The Quantum Lattice Boltzmann Method in Intel Quantum SDK

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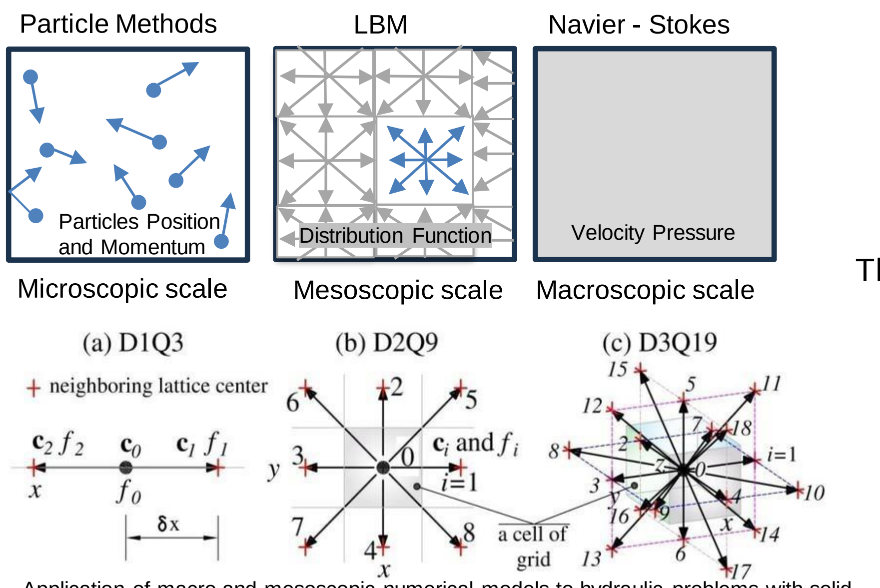
Abstract

Partial Differential Equations (PDEs) serve as the mathematical framework for modeling various physical phenomena, such as fluid dynamics. Traditionally, these PDEs are discretized into algebraic equations that are computationally solved at discrete points in space. The complexity of the physical model directly increases the algebraic system's dimensionality, demanding greater computational resources. Quantum computing presents a potential solution to this computational challenge. In our study, we explore the implementation of the advection-diffusion equation with one, two, and three dimensions using the Quantum Lattice Boltzmann Method (LBM) by the Intel Quantum Software Development Kit (SDK), emphasizing the integration of the quantum walk process.

Keywords: **Lattice Boltzmann Method, Quantum Computing, Computational Fluid Dynamics, Quantum Walk, Intel Quantum SDK**

1. Introduction

The Lattice Boltzmann Method (LBM) offers a novel approach to fluid dynamics problems. Rather than solving the Navier-Stokes equations directly, LBM simulates the macroscopic behavior of fluids through a mesoscopic lens. This is achieved by modeling fluids as collections of fictitious particles, which undergo successive propagation and collision processes across a discrete lattice grid.



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CLASSICAL METHOD

The collision of fictive particles is formulated as

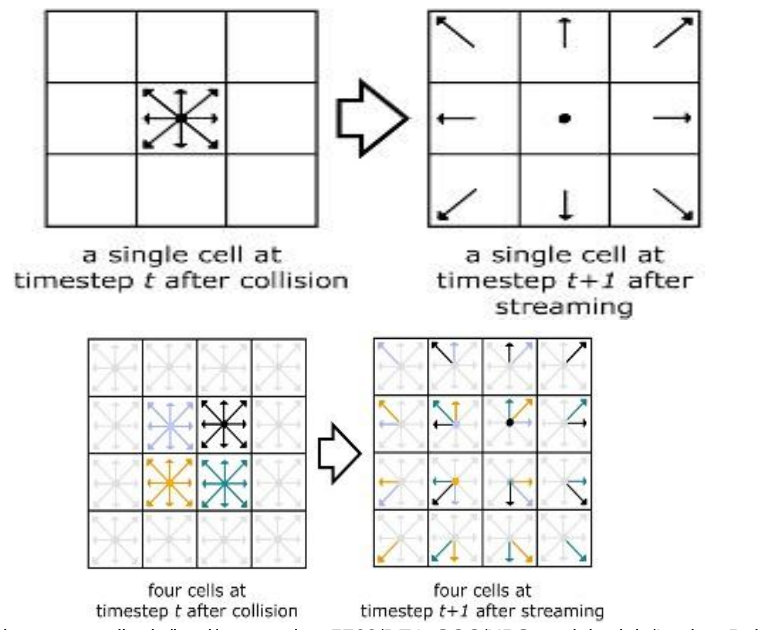
$$f^*(\vec{x},t) = f_a(\vec{x},t) - \frac{\Delta t}{\tau}\left(f_a(\vec{x},t) - f_a^{eq}(\vec{x},t)\right)$$

