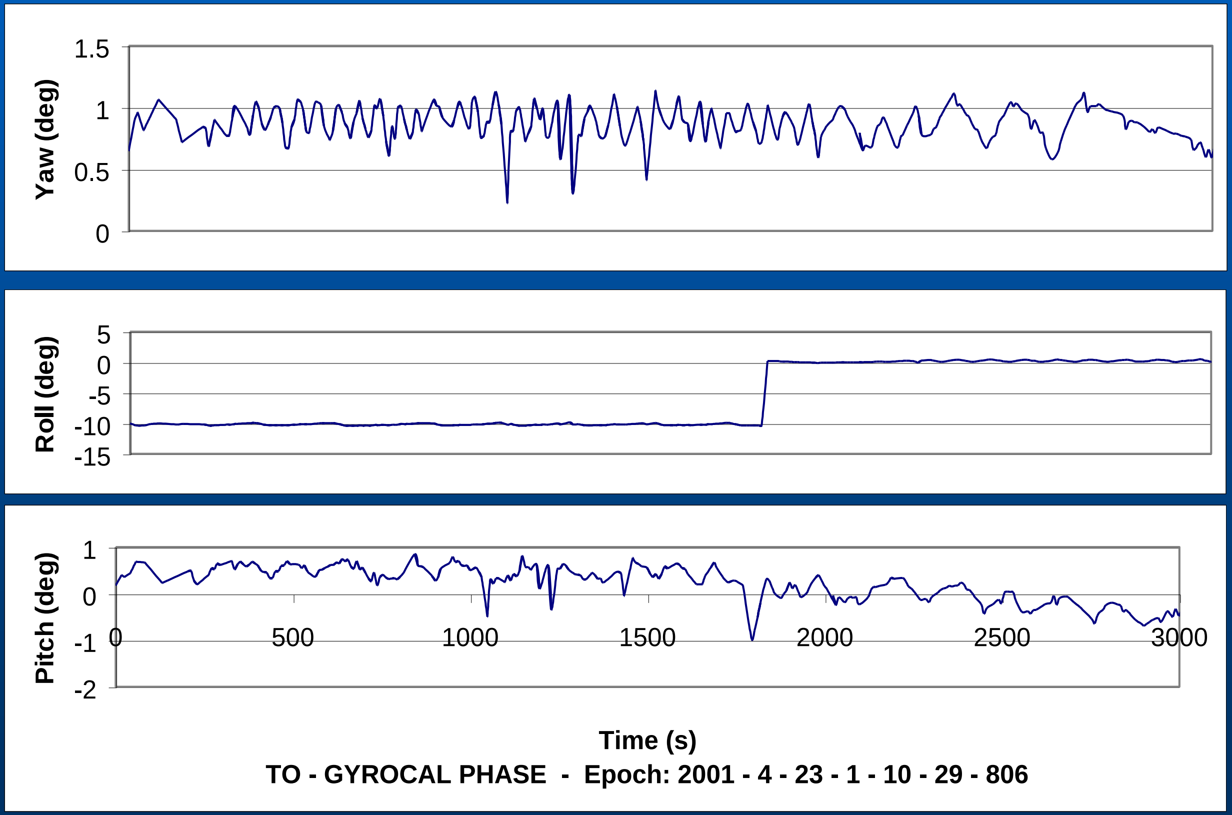
QUALIS - GSAT 1 EXPERIENCES - CONTINUED
SUCCESSFULLY IDENTIFIED STARS (TO- APR 23)

SL NO.	STAR ID	STAR MAGNITUDE
1.	530	0.90
2.	587	2.32
3.	548	3.20
4.	675	2.75
5.	624	2.62
6.	817	2.74
7.	757	2.61
8.	949	2.65
9.	738	2.56
10.	683	2.43
11.	984	3.20

