

Abstraction Using Symmetries in Markov Decision Processes

A Project Report

submitted by

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THESIS CERTIFICATE

This is to certify that the thesis entitled **Abstraction Using Symmetries in Markov Decision Processes**, submitted by **Shravan Matthur Narayanamurthy**, to the Indian Institute of Technology, Madras, for the award of the degree of **Master of Technology**, is a bona fide record of the research work carried out by him under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

KEYWORDS: Markov Decision Processes, Symmetries, Abstraction

Current approaches to solving Markov Decision Processes (MDPs), the de-facto standard for modeling stochastic sequential decision problems, scale poorly with the size of the MDP. When used to model real-world problems though, MDPs exhibit considerable implicit redundancy, especially in the form of symmetries. However, existing model minimization approaches do not efficiently exploit this redundancy due to symmetries. These approaches involve constructing a reduced model first and then solving them. Hence we term these as “explicit minimization” approaches.

In the first part of this work, we address the question of finding symmetries of a given MDP. We show that the problem is *Isomorphism Complete*, that is, the problem is polynomially equivalent to verifying whether two graphs are isomorphic. Apart from the theoretical importance of this result it has an important practical application. The reduction presented can be used together with any off-the-shelf Graph Isomorphism solver, which performs well in the average case, to find symmetries of an MDP. In fact, we present results of using NAutY (the best Graph Isomorphism solver currently available), to find symmetries of MDPs. We next address the issue of exploiting the symmetries of a given MDP. We propose the use of an explicit model minimization algorithm called the G-reduced image algorithm that exploits symmetries in a time efficient manner. We present an analysis of the

algorithm and corroborate it with empirical results on a probabilistic GridWorld domain and a single player GridWorld Soccer domain. We also present results of integrating the symmetry finding with the explicit model minimization approach to illustrate an end-to-end approach for “Abstraction using Symmetries in MDPs”.

We then note some of the problems associated with this explicit scheme and as a solution present an approach wherein we integrate the symmetries into the solution technique implicitly. However, we should select a suitable solution technique so that the overheads due to integration do not outweigh the improvements. We validate this approach by modifying the Real Time Dynamic Programming (RTDP) algorithm and empirically demonstrate significantly faster learning and reduced overall execution time on several domains.

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ABBREVIATIONS

IITM	Indian Institute of Technology, Madras
RTFM	Read the Fine Manual

NOTATION

r	Radius, m
α	Angle of thesis in degrees
β	Flight path in degrees

CHAPTER 1

INTRODUCTION

This document provides a simple template of how the provided `iitmdiss.cls` \LaTeX class is to be used. Also provided are several useful tips to do various things that might be of use when you write your thesis.

Before reading any further please note that you are strongly advised against changing any of the formatting options used in the class provided in this directory, unless you are absolutely sure that it does not violate the IITM formatting guidelines. *Please do not change the margins or the spacing.* If you do change the formatting you are on your own (don't blame me if you need to reprint your entire thesis). In the case that you do change the formatting despite these warnings, the least I ask is that you do not redistribute your style files to your friends (or enemies).

It is also a good idea to take a quick look at the formatting guidelines. Your office or advisor should have a copy. If they don't, pester them, they really should have the formatting guidelines readily available somewhere.

To compile your sources run the following from the command line:

```
% latex thesis.tex  
% bibtex thesis  
% latex thesis.tex  
% latex thesis.tex
```

Modify this suitably for your sources.

To generate PDF's with the links from the `hyperref` package use the following command:

```
% dvipdfm -o thesis.pdf thesis.dvi
```

1.1 Package Options

Use this thesis as a basic template to format your thesis. The `iitmdiss` class can be used by simply using something like this:

```
\documentclass[PhD]{iitmdiss}
```

To change the title page for different degrees just change the option from `PhD` to one of `MS`, `MTech` or `BTech`. The dual degree pages are not supported yet but should be quite easy to add. The title page formatting really depends on how large or small your thesis title is. Consequently it might require some hand tuning. Edit your version of `iitmdiss.cls` suitably to do this. I recommend that this be done once your title is final.

To write a synopsis simply use the `synopsis.tex` file as a simple template. The `synopsis` option turns this on and can be used as shown below.

```
\documentclass[PhD,synopsis]{iitmdiss}
```

Once again the title page may require some small amount of fine tuning. This is again easily done by editing the class file.

This sample file uses the `hyperref` package that makes all labels and references clickable in both the generated DVI and PDF files. These are very useful when reading the document online and do not affect the output when the files are printed.

1.2 Example Figures and tables

Figure 1.1 shows a simple figure for illustration along with a long caption. The formatting of the caption text is automatically single spaced and indented. Table 1.1 shows a sample table with the caption placed correctly. The caption for this should always be placed before the table as shown in the example.

Figure 1.1: Two IITM logos in a row. This is also an illustration of a very long figure caption that wraps around two two lines. Notice that the caption is single-spaced.

1.3 Bibliography with BIB \TeX

I strongly recommend that you use BIB \TeX to automatically generate your bibliography. It makes managing your references much easier. It is an excellent way to

Table 1.1: A sample table with a table caption placed appropriately. This caption is also very long and is single-spaced. Also notice how the text is aligned.

x	x^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64

organize your references and reuse them. You can use one set of entries for your references and cite them in your thesis, papers and reports. If you haven't used it anytime before please invest some time learning how to use it.

I've included a simple example BIB_T_EX file along in this directory called `refs.bib`. The `iitmdiss.cls` class package which is used in this thesis and for the synopsis uses the `natbib` package to format the references along with a customized bibliography style provided as the `iitm.bst` file in the directory containing `thesis.tex`. Documentation for the `natbib` package should be available in your distribution of L_AT_EX. Basically, to cite the author along with the author name and year use `\cite{key}` where `key` is the citation key for your bibliography entry. You can also use `\citet{key}` to get the same effect. To make the citation without the author name in the main text but inside the parenthesis use `\citep{key}`. The following paragraph shows how citations can be used in text effectively.

More information on BIB_T_EX is available in the book by Lamport [1994]. There are many references [Lamport, 1994; Kopka and Daly, 2003; Griffiths and Higham, 1997] that explain how to use BIB_T_EX. Read the `natbib` package documentation for more details on how to cite things differently. Different type of citations are

book Bellman [1957], one article in the book Amarel [1968], thesis Manning [1990], technical report Ravindran and Barto [2001], journal Barto *et al.* [1995], conference Knoblock [1990] and other like url Crawford [1992].

1.4 Other useful L^AT_EX packages

The following packages might be useful when writing your thesis.

- It is very useful to include line numbers in your document. That way, it is very easy for people to suggest corrections to your text. I recommend the use of the `lineno` package for this purpose. This is not a standard package but can be obtained on the internet. The directory containing this file should contain a `lineno` directory that includes the package along with documentation for it.
- The `listings` package should be available with your distribution of L^AT_EX. This package is very useful when one needs to list source code or pseudo-code.
- For special figure captions the `ccaption` package may be useful. This is specially useful if one has a figure that spans more than two pages and you need to use the same figure number.
- The notation page can be entered manually or automatically generated using the `nomencl` package.

More details on how to use these specific packages are available along with the documentation of the respective packages.

APPENDIX A

A SAMPLE APPENDIX

Just put in text as you would into any chapter with sections and whatnot. Thats the end of it.

Publications

1. S. M. Narayanamurthy and B. Ravindran (2007). Efficiently Exploiting Symmetries in Real Time Dynamic Programming. *IJCAI 2007, Proceedings of the 20th International Joint Conference on Artificial Intelligence*, pages 2556–2561.

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