The propagation step is nothing more than a shift in space mathematically

$$f^*(\vec{x} + \vec{c}_a\Delta t, t + \Delta t) = f_a^*(\vec{x},t)$$

Classical variables like density velocity are obtained by a simple summation on each lattice site

$$\rho(\vec{x},t) = \sum a\, f_a(\vec{x},t), \rho(\vec{x},t)\vec{u}(\vec{x},t) = \sum a\, f_a(\vec{x},t)c_a$$



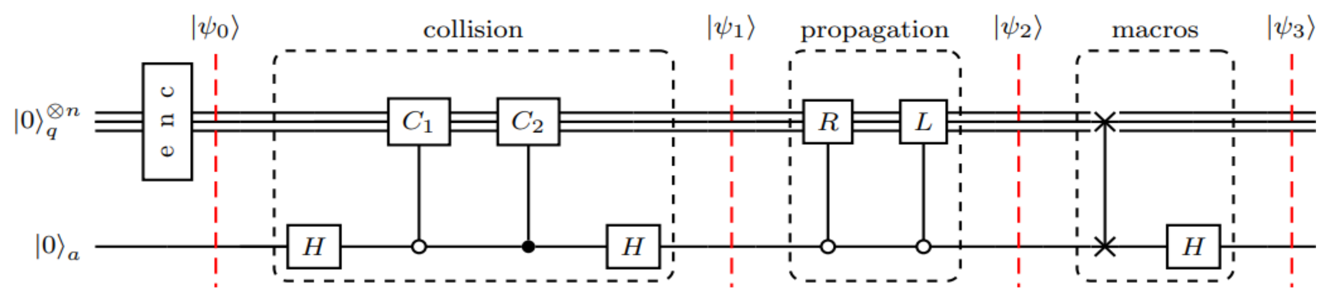
https://people.ece.cornell.edu/land/courses/ece5760/DE1_SOC/HPS_peripherials/Lattice_Boltzmann_index.html

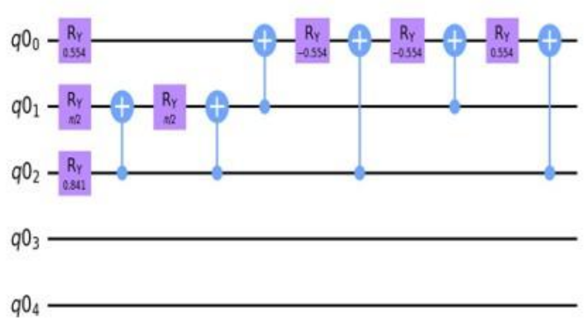
2. Quantum Lattice Boltzmann Method

Classical and Quantum Systems: An Analogy

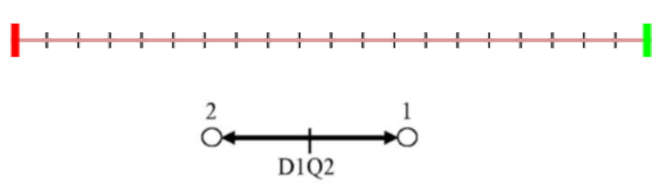
- Components of fluid density are encoded into the amplitudes of quantum states.
- Collision state represents an evolution for quantum state, driven by a diagonal operator.
- The propagation step is executed through the Quantum Walk procedure.