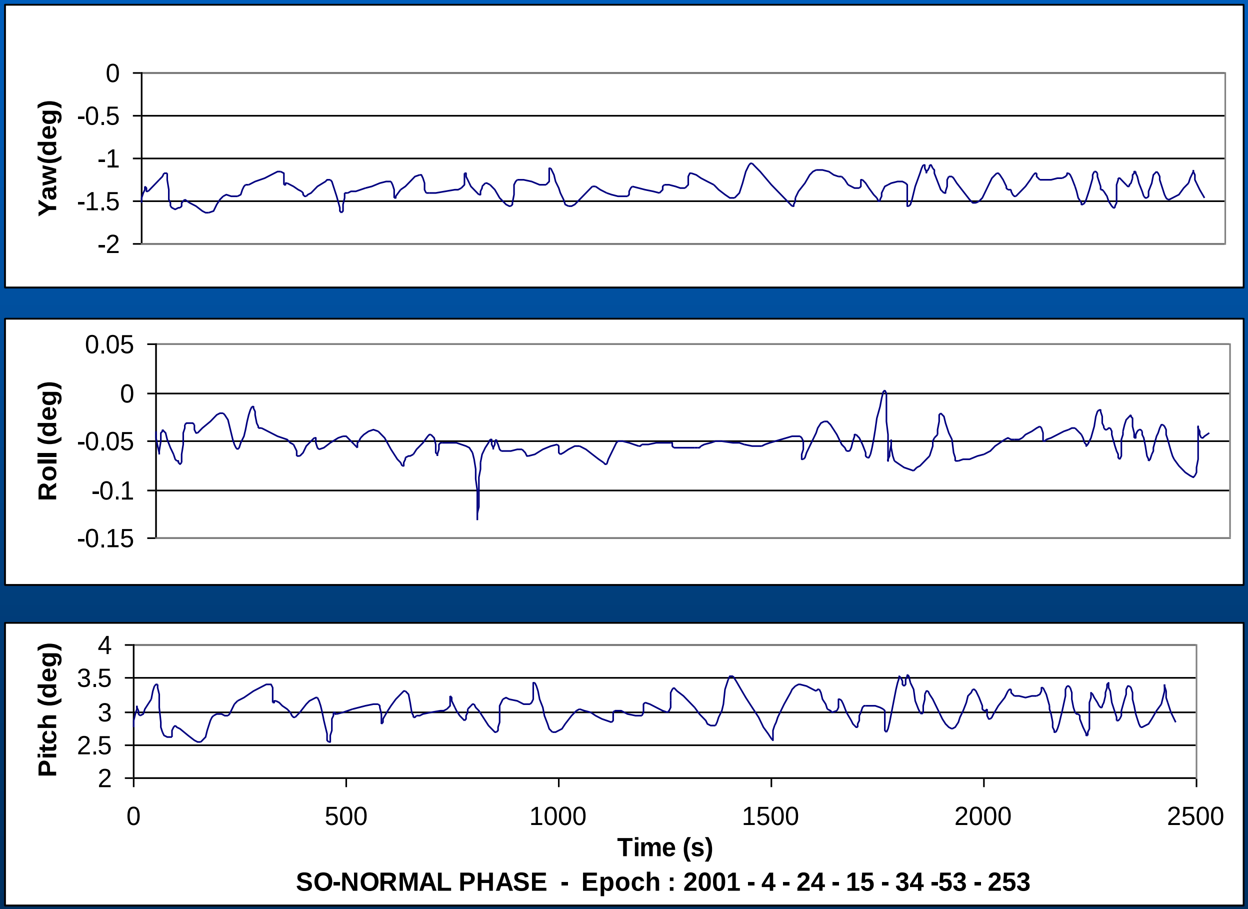
QUALIS - GSAT 1 EXPERIENCES - CONTINUED
SUCCESSFULLY IDENTIFIED STARS (SO- APR 24)

SL NO.	STAR ID	STAR MAGNITUDE
1.	1562	1.80
2.	1517	2.37
3.	1447	3.14
4.	1496	2.44
5.	1485	3.17
6.	1514	1.77
7.	1408	3.01

QUALIS OUTPUT - TO PHASE - APR 23



QUALIS OUTPUT - SO PHASE - APR 24

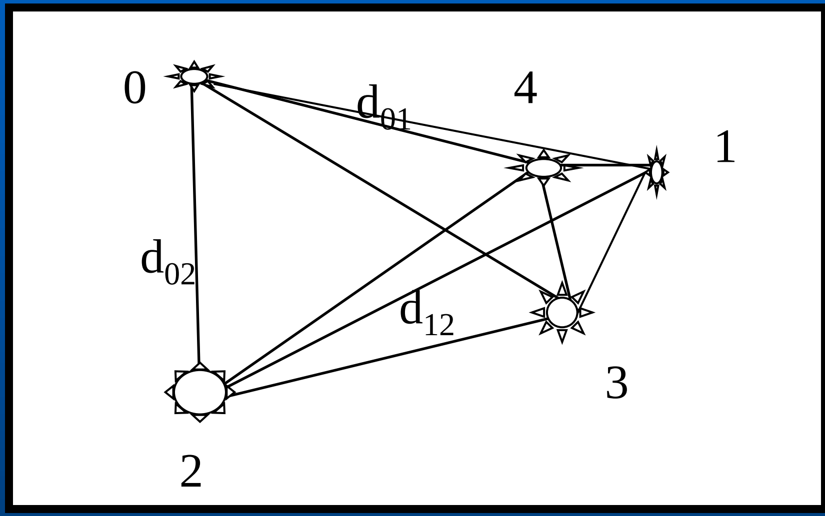


Star TRiplet IDEntification (STRIDE)

