Spike Train Analysis: Estimating the firing rate of a simple model neuron from its spike train

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Overview

Introduction to Computational Neuroscience

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- Computers are therefore widely used to aid the analysis of neural data.
- Mathematicians, physicists and engineers have looked to use their skills to analyse data and model phenomena.

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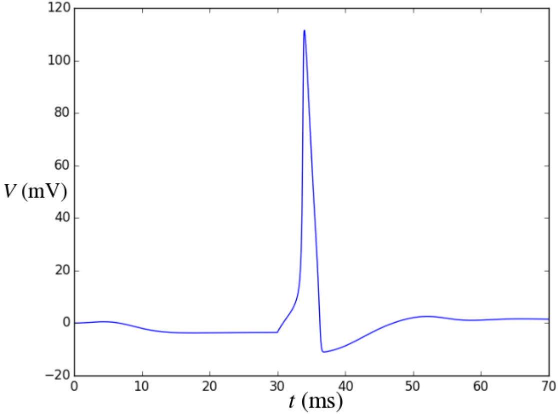
- They consist of a rapid depolarisation and subsequent hyper-polarisation of the membrane potential.
- A neuron either spikes or it doesn't, there are no fractional spikes.
- A neuron's spikes tend to have a typical voltage profile.

Example spike



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Spike trains are typically treated as point processes in computational neuroscience.

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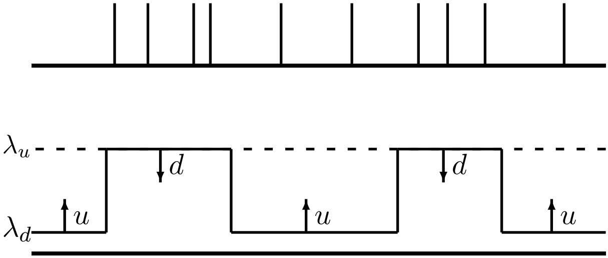
- It is assumed that the features are very specific, so most neurons would be 'down' more than they are 'up'.
- This is typical of *sparse coding*, which has been shown to be an energy-efficient coding scheme in the brain.

The Model



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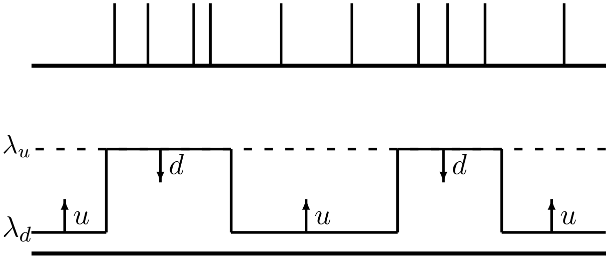

The model assumes that the neuron is a bimodal Poisson process, with constant firing rates λ_u and λ_d. The ‘switching rates’, u and d, are also assumed to be constant Poisson rates.

Markov Chains

- The on/off - states form a *Markov chain*; there is no extra information to be gained by knowing the history of the process.

$$P\left(x(t_0+h)=A\,|x(t),t\leq t_0\right)=P\left(x(t_0+h)=A\,|x(t_0)\right)$$

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For the up/down-states, the Markov chain is described as:

$$P'(t)=P(t)Q,\ \text{where}\ Q=\begin{pmatrix}-d & d\\ u & -u\end{pmatrix}$$

Markov Chains

So, if the probability, $p_u(t_0)$, of being in the up-state is known at a time t_0, then:

$$P(t) = P(t_0)e^{(t-t_0)Q}$$

and

$$p_u(t) = \frac{u}{u+d}\left(1 - e^{-(u+d)(t-t_0)}\right) + p_u(t_0)e^{-(u+d)(t-t_0)}$$

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However, this ignores the differing spiking rates of the two states.

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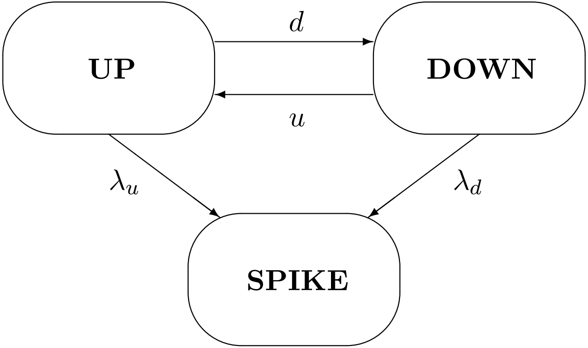
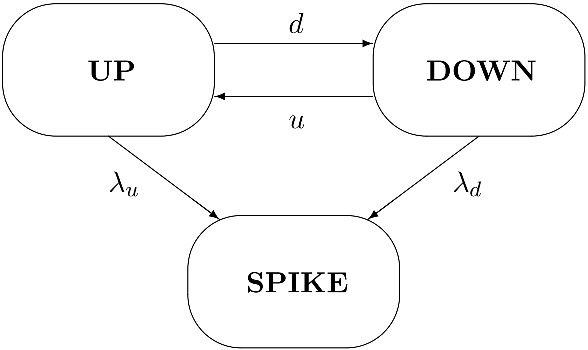
This has transition matrix Q_s:

$$Q_s = \begin{pmatrix} -d-\lambda_u & d & \lambda_u \\ u & -u-\lambda_d & \lambda_d \\ 0 & 0 & 0 \end{pmatrix}$$