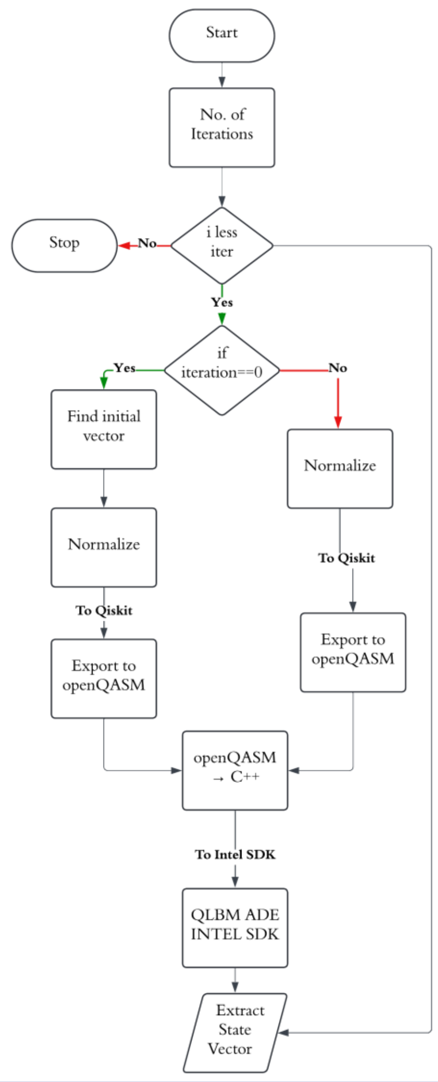
The D1Q2 model





1D Grid with M points





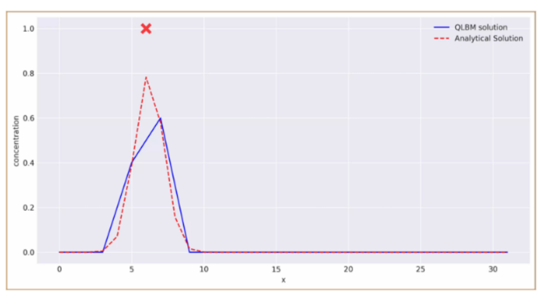
Encoding Phase:
Initially, the fluid distribution is vectorized and encoded into a quantum state via a reverse iterative process.

Collision Dynamics:
This stage utilizes a non-unitary, block-diagonal matrix, with the evolution operated by a linear combination of unitary transformations.

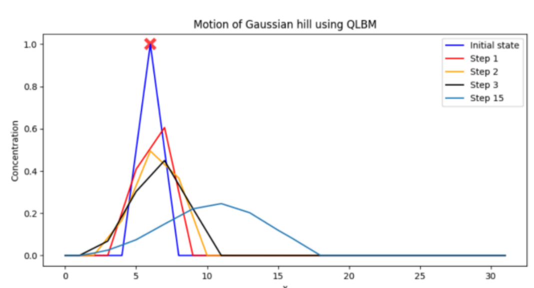
Quantum Propagation:
The propagation of distribution functions is executed via a Quantum Walk, enabling the discrete spatial shift of these functions.

Macroscopic Quantities:
Macroscopic variables are determined using swap and Hadamard gates to renormalize the post-selected state, facilitating the computation of the system's state at the next time step t+1.