- ASSUMPTION - RELATIVE MAGNITUDE BETWEEN STARS PRESERVED BY STAR CAMERA
- WHEN THE DYNAMIC RANGE IS LARGE, SIZE OF TRIPLET STAR CATALOG GROWS EXPONENTIALLY
- SEARCH PROCESS IS TEDIOUS - ON GROUND FINE - ONBOARD LARGER MEMORY REQUIRED
- FAILURE RATE IS MINIMUM

STRIDE Algorithm Details

- SKY 2000 / SAO CATALOG IS USED
- IF ANGULAR DISTANCE < 0.3° BETWEEN STARS THE FAINTEST STAR REMOVED - TO AVOID MIS IDENTIFICATION
- THE CATALOG ORDERED IN ASCENDING ORDER OF MAGNITUDE
- TRIPLET DATA BASE GENERATION THAT OCCURS WITHIN THE FOV - UNIQUE TRIPLETS FORMED



STAR TRIPLETS IN CCD FRAME

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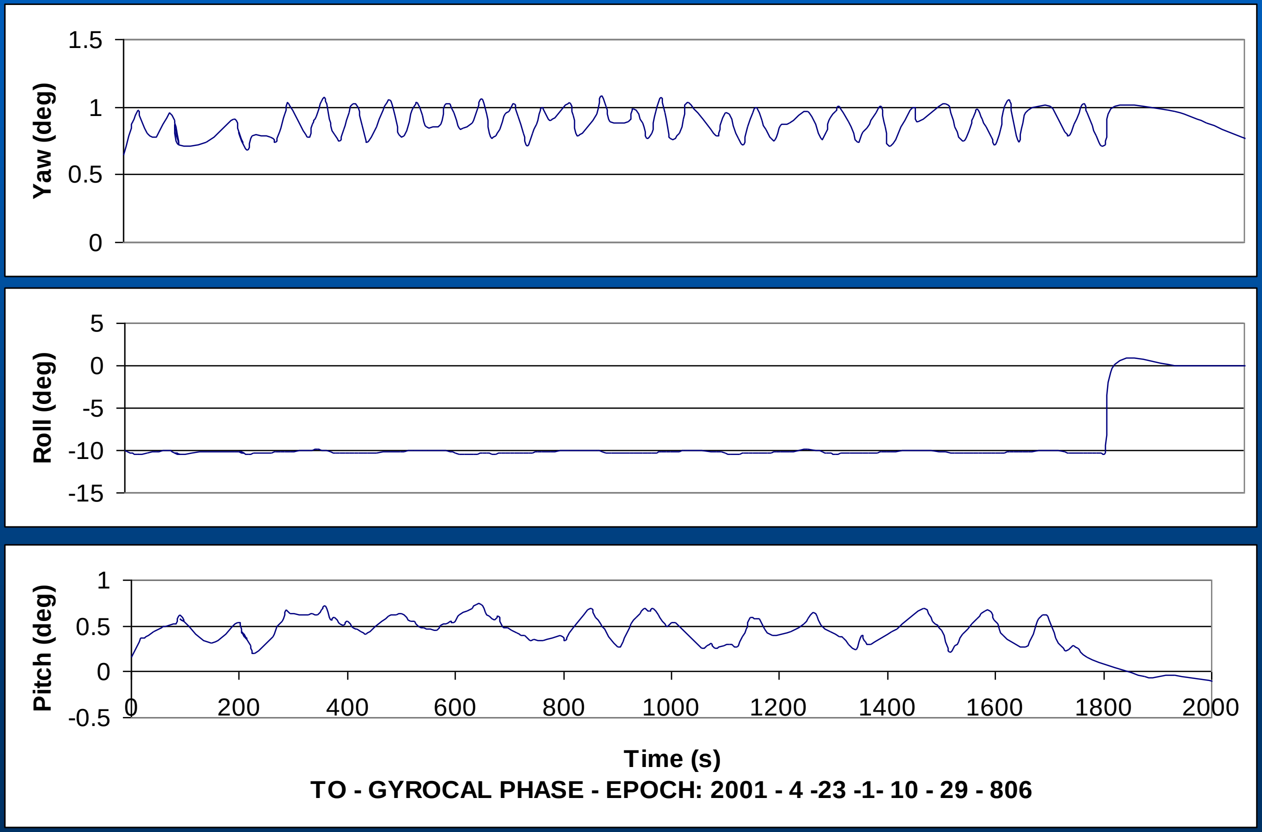
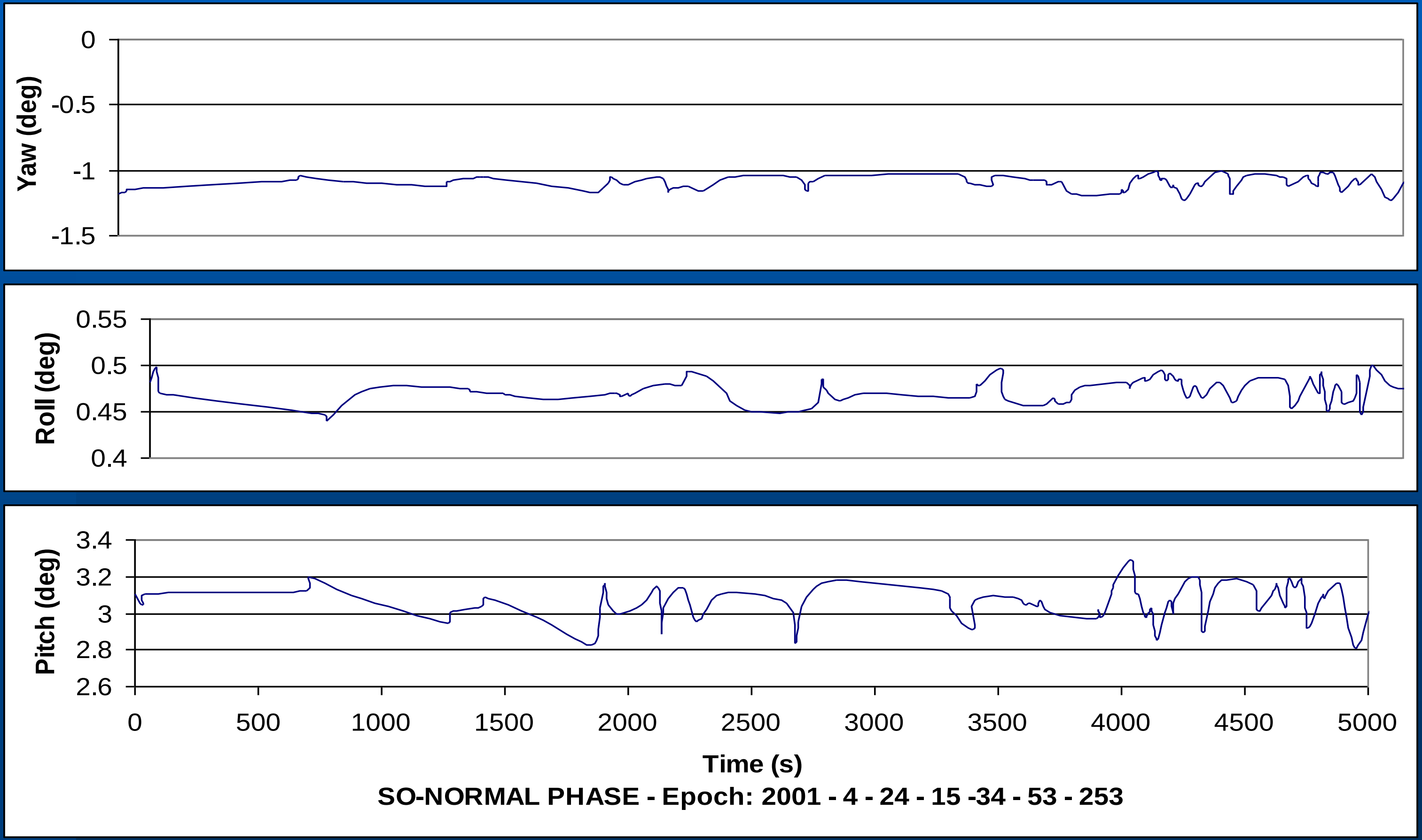
STRIDE OUTPUT - TO PHASE - APR 23



STRIDE OUTPUT - SO PHASE - APR 24



AD using Vector Observations

- Three axis AD is obtained when two or more vectors are measured in s/c body and their corresponding inertial vectors are known
- Two Algorithms
  - The TRIAD Algorithm
  - The QUEST Algorithm

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The TRIAD Algorithm

Let $\widehat{W}_1$ and $\widehat{W}_2$ are observation unitvectors

Let $\widehat{V}_1$ and $\widehat{V}_2$ are corresponding reference unitvectors such that

$A\widehat{V}_1 = \widehat{W}_1$ and $A\widehat{V}_2 = \widehat{W}_2$

Construct two triads of orthonormal references

$$\widehat{r}_1 = \widehat{V}_1 \qquad \widehat{r}_2 = (\widehat{V}_1 \times \widehat{V}_2)/\left|\breve{V}_1 \times \widehat{V}_2\right|$$

$$\widehat{r}_3 = (\widehat{V}_1 \times (\widehat{V}_1 \times \widehat{V}_2))/\left|\breve{V}_1 \times \widehat{V}_2\right|$$

$$\widehat{s}_1 = \widehat{W}_1 \qquad \widehat{s}_2 = (\widehat{W}_1 \times \widehat{W}_2)/\left|\widehat{W}_1 \times \widehat{W}_2\right|$$

$$\widehat{s}_3 = (\widehat{W}_1 \times (\widehat{W}_1 \times \widehat{W}_2))/\left|\widehat{W}_1 \times \widehat{W}_2\right|$$

$$M_{ref} = \left[\widehat{r}_1 \quad \widehat{r}_2 \quad \widehat{r}_3\right] \qquad M_{obs} = \left[\widehat{s}_1 \quad \widehat{s}_2 \quad \widehat{s}_3\right]$$

then $A = M_{obs} M_{ref}^T$

The QUEST Algorithm

If there are n number of observations V and corresponding reference vectors W we can use the QUEST method which is optimal. Then optimal attitude matrix A is obtained when the loss function is minimized. This problem was posed by Wahba as

$$L(A) = \frac{1}{2}\sum_{i=1}^{n} a_i \left| \hat{W}_i - A\hat{V}_i \right|^2 \text{ such that } \sum_{i=1}^{n} a_i = 1$$

Several methods were used in obtaining this solution. Most of them Numerical in nature. However, the solution by Mortari in closed form is used these days onboard.

ESTIMATOR OF OPTIMAL QUATERNION (ESOQ)

- Wahba problem is converted into Eigenvalue problem when quaternion is used in the formulation.
- The resultant eigenvalue problem requires solving for eigenvalues of a 4×4 [K] matrix.
- The eigen vector corresponds to the maximum eigenvalue is the required quaternion.

To find λ_{max} the following Quartic Algebraic Equation is solved:

$$\lambda^4 + a\,\lambda^3 + b\lambda^2 + c\lambda + d = 0$$

$$\text{where } a = \mathrm{tr}[\mathbf{K}] = 0,\ b = -2(\mathrm{tr}[\mathbf{B}]) + \mathrm{tr}\left[\mathrm{adj}\,(B+B^T)\right] - z^T z,$$

$$c = -\,\mathrm{tr}\left[\mathrm{adj}(\mathbf{K})\right],\ \text{and } d = \det[\mathbf{K}]$$

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ESOQ - Continued

When the number of observations n = 2 :

$$\lambda_{\max} = \frac{1}{2}\left(\sqrt{2\sqrt{\mathrm{d}} - b} + \sqrt{-2\sqrt{\mathrm{d}} - b}\right)$$

When the number of observations n > 2 : λ_{max} Computed Using Newton – Raphson iterative method

$$\lambda_{\mathrm{i}+1} = \lambda_{\mathrm{i}} - \frac{\lambda_{\mathrm{i}}^4 + b\lambda_i^2 + c\lambda_i + d}{4\lambda_{\mathrm{i}}^3 + 2b\lambda_i + c}$$

The Optimal Quaternion is given by the following expression

$$\mathbf{q} = \begin{Bmatrix} (\lambda_{\max} - \mathrm{tr}\,[\mathbf{B}])\,\mathbf{e}_{\mathrm{k}} \\ \mathbf{z}^{\mathrm{T}}\,\mathbf{e}_{\mathrm{k}} \end{Bmatrix}$$

Where B = $\sum_i \alpha_i b_i r_i$ is the attitude profile matrix formed by measurement vectors and e_k is the Eigenvector. The Sequential Rotational Technique is used for singularity avoidance.

END OF
ACTIVE
STABILIZATION METHODS