UP
DOWN
SPIKE
d
u
λ_u
λ_d



Between any two spikes, this Markov chain determines the probability of being in the up-state or the down-state. By setting $t_0 = 0$, this has solution:

$$P(t) = P(0)e^{tQ_s}$$

Estimated Firing Rate

Then the firing rate is calculated as:

$$r(t) = \lambda_u p_u(t) + \lambda_d \left(1 - p_u(t)\right) = (\lambda_u - \lambda_d) p_u(t) + \lambda_d$$

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If the firing rate at $t = 0$ is r_0, then this has the very nice form:

$$r(t) = \frac{\alpha \left(\beta - r_0\right) e^{-\alpha t} + \beta \left(r_0 - \alpha\right) e^{-\beta t}}{\left(\beta - r_0\right) e^{-\alpha t} + \left(r_0 - \alpha\right) e^{-\beta t}} \tag{1}$$

where $-\alpha, -\beta$ are the non-zero eigenvalues of Q_s.

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$$P(\text{up-state}|\text{spike}) = \frac{P(\text{spike}|\text{up-state})P(\text{up-state})}{P(\text{spike})}$$
$$= \frac{\lambda_u p_u(t)}{(\lambda_u - \lambda_d)p_u(t) + \lambda_d}$$

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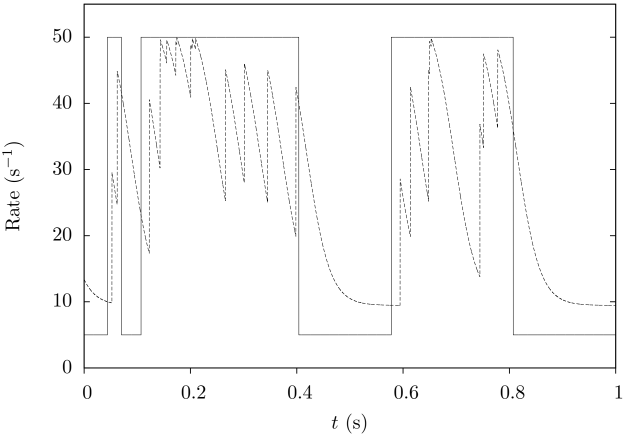
&= \frac{\lambda_u p_u(t)}{(\lambda_u - \lambda_d)p_u(t) + \lambda_d}
\end{aligned}
$$

In term of the firing rate, get:

$$
r(t) \mapsto (\lambda_u + \lambda_d) - \frac{\lambda_u \lambda_d}{r(t)} \tag{2}
$$

Firing Rate Estimate



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Given an inhomogeneous Poisson rate $f(t)$, the probability density function (pdf) of the associated ISI distribution can be calculated:

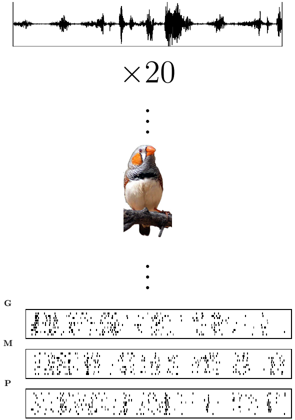
$$p_{ISI}(t) = f(t)e^{\int_0^t f(s)\,ds}$$

With the rate function $r(t)$ as above, get:

$$p_{ISI}(t) = \rho e^{-\alpha t} + (1 - \rho) e^{-\beta t} \tag{3}$$

This is the pdf of the hyper-exponential distribution H_2.

Zebra Finch Data



20 conspecific songs.
Each is played ten times.

The electrode is in the area L1.

The output is recorded.

Notes on Data

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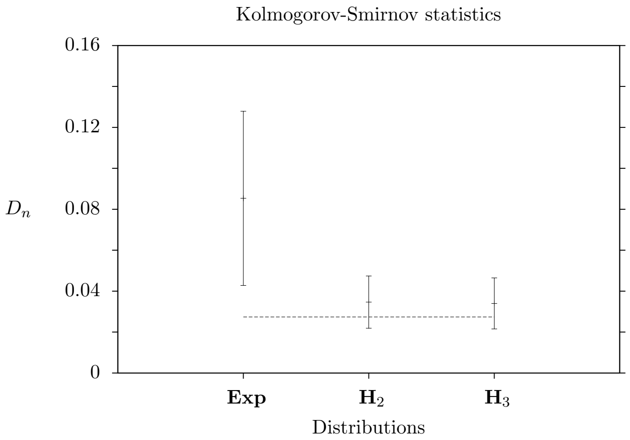
- Instead, the ten presentations of each stimulus are superimposed to get an 'average response'.
- The data is trained on 16 of the songs and tested on the remaining four songs.

Results


Kolmogorov-Smirnov statistics
D_n
0.16
0.12
0.08
0.04
0
Exp
$\mathbf{H}_2$
$\mathbf{H}_3$
Distributions

Conclusions

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- Evidence of sparse coding in the auditory forebrain of zebra finches.

Thanks

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the School of Maths here in TCD,

and the University of Bristol.

Questions?

Thank you for listening!

Any questions?