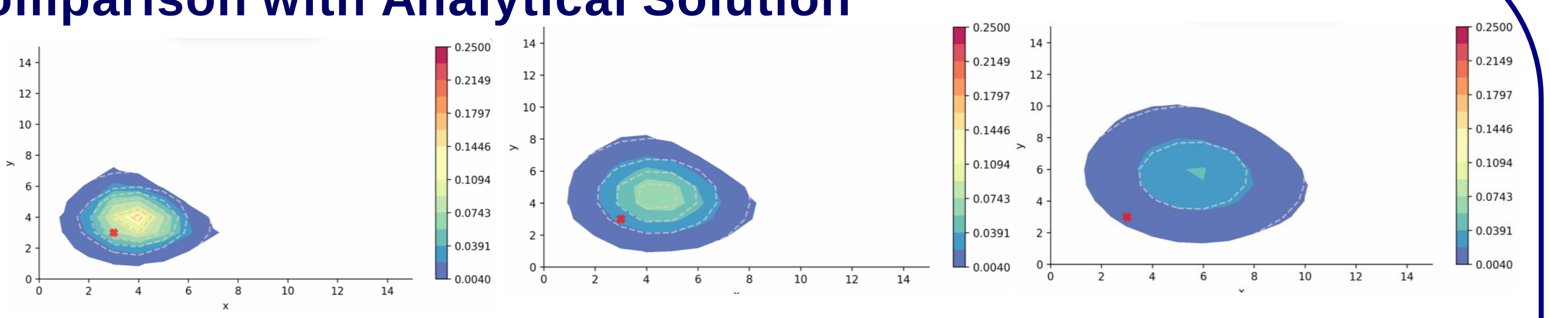
3. Outcome and Comparison with Analytical Solution



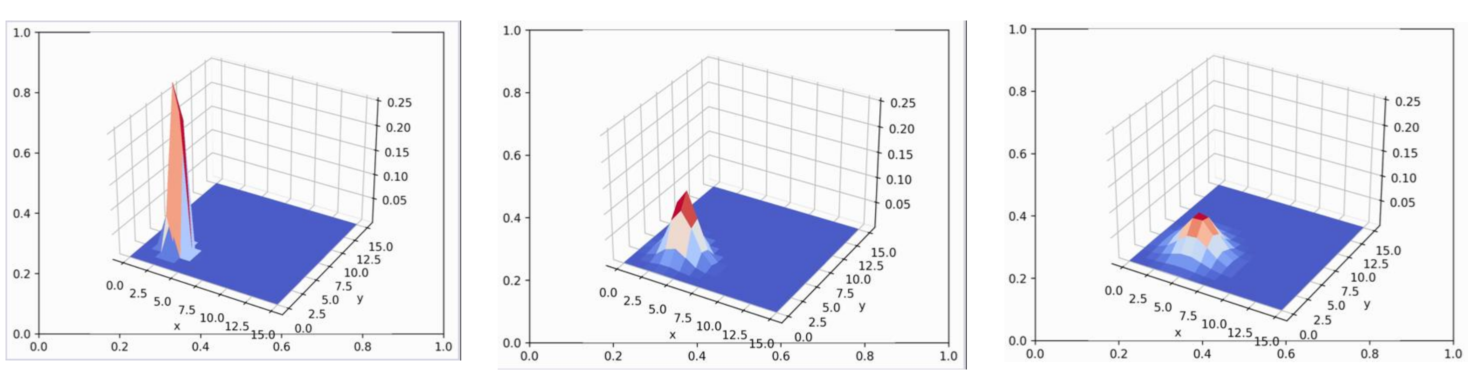
Comparative Analysis of QLBM and Analytical Solution Concentration



The motion of a 1D Gaussian Hill



The motion of a 2D Gaussian Hill



The motion of a 3D Gaussian Hill

In conclusion, our research provides a comprehensive comparison between classical and quantum methodologies applied within the Lattice Boltzmann Method framework, underscoring the analogous strategies in fluid dynamics modeling. We have developed and implemented a novel quantum algorithm capable of solving the advection-diffusion equation through the Intel Quantum SDK, potentially paving the way for more efficient computational fluid dynamics in quantum computing paradigms.

Reference

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[2] Khalate, P., Wu, X., Premaratne, S., Hogaboam, J., Holmes, A., Schmitz, A., Guerreschi, G. G., Zou, X. & Matsuura, A. Y., https://arxiv.org/abs/2202.11142