U

UDACITY

5 — Standardizing

5.1 Z score

> **Definition 5.1 — Standard Score.** Given an observed value x, the Z score finds the number of Standard deviations x is away from the mean.
>
> $$Z = \frac{x - \mu}{\sigma}$$

5.1.1 Standard Normal Curve

The standard normal curve is the curve we will be using for most problems in this section. This curve is the resulting distribution we get when we standardize our scores. We will use this distribution along with the Z table to compute percentages above, below, or in between observations in later sections.
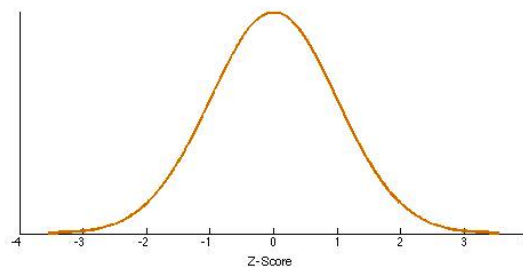

Figure 5.1: The Standard Normal Curve

5.2 Examples

5.2.1 Finding Standard Score

■ **Example 5.1**   The average height of a professional basketball player was 2.00 meters with a standard deviation of 0.02 meters. Harrison Barnes is a basketball player who measures 2.03 meters. How many standard deviations from the mean is Barnes' height?

First we should sketch the normal curve that represents the distribution of basketball player heights.
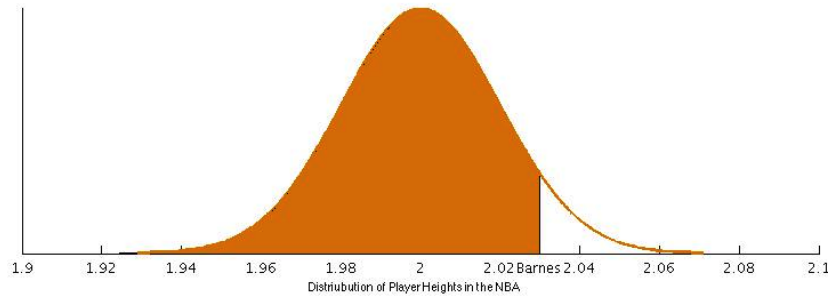

Figure 5.2: Notice we place the mean height 2.00 right in the middle and make tick marks that are each 1 standard deviation or 0.02 meters away in both directions.

Next we should compute the standard score (i.e. z score) for Barnes' height. Since $\mu = 2.00$, $\sigma = 0.02$, and $x = 2.03$ we can find the z-score

$$\frac{x - \mu}{\sigma} = \frac{2.03 - 2.00}{0.02} = \frac{0.03}{0.02} = 1.5$$

Ⓡ   Finding 1.5 as the z score tells us that Barnes' height is 1.5 standard deviations from the mean, that is $1.5\sigma + \mu = $ Barnes' Height

                                                                                                                        ■

■ **Example 5.2**   The average height of a professional hockey player is 1.86 meters with a standard deviation of 0.06 meters. Tyler Myers, a professionally hockey, is the same height as Harrison Barnes. Which of the two is taller in their respective league?

To find Tyler Myers standard score we can use the information: $\mu = 1.86$, $\sigma = 0.06$, and $x = 2.03$. This results in the standard score:

$$\frac{x - \mu}{\sigma} = \frac{2.03 - 1.86}{0.06} = \frac{0.17}{0.06} = 2.833$$

Comparing the two z-scores we see that Tyler Myers score of 2.833 is larger than Barnes' score of 1.5. This tells us that there are more hockey players shorter than Myers than there are basketball players shorter than Barnes'.   ■

5.3 **Practice Problems**

Find the Z-score given the following information

Problem 5.1 $\mu = 54$, $\sigma = 12$, $x = 68$

Problem 5.2 $\mu = 25$, $\sigma = 3.5$, $x = 20$

Problem 5.3 $\mu = 0.01$, $\sigma = 0.002$, $x = 0.01$

Problem 5.4   The average GPA of students in a local high school is 3.2 with a standard deviation of 0.3. Jenny has a GPA of 2.8. How many standard deviations away from the mean is Jenny's GPA?

Problem 5.5   Jenny's trying to prove to her parents that she is doing better in school than her cousin. Her cousin goes to a different high school where the average GPA is 3.4 with a standard deviation of 0.2. Jenny's cousin has a GPA of 3.0. Is Jenny performing better than her cousin based on standard scores?

Problem 5.6   Kyle's score on a recent math test was 2.3 standard deviations above the mean score of 78%. If the standard deviation of the test scores were 8%, what score did Kyle get on